ISSN 1063-7796, Physics of Particles and Nuclei, 2024, Vol. 55, No. 4, pp. 716–724. © Pleiades Publishing, Ltd., 2024.

Neutron Stars and Black Holes as Natural Laboratories of Fundamental Physics

A. F. Zakharov[a, b, *]

[a] *National Research Center "Kurchatov Institute," Moscow, 123182 Russia*

[b] *Bogoliubov Laboratory for Theoretical Physics, JINR, Dubna, 141980 Russia*

**e-mail: alex.fed.zakharov@gmail.com*

Received November 30, 2023; revised December 10, 2023; accepted December 14, 2023

Abstract—The statistics of particles with half-integer spin was constructed in 1926 in the works of E. Fermi and P. A. M. Dirac. Soon after, it was realized that these statistics are extremely important for building a theory of such compact objects as white dwarfs. In this case, there is a limit to the mass of such objects, which is called the Chandrasekhar's limit. The neutron was discovered by Chadwick in 1932, and already in 1933 Baade and Zwicky suggested that there are neutron stars that arise as a result of supernova explosions and the collapse of a massive core. Pulsars were discovered in 1968 and it was soon realized that pulsars are neutron stars with giant magnetic fields. Binary neutron stars (both in the binary pulsar system and in the kilonova explosion event GW170817) played a key role in the detection of gravitational radiation predicted by general relativity. In 1963, quasars were discovered—fairly compact objects with a gigantic energy release and located at a cosmological distance. It was soon realized that the most natural model of quasars involved a supermassive black hole. Observations of the motions of bright stars in the vicinity of the Galactic center and reconstruction of shadows in the center of the M87 galaxy and the center of our Galaxy based on observations of synchrotron radiation at a wavelength of 1.3 mm provide additional confirmation of the presence of supermassive black holes in the centers of these galaxies.

DOI: 10.1134/S1063779624700060

1. COMPACT STARS AS THE FIRST APPLICATIONS OF QUANTUM THEORY FOR ASTRONOMICAL OBJECTS

An initial development of quantum mechanics was extremely rapid. In 1925 W. Pauli introduced his exclusion principle [1] while at the same year a spin of electron was discovered by Uhlenbeck and Goudsmit. A report about this result was published in one page article [2]. An interesting history of this discovery is given in the Goudsmit's lecture in 1965 [4] and in interviews of G. Uhlenbeck[1] and S.A. Goudsmit[2] given to T.S. Kuhn. A biography of S. Goudsmit was extremely interesting, for instance, during WWII he was an active member of ALSOS team which was organized to prevent a creation of a German Atomic bomb [5] and later he was a rather successful editor of the Physical Reviews Journal).

Based on the Pauli's result and Uhlenbeck–Goudsmit achievements E. Fermi [6] and P.A.M. Dirac [7] obtained a quantum distribution for electrons. At this period Sir Ralph Howard Fowler was the Dirac supervisor (between 1922 and 1939 R.H. Fowler supervised fifteen Fellows of the Royal Society among them G. Birkhoff, D.R. Hartree, H.J. Bhabha and three Nobel Laureates such as P.A.M. Dirac, S. Chandrasekhar and N.F. Mott) and as FRS Fowler had an opportunity to communicate the Dirac's paper in Proceedings of the Royal Society A (as it was really done). Actually, E. Fermi was the first author who obtained the distribution for electrons and he wrote a letter to P.A.M. Dirac and he recognized the Fermi's priority[3]. First applications of Fermi–Dirac statistics for a theory of white dwarfs were considered in [8, 9][4]. Soon after that, E.C. Stoner [11, 12] found a maximal mass limit for white dwarfs assuming an uniform mass distribution. Soon after that S. Chandrasekhar obtained a maximal mass limit assuming polytropic mass distribution for the relativistically degenerate electron Fermi-gas [13, 14]. Later, he generalized his results for stellar configurations with degenerate cores [15] (in 1930s Chandrasekhar visited the Soviet Union, he had numerous meetings with Soviet astronomers and during one of these conversations V.A. Ambartsumian

[1] www.aip.org/history-programs/niels-bohr-library/oral-histories/4922-2.

[2] www.aip.org/history-programs/niels-bohr-library/oral-histories/4640-1.

[3] https://museum.cref.it/en/virtual-tour-en/fermions-and-bosons/.

[4] A detailed discussion of Frenkel's contribution in theory of white dwarfs was done in [10].

proposed Chandrasekhar to solve this problem to convince people that results about a mass limit of white dwarfs are rather general). In 1932 L.D. Landau also obtained a correct mass limit for white dwarfs [16], however, at the end of this paper he noted that the conclusion looks so strange that in stars there are regions where laws of quantum mechanics and quantum statistics should be violated. A detailed analysis of this paper and its connection with a result on mass limit for neutron stars were done in [17]. In 1932 J. Chadwick discovered neutron [18]. Soviet physicist S.E. Frisch was reminding about this time (he worked on Optical Institute of Soviet Academy of Sciences in 1930s) [19]: "I told about the last major achievement—the discovery of neutrons. This speech of mine was then discussed not only among young people, but also in the party committee and it was qualified as an attempt to distract the attention of the Optical Institute researchers from the important practical tasks facing them with stories about the discovery of bourgeois physicists having fun finding useless particles". These reminiscences show that in the thirties there was no broad understanding of the importance of discoveries in the field of fundamental physics, and especially, in elementary particle physics. Despite this, new rapidly developing branches of physics attracted the interest of Soviet scientists. Currently, we know that the knowledge and experience of Soviet scientists in the field of atomic and nuclear physics were extremely in demand during the successful implementation of the Soviet atomic project. Soon after the neutron discovery, W. Baade and F. Zwicky declared that neutron stars exist and they suggested that supernovae explosions represent the transitions of ordinary stars into neutron stars [20, 21]. In Newtonian approximation for gravitational field a mass limit for neutron stars was found by G. Gamow in one of first monograph on nuclear physics [22] (later it was found that the Gamow's expression for maximal mass of neutron stars was not right and it was corrected in [23]). In general relativistic approach a maximal mass bound for neutron stars was obtained in [24]. Later, these results were clarified in [25] where the authors established that the maximum mass of the equilibrium configuration of a neutron star cannot be larger than $3.2M_\odot$ (it would be reasonable to note that the authors used the Pontryagin's maximum principle [26] to obtain their result). However, these bounds on maximal mass of neutron stars were obtained for spherical symmetric static configurations of neutron stars while a rotation could significantly change these conclusions on maximal masses of neutron stars [27].

2. CRAB NEBULA AS A UNIQUE ASTRONOMICAL OBJECT TO TEST FUNDAMENTAL PHYSICS LAWS

Astronomers observed the Crab nebula which is actually a supernova remnant. This object corresponds to the supernova explosion observed and the results recorded by Chinese and Japanese astronomers in 1054 (the corresponding supernova is called SN 1054). As a famous Soviet astronomer I.S. Shklovsky noted the Crab Nebula was one of the most valuable astronomical object which stimulated a development of new ideas and methods of modern astrophysics [28] (see also [29]). In addition, we have to note that it is a neutron star inside the nebula but it was understood only in 1960s. The Crab nebula was firstly observed by J. Bevis in 1731 and later it was re-discovered by C. Messier (initially he thought that he observed the Halley's comet because the object was located near an expected position of the Halley's comet) [30] (in this paper C. Messier noted that this object was firstly observed by Dr. Bevis around 1731 and it was published in the English sky atlas where this object was listed as M1 in the Messier catalogue). William Parsons, 3rd Earl of Rosse[5] built two large telescopes at his ancestrial set at Birr Castle (Ireland): a 36-inch reflector started to operate in 1839, later a 72-inch (180 cm) telescope popular known as the Leviathan of Parsonstown began to work in 1845 (at this time it was the largest telescope in the world). In 1844 W. Parsons observed M1, drew its image and published the result among a few other nebulae and initially he thought that nebula image consists of a cluster of stars [32]. In 1840s the Scottish astronomer John Pringle Nichol visited Birr and in 1846 he reproduced Rosse's drawing (looking more like a tadpole) in [33] where he captioned the picture as "Lord Rosse Crab Nebula"[6]. Later, J.P. Nichol used the Crab nebula name for the object M1 from the Messier catalogue and this name has become generally accepted.

In 1921 C.O. Lampland and J.C. Duncan pointed out changes observed in the Crab nebula [34, 35]. In 1928 E. Hubble noted that the Crab nebula is very close to an ancient SN 1054 described in Chinese annals [36]. Analyzing velocities in the Crab Nebula it was started to be possible to estimate an age of the object. For instance, V. Trimble analyzed the proper motions of 132 elements on Baade's photographs obtained during 1939–1966 and she concluded that explosion date of 1140 ± 10 A. D. assuming that there was no acceleration or/and curvature due to presence of magnetic fields and turbulence [37]. In 1942 Dutch sinologist J.J.L. Duyvendak translated Chinese and Japanese ancient records on the guest star visible in 1054 (a correspondence between names of constellations in Chinese and Western astronomical literatures and a correspondence between Chinese and Western

[5] William Parsons, 3rd Earl of Rosse (1800–1867) built the telescopes which were greatest ones in the XIX century. He was ready to share his a great telescope with others [31]. He was president of the Royal Society (UK) from 1849 to 1854 and he was elected a member of the Imperial Academy of Sciences of St. Petersburg in 1853.

[6] ianridpath.com/startales/rosse-crab.html.

calendars were established in this translation) and he identified the Crab nebula with the SN 1054 [38]. Astronomical aspects of this identification were discussed by N.U. Mayall and J.H. Oort [39] and the authors confirmed that the Crab Nebula may be identified with the 1054 supernova. This identification caused an additional attention to the Crab nebula. W. Baade got photographs of the Crab nebula and concluded that the nebulosity consists of two distinct parts, namely, an outer system of filaments and an inner mass of amorphous structure [40]. He showed that regions with H_α emission are localized in the envelope of filaments, the continuum was emitted in the amorphous mass. R. Minkowski made spectral observations of the Crab nebula and concluded that his observations supported the Chandrasekhar's suggestion that supernovae of type I are due to transition of stars heavier than the limiting mass into the degenerate state [41]. In 1950s B. Miller and H. Abt found petroglyphs in Arizona which represent the Crab nebula and Moon in the spring 1954 according to opinions of the authors [42, 43].

In 1948 J. Bolton discovered four new radio sources including Taurus A, however, angular width was not small (<30°) around NGC 1952 (Messsier 1) [44]. Soon after that, the location of the radio source was much more significantly bounded ($\delta = \pm 30^s$, $\alpha = \pm 7'$) [45].

After observations of the Crab nebula in radio and optical bands it was started to be clear that it is very hard to explain an emission in this object as a thermal emission. In 1953 a famous Soviet astrophysicist I.S. Shklovskii proposed a synchrotron emission[7] as a key mechanism to explain observational data in a wide range of frequency band from radio to optics [50]. Later the Shklovskii's model was confirmed with consequent X-ray data. In his memoirs Shklovskii noted the idea to use a synchrotron mechanism for the Crab nebula was among of his brightest insights [51].

Galactic X-rays started to observe with space telescopes since early 1960s and from this time the Galactic X-ray flux was associated with known sources including the Crab nebula [52] but later during a lunar occultation on July 7, 1964 it was found that X-ray source at the Crab is rather compact [53].

In 1968 it was reported on a discovery of rapidly pulsating radio source [54] (the corresponding data were obtained in Mullard Radio Astronomy Observatory in July 1967). After these observations in July 1967 rumors spread about the discovery of a new phenomenon basically among British and American researchers. For instance, J.A. Wheeler was reminded [55]: "In the fall of 1967, Vittorio Canuto, administrative head of NASA's Goddard Institute for Space Studies, invited me to a conference to consider possible interpretations of the exciting new evidence just arriving from England on pulsars. What were these pulsars? Vibrating white dwarfs? Rotating neutron stars?" One of the first model for pulsars was proposed by F. Pacini [56] (the paper was submitted October 3, 1967). In this model Pacini developed the model proposed by Hoyle, Narlikar and Wheeler [57] where the authors considered a neutron star as an oblique rotator with a high magnetic field and such a star might directly emit electromagnetic waves. These ideas were adopted F. Pacini for the pulsar model in [58]. A similar model was proposed by T. Gold[8] [59] (this paper was submitted on May 20, 1968 and it was accepted on 25 May, 1968). However, the initial reception of these ideas was not positive. As an illustration T. Gold noted that when he planned to participate at the The First International Conference on Pulsars organized by the NASA Goddard Institute for Space Studies in New York Yeshiva University for May 20 and 21, only three months after the first announcement of the discovery. T. Gold reminded on these times [60]: "I had sent my paper to the organizers, prior to its publication in Nature, with the request for a brief time slot at the conference to present these considerations. Their response was, "Your suggestion is so outlandish that if we admit this for presentation to the conference, there would be no end to the number of other equally crazy suggestions that we would have to admit". I was not allowed to speak formally, though I got in a few words from the floor". Initially, models of radially pulsating white dwarf stars were the most popular theoretical models for pulsars [60]. In spite of that Pacini and Gold developed their models of rotating neutron stars with very high magnetic fields around 10^{12}–10^{13} G [58, 61] (and so huge magnetic fields do not look realistic at this time).

Soon after the discovery of the first pulsar two other pulsars (NP 0527 and NP 0532) were discovered near the Crab nebula using 91-m Green Bank antenna [62], later it was established that the pulsar the NP 0532 is located in the Crab nebula [63]. Using 1000 foot antenna at the Arecibo the heliocentric period determined from data taken on November 15, 1968 is 33.09112 ± 0.00003 ms [64][9]. In 1969 it was established that there is an X-ray pulsar in the Crab Nebula with interpulses separated by about 12 ms [66].

[7] The synchrotron radiation was discovered by Pomeranchuk and his co-authors in 1940s [46–48]. In 1947 I.S. Shklovsky and V.L. Ginzburg participated in the Brazilian expedition of USSR Academy of Sciences to observe the Solar eclipse in optics and radio and there S.E. Khaikin and B.M. Chikhachev discovered radio emission from Solar corona. Earlier Shklovsky predicted that radio emission should be generated in the Solar corona [49].

[8] Both researchers (F. Pacini and T. Gold) worked at the Center for Radiophysics and Space Research in Cornell University at the end of 1960s when the first their papers on pulsar models were published.

[9] The discovery of the Crab Nebula Pulsar was described in [65].

Considering the Crab nebula case we should note that great efforts of many experts in different fields (including ancient astronomers and historians, linguists and sinologists, physicists (experimentalists and theorists), astrophysicists gave an opportunity to verify the Baade–Zwicky hypothesis that neutron stars were born in supernova explosions. Therefore, contributions of all these people were extremely valuable for our current understanding the processes where nuclear matter could be formed in nature. Ancient astronomers did not know that they participated in investigations of a fundamental physical problem they simply honestly observed sky and recorded results, archivists carefully kept these records, sinologists established correspondences between Eastern and Western chronologies and astronomical terminologies. Only at the instant when the tiling puzzle was solved experts could be confirmed that they really contributed in a solution of a fundamental problem of nature. A confirmation that ancient Eastern astronomers contributed in fundamental physical problem solution came after several hundred years as in the case of the Crab nebula. Generally, if at least one action in our movement to solve the problem is missed, the solution of the puzzle may be unattainable.

One of the main aim of mega-science project NICA is the study of nuclear matter in a high baryonic density. Such a matter exists in interiors of neutron stars. Therefore, observations of neutron stars and analysis of observational data are complimentary to accelerator experiments in laboratory and astrophysical data could significantly constrain models for equation of state for nuclear matter. A topical issue on exotic matter in neutron stars was published in the European Physical Journal A [67], in particular, astrophysical constraints for dense matter phases were considered in the collected papers.

A more detailed information on equation of state for neutron stars and on their structure could be found in [68], astrophysics of neutron stars was presented in [69].

3. NEUTRON STARS AND GRAVITATIONAL WAVES

Neutron stars played exclusively important role in discoveries of gravitational radiation. In July 1974 59-ms pulsar, PSR 1913 + 16 was detected at the Arecibo Observatory in Puerto Rico [70]. Orbital period of binary system is $P_b = (27\,908 \pm 7)$ s, eccentricity is $e = 0.615 \pm 0.010$, the mass function is $f(m) = (M_2 \sin i)^3 / (M_1 + M_2)^2$, i is the the inclination between the orbit and the plane of the sky, M_1 and M_2 are masses of the components in the binary system. The discovery of the close binary system gave an opportunity to investigate special and general relativistic effects. Later, parameters of the binary were clarified, in particular, the pulsar mass is $m_P = (1.42 \pm 0.06)M_\odot$, the companion mass is $m_c = (1.41 \pm 0.06)M_\odot$, the total mass was evaluated with a better accuracy $M = m_P + m_c = (2.8275 \pm 0.007)M_\odot$ the inclination is determined from the relation $\sin i = 0.72 \pm 0.03$ [71]. Since the close binary system emits gravitational radiation the orbit is shrinking while the orbital period is decreasing and in this case we could use the simplest expression for the orbital period changes derived in [72]. For binary system PSR 1913 + 16 the theoretical estimate or the orbital period derivative is $\dot{P}_b = (-2.403 \pm 0.005) \times 10^{-12}$ (this quantity is dimensionless). Observations yielded the measured value $\dot{P}_b = (-2.30 \pm 0.22) \times 10^{-12}$ [71]. This nice agreement provided an evidence of the existence of gravitational radiation. Therefore, it was obtained a confirmation of general relativity predictions.

It was known that gravitational waves were firstly detected from the binary black hole merger (GW150914) and signals were obtained from two LIGO detectors located at Hanford (Washington) and Livingston (Louisiana) [73]. In this paper the authors reported that they discovered gravitational waves, binary black holes and in addition, they constrained graviton mass. Moreover, GR predictions in strong gravitational field approximation were confirmed in coincidence of detected gravitational wave signal and theoretically calculated one. In spite of a beautiful coincidence of theoretical simulations and observations at a significance level greater than 5.1σ a solid confirmation that LIGO–Virgo collaborations indeed detected GW signal but not noise with unknown origin was found in the detection of binary neutron star merger GW170817 [74]. Observations of electromagnetic counterpart with ground and space-based facilities strongly supported these conclusions [75, 76]. These achievements launched a new stage of astronomical observations with multi-messenger facilities.

Pulsars represent ideal natural clocks in space since an observer detects times of arrival (TOA) of electromagnetic pulses. In 1978 M.V. Sazhin proposed a way to detect ultra long gravitational waves generated by binary supermassive black holes analyzing perturbations for TOA [77]. Soon after that this idea was polished and promoted in [78]. Constraints of stochastic gravitational wave background were discussed in [79]. In Arecibo Observatory a pulsar timing array started in 1980s [80]. A physical sense of Hellings–Downs curve was explained in [81]. The NANOGrav collaboration monitored 67 pulsars for around 15 yr using Arecibo Observatory Telescope, Green Bank Telescope (GBT) and Very Large Array (VLA) and the collaboration declared that they found stochastic gravitational-wave background since correlations follow the Hellings–Downs pattern with probability $p = 10^{-3}$ ($\approx 3\sigma$) [82]. According to estimates of these authors a typical amplitude for gravitational-wave background (GWB) at reference frequency $1\,\mathrm{yr}^{-1}$ is $A_{\mathrm{GWB}} = 2.4^{+0.7}_{-0.6} \times 10^{-15}$ and it corresponds to integrated energy of $\Omega_{\mathrm{gw}} = 9.3^{+5.8}_{-4.0} \times 10^{-9}$ (in critical density units).

4. TESTS OF GENERAL RELATIVITY

General relativity (GR) was created more than a century ago and since these times it successfully passed all experimental tests and it was confirmed in numerous observations. In one of the first works on GR, A. Einstein successfully solved the problem of explaining the anomaly of Mercury's motion (now this anomaly is called the Schwarzschild precession) and calculated the angle of deflection of the light near the gravitating body. This prediction was confirmed by observations during Solar eclipse in 1919 and this test concerning deflection of light during solar eclipse was very impressive because GR better explained observational data visible displacements of foreground stars than Newtonian theory of gravity. However, it is necessary to note that a majority of these tests were realized for weak gravitational field cases.

Recently, gravity laws were checked for antimatter in CERN. Namely, the ALPHA-g collaboration tested relativistic prediction for antihydrogen and the authors claimed that within the stated errors the detected value is consistent with downward acceleration of $1g$ for antihydrogen and this result is in correspondence with GR expectations.

Currently, there are two opportunities to test gravity laws in a strong gravitational field where we really have to solve Einstein equations. The first case is the early Universe and we do not discuss the case in the paper. Astrophysical black holes represent another case and we will discuss the case briefly in the next section.

5. ASTROPHYSICAL BLACK HOLES

As it was noted earlier, the LIGO collaboration discovered gravitational waves from binary black hole merger (gravitational wave event GW150914). After data analysis for the first three LIGO–Virgo observing runs between September 15 and March 2020 [84]. The Gravitational-Wave Transient Catalog (GWTC-3) consists of 90 mergers binaries comprised of black holes (BHs) and neutron stars (NSs). Majority of these events are binary black hole mergers, however, there are two binary neutron star mergers (GW170817 and GW190425) but electromagnetic counterpart was found only for GW170817. It was found four NSBH binary systems (GW200105, GW200115, GW190426, GW190917) and for them electromagnetic counterparts were not found as well. We also should note that theoretically and numerically calculated templates of gravitational waves are in remarkable accordance with observational data. It means that we have an additional GR test in a strong gravitational field approximation.

In order to find a suitable model of the gravitational potential for an astronomical object, it is reasonable to analyze motions of test particles in the vicinity of the object. In recent years, there has been tremendous progress in the observational capabilities of modern large telescopes. One of the features of a number of large telescopes is the adaptive optics system. Equipping telescopes with adaptive optics systems can significantly improve the resolution of ground-based telescopes, since it can significantly reduce the influence of atmospheric turbulence which usually spoils images of astronomical objects. Two groups of astronomers monitored a vicinity of the Galactic Center (GC) for decades. In particular, they observed trajectories of bright stars near GC. One group led by Andrea Ghez uses the 10-m Keck twin telescope in Hawaii, another group led by Reinhard Genzel uses four VLT telescopes in Chili. Now four VLTs could act as an interferometer, which is called GRAVITY, perspectives of current and future interferometric observations in optical and infrared bands are discussed in [85]. These telescopes were equipped with adaptive optics facilities. Analyzing the trajectories of bright stars astronomers from both groups concluded that the gravitational field at GC is basically determined by supermassive black hole with mass around $4 \times 10^6 M_\odot$ while extended mass component inside the S2 star orbit must be less than $3000 M_\odot$ at 1σ level [86]. In May 2018 the S2 star was passing the pericenter of its orbit and analyzing results of spectral observations GRAVITY and Keck collaborations declared that observational data for gravitational redshifts are in concordance with theoretical estimates [87–89]. The GRAVITY collaboration analyzed hot spot motions near the event horizon of the supermassive black hole at the GC [90, 91] and compared these data with theoretical fits based on calculations of geodesics in Kerr metric, therefore, we have an opportunity to investigate GR predictions in a strong gravitational field limit. Recently, the GRAVITY collaboration found that the Schwarzschild precession (relativistic advance) corresponds to its estimate done in the first post-Newtonian approximation [92]. Based on observational data concerning the Schwarzschild precession constraints on Yukawa gravity and graviton mass were found [93, 94].

About 100 yr ago, when quantum mechanics and the theory of relativity were created, thought experiments were quite popular when discussing various problems. At the same time, the possibility of implementing these experiments was not usually discussed. However, as new technologies and experimental techniques develop, it becomes possible to realize thought experiments in practice. In the case of astronomical observations, in order to implement thought experiments (observations) into real ones, it is necessary to answer three questions: which object needs to be observed, which observational instruments need to be used, and what needs to be discovered as a result of such observations. Thus, the concept of the shadow of a black hole has evolved from a theoretical category into a quantity that can be obtained from observations. The shadow concept was proposed by J.M. Bardeen [95]. He assume that there is a bright screen behind a Kerr black hole and photons are freely propagate near this black hole. In this case an observer can see a dark spot (a shadow) on the background of a bright screen, but Bardeen did not discuss an opportunity to observe a shadow since first there is no bright screen behind black holes, second, for known distances and masses angular sizes of shadows are too small to be detected. However, observational facilities are improving in particular, interferomeric systems are developing not only in radio band, but also for shorter waves including optical and X-ray bands. In the 1980s, L.I. Matveenko proposed launching a space radio telescope, but the implementation of this project was delayed for economic reasons. An active work on the project, which was called Radioastron, was resumed in the 2000s (the interferometer was in operation from 2011 to 2019). The angular resolution of the interferometer with a ground-based space base was 8 μas at the shortest wavelength of 1.3 cm. The black hole in the Galactic Center has a mass of $4\times 10^6 M_\odot$, it is located at a distance of about 8 kpc, thus the angular size of the Schwarzschild radius is comparable to the angular resolution of the interferometer. Therefore, it makes sense to consider relativistic effects in the vicinity of a black hole in the Galactic Center. Falcke et al. considered a numerical model do observe the black hole shadow at the Galactic Center [96] where it was also noted that a shadow could be detected in mm band while in cm band a scatter of photons by electrons is smearing shadows. In [97] it was discussed an opportunity to detect a shadow around the black hole at GC with ground (or space—ground) VLBI acting in mm and sub-mm bands since the shadow size is around $50\,\mu as$. This prediction was remarkably confirmed when the Event Horizon Telescope (EHT) Collaboration reconstructed the shadow at GC [98] (earlier the EHT Collaboration reconstructed the shadow for the black hole at M87* [99]). In [100] it was shown that in the case of Reissner–Nordström metric there is an analytical expression for shadow size as a function of electric charge. Later, this relation the relations were generalized for a tidal charge case (in this case q^2 may be negative where q is a charge) [101, 102]. Since the EHT collaboration constrained shadow sizes for M87* and Sgr A*, in [103–106] it was shown an opportunity to constrain a tidal charge (or a corresponding parameter of scalar-tensor Horndeski theories).

6. CONCLUSIONS

Investigating different aspects of neutron star astrophysics a number of remarkable discoveries have been done. Studies of nuclear matter is among the most interesting problems of fundamental physics for more than 90 yr. Observations of bright stars near the Galactic center and their theoretical analysis give an opportunity to check GR predictions and constrain alternative theories of gravity. Due to enormous progress of observational facilities some theoretical concepts transform to observational quantity which may be obtained from astronomical observations. For instance, now the shadow is not only theoretical concept since currently it may be obtained from shadow reconstructions for M87* and Sgr A* [107].

ACKNOWLEDGEMENTS

The author thanks the organizers of the 25th International Seminar on High Energy Physics Problems “Relativistic Nuclear Physics and Quantum Chromodynamics” (JINR, Dubna) for the invitation to present an invited talk at this event and their attention to the research presented in the paper. The author appreciates the referee for constructing criticism and useful remarks which helped to revise the manuscript.

FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest. In a recent paper [108] we constrained a graviton mass based on C. Will approximation of gravitational potential while in [109] new tests of GR were considered. A remarkable review on astrophysical black holes was published recently in [110].

REFERENCES

1. W. Pauli, “Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren,” Z. Phys. **31**, 765–783 (1925).
2. G. E. Uhlenbeck and S. Goudsmit, “Ersetzung der Hypothese vom unmechanischen Zwang durch eine Forderung bezüglich des inneren Verhaltens jedes einzelnen Elektrons,” Die Naturwissenschaften **13**, 953–954 (1925).
3. G. E. Uhlenbeck and S. Goudsmit, “Spinning electrons and the structure of spectra,” Nature **117**, 264–265 (1926).
4. S. A. Goudsmit, “The discovery of the electron spin.” https://www.lorentz.leidenuniv.nl/history/spin/goudsmit.html.

5. S. Kean, *The Bastard Brigade: The True Story of the Renegade Scientists and Spies Who Sabotaged the Nazi Atomic Bomb* (Little, Brown and Co., New York, 2019).
6. E. Fermi, "Sulla quantizzazione del gas perfetto monoatomico," Rendiconti Lincei **3**, 145—149 (1926). Translated by A. Zannoni, "On the quantization of the monoatomic ideal gas," arXiv:cond-mat/9912229.
7. P. A. M. Dirac, "On the theory of quantum mechanics," Proc. R. Soc. A **112**, 661–677 (1926).
8. R. H. Fowler, "On dense matter," Mon. Not. R. Astron. Soc. **87**, 114–122 (1926).
9. J. Frenkel, "Anwendung der Pauli-Fermischen Elektronengastheorie auf das Problem der Kohäsionskräfte," Z. Phys. **50**, 234–248 (1928).
10. D. G. Yakovlev, "The article by Ya. I. Frenkel' on 'binding forces' and the theory of white dwarfs," Sov. Phys. Usp. **37**, 609–612 (1994).
11. E. C. Stoner, "The limiting density in white dwarf stars," London, Edinburgh and Dublin Philos. Mag. J. Science. (Ser. 7) **7**, 63—70 (1929).
12. E. C. Stoner, "The limiting density in white dwarf stars," London, Edinburgh and Dublin Philos. Mag. J. Science. (Ser. 7) **9**, 944—963 (1930).
13. S. Chandrasekhar, "The maximum mass of ideal white dwarfs," Astrophys. J. **74**, 81–82 (1931).
14. S. Chandrasekhar, "The highly collapsed configurations of a stellar mass," Mon. Not. R. Astron. Soc. **91**, 456–466 (1931).
15. S. Chandrasekhar, "Stellar configurations with degenerate cores," Observatory **57**, 373–377 (1934).
16. L. D. Landau, "On the theory of stars," Phys. Z. Sowjetunion **1**, 285–288 (1932).
17. D. G. Yakovlev, P. Haensel, G. Baym, and C. Pethick, "Lev Landau and the concept of neutron stars," Phys. Usp. **56**, 289–295 (2013).
18. J. Chadwick, "Possible existence of neutron," Nature **129**, 312 (1932).
19. S. E. Frisch, *Through the Prism of Time* (Politizdat, Moscow, 1991) [in Russian].
20. W. Baade and F. Zwicky, "Remarks on super-novae and cosmic rays," Phys. Rev. **46**, 76–77 (1934).
21. W. Baade and F. Zwicky, "Cosmic rays from super-novae," Proc. Natl. Acad. Sci. **20**, 259–263 (1934).
22. G. Gamow, *Structure of Atomic Nuclei and Nuclei Transformations* (Gos. Tekh.-Teor. Izd., Moscow, 1932; Oxford Press, Oxford, 1937).
23. H. Ludwig and R. Ruffini, "Gamow's calculation of the neutron star's critical mass revisited," J. Kor. Phys. Soc. **65**, 892–896 (2014).
24. J. R. Oppenheimer and G. M. Volkoff, "On massive neutron cores," Phys. Rev. **55**, 374–381 (1939).
25. C. E. Rhoades and R. Ruffini, "Maximum mass of a neutron star," Phys. Rev. Lett. **32**, 324–327 (1974).
26. L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mischenko, *The Mathematical Theory of Optimal Processes* (Nauka, Moscow, 1983; Interscience, New York, 1962).
27. G. B. Cook, S. L. Shapiro, and S. A. Teukolsky, "Rapidly rotating neutron stars in general relativity: Realistic equations of state," Astrophys. J. **424**, 823–845 (1994).
28. I. S. Shklovsky, *Supernovae* (Nauka, Moscow, 1966; Wiley, New York, 1968).
29. "A survey of current research," in *Proceedings of the NATO Advanced Study Institute at Cambridge, U. K., 1981*, Ed. by M. Rees and R. J. Stoneham (Dordrecht Reidel, Boston, 1982).
30. M. Messier and J. Bevis, "Extract of two letters from M. Messier, of the Royal Academy of Sciences, and F. R. S. to M. De Magalhaens, on a New Comet: Translated by Dr. Bevis, F. R. S.," Phil. Trans. **61**, 104–106 (1771).
31. "*Earl of Rosse*, William Parsons, the third Earl of Rosse," Hermathena **107**, 5—13 (1868).
32. Earl of Rosse (Parsons W.), "Observations of some of the nebulae," Philos. Trans. R. Soc. London **134**, 321–324 (1844).
33. J. P. Nichol, *Thoughts on Some Important Points Relating to the System of the World* (William Tate, Edinburg, 1846).
34. C. O. Lampland, "Observed changes in the structure of the "Crab" Nebula (NGC 1952)," Publ. Astron. Soc. Pacific **33**, 79–84 (1921).
35. J. C. Duncan, "Changes observed in the Crab Nebula in Taurus," Proc. Natl. Acad. Sci. **7**, 179–180 (1921).
36. E. P. Hubble, "Novae or temporary stars," Astron. Soc. Pacific Leaflets **7**, 105–112 (1955).
37. V. Trimble, "Motions and structure of the filamentary envelope of the Crab Nebula," Astron. J. **73**, 535–658 (1968).
38. J. J. L. Duyvendak, "Further data bearing on the identification of the Crab Nebula with the supernova of 1054 AD. Part I. The ancient oriental chronicles," Publ. Astron. Soc. Pacific **54**, 91–94 (1942).
39. N. U. Mayall and J. H. Oort, "Further data bearing on the identification of the Crab Nebula with the supernova of 1054 AD. Part II. The astronomical aspects," Publ. Astron. Soc. Pacific **54**, 95–104 (1942).
40. W. Baade, "The Crab Nebula," Astrophys. J. **96**, 188–198 (1942).
41. R. Minkowski, "The Crab Nebula," Astrophys. J. **96**, 199–213 (1942).
42. W. C. Miller, "Two prehistoric drawing of possible astronomical significance," Astron. Soc. Pacific Leaflets **7**, 105–112 (1955).
43. H. A. Abt and J. W. Fountain, "An earlier explosion date for the Crab Nebula supernova," Res. Astron. Astrophys **18**, 37 (2018).
44. J. G. Bolton, "Discrete sources of galactic radio frequency noise," Nature **162**, 141–142 (1948).
45. J. G. Bolton, G. J. Stanley, and O. B. Slee, "Positions of three discrete sources of galactic radio-frequency radiation," Nature **164**, 101–102 (1949).
46. I. Pomeranchuk, "The maximum energy that primary cosmic ray electrons can have on the Earth's surface due to radiation in the Earth's magnetic field," J. Phys. USSR **2**, 65–71 (1940).
47. D. Iwanenko and I. Pomeranchuk, "On the maximal energy attainable in a betatron," Phys. Rev. **65**, 343 (1944); Dokl. Akad. Nauk SSSR **44**, 343 (1944).

48. L. A. Artsimovich and I. Pomeranchuk, “Radiation of rapid electrons in magnetic field,” J. Phys. USSR **9**, 267–280 (1945); Zh. Eksp. Teor. Fiz. **16**, 379–392 (1946).
49. I. S. Shklovsky, “The current state of the question about the nature of the solar corona,” Sov. Phys. Usp. **30**, 63–102 (1946).
50. I. S. Shklovskii, “On the nature of the glow of the Crab Nebula,” Dokl. Akad. Nauk SSSR **90**, 983–986 (1953).
51. I. S. Shklovsky, *Five Billion Vodka Bottles to the Moon: Tales of a Soviet Scientist* (W.W. Norton, New York, 1991).
52. H. Gursky, R. Giacconi, F. R. Paolini, and B. Rossi, “Further evidence for the existence of galactic X rays,” Phys. Rev. Lett. **11**, 530–535 (1963).
53. H. Gursky, R. Giacconi, F. R. Paolini, and B. Rossi, “Lunar occultation of X-ray emission from the Crab Nebula,” Science **146**, 912–917 (1963).
54. A. Hewish, S. J. Bell, J. D. H. Pilkington, P. F. Scott, and R. A. Collins, “Observation of a rapidly pulsating radio source,” Nature **217**, 709–713 (1948).
55. K. W. Ford and J. A. Wheeler, *Geons, Black Holes and Quantum Foam: A Life in Physics* (W. W. Norton, New York, 2000).
56. F. Pacini, “Observation of a rapidly pulsating radio source,” Nature **216**, 567–568 (1967).
57. F. Hoyle, J. Narlikar, and J. A. Wheeler, “Electromagnetic waves from very dense stars,” Nature **203**, 914–916 (1968).
58. F. Pacini, “Rotating neutron stars, pulsars and supernova remnants,” Nature **219**, 145–146 (1968).
59. T. Gold, “Rotating neutron stars as the origin of the pulsating radio sources,” Nature **218**, 731–732 (1968).
60. T. Gold, *Taking the Back Off the Watch: A Personal Memoir*, Ed. by S. Mitton (Springer, Berlin, 2012).
61. T. Gold, “Rotating neutron stars and the nature of pulsars,” Nature **221**, 25–27 (1969).
62. D. H. Staelin and E. C. Reifenstein, III, “Pulsating radio sources near the Crab Nebula,” Science **162**, 1481–1483 (1968).
63. E. C. Reifenstein, III, W. D. Brundage, and D. H. Staelin, “Crab Nebula pulsar NP 0527,” Phys. Rev. Lett. **22**, 311–311 (1969).
64. J. M. Comella, H. D. Craft, Jr., R. V. E. Lovelace, and J. M. Sutton, “Crab Nebula pulsar NP 0527,” Nature **221**, 453–454 (1969).
65. R. V. E. Lovelace and G. L. Tyler, “On the discovery of the period of the Crab Nebula pulsar,” Observatory **132**, 186–188 (2012).
66. G. Fritz, R. C. Henry, J. F. Meekins, T. A. Chubb, and H. Friedman, “X-ray pulsar in the Crab Nebula,” Science **164**, 709–712 (1969).
67. D. Blaschke, J. Schaffner-Bielich, and H.-J. Schulze, “Topical issue on exotic matter in neutron stars,” Eur. Phys. J. A **52**, 71 (2016).
68. P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, *Neutron Stars I: Equation of State and Structure* (Springer, Berlin, 2007).
69. V. M. Lipunov, *Astrophysics of Neutron Stars* (Nauka, Moscow, 1987; Springer, Berlin, 1992).
70. R. A. Hulse and J. H. Taylor, “Discovery of a pulsar in a binary system,” Astrophys. J. Lett. **195**, L51–L53 (1969).
71. J. H. Taylor and J. M. Weisberg, “A new test of general relativity: Gravitational radiation and binary pulsar PSR 1913+16,” Astrophys. J. **253**, 908–920 (1982).
72. P. C. Peters and J. Mathews, “Gravitational radiation from point masses in a Keplerian orbit,” Phys. Rev. **131**, 435–440 (1963).
73. B. P. Abbott et al. (LIGO Sci. Collab. and Virgo Collab.), “Observation of gravitational waves from a binary black hole merger,” Phys. Rev. Lett. **116**, 061102 (2016).
74. B. P. Abbott et al. (LIGO Sci. Collab. and Virgo Collab.), “GW170817: Observation of gravitational waves from a binary neutron star in spiral,” Phys. Rev. Lett. **119**, 161101 (2017).
75. B. P. Abbott et al. (LIGO Sci. Collab. and Virgo Collab.), “Multi-messenger observations of binary neutron star merger,” Astrophys. J. Lett. **848**, L12 (2017).
76. B. P. Abbott et al. (LIGO Sci. Collab. and Virgo Collab.), “A gravitational-wave standard siren measurement of the Hubble constant,” Nature **551**, 85–88 (2017).
77. M. V. Sazhin, “Opportunities for detecting ultra-long gravitational wave,” Sov. Astron. **22**, 36–38 (1978).
78. S. Detweiler, “Pulsar timing measurements and the search for gravitational waves,” Astrophys. J. **234**, 1100–1104 (1979).
79. R. W. Hellings and G. S. Downs, “Upper limits on the isotropic gravitational radiation background from pulsar timing analysis,” Astrophys. J. Lett. **265**, L39–L42 (1983).
80. R. S. Foster and D. C. Backer, “Constructing a pulsar timing array,” Astrophys. J. **361**, 300–308 (1990).
81. F. A. Jenet and J. D. Romano, “Understanding the gravitational-wave Hellings–Downs curve for pulsar timing array in terms of sound and electromagnetic waves,” Am. J. Phys. **83**, 635–646 (2015).
82. G. Agazie et al. (NANOGrav Collab.), “NANOGrav 15 yr data set: Evidence for a gravitational-wave background,” Astrophys. J. **951**, L8 (2023).
83. E. K. Anderson, C. J. Baker, W. Bertsche, et al., “Observation of the effect of gravity on the motion of antimatter,” Nature **621**, 716–738 (2023).
84. R. Abbott et al. (LIGO Sci. Collab., Virgo Collab., KAGRA Collab.), “Population of merging compact binaries inferred using gravitational waves through GWTC,” Phys. Rev. X **13**, 011048 (2023).
85. F. Eisenhauer, J. D. Monnier, and O. Pfuhl, “Advances in optical/infrared interferometry,” Ann. Rev. Astron. Astrophys. **61**, 237–285 (2023).
86. R. Abuter et al. (GRAVITY Collab.), “Mass distribution in the Galactic center based on interferometric astrometry of multiple stellar orbits,” Astron. Astrophys. **657**, L12 (2022).
87. R. Abuter et al. (GRAVITY Collab.), “Detection of the gravitational redshift in the orbit of the star S2 near the Galactic centre massive black hole,” Astron. Astrophys. **615**, L15 (2018). arXiv:1807.09409.

88. R. Abuter et al. (GRAVITY Collab.), "A geometric distance measurement to the galactic center black hole with 0.3% uncertainty," Astron. Astrophys. **625**, L10 (2019). arXiv:1904.05721.
89. T. Do, A. Hees, A. Ghez, et al., "Relativistic redshift of the star S0-2 orbiting the galactic center supermassive black hole," Science **365**, 664–668 (2019). arXiv: 1907.10731v1 [astro-ph.GA].
90. R. Abuter et al. (GRAVITY Collab.), "Detection of orbital motions near the last stable circular orbit of the massive black hole Sgr A*," Astron. Astrophys. **618,** L10 (2018). arXiv:1810.12641.
91. M. Bauböck et al. (GRAVITY Collab.), "Modeling the orbital motion of Sgr A*'s near-infrared flares," Astron. Astrophys. **635**, A143 (2018).
92. R. Abuter et al. (GRAVITY Collab.), "Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic Centre massive black hole," Astron. Astrophys. **636**, L5 (2020). arXiv:2004.07187.
93. P. Jovanović et al., "Constraints on Yukawa gravity parameters from observations of bright stars," J. Cosm. Astropart. Phys **03**, 056 (2023). arXiv:2211.12951.
94. P. Jovanović et al., "Improvement of graviton mass constraints using GRAVITY's detection of Schwarzschild precession in the orbit of S2 star around the Galactic Center," Phys. Rev. D **109**, 064046 (2024). arXiv:2305.13448.
95. J. Bardeen, "Timelike and Null Geodesics in the Kerr Metric." In *Black Holes (Les Astres Occlus)*, Ed. by S. DeWitt and C. DeWitt-Morette (Gordon and Breach, New York, 1973)), pp. 215–239.
96. H. Falcke, F. Melia, and E. Agol, "Viewing the shadow of the black hole at the Galactic center," Astrophys. J. **528**, L13–L16 (2000). arXiv:astro-ph/9912263.
97. A. F. Zakharov, A. A. Nucita, F. De Paolis, and G. Ingrosso, "Measuring the black hole parameters in the Galactic center with RADIOASTRON," New Astron. **10**, 479–489 (2005).
98. K. Akiyama, A. Alberdi, W. Alef, et al. (Event Horizon Telescope Collab.), "First Sagittarius A* Event Horizon Telescope results. I. The shadow of the supermassive black hole in the center of the Milky Way," Astrophys. J. Lett. **930**, L12 (2022).
99. K. Akiyama, A. Alberdi, W. Alef, et al. (Event Horizon Telescope Collab.), "First M87 Event Horizon Telescope results. I. The shadow of the supermassive black hole," Astrophys. J. Lett. **875**, L1 (2019). arXiv: 1906.11238.
100. A. F. Zakharov, F. De Paolis, G. Ingrosso, and A. A. Nucita, "Direct measurements of black hole charge with future astrometrical missions," Astron. Astrophys. **442**, 795–799 (2005). arXiv:astro-ph/0505286.
101. A. F. Zakharov, A. A. Nucita, F. De Paolis, and G. Ingrosso, "Shadows as a tool to evaluate black hole parameters and a dimension of spacetime," New Astron. Rev. **56**, 64–73 (2012).
102. A. F. Zakharov, "Constraints on a charge in the Reissner-Nordström metric for the black hole at the Galactic center," Phys. Rev. D **90**, 062007 (2014). arXiv: 1411.5911 [gr-qc].
103. A. F. Zakharov, "Constraints on a tidal charge of the supermassive black hole in M87* with the EHT observations in April 2017," Universe **8**, 141 (2022). arXiv: 2108.01533.
104. A. F. Zakharov, "Shadows around Sgr A* and M87* as a tool to test gravity theories," Astron. Astrophys. Trans. **33**, 285–296 (2022). arXiv:2208.06805[astro-ph.GA].
105. A. F. Zakharov, "The Galactic center and M87*: Observations and interpretations," Phys. Part. Nucl. Lett. **20**, 538—543 (2023).
106. A. F. Zakharov, "Trajectories of bright stars and shadows around supermassive black holes as tests of gravity theories," Phys. Part. Nucl. **54**, 889–895 (2023).
107. A. F. Zakharov, "Shadows near supermassive black holes: From a theoretical concept to GR test," Intern. J. Mod. Phys. D **54,** 2340004 (2023). arXiv: 2308.01301.
108. P. Jovanović et al., "Constraints on graviton mass from Schwarzschild precession in the orbits of S-Stars around the Galactic Center," Symmetry **16**, 397 (2024). arXiv:2404.09795.
109. A. F. Zakharov, "New tests of general relativity", In Proc. of the Conference on Particle Physics and Cosmology (Rubakov Conference), Proceedings of Science (ICPPCRubakov2023) **455**, 030 (10 pp.)
110. R. Genzel, F. Eisenhauer, and S. Gillessen, "Experimental studies of black holes: status and future prospects", Astron. Astrophys. Rev. **32**:3 (2024). arXiv: 2404.03522.

Publisher's Note. Pleiades Publishing remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.