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Nuclear Decay Oscillations and Nonlinear Quantum Dynamics

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Abstract—Several experimental groups reported the evidence of periodic modulations of nuclear decay constants which amplitudes are of the order 10^{-3} and periods of one year, 24 h or about one month. We argue that such deviations from radioactive decay law can be described in nonlinear quantum mechanics framework, in which decay process obeys to nonlinear Shrödinger equation with Doebner–Goldin terms. Possible corrections to Hamiltonian of quantum system interaction with gravitation field considered, it's shown that they correspond to some emergent gravity theories, in particular, bilocal field model. It's shown that proposed model describes decay parameter variations which agree with experimental results for Po-214 α-decay life-time oscillations.

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1. INTRODUCTION

Natural radioactivity law is one of most fundamental laws of modern physics, in accordance with it, nuclear decay parameters are time-invariant and practically independent of environment [1]. However, some recent experiments have reported the evidence of periodic modulations of nuclear α- and β-decay parameters of the order of 10^{-3} and with typical periods of one year, one day or about one month [2–6]. First results on essential deviations for β-decay rate were obtained during the measurement of ^{32}Si isotope life-time [2]. Sinusoidal annual oscillations with the amplitude 6×10^{-4} relative to total decay rate and maximal rate at the end of February were found. Since then, annual oscillations of β-decay rate for different nuclei from Ba to Ra were reported, for all of them the oscillation amplitude is of the order 5×10^{-4} with its maximum on the average at mid-February [3]. Annual oscillations of ^{238}Pu α-decay rate with the amplitude of the order 10^{-3} also were reported [4]. Life-time of short-living α-decayed isotope ^{214}Po was measured directly, the annual and daily oscillations with amplitude of the order 9×10^{-4}, with annual maxima at mid-March and daily maxima around 6 a.m. were found during three years of measurements [5]. Small oscillations of decay electron energy spectra with period 6 months were found in Tritium β-decay [6]. However, some other β-decay experiments exclude any decay constant modulations as large as reported ones [7].

Until now theoretical discussion of these results had quite restricted character. In particular, oscillations of β-decay rate was hypothized as anomalous interaction of Sun neutrino flux with nuclei or seasonal variations of fundamental constants [3]. Yet, neither of these hypothesis can't explain α-decay parameter oscillations of the same order, because nucleus α-decay stipulated by strong interaction and should be insensitive to neutrino flux or other electroweak processes. Therefore, observations of parameter oscillations for both decay modes supposes that some universal mechanism independent of particular type of nuclear interactions induces the decay parameter oscillations. Nowadays, the most universal microscopic theory is quantum mechanics (QM), in its framework, radioactive nucleus treated as metastable quantum state [8, 9]. Evolution of such states was the subject of many investigations and its principal features are now well understood [9]. Notorious example is Gamow theory of α-decay which describes it as quantum tunneling of α-particle through the potential barrier constituted by nuclear shell and nucleus Coulomb field [10, 11]. However, in its standard formulation, Gamow theory excludes any significant variations of decay parameters under influence of Sun gravity and similar astrophysical factors. In this paper, it's argued that such influence can appear, if one applies for α-decay description the nonlinear modification of standard QM, which developed extensively in the last years [12, 13]. In particular, we shall use Doebner–Goldin formalism modified to account nonlinear interaction of quantum systems with gravity [14, 15]. Basing on it, Gamow α-decay theory with nonlinear Hamiltonian constructed, and model calculations compared with experimental results for ^{214}Po α-decay life-time variations [5]. In its framework, influence of Sun gravity on α-decay presumably correspond to results of some emergent gravity theories [16].

2. NONLINEAR QM MODEL

Interest to nonlinear evolution equations can be dated back to the early days of quantum physics, but at that time they appeared in effective theories describing collective effects [8]. Now it's acknowledged also that nonlinear corrections to standard QM Hamiltonian can exist also at fundamental level [18, 19]. Significant progress in the studies of such nonlinear QM formalism was achieved in the 80s, marked by the seminal papers of Bialynicki–Birula and Mycielcki (BM), and Weinberg [12, 13]. Since then, many variants of nonlinear QM were considered in the literature (see [15] and refs. therein). Some experimental tests of QM nonlinearity were performed, but they didn't have universal character, rather they tested Weinberg and BM models only [19]. Currently, there are two different approaches to the nature of QM nonlinearity. The main and historically first one supposes that dynamical nonlinearity is universal and generic property of quantum systems [12, 13]. In particular, it means that nonlinear terms can influence their free evolution, inducing soliton-like corrections to standard QM wave packet [12]. Alternative concept of QM nonlinearity which can be called interactive, was proposed by Kibble, it supposes that free system evolution should be linear, so that nonlinear dynamics related exclusively to the system interactions with external fields [20]. Until now, detailed calculations of such nonlinear effects were performed only for hard processes of particle production in the strong fields [21, 22], this formalism can't be applied directly to nonrelativistic processes, like nucleus decay. Due to it, to describe the interaction of metastable state with external field, we'll start from consideration of universal nonlinear model and develop its modification, which can incorporate particle-field and nucleus-field interactions at low energies.

In universal approach to QM nonlinearity, it supposed that particle evolution described by nonlinear Schrödinger equation of the form [8]

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{r},t)\psi + F(\psi,\overline{\psi})\psi, \qquad (1)$$

where m is particle mass, V is external potential, F is arbitrary functional of system state. Currently, the most popular and elaborated nonlinear QM model of universal type is by Doebner and Goldin (DG) [14, 15] with nonlinear term

$$F = \hbar^2\lambda\left(\nabla^2 + \frac{|\nabla\psi|^2}{|\psi|^2}\right), \qquad (2)$$

where λ is imaginary or real constant. With the notation

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)$$

we abbreviate (1) to $i\hbar\partial_t\psi = H_0\psi + F\psi$. In fact, general DG model describes six-parameter family of nonlinear Hamiltonians, but the action of all its nonlinear terms on realistic quantum systems is similar to F of (2), hence for the start only this ansatz will be used in our calculations [15]. The choice of λ of (2) to be imaginary prompted by results of nonrelativistic current algebras [14], but they doesn't have mandatory character; below we'll consider F terms both with imaginary and real λ for $|\lambda| \ll 1$. Main properties of Eq. (1) for imaginary λ were studied in [14], they can be promptly extended on real λ and summarized for both cases as follows: (a) The probability is conserved. (b) The equation is homogeneous. (c) The equation is Euclidian- and time-translation invariant for $V = 0$. (d) Noninteracting particle subsystem remain uncorrelated (separation property). Distinct values of λ can occur for different particle species. (e) Writing $\langle Q\rangle = \int\overline{\psi}\hat{Q}\psi d^3x$ for operator expectation value, in particular, since $\int\overline{\psi}F\psi d^3x = 0$, the energy functional for a solution of (1) is $\langle i\hbar\partial_t\rangle = \langle H_0\rangle$. For $V = 0$, plane waves $\psi = \exp[i(k_0\vec{r} - \omega t)]$ with $\omega = E\hbar$, $|\vec{k}_0|^2 = 2mE/\hbar$ are solutions both for real and imaginary λ.

As was mentioned above, simple quantum model of metastable state decay describes it as particle tunneling through the potential barrier with suitable parameters [8]. It's natural to expect that for small λ the tunneling mechanism doesn't change principally, and resulting state difference from standard QM solutions is small. Hence such linear solutions can be treated as consistent approximations for incoming states to nonlinear solutions for the same system parameters. To illustrate the influence of nonlinear DG term on particle tunneling, consider 1-dimensional plane wave tunneling through the potential barrier. Suppose that rectangular barrier of the height V_0 located between $x = 0$ and $x = a$, and plane wave particle state with energy $E < V_0$ spreads from $x = -\infty$. Long-living metastable states appear for small transmission coefficient $D \to 0$, which corresponds to barrier width $a \to \infty$ for fixed E, V_0. For example, for ^{238}U α-decay $D \approx 10^{-37}$ [10]. We'll study stationary solutions of Eq. (1) basing on its asymptotic in this limit. Standard QM stationary solution for $x < 0$

$$\psi_0(x) = \exp(ikx) + A\exp(-ikx),$$

with $k = \frac{1}{\hbar}(2mE)^{\frac{1}{2}}$; for $a \to \infty$ it gives $|A| \to 1$, i.e. nearly complete wave reflection from the barrier.

Hence ψ_0 can be decomposed as $\psi_0 = \psi_\infty + \psi_d$ where the asymptotic state $\psi_\infty = 2\cos(kx - \alpha_0)$, $\alpha_0 = \arctan \chi_0/k$ where

$$\chi_0 = \frac{1}{\hbar}[2m(V_0 E)]^{\frac{1}{2}}. \quad (3)$$

Then, $\psi_d \simeq A_d \exp(-ikx)$ where $|A_d| \simeq \exp(-2\chi_0 a)$, i.e. is exponentially small. In distinction, for DG nonlinear Hamiltonian the incoming and reflected waves suffer rescattering, so stationary state $\psi \neq \psi_0$ for $x < 0$. For real λ, the stationary solution can be obtained performing nonolinear transformation of solution of adjoined linear equation [14, 15]. Namely, for real solution $\eta(x)$ of such Schrödinger equation, the solution of nonlinear equation

$$\psi = \eta \exp(\gamma \ln \eta^2),$$

where

$$\gamma = \frac{\Gamma}{1 - 4\Gamma}$$

and $\Gamma = \lambda m$. Below for brevity such exponential ansatz replaced by corresponding function rate. In particular, asymptotic solution for $x \le 0$ and $a \to \infty$

$$\psi_N 2[\cos(qx - \alpha)]^{\frac{1-2\Gamma}{1-4\Gamma}}$$

with $\alpha \approx \alpha_0$ and

$$q = \frac{1}{\hbar}\frac{[2mE(1 - 4\Gamma)]^{\frac{1}{2}}}{1 - 2\Gamma}.$$

Thus, asymptotic solution ψ_N differs from standard QM one, for finite a the correction to it can be taken to be equal to ψ_d, i.e. $\psi \approx \psi_N + \psi_d$. For imaginary λ the corresponding nonlinear transformation given in [14, 15], however, for complete wave reflection from the barrier the consistent asymptotic solution for ψ_N doesn't exist, because ψ_N phase singularities appear at its nodes. In this case, the linear QM solution $\psi_\infty(x)$ for the same system parameters can be used as its approximation. For $x > a$, both for real and imaginary λ, the solution is $\psi^+(x) = C^+ \exp(ikx)$, C^+ value calculated below.

To describe the tunneling, it's necessary first to find solution for $0 < x < a$ with $\psi(a) \to 0$ for $a \to \infty$, which is main term of tunneling state. For real λ, such solution of Eq. (1) is $\psi_1(x) = B_1 \exp(-\chi x)$ where

$$\chi = \frac{1}{\hbar}\frac{[2m(V_0 - E)]^{\frac{1}{2}}}{(1 - 4\Gamma)^{\frac{1}{2}}}.$$

In the linear QM formalism, for $a \to \infty$, it follows that

$$B_1 = -\frac{2k(k - i\chi_0)}{k^2 + \chi_0^2}.$$

For small $|\lambda|$ this formulae can be used with good accuracy also in nonlinear case substituting in it χ_0 of (3) by χ. Analogously to standard QM, another solution, which describes the secondary term, is $\psi_2(x) = B_2 \exp(\chi x)$, yet $|B_2| \sim \exp(-2\chi a)$, so ψ_2 is exponentially small in comparison with ψ_1. Therefore, transmission coefficient can be estimated with good accuracy accounting only main term ψ_1, it gives $D_1 = |B_1|^2 \exp(-2\chi a)$, so that D_1 exponentially depends on λ.

Due to dynamics nonlinearity, the superpositions of two terms, in general, aren't solutions. Analytic solutions, which correspond to such superpositions, exist in two cases only, defined by $b_s = B_2 B_2/B_1$ ratio. First, for imaginary b_s the solution is just $\psi = \psi_1 + \psi_2$; second, for b_s real

$$\psi(x) = B_1^{\frac{1-2\Gamma}{1-4\Gamma}}[\exp(-\chi_b x) + b_s \exp(\chi_b x)^{\frac{1-2\Gamma}{1-4\Gamma}},$$

where

$$\chi_b = \frac{1}{\hbar}\frac{[2m(V_0 - E)(1 - 4\Gamma)]^{\frac{1}{2}}}{1 - 2\Gamma}. \quad (4)$$

Other solutions, corresponding to complex b_s, can be approximated as the linear interpolation between these two solutions. In the linear approximation

$$B_2 = B_1 \frac{\chi^2 - k^2 + 2ik\chi}{\chi^2 + k^2}\exp(-2\chi a).$$

For typical α-decay parameters $E \approx V_0/2$, it correspond to χ, k values such that $\chi \approx k$. Therefore, b_s can be taken to be imaginary, and resulting ψ will be $\psi_{1,2}$ superposition. In this case, transmission coefficient $D_s = 2D_1$, so that D_s also has exponential dependence on λ. For imaginary λ, the main term $\psi_1 = B_1 \exp(-\chi_0 \varpi x)$ where

$$\varpi = \frac{1 + 2m\lambda}{(1 + 4m^2|\lambda|^2)^{\frac{1}{2}}}.$$

Transmission coefficient for main term is equal to $D_1 = |B_1|^2 \exp(-2\chi_0 \upsilon a)$ where $\upsilon = \mathrm{Re}\varpi$. It supposes that D_1 dependence on λ is less pronounced than for real λ. Then, secondary term $\psi_2 = \exp(\chi_0 \varpi x)$. Both for real and imaginary λ, $C^+ = \psi(a)\exp(-ika)$, which defines $\psi = \psi^+$ for $x > a$. It's notable that considered nonlinear Hamiltonian term F influences mainly the transitions between degenerate states, as property (e) demonstrates. Due to it, tunneling transmission coefficients and related decay rates are sensitive to the presence of nonlinear terms in evolution equation, hence the study of such processes can be important method of quantum nonlinearity search.

3. α-DECAY OSCILLATION MODEL

Gamow theory of nucleus α-decay supposes that in initial nuclei state, free α-particle already exists inside the nucleus, but its total energy E is smaller than maximal height of potential barrier constituted by nuclear forces and Coulomb potential [10]. Hence α-particle can leave nucleus volume only via quantum tunneling through this barrier. For real nucleus, barrier potential isn't rectangular, but has complicated form described by some function $V(r)$ defined experimentally [10, 11]. In this case, to calculate transmission rate in our model, WKB approximation for Hamiltonian of (1) used [8]; its applicability to our nonlinear Hamiltonian can be easily checked. The calculations described here only for real λ, for imaginary λ they are similar. In this ansatz, 3-dimensional α-particle wave function reduced to $\psi = \frac{1}{r}\exp(i\sigma/\hbar)$; function $\sigma(r)$ can be decomposed in $\hbar$ order $\sigma = \sigma_0 + \sigma_1 + \ldots$ [8]. Given α-particle energy E, one can find the distances R_0, R_1 from nucleus centre at which $V(R_{0,1}) = E$. Then, for our nonlinear Hamiltonian the equation for σ_0 is

$$\left(\Lambda - \frac{1}{2m}\right)\left(\frac{\partial\sigma_0}{\partial r}\right)^2 = E - V(r), \quad (5)$$

where $\Lambda = 2\lambda$ for $R_0 \le r \le R_1$, $\Lambda = 0$ for $r < R_0$, $r > R_1$. Its solution for $R_0 \le r \le R_1$ can be written as

$$\psi = \frac{1}{r}\exp(i\sigma_0/\hbar) = \frac{C_r}{r}\exp\left[-\frac{1}{\hbar}\int_{R_0}^{r} p(\varepsilon)d\varepsilon\right],$$

where C_r is normalization constant,

$$p(r) = \frac{1}{\hbar}\left[\frac{2m(V(r)-E)}{1-4\Gamma}\right]^{\frac{1}{2}},$$

where $\Gamma(t) = m\lambda(t)$. Account of higher order σ terms doesn't change transmission coefficient which is equal to

$$D = \exp\left[-\frac{2}{\hbar}\int_{R_0}^{R_1} p(\varepsilon)d\varepsilon\right] = \exp\left[-\frac{\varphi}{(1-4\Gamma)^{\frac{1}{2}}}\right], \quad (6)$$

here φ is constant, whereas Γ can change in time. For imaginary λ the calculations result in the same D ansatz, but with

$$p(r) = \frac{1}{\hbar}\left[\frac{2m(V-E)}{1+4m^2|\lambda|^2}\right]^{\frac{1}{2}}.$$

To calculate nucleus life-time, D should be multiplied by the number of α-particle kicks into nucleus potential wall per second [10].

To study decay parameter variations in external field, we'll suppose now that nonlinear Hamiltonian term F depends on external field A. In our model, such field is gravity, characterized usually by its potential $U(\vec{R},t)$. In this case, U should be accounted in evolution equation twice. First, mU should be added to H_0, so that it changed to $H_0' = H_0 + mU$; second, nonlinear H term F can depend on U or some its derivative. For minimal modification of DG model we'll assume that for F ansatz of (2) its possible dependence on external field is restricted to parameter λ dependence: $\lambda = f(U)$, so now λ isn't constant, but the function of $\vec{R}$ and t. It supposed also that $f \to 0$ for $U \to 0$, so that free particle evolution is linear. Considered model doesn't permit to derive λ dependence on Sun gravity, but it can be obtained from its comparison with experimental results for ^{214}Po α-decay. We'll suppose that λ is function of gravitation potential $U(\vec{R}_n,t)$ where $\vec{R}_n$ is nucleus coordinate in Sun reference frame (SRF). As follows from Eq. (6) for small λ

$$D \approx (1 + 2\phi\Gamma)\exp(-\phi).$$

For ^{214}Po decay, its life-time $\tau_0 = 16.4 \times 10^{-6}$ s, model estimate gives $\phi \approx 60$. For annual τ variation the best fit for 3 yr exposition has main harmonics

$$\tau_a(t) = \tau_0[1 + A_a \sin w_a(t + \varphi_a)],$$

where t defined in days, $A_a = 9.8 \times 10^{-4}$, $w_a = 2\pi/365$, $\varphi_a = 174$ days [5]. Remind that Earth orbit is elliptic, the minimal distance from Sun is at about January 3 and maximal at about July 5, maximal/minimal orbit radius difference is about 3×10^{-2} [23]. Plainly, the minima and maxima of U time derivative $\partial_t U$ will be located approximately in the middle between these dates, i.e. about April 5 and October 3, correspondingly. In general, this dependence described as

$$\partial_t U = K^a \sin w_a(t + \varphi_u),$$

here $\varphi_u = 185$ days, $K^a = 1.5$ m^2/s, as the result, such model φ_u value in a good agreement with experimental φ_a value. Thereon, it means that the plausible data fit is $\lambda(t) = g\partial_t U$, where g is interaction constant, which can be found from the data for ^{214}Po decay. It follows from the assumed equality of oscillation amplitudes $A_a = 2\phi mgK^a$ that the resulting $g = .35\times10^{-8}\ \frac{\text{s}^3}{\text{m}^2\ \text{MeV}}$.

Another experimentally found harmonic corresponds to daily variations with best fit

$$\tau_d(t) = \tau_0[1 + A_d \sin w_d(t + \varphi_d)]$$

where t defined in hours, $A_d = 8.3\times10^{-4}$, $w_d = 2\pi/24$, $\varphi_d = 12$ h [5]. Such oscillation can be attributed to variation of Sun gravity due to daily lab. rotation around Earth axe. It's easy to check that nucleus lifetime dependence also coincides with $\partial_t U$ time dependence with high precision. Really, it described as

$$\partial_t U = K^e \sin w_d(t + \varphi_e)$$

with $\varphi_e = 12$ h, $K^e = 0.9$ m^2/s [23]. It follows that $A_d = 2\phi mgK^e$; if to substitute in this equality g value, calculated above, it gives $A_d = 5.5\times10^{-4}$, which is in a reasonable agreement with its experimental value.

4. NONLINEARITY, NONLOCALITY AND CAUSALITY

In this paper, we studied hypothetical nonlinear corrections to standard QM description of system interaction with external fields. Such terms have strictly quantum origin and disappear in classical limit, their existence should be verified in dedicated experiments. To study their general features, we considered the simple nonrelativistic model, which includes the additional terms for the interaction of quantum systems with gravitational field. It was argued earlier that QM nonlinearity violates relativistic causality for multiparticle systems, in particular, it permits superluminal signaling for EPR-Bohm pair states [24, 25]. However, this conclusion was objected and still disputed [19]. Plainly, these arguments would be even more controversial, if nonlinear effects exist only inside the field volume. In particular, the metastable state in external field can be considered as open system, yet it was shown that superluminal signaling between such systems is impossible [26]. Moreover, heavy nucleus is strongly-bound system, so it's unclear whether it's possible to prepare entangled state of two α-particles located initially inside two different nuclei.

It was supposed earlier that gravity is emergent (induced) theory and originates, in fact, from some nonlocal field theory [16, 17]. It follows then, that gravity effects can be effectively described by multilocal (collective) field Φ_n or by several multilocal field modes $\{\Phi_1,\dots,\Phi_n,\dots\}$. It was shown, in particular, that bilocal scalar field $\Phi_2(x_1, x_2)$ reproduces classical gravity effects up to the second order [16]. Such bilocal field Φ_2 supposedly can interact with bilocal operators of massive fields, in particular, it can be nonrelativistic particle systems. Such interaction doesn't violate causality, if for the pair of separated quantum objects it influence only their bilocal observables of EPR-Bohm type [18, 24]. The simple example of such observable is spin projection difference of two fermion system.

In our model, it assumed that in infrared limit the gravity described by two terms $\{\Phi_1, \Phi_2\}$ where $\Phi_1 = U(\vec r)$ is standard local potential. Denote as $\vec r_1$ the coordinate of α-particle, $\vec r_2$ the coordinate of remnant nucleus centre of mass, and $\vec r_s = \vec r_1 - \vec r_2$. For considered α-decay model, the joint state of remnant nucleus and α-particle is entangled, their bilocal observable $\vec r_s$ is of EPR-Bohm type. It's notable that it's equivalent to basic coordinates of bilocal field which described by them as $\Phi_2(\vec R_a, \vec r_s)$ where $\vec R_a = \frac{1}{2}(\vec r_1 + \vec r_2)$ [16]. If gravity field is local then for $\langle|\vec r_s|\rangle \ll |\vec R_n|$ it will act mainly on nucleus total state $\Psi(\vec R_n)$, but not on nucleus internal state $\psi(\vec r_s)$. Only bilocal field can influence it, and as follows from our analysis of ^{214}Po α-decay, it's plausible that for bilocal scalar field $\Phi_2 \sim \partial_t U(\vec R_n)$. In accordance with it, for D-G ansatz with $\lambda = f(\Phi_2)$ the corresponding nonlinear term of our Hamiltonian for nucleus

$$F = k_b \partial_t U(\vec R_n)\left(\frac{\partial^2}{\partial \vec r_s^{\,2}} + \frac{1}{\psi^2}\left|\frac{\partial\psi}{\partial \vec r_s}\right|^2\right),$$

where k_b is arbitrary constant and $\langle|\vec r_s|\rangle \ll |\vec R_n|$. It means that Φ_2 interaction with nucleus described by nonlinear operator; as the result, α-particle transmission coefficient $D(t)$ can oscillate around its constant value for linear Gamow theory. For large $|\vec r_s|$ it can be supposed that

$$\Phi_2 = \frac{1}{2^{\frac{1}{3}}}\{\partial_t U(\vec r_1)\partial_t U(\vec r_2)[\partial_t U(\vec r_1) + \partial_t U(\vec r_2)]\}^{\frac{1}{3}}. \qquad (7)$$

As follows from equivalence principle, in lab. reference frame, located on Earth surface, Sun gravitation potential $U'(\vec R_c) \approx 0$, yet $\partial_{\vec r} U' \neq 0$. It supposes that nuclear decay process violates equivalence principle, however, some theories of emergent gravity predict that it can be violated in quantum processes [16, 27]. In addition, other results for ^{214}Po α-decay seems to support such conclusion. Namely, beside described life-time oscillations, these data contains also harmonics with period 24 h 50 min, which is equal to lunar day duration and so can be related to moon gravity effect [5]. Studies in quantum gravity supposes that this theory can be similar to QFT with massless messenger called graviton. Notorious example of massless messenger formalism represents QED. It's well known yet that in its nonrelativistic limit there are some electromagnetic effects, like Casimir effect or Lamb shift, which can't be described by Scroedinger equation, but only via accounting higher order QED terms. It seems possible that the observed decay oscillations can have analogous origin corresponding to the nonzero infrared limit of some hypothetical quantum gravity terms.

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