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Self-Consistent Approach to β-Decay and Delayed Multi-Neutron Emission[1]

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Abstract—The beta-decay half-lives and delayed multi- neutron emission branchings for the nuclei near the new neutron shell N = 34 are treated within self-consistent Density Functional + Continuum QRPA model (DF + CQRPA). A comparison with the recent self-consistent calculations from relativistic QRPA and standard (semi-microscopic) FRDM is performed.

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INTRODUCTION

The studies of nuclear beta decay are indispensable for our understanding of nuclear structure far from β-stability line, planning and interpretation of the radioactive ion-beam experiments and astrophysical *r*-process modeling. Accurate β-decay data on fission products are important for safety studies of advanced nuclear reactors. For the nuclei far from the stability-line, often only the integral quantities, as β-decay half-lives and β-delayed neutron emission probabilities are experimentally available but they give us an important insight to nuclear spin-isospin properties. A combined analysis of total β-decay half-lives and multi-neutron emission rates (Pxn-values) enables one to reconstruct the beta decay strength functions carrying back information on the time-odd components of nuclear density functional at high isospin-asymmetry regime.

A substantial amount of β-decay data on the fission and fragmentation products come from the acting radioactive beam facilities: ISOLDE-CERN, ALTO, RIKEN, TRIUMF, NSCL, and more information is expected from constructed FAIR, Spiral-2, HIE-CERN facilities. The recent beta-decay experiments have already shown that not only the neutron-halo nuclei near the drip-line reveal the features of weakly bound open quantum systems. Thus, related effects of ground state spin inversion and/or neutron-halo may arise in very neutron-rich nuclei because of “shell-erosion” originated from proton-neutron tensor interaction, as well as three nucleon (3N) forces.

Our approach is based on on the general principles of the theory of finite Fermi systems (TFFS) [1, 2] augmented with the self-consistency conditions [3]: a well-known alternative to the Skyrme Hartree–Fock method. Within this approach, effects generated by the momentum and energy dependences of effective forces are considered on an equal footing. In constructing the Fayans functional [4] energy-density functional (DF), explicit energy dependence was excluded. This results in more sophisticated density dependence than the one of standard Skyrme functional arising from an effective account of the 3N correlations. Based on the self-consistent description of the ground state properties with the Fayans energy–density functional, the framework allowing large-scale continuum QRPA calculations of the Gamow-Teller (GT) and first-forbidden (FF) beta-decay properties (DF + CQRPA) has been developed in [5].

In this brief report, the performance of self-consistent DF + CQRPA is demonstrated in calculations of the ground state and beta-decay properties for (near) spherical nuclides in the Ca region with protons filling the sd shell and neutrons pf shell. Our DF + CQRPA calculations are compared with the recent large-scale calculations from the relativistic RHB + QRPA [6] (both frameworks are self-consistent and include the allowed Gamow–Teller and first-forbidden beta decays). An emphasis is made on the possible constraints imposed on the density functional by the beta-decay strength functions, half-lives as well as delayed neutron-emission rates.

2. SELF-CONSISTENT MODELS OF β-DECAY STRENGTH FUNCTION

In table fully microscopic models are presented which have been used for large-scale calculations of the beta decay properties (Table 1). Self-consistent models are of special importance for reliable extrapo-

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Table 1. Self-consistent microscopic models of beta-decay strength function

Model	The ground state (g.s.)	*NN*-interaction	Details
Shell-model [7]	Spherical & Deformed g.s. Self—consistent.	$N = 50$-by K. Sieza,F. Nowacki $N = 82$ with 88Sr-core	Nuclei near the closed $N = 50$, 82, 126 shells. Fitting of (g_A/G_V)
DF + CQRPA [5]	Spherical g.s. Full ph basis. Self-consistent. DF3, DF3a functionals. A-dependent, density dependent BCS pairing	*ph*: δ-function + π + ρ meson exchange *pp*: δ-function	400 (quasi-)spherical nuclei No fitting to $T_{1/2}$
RHB + RQRPA [6]	Spherical g.s. Relativistic Hartree—Bogolubov. Self-consistent. D3C* functional. Finite range pairing: Gogny D1S	*ph*: D3C* *pp*: (N—Z)-dependent, finite range: Gogny D1S.	All nuclei. Fitting to $T_{1/2}$ of the sample nuclei
F A M [8]	Spherical & Deformed g.s. Self-consistent. Skyrme functional SkO’	*ph*: SkO’ fitted to Landau—Migdal interaction pp: density dependent δ-function	All nuclei. Fitting to $T_{1/2}$ of 44 sample nuclei

lation of nuclear data to extreme N/Z ratios. The first one is the **Shell-Model**, its main features are: (a) self-consistent description of the ground state, (b) deformation is treated in “natural” way, (c) an impact of the complex many particle-many hole (np-nh) configurations is taken care of. The practical applications are restricted by computational reasons: a model space limiting the rank of the QRPA matrix. Additional free parameters (the GT and FF quenching factors) are used in order to account for restricted model space. Thus, the recent multi-configuration shell-model study including the GT + FF decays [7] has been applied only to the nuclei near the closed $N = 50$, 82 shells.

In a wide region of spherical nuclei including the superheavy ones, the method of choice is the quasiparticle random phase approximation (QRPA) based on the density functional approach (DF). It successfully describes a gross structure of beta-decay strength functions. Within **DF + CQRPA** β-decay strength function is calculated within the finite Fermi system theory framework. For the spin—isospin effective NN-interaction in one-particle-one-hole (ph) channel, the Landau—Migdal interaction, and nuclear medium modified one-π and one-ρ exchange terms are included [5]. The framework is applied for nuclei with pairing correlations: density-dependent, *A*-dependent, zero-range $T = 1$ pairing force is used. Zero-range interaction with constant strength is used in the isoscalar $T = 0$ channel (dynamic pairing). The correlations beyond the QRPA are included by re-scaling the spin-dependent multipole operators by the same energy-independent quenching factor $Q^{1/2} = (g_A/G_A)$. The one-pion component of the residual interaction is quenched by the same factor Q. allowed and first-forbidden transitions treated in terms of the reduced multipole operators depending on the space and spin variables [3]. The allowed Gamow—Teller (GT) and first-forbidden (FF) beta decays are treated in the one and the same scheme. The full first-forbidden operators set was worked out with the relativistic operators α, Υ_5 reduced to their space-dependent counterparts via CVC and PCAC relations [5]. This gives a possibility to benefit from the full ph-basis continuum pnQRPA framework. A deficiency of the present version of the method is neglect of the deformation and *np-nh* configurations.

Relativistic Density Functional+QRPA [6] is based on the relativistic spherical Hartree-Bogolubov model with density dependent meson-nucleon interactions. The relativistic density functional DD-ME2 is used in the particle-hole (ph) channel. The finite-range Gogny interaction D1S is employed for the $T = 1$ ground state pairing, while the (N–Z)-dependent strength is assumed for the dynamic $T = 0$ pairing. RHB + QRPA is a spherical density functional approach but applied globally to entire nuclear chart including the deformed nuclei. The Gamow—Teller and full first-forbidden operators are included.

The computational limitation made treatment of spin-isospin response in deformed nuclei a tough problem for the self-consistent density-functional methods. **Finite Amplitude Method (FAM)** [8] offers an efficient way avoiding the problems imposed by

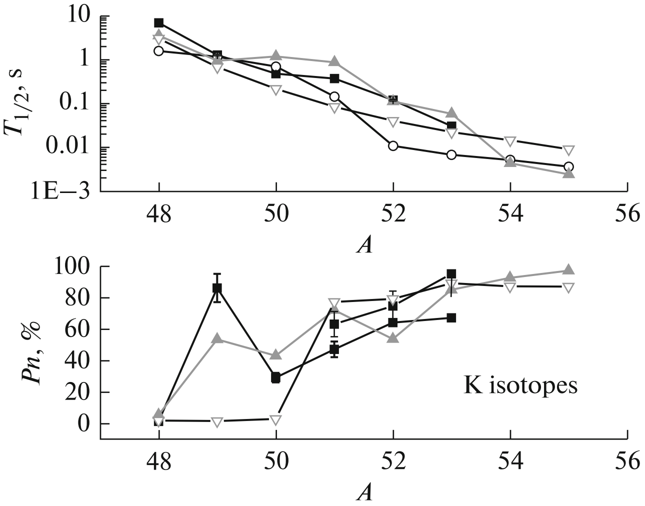

Fig. 1. (left) The experimental half-lives $T_{1/2}$ (squares) and total *Pn* values of K isotopes (NUBASE [17]) vs the ones from DF + CQRPA(up triangles), FRDM [10] (open cirles), RHB + QRPA [8] (down triangles).

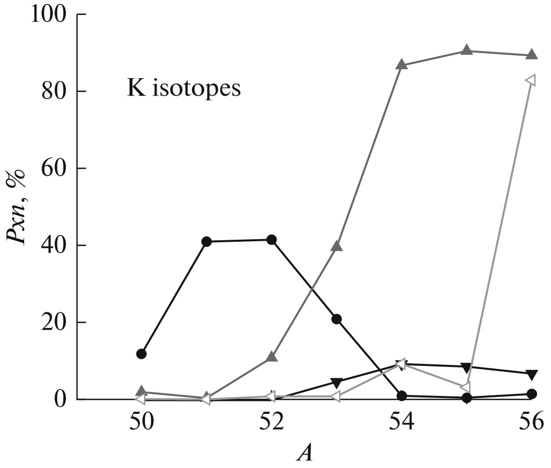

Fig. 2. The *P*1*n* values (circles), *P*2*n* (up triangles), *P*3*n* (down triangles) from DF + CQRPA vs *P*2*n* from RHB + QRPA [8] (open triangles) for K isotopes.

huge QRPA matrix in deformed nuclei. FAM was suggested by Nakatsukasa et al. [9], as a method of finding the nuclear linear response bypassing the QRPA eigenvalue problem. What is needed are the one-body operators and appropriate iterative technics. Instead of constructing the QRPA matrix of the eigenvalue problem one can solve nonlinear equations for unknown amplitudes X and Y using the variances of Hamiltonian matrices (first variances of the density functional on normal and anomalous densities). Such method facilitates drastically self-consistent pnQRPA problem for deformed nuclei.

3. MULTI-NEUTRON EMISSION IN DF + CQRPA AND RQRPA

Below the performance of the DF + CQRPA approach is briefly exemplified in comparison with results of relativistic QRPA model [6]. The β-decay and multi-neutron emission rates are analysed for the K isotopes near the magic shell-closures $Z = 20$, $N = 28$, 32, 34. The calculated values of $Q_{\beta n} = Q_\beta - S_n(Z + 1)$ in $^{42-54}$K reveal a tenfold growth of the $Q_{\beta n}$ only 6 mass units away from $N = 28$ (not shown) which is a strong argument in favor of the $N = 32$, 34 (sub)shells. The calculated Gamow–Teller strength function show an appearance of the strong transition in ^{54}K with the transition energy $\omega = 15.1$ MeV (Ex = 6.3 MeV in ^{54}Ca) related to the 1*nf*7/2–1*pf*7/2 configuration. It is responsible for a corresponding reduction of the half-life (Fig. 1). The DF + CQRPA and relativistic QRPA describe available half-lives better than the standard FRDM model [10] often used in astrophysical r-process modeling. The total delayed neutron emission probabilities in both models (Fig. 1) increase smoothly from ^{51}K to ^{54}K. However, the mass dependence of the delayed multi-neutron emission probabilities in DF + CQRPA and RHB + QRPA (Fig. 2) is very different for A > 50. The DF + CQRPA predicts high *P*2*n* ($A = 53$) = 40% and giant *P*2*n* >80% for $A > 53$ isotopes while due to the differences in $Q_{\beta n}$ values in RHB + QRPA the high *P*2*n* values start at $A = 56$. These theoretical predictions will be verified in the measurements planned within the recent addendum to the CERN-ISOLDE&TOF Project [11].

CONCLUSIONS

After the seminal paper of Goldansky [12] predicting a possibility of two-neutron emission in very neutron-rich nuclei and first microscopic studies of multi-neutron emission [13], the field still offers new puzzles. New intriguing aspect of the problem is related to the claimed observation of unstable di-neutron resonance state with internal excitation energy about 70 keV [14] and continuous quest for hypothetical tetraneutron resonance (0.8 MeV) [15]. The structure of (quasi)spherical nuclei with high isospin asymmetry is properly treated within self-consistent nuclear beta-decay approach [5–9]. It reliably describes both ground state properties and small amplitude spin excitations. Microscopic treatment of the GT and FF beta-decays within the single DF+CQRPA framework makes it possible to explain effects of the mean-field evolution and its impact on beta-decay and delayed multi-neutron emission of very neutron-rich nuclei near and beyond the closed neutron shells. The constraints on nuclear density functional from the beta decay observables obtained in the experiments at JINR, GSI, ALTO, HRIBF и RIKEN are of great value for better understanding the structure of nuclei far from stability. Acquiring the well-studied proton shell at $Z = 20$ and neutron shells at $N = 20$, 28, as well the recently confirmed "new" ones at $N = 32$, 34 the Ca region presents unique testing ground both for chiral effective field theory with 3N forces and density functional approaches. First of all, it would be important to find out possible correlation between the prediction of giant two-neutron emission probability of in $^{53-56}$K and unexpectedly large charge radii recently found in ^{51,52}Ca [16].

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