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ELEMENTARY PARTICLES AND FIELDS
Theory

Measurement of the Weinberg Angle in an Experiment at the Super Charm Tau Factory with a Polarized Beam

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Abstract—Measurement of the effective weak mixing angle θ_{eff} in an experiment at the Super Charm Tau Factory is discussed for the case of a longitudinal polarization of electrons. A method for measuring the average electron beam polarization by means of analyzing the differential cross section for the decay process $J/\psi \to [\Lambda \to p\pi^-][\bar{\Lambda} \to \bar{p}\pi^+]$ is proposed. At a collider luminosity of 10^{35} cm^{-2}s^{-1} and a polarization degree of 0.8, the parameter $\sin^2\theta_{\text{eff}}$ can be measured to a relative precision better than 1%, which will be sufficient for observing the deviation of $\sin^2\theta_{\text{eff}}$ from its value at the Z-boson peak.

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INTRODUCTION

In the $SU(2)_L \times U(1)_Y$ electroweak-interaction model [1], the Weinberg angle θ_{W} is a parameter that determines the coupling of the photon and Z-boson fields to the fields of the gauge B and W^3 bosons; that is,

$$A \equiv B\cos\theta_{\text{W}} + W^3\sin\theta_{\text{W}}, \tag{1}$$
$$Z \equiv -B\sin\theta_{\text{W}} + W^3\cos\theta_{\text{W}}.$$

The Weinberg angle appears in the vector part of neutral weak interaction:

$$g_V^f \equiv I_3^f - 3Q_f\sin^2\theta_{\text{W}}. \tag{2}$$

Here, I_3 and Q_f are, respectively, the weak isospin and electric charge of the fermion field f. Corrections to the dominant contribution result in experimentally observing the effective value

$$\sin^2\theta_{\text{eff}} \equiv \kappa_Z^f\sin^2\theta_{\text{W}}, \tag{3}$$

where the coefficient κ_Z^f depend on the momentum transfer and is calculable quite precisely: the uncertainty at low momenta is 2×10^{-5} [2].

The value $\sin^2\theta_{\text{eff}}$ was measured to a relative precision of 0.1% at the Z-boson peak in experiments at the Large Electron–Positron (LEP) collider (CERN) and the Stanford Linear Collider (SLC, SLAC) [3]. A 4% deviation of $\sin^2\theta_{\text{eff}}$ from its value at the Z-boson peak is expected at energies around $\mathcal{O}(1\text{ GeV})$ and below [4]. Measurements of $\sin^2\theta_{\text{eff}}$ at low energies were performed on the basis of various procedures: by parity violation in atoms, by Møller scattering, by Mott scattering, and by the deep-inelastic scattering of neutrinos and electrons on nuclei of atoms [5]. The results of these measurements agree with the Standard Model prediction, but the accuracy of the experiments of low energies is substantially inferior at the present time to the accuracy of the results obtained at the Z-boson peak.

Measurement of $\sin^2\theta_{\text{eff}}$ at low energies is of interest from the point of view of testing the electroweak model. Precision measurements are sensitive to nonstandard contributions to κ_Z^f—for example, to the extended electroweak model featuring extra gauge bosons.

EXPERIMENT AT SUPER CHARM TAU FACTORY

Projects of super charm tau factories, which are symmetric electro–positron collides rated to a high luminosity of 10^{35} cm^{-2}s^{-1} and values of the c.m. collision energy $\sqrt{s}$ between 2 and 6 GeV [6, 7]. The projects are aimed at providing a high longitudinal electron polarization at the collision point. In an experiments at such colliders, the parameter $\sin^2\theta_{\text{eff}}$ can be measured by the asymmetry of the total cross section at the J/ψ-particle peak; that is,

$$\mathcal{A}_{LR} = \mathcal{P}_e \cdot \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \equiv \mathcal{P}_e \cdot \mathcal{A}_{LR}^0, \tag{4}$$

where σ_R (σ_L) is the total cross section for electrons that have a right-hand (left-hand) polarization and $\mathcal{P}_e$ stands for the degree of electron polarization ($0 \leq$

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$\mathcal{P}_e \leq 1$). In order to measure the cross sections σ_R and σ_L, one can make use of the hadronic decays of J/ψ particles. The number of detected hadronic decays of J/ψ particles at a specific polarization (N_L or N_R) is related to the respective cross section by the equation

$$\sigma_\alpha = \frac{N_\alpha}{\mathcal{L}_\alpha \varepsilon_\alpha}, \quad \alpha \in \{L, R\}, \tag{5}$$

where the parameters ε_α describe the efficiency of event reconstruction in the detector and the branching ratio for J/ψ decay to the hadron state s being considered and $\mathcal{L}_\alpha$ stand for the respective luminosity integrals. Characteristic values of N_α in experiments at a Super Charm Tau Factory are on the order of 10^{12}. The luminosity integrals $\mathcal{L}_\alpha$ should then be known with a relative statistical accuracy not poorer than 10^{-6}. For a discussion on the approaches to a precision measurement of the luminosity integral in such experiments, the interested reader is referred to [8].

The left–right asymmetry $\mathcal{A}^0_{LR}$ arises because of the interference between the processes $e^+e^- \to \gamma^* \to c\bar{c}$ and $e^+e^- \to Z^* \to c\bar{c}$, and one can express it in terms of $\sin^2\theta_{\text{eff}}$ as [9]

$$\mathcal{A}^0_{LR} = \frac{-\sin^2\theta_{\text{eff}} + 3/8}{2\sin^2\theta_{\text{eff}}(1-\sin^2\theta_{\text{eff}})}\left(\frac{m_{J/\psi}}{m_Z}\right)^2 \approx 4.7 \times 10^{-4}. \tag{6}$$

In order to obtain $\sin^2\theta_{\text{eff}}$, it is necessary to measure, the left-right asymmetry $\mathcal{A}_{LR}$ and the polarization degree $\mathcal{P}_e$. From expressions (4) and (6), we obtain a relation for the relative uncertainties in the form

$$\frac{\sigma\left(\sin^2\theta_{\text{eff}}\right)}{\sin^2\theta_{\text{eff}}} = C_{\mathcal{A}_{LR}}\frac{\sigma\left(\mathcal{A}_{LR}\right)}{\mathcal{A}_{LR}} \oplus C_{\mathcal{P}_e}\frac{\sigma\left(\mathcal{P}_e\right)}{\mathcal{P}_e} \approx 0.3\%, \tag{7}$$

where $C_{\mathcal{P}_e} = -C_{\mathcal{A}_{LR}} \approx 0.44$. The value of 0.3% was obtained for one experimental run at the polarization degree of $\mathcal{P}_e = 0.8$ and under the assumption that the accuracy of the measurement is restricted by the statistical uncertainty in measuring the left-right asymmetry $\mathcal{A}_{LR}$. The average electron-polarization degree $\mathcal{P}_e$ should then be monitored to a relative precision better 0.1%.

Laser polarization monitors may provide a sufficient statistical precision, but a systematic uncertainty may turn out to be a serious problem. A determination of $\mathcal{P}_e$ from an analysis of the same data in which one measures the left–right asymmetry $\mathcal{A}_{LR}$ is an alternative solution to the problem. This approach is likely to be optimal from the point of view of controlling systematic uncertainties. In the following, it will be shown that the process

$$e^+e^- \to J/\psi \to [\Lambda \to p\pi^-][\bar{\Lambda} \to \bar{p}\pi^+] \tag{8}$$

can be used to monitor the electron polarization $\mathcal{P}_e$.

THE DIFFERENTIAL CROSS SECTION FOR THE PROCESS $e^+e^- \to J/\psi \to [\Lambda \to p\pi^-][\bar{\Lambda} \to \bar{p}\pi^+]$

The process $e^+e^- \to J/\psi \to [\Lambda \to p\pi^-][\bar{\Lambda} \to \bar{p}\pi^+]$ can be represented in the form the diagram in Fig. 1. This process is described in terms of the following ingredients:

- the lepton current featuring a polarized electron and having the form

$$j^\mu_e \equiv \bar{v}_{-\xi}\gamma^\mu u_\xi = \sqrt{s}\,(0, \xi\cos\theta, i, -\xi\sin\theta)\,, \tag{9}$$

  where $\xi = \pm 1$ is the doubled electron helicity and the direction of the z axis coincides with the direction of the Λ momentum;

- the $J/\psi \to \Lambda\bar{\Lambda}$ decay vertex described by two form factors as

$$-ie_g\bar{u}_\Lambda(p_1)\Big[G^\psi_M\gamma^\mu - \frac{2m_\Lambda}{Q^2}\left(G^\psi_M - G^\psi_E\right)Q^\mu\Big]v_{\bar{\Lambda}}(p_2), \tag{10}$$

  where p_1 and p_2 stand for, respectively, the Λ and $\bar{\Lambda}$ momenta and $Q \equiv p_1 - p_2$;

- the $\Lambda \to p\pi^-$ ($\bar{\Lambda} \to \bar{p}\pi^+$) decay vertex

$$\bar{u}_p\left[A + B\gamma^5\right]u_\Lambda, \quad (\bar{v}_{\bar{\Lambda}}\left[A' + B'\gamma^5\right]v_{\bar{p}}). \tag{11}$$

The differential cross section is obtained as the contraction of the lepton and hadron tensors, that is,

$$\frac{d\sigma}{d\zeta} \propto L^{\mu\nu}H_{\mu\nu} \propto a(\zeta) + \xi b(\zeta), \tag{12}$$

where ζ is a set of five kinematical parameters whose number corresponds to the dimensionality of the relevant phase space. The symmetric part of the lepton tensor is independent of the polarization, while its antisymmetric part is proportional to the electron helicity; that is,

$$L^{\mu\nu} \equiv (j^\nu_e)^\dagger j^\mu_e = k^\mu_+ k^\nu_- + k^\mu_- k^\nu_+ - \frac{s}{2}g^{\mu\nu} - \xi i\varepsilon^{\mu\nu\alpha\beta}k_{-\alpha}k_{+\beta}, \tag{13}$$

where k_- (k_+) is the electron (positron) momentum. The details of the calculations are given in [8]. Therefore, the ultimate result is presented immediately below.

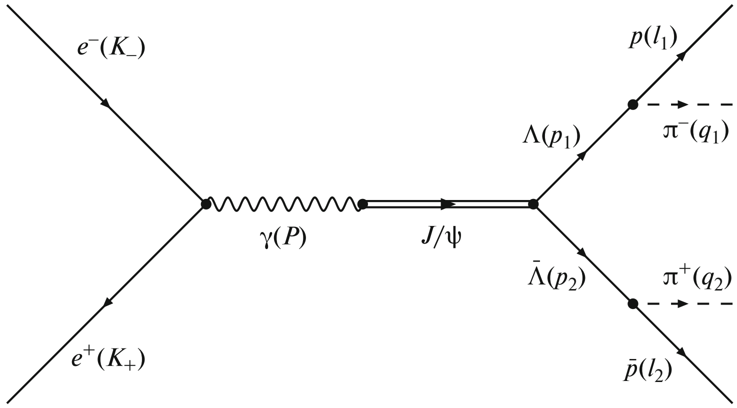

Fig. 1. Diagram for the process $J/\psi \to [\Lambda \to p\pi^-][\bar{\Lambda} \to \bar{p}\pi^+]$.

The differential cross section (12) depends on four parameters; that is,

$$\alpha \equiv \frac{s\left|G_M^\psi\right|^2 - 4m_\Lambda{}^2\left|G_E^\psi\right|^2}{s\left|G_M^\psi\right|^2 + 4m_\Lambda{}^2\left|G_E^\psi\right|^2}, \tag{14}$$

$$\Delta\Phi \equiv \arg\left(\frac{G_E^\psi}{G_M^\psi}\right), \quad \alpha_1, \quad \alpha_2,$$

where α_1 and α_2 are the parameters of the decays $\Lambda \to p\pi^-$ and $\bar{\Lambda} \to \bar{p}\pi^+$, respectively. These parameters were measured in the BESIII experiment [10]. The results are the following:

$$\Delta\Phi = (42.4 \pm 0.6 \pm 0.5)^\circ, \tag{15}$$
$$\alpha = 0.461 \pm 0.006 \pm 0.007,$$
$$\alpha_1 = 0.750 \pm 0.009 \pm 0.004,$$
$$\alpha_2 = -0.758 \pm 0.010 \pm 0.007.$$

It is convenient to write the explicit expression for the cross section (12) in the combined reference frame shown in Fig. 2 and to choose the following kinematical variables: the polar angle (θ) of the Λ momentum in the center-of-mass (c.m.) frame; the polar and azimuthal angles (θ_1 and ϕ_1, respectively) of the proton momentum in the Λ rest frame; and the analogous parameters (θ_2 and ϕ_2) for the antiproton that are specified in the $\bar{\Lambda}$ rest frame. Thus, we have

$$\zeta = \{\cos\theta, \cos\theta_1, \phi_1, \cos\theta_2, \phi_2\}, \tag{16}$$
$$d\zeta = d\cos\theta d\Omega_1 d\Omega_2, \quad \Omega_i = d\cos\theta_i d\phi_i.$$

In terms of these variables, the functions a and b have the form

$$a(\zeta) = \mathcal{F}_0 + \alpha\mathcal{F}_5 \tag{17}$$
$$+ \alpha_1\alpha_2\left(\mathcal{F}_1 + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\mathcal{F}_2 + \alpha\mathcal{F}_6\right)$$
$$+ \sqrt{1-\alpha^2}\sin(\Delta\Phi)\left(\alpha_1\mathcal{F}_3 + \alpha_2\mathcal{F}_4\right),$$

where

$$\mathcal{F}_0 = 1, \quad \mathcal{F}_1 = \sin^2\theta\sin\theta_1\sin\theta_2\cos\phi_1\cos\phi_2$$
$$+ \cos^2\theta\cos\theta_1\cos\theta_2,$$
$$\mathcal{F}_2 = \sin\theta\cos\theta(\sin\theta_1\cos\theta_2\cos\phi_1$$
$$+ \cos\theta_1\sin\theta_2\cos\phi_2),$$
$$\mathcal{F}_3 = \sin\theta\cos\theta\sin\theta_1\sin\phi_1,$$
$$\mathcal{F}_4 = \sin\theta\cos\theta\sin\theta_2\sin\phi_2,$$
$$\mathcal{F}_5 = \cos^2\theta, \quad \mathcal{F}_6 = \cos\theta_1\cos\theta_2$$
$$- \sin^2\theta\sin\theta_1\sin\theta_2\sin\phi_1\sin\phi_2,$$

and

$$b(\zeta) = (1+\alpha)(\alpha_1\mathcal{G}_1 + \alpha_2\mathcal{G}_2) \tag{19}$$
$$+ \sqrt{1-\alpha^2}\cos(\Delta\Phi)(\alpha_1\mathcal{G}_3 + \alpha_2\mathcal{G}_4)$$
$$+ \sqrt{1-\alpha^2}\alpha_1\alpha_2\sin(\Delta\Phi)\mathcal{G}_5,$$

where

$$\mathcal{G}_1 = \cos\theta\cos\theta_1, \quad \mathcal{G}_2 = \cos\theta\cos\theta_2,$$
$$\mathcal{G}_3 = \sin\theta\sin\theta_1\cos\phi_1, \quad \mathcal{G}_4 = \sin\theta\sin\theta_2\cos\phi_2,$$
$$\mathcal{G}_5 = \sin\theta\left(\sin\theta_1\cos\theta_2\sin\phi_1 + \cos\theta_1\sin\theta_2\sin\phi_2\right).$$

The result presented in (17) was obtained in [11]; that part of the differential cross section in (19) which is associated with the electron polarization was first published in [8].

Over one year, an experiment at the Super Charm Tau Factory will record about $0.8 \times 10^9\varepsilon_{\text{det}}$ events of the process in (8), where ε_{det} is the detection efficiency. Describing the measured angular distribution by expression (12), one can determine the form factors (14) and the polarization degree $\mathcal{P}_e$. The idea of this approach was implemented by means of a simple Monte Carlo simulation. The model parameters were optimized by the unbinned maximum-likelihood method. Table 1 gives the result emerging from the application of the three procedures considered here:

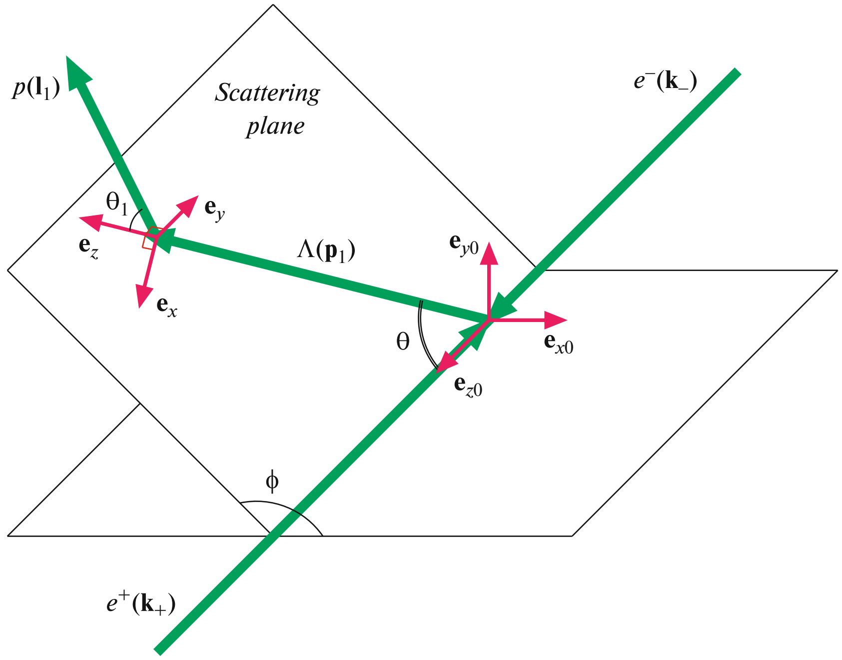

Fig. 2. Combined reference frame. The $(e_{x_0}, e_{y_0}, e_{z_0})$ basis is specified in the center-of-mass frame and is fixed; the e_{z_0} unit vector is directed along the electron-beam axis. The (e_x, e_y, e_z) basis is defined in the Λ rest frame as follows: $e_z = p_1/|p_1|$, $e_y = \frac{1}{\sin\theta}\left(e_z \times \frac{k_-}{|k_-|}\right)$, and $e_x = e_y \times e_z$.

1. Five-dimensional analysis without polarization ($\mathcal{P}_e = 0$);
2. Five-dimensional analysis with polarization ($\mathcal{P}_e = 0.8$);
3. Three-dimensional analysis with polarization ($\mathcal{P}_e = 0.8$).

Procedure 3 is based on the possibility of performing an analysis where events are reconstructed partly, in which case only one of the Λ baryons is detected (the recoil mass can be used to select the required events). One then arrives at the three-dimensional differential cross section

$$\frac{d\sigma}{d\cos\theta d\Omega_1} \propto 1 + \alpha\cos^2\theta \tag{21}$$
$$+ \alpha_1\sqrt{1-\alpha^2}\sin(\Delta\Phi)\sin\theta\cos\theta\sin\theta_1\sin\phi_1$$
$$+ \xi\Big[(1+\alpha)\,\alpha_1\cos\theta\cos\theta_1$$
$$+ \alpha_1\sqrt{1-\alpha^2}\cos(\Delta\Phi)\sin\theta\sin\theta_1\cos\phi_1\Big],$$

which arises after integration of expression (12) over the solid angle Ω_2.

Procedure 2 provides a statistical accuracy of measurement of the degree of polarization $\mathcal{P}_e$ at a level of 10^{-4}, which is obviously sufficient for measuring $\sin^2\theta_{\text{eff}}$. Procedure 3 also makes it possible to reach a sufficient accuracy of measurement of $\mathcal{P}_e$. It is noteworthy that only in the presence of polarization does procedure 3 enable one to determine all four parameters in (14).

In the presence of polarization, the accuracy of measurement of the form factors in (14) within procedure 2 is higher that that within procedure 1. The uncertainties in the parameters α_1 and α_2 decrease most strongly, which has an obvious explanation. In the absence of polarization, Λ and $\bar{\Lambda}$ play the role of polarimeters for each other; in the case of a polarized beam, the differential cross section for the decay of each baryon carries more comprehensive information, making it possible to disentangle correlations and to improve the accuracy. Indeed, the α_1–α_2 correlation coefficient is 0.9 within procedure 1 and −0.1 within

Table 1. Statistical accuracy reached in measuring the degree of polarization $\mathcal{P}_e$ and the form factors (14) after one year of operation of the experiment at the Super Charm Tau Factory

Procedure	$\sigma\,(10^{-4})$			
	$\mathcal{P}_e$	α	$\Delta\Phi$, rad	α_i
5D analysis at $\mathcal{P}_e = 0$	–	1.5	3.1	2.8
5D analysis at $\mathcal{P}_e = 0.8$	1.3	1.2	1.6	0.9
3D analysis at $\mathcal{P}_e = 0.8$	4.3	1.2	2.4	3.4

procedure 2. Thus, the polarization enhances substantially sensitivity to the CP-violating parameter

$$A_\Lambda \equiv \frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2}. \qquad (22)$$

Within the Standard Model. we have $A_\Lambda \lesssim 0.5 \times 10^{-4}$. Over one year of operation of the experiment at $\mathcal{P}_e = 0.8$, a limit at a level of 1.2×10^{-4} can be set on this parameter.

CONCLUSIONS

Experiments at the Super Charm Tau Factory with a polarized beam would provide a unique possibility of measuring the weak interaction of the c quark at the momentum transfer $m_{J/\psi}c$. A method for measuring $\sin^2\theta_{\text{eff}}$ at the J/ψ-meson peak has been described in the present article. A relative precision of 0.3% can be attained by this method. So precise a measurement raises questions concerning control of systematic uncertainties. A detailed analysis of the factors that should be taken into into account has yet to be performed. A discussion of these issues was begun in [8].

The decay process in (8) may serve as a tool for precisely monitoring the average electron polarization. At the same time, measurement of the form factors of baryons and searches for CP violation in their decays are of interest in and of themselves. The presence of a polarized beam strengthens this part of the physics program of the experiment. A further investigation of baryon physics with a polarized beam, including cascade decays and decays of charmed baryons, would become a natural development of the results described in the present article.

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