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Theory

Role of Gravitation in Elementary Particle Structure

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Abstract—It is shown in the tetrad representation that there are Reissner–Nordström solutions describing systems with a finite action and total inertial mass equal to the gravitational mass. These systems consist of entirely electromagnetic and gravitational fields. There are not any massive point-charges inside them. The system is called "the classical electron" if its charge is equal to -4.80×10^{-10} esu. Its electromagnetic field is localized in a space region of about 10^{-34} cm. The total stress tensor for the classical electron is shown to be identically zero. This means that there is no need in an additional surface-tension of unknown nature preventing the system disintegration, as it takes place in the Lorentz electron model. The hypothesis that gravitation can play a crucial role in the structure of all elementary particles is discussed.

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1. INTRODUCTION

As it is well known, quantum field theory can describe physical phenomena with high precision using the infinite charge and mass renormalization procedure to get experimentally measured observables (see for instance [1–3]). The physicists faced with the problems of the infinite renormalization for the first time in quantum electrodynamics (QED). One of reasons which lead to divergent integrals in QED is the point-like nature of the fundamental particles (leptons and quarks) being the source of the electromagnetic field. There are two radically different ways to avoid the divergences. The former way is to consider the extended objects instead of the point-like fundamental particles. The latter is to consider theories in which contributions of singularities are canceled due to an internal symmetry of the lagrangian.

In the model of a classical electron proposed by Lorentz [4], the electron is considered as a drop of a charged liquid with a radius a. This model gives the finite mass and charge of the electron. Since the drop is exploded by electrostatic forces, the additional surface-tension of the liquid has to be introduced to prevent disintegration of the electron. But the nature of this surface-tension is unknown that is the plague of the model. If one considers the electron as a point-like particle the problem of the nature of the surface-tension disappears. But as is well known the electromagnetic mass of any point-charge in classical electrodynamics (CE) is positive and tends to infinity if $a \to 0$. Indeed, the energy density of the electromagnetic field, t_{00} goes to infinity at $r \to 0$ as r^{-4}. Here r denotes the distance between the field observation point and the position of the point-charge. As a result, the integral $\int t_{00} 4\pi r^2 dr$ for the electromagnetic energy is divergent in CE as dr/r^2.

In order to make simple qualitative estimates of the gravitation contribution to the total mass of the point-like particle, let us add the relation $dm = d\mathcal{E}/c^2$ to Newton's law of gravitation. Here, dm denotes the mass of the small subsystem of a system under consideration interacting with other subsystems due to gravitation and $d\mathcal{E}$ is its energy. Let us consider a system when matter consists of the electromagnetic field only. Then the contribution to the total energy of the gravitational attraction between three-dimensional space regions filled with the electromagnetic field is negative in the Newtonian theory of gravitation. It is small since its contribution is proportional to the small gravitational constant k. But gravitation contributes to the total energy quadratically with respect to t_{00} which tends to infinity at $r \to 0$ for the electric field corresponding to the point-charge. Note that the electromagnetic contribution is linear with respect to the component t_{00} of the energy-momentum tensor t_{ik}. Therefore at some large energy density, the gravitational and electromagnetic contribution can be of the same order of magnitude. Since these two contributions cancel each other the total energy (mass) of the point-charge can, in principle, be finite even in this toy model.

It will be shown in the framework of general relativity that the gravitational contribution can make the total mass of the classical electron (with the electric field strength $|\mathbf{E}| = |e|/r^2$) finite. This result is in

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contradiction with the usual opinion that gravitation cannot play any role in the elementary particle structure since the gravitational interaction is much weaker than even electromagnetic one. Indeed, the electrostatic repulsion between two electrons is larger than their gravitational attraction by a factor of 4.2×10^{42}. This ratio even for heavy leptons and quarks with masses of a few GeV/c^2 is smaller by a factor of about 10^6, nevertheless this ratio is huge.

The consideration in the present paper is performed within the framework of the classical physics only since it is not well understandable now, how quantum effects can be taken into account for gravitation. The exact Reissner–Nordström (RN) solution of the Einstein-Maxwell equations is used to describe the classical electron. It turns out that for some relation between the parameters e and m of the RN solution, its total inertial mass and the action become finite. The equivalence principle is also valid for the system called the classical electron. Moreover, there is no need in additional nonelectromagnetic surface-tension. The classical electron has the experimental electrical charge of the real electron e but the huge mass $m_{\rm cl} = 1.86 \times 10^{-6}$ g. It represents the smallest possible charged black hole with the horizon radius of the order of 10^{-34} cm. The idea that the elementary particles are black holes was discussed for the first time by A. Einstein and N. Rosen in [5]. The classical electron is the system of the electromagnetic and gravitational fields. There is no point-like particle with a bare mass m_b and the electrical charge e (usually called the electron) inside the system under discussion. The proofs of the above statements and details of calculations can be found in [6].

2. REISSNER–NORDSTRÖM SOLUTION

Let us consider the spherical coordinates $(x^0, x^1, x^2, x^3) = (ct, r, \theta, \varphi)$ where c is the velocity of light in empty space and t denotes time, r is a radius, while θ, φ denote polar and azimuthal angles. The stationary solution of the Maxwell and Einstein equations, depending only on r, was found independently by H. Reissner, H. Weyl, G. Nordström, and G.B. Jeffery [7–10], nevertheless it is called usually the Reissner–Nordström solution. For this solution, the differential of the spacetime interval ds is given by

$$ds^2 = g_{ik} dx^i dx^k = \Lambda (dx^0)^2 - dr^2/\Lambda - r^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where g_{ik} denote the covariant components of the metric tensor. Summing over any pair of identical covariant and contravariant Latin indexes is assumed in Eq. (1) and in all below formulas. All Latin indexes can be equal to 0, 1, 2, 3, while the Greek indexes can be equal to 1, 2, 3. Summing over any pair of identical Greek indexes will also be assumed. For exclusions, the absence of a sum will be specially stressed. The function Λ in Eq. (1) is

$$\Lambda = 1 - \frac{r_g}{r} + \frac{r_e^2}{r^2} = (r^2 - r r_g + r_e^2)/r^2. \quad (2)$$

Here, r_g denotes the Schwarzschild radius [11]

$$r_g = \frac{2km}{c^2}, \quad (3)$$

where $k = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2} is the gravitational constant, e and m are respectively the electric charge and mass of the considered system, while the radius r_e is

$$r_e = \frac{\sqrt{ke^2}}{c^2}. \quad (4)$$

As is obvious from Eq. (2) Λ is not negative when $r_0^2 \geq 0$, where

$$r_0^2 = r_e^2 - r_g^2/4 \equiv r_e^2 \left(1 - \frac{km^2}{e^2}\right). \quad (5)$$

Since $g_{00} = \Lambda > 0$ for $r_0^2 > 0$, there is no horizon of events for which g_{00} has to be zero. According to Eq. (5) a horizon is absent if

$$m < \sqrt{\frac{e^2}{k}}. \quad (6)$$

The component g_{00} of the metric tensor is singular at $r = 0$, while all other covariant components are regular and nonzero for $0 < r < \infty$ and $\sin\theta > 0$.

It is easy to check that for the coordinates ξ^i with $\xi^0 = x^0$ and

$$\xi^1 = r\sin\theta\cos\varphi, \quad \xi^2 = r\sin\theta\sin\varphi, \quad \xi^3 = r\cos\theta, \quad (7)$$

the covariant components of the metric tensor are singular at $r = 0$, while the determinant of the matrix g_{ik} is always unity. The coordinates ξ^i tend to Euclidean limits when $r \to \infty$ and g_{ik} goes to the Minkowski limit $g_{ik} \to \eta_{ik}$, where η_{ik} is the diagonal matrix $\eta_{ik} = \text{diag}(1,-1,-1,-1)$. The only inconvenience of these coordinates is the following: the metric tensor is not diagonal for $r \sim r_e$.

It is more convenient for many calculations to apply the isotropic coordinates for which the metric tensor is diagonal. In order to introduce them, we define the new radial variable ρ with the relations

$$r = \rho \mathcal{D}(\rho), \quad (8)$$

$$\mathcal{D}(\rho) = 1 + \frac{r_g}{2\rho} - \frac{r_0^2}{4\rho^2} \equiv \left[1 + \frac{r_g}{4\rho}\right]^2 - \frac{r_e^2}{4\rho^2}, \quad (9)$$

where r_g, r_e^2, and r_0^2 are given by Eqs. (3), (4), and (5), respectively. Defining

$$\mathcal{N}(\rho) \equiv \frac{dr}{d\rho} = 1 + \frac{r_0^2}{4\rho^2}, \tag{10}$$

we get the formula for Λ

$$\Lambda = \frac{\mathcal{N}^2}{\mathcal{D}^2}. \tag{11}$$

Let us introduce the pseudo-Euclidean coordinates $\rho^0 = x^0$, $\rho_x \equiv \rho^1$, $\rho_y \equiv \rho^2$, $\rho_z \equiv \rho^3$ (called the uniform coordinates) with the relations

$$\rho_x = \rho \sin\theta \cos\varphi, \quad \rho_y = \rho \sin\theta \sin\varphi, \quad \rho_z = \rho\cos\theta. \tag{12}$$

Now, the formula for the spacetime interval looks like

$$ds^2 = g_{ik} d\rho^i d\rho^k = \frac{\mathcal{N}^2}{\mathcal{D}^2}(d\rho^0)^2 - \mathcal{D}^2(d\rho_x^2 + d\rho_y^2 + d\rho_z^2). \tag{13}$$

It defines both the covariant and contravariant components of the metric tensor for the uniform coordinates since g_{ik} is diagonal and $g^{ii} = 1/g_{ii}$ (no sum over i).

According to Eq. (10) $\mathcal{N} \geq 1$ for $r_0^2 \geq 0$, hence r increases with increasing ρ monotonically. For asymptotically large $\rho \to \infty$, $\mathcal{D} \to 1$ in accordance with Eq. (9), therefore $r/\rho \to 1$ due to Eq. (8). The minimal value of r equal to zero corresponds to the minimal possible positive value of $\rho = \rho_{\text{min}}$. This value is the maximal root of the equation $\mathcal{D}(\rho) = 0$ and is equal to

$$\rho_{\text{min}} = r_e/2 - r_g/4. \tag{14}$$

This means that the sphere in the three-dimensional space (ρ_x, ρ_y, ρ_z) with the radius $\rho = \rho_{\text{min}}$ corresponds to the point $r = 0$. There is no contradiction in this respondency since according to Eq. (13) the distance between any two points on the sphere is zero due to the relation $\mathcal{D}(\rho_{\text{min}}) = 0$.

3. TETRAD REPRESENTATION

In the tetrad representation proposed in [12], the fundamental variables of the gravitational field are four unit four-vectors $h_{(a)}$ ($a = 0, 1, 2, 3$ is a counting number of the four-vector) being functions of the coordinates of points in the four-dimensional spacetime. The four-vector with $a = 0$ is chosen time-like, while all others are space-like, namely

$$h_{(a)i} h^i_{(b)} = \eta_{ab} \tag{15}$$

with the diagonal matrix $\eta_{ab} = \text{diag}(1,-1,-1,-1)$. Defining η^{ab} equal to η_{ab} the vectors $h^{(a)}$ can be expressed in terms of $h_{(a)}$ [12, 13]

$$h^{(a)i} = \eta^{ab} h^i_{(b)}, \quad h^{(a)}_i = \eta^{ab} h_{(b)i}. \tag{16}$$

As follows from Eqs. (15) and (16) the orthogonality conditions look like [12, 13]

$$h_{(a)i} h^{(b)i} = \delta^b_a, \tag{17}$$

$$h_{(a)i} h^{(a)k} = \delta^k_i, \tag{18}$$

where δ^k_i and δ^b_a denote the Kronecker symbols. The fundamental relations between the covariant (contravariant) tetrad components, $h_{(a)i}$ ($h^j_{(a)}$) and the metric tensor components are [12, 13]

$$h_{(a)i} h^{(a)}_l = g_{il}, \tag{19}$$

$$h^j_{(a)} h^{(a)k} = g^{jk}. \tag{20}$$

The partial derivative of $h^{(a)}_i$ over the uniform coordinate ρ^j is denoted by $h^{(a)}_{i,j}$, while $h^{(a)i}_{\ ;j}$ is the covariant derivative of $h^{(a)i}$. The relations between them are the following [6, 12]:

$$h^{(b)k}_{\ ;l} = \frac{1}{2} h^{(b)m} h^k_{(a)} (h^{(a)}_{l,m} - h^{(a)}_{m,l}) + \frac{1}{2} h^{(c)i} h^k_{(c)} \times [(h^{(b)}_{i,l} - h^{(b)}_{l,i}) + h^{(b)m} h_{(a)l} (h^{(a)}_{i,m} - h^{(a)}_{m,i})]. \tag{21}$$

4. LAGRANGIAN FOR THE REISSNER–NORDSTRÖM SOLUTION

The formula for the total lagrangian density reads

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{g}} + \mathcal{L}_{\text{em}}, \tag{22}$$

where $\mathcal{L}_{\text{g}}$ is the lagrangian density of gravitational field, while $\mathcal{L}_{\text{em}}$ denotes the lagrangian density of electromagnetic field. In the tetrad representation, the former is given by the relation [12, 14]

$$\mathcal{L}_{\text{g}} = \frac{|h|}{2\kappa} \left(h^k_{(a);l} h^{(a)l}_{\ ;k} - h^k_{(a);k} h^{(a)l}_{\ ;l} \right), \tag{23}$$

where $\kappa = 8\pi k/c^4$ and $|h|$ denotes the determinant of the 4×4 matrix $h_{(a)i}$. As is demonstrated by Eq. (21) the covariant derivative of the tetrad depends only on the tetrad components and linearly on their first partial derivatives with respect to the space-time coordinates. Therefore the lagrangian $\mathcal{L}_{\text{g}}$ in Eq. (23) depends also on the tetrads and their first partial derivatives.

According to Eq. (13) and relation [12, 14]

$$g = -|h|^2, \tag{24}$$

the formula for $|h| = \sqrt{-g}$ for the uniform coordinates for the RN solution is

$$|h| = \mathcal{N}\mathcal{D}^2. \tag{25}$$

For the tetrads with the covariant components

$$h_{(0)k} = h_k^{(0)} = \frac{\mathcal{N}}{\mathcal{D}}\delta_k^0, \quad h_{(\mu)k} = -h_k^{(\mu)} = \mathcal{D}\delta_k^\mu \tag{26}$$

for any k and μ, relation (19) is fulfilled. Formulas for the contravariant components are

$$h^{(0)i} = h^i_{(0)} = g^{00}h_{(0)0}\delta_0^i = \delta_0^i\mathcal{D}/\mathcal{N}, \tag{27}$$

$$h^i_{(\mu)} = -h^{(\mu)i} = g^{ik}h_{(\mu)k} = -\delta_\mu^i/\mathcal{D}. \tag{28}$$

These components obey Eq. (20). Calculating formulas for $h^{(a)k}_{\;;l}$ using Eq. (21) one gets [6] from Eq. (23)

$$\mathcal{L}_g = \frac{\mathcal{N}\mathcal{D}'}{\kappa\mathcal{D}}\left\{2\frac{\mathcal{N}'}{\mathcal{N}} - \frac{\mathcal{D}'}{\mathcal{D}}\right\}, \tag{29}$$

where $\mathcal{N}'$ and $\mathcal{D}'$ denote derivatives of $\mathcal{N}(\rho)$ and $\mathcal{D}(\rho)$ with respect to ρ.

The general formula for the lagrangian density of the electromagnetic field reads [13]

$$\mathcal{L}_{\text{em}} = -\frac{|h|}{16\pi}g^{im}g^{kn}F_{ik}F_{mn}$$
$$= -\frac{|h|}{16\pi}h^i_{(a)}h^{(a)m}h^k_{(b)}h^{(b)n}F_{ik}F_{mn}, \tag{30}$$

where the electromagnetic tensor F_{ik} is expressed through the four-potential A_m

$$F_{ik} = \frac{\partial A_k}{\partial\rho^i} - \frac{\partial A_i}{\partial\rho^k}. \tag{31}$$

For the uniform coordinates, the nonzero components of the electromagnetic tensor are [6]

$$F_{0\lambda} = -F_{\lambda 0} = \frac{e}{r^2}\mathcal{N}n_\lambda = \frac{e\mathcal{N}}{\rho^2\mathcal{D}^2}n_\lambda, \tag{32}$$

where $n_\lambda = \rho_\lambda/\rho$ denotes the unit three-vector. If we substitute Eqs. (32) and (26)–(28) into Eq. (30), the final formula for $\mathcal{L}_{\text{em}}$ is obtained

$$\mathcal{L}_{\text{em}} = \frac{e^2\mathcal{N}}{8\pi\rho^4\mathcal{D}^2} = \frac{\mathcal{N}r_e^2}{\kappa\rho^4\mathcal{D}^2}. \tag{33}$$

In the second representation for $\mathcal{L}_{\text{em}}$ in Eq. (33), the formula $e^2/(8\pi) = r_e^2/\kappa$ is taken into account that follows from Eq. (4) and relation between k and κ.

Substituting $\mathcal{L}_{\text{em}}$ given by Eq. (33) and $\mathcal{L}_g$ from Eq. (29) into Eq. (22) and using Eqs. (9) and (10) respectively for $\mathcal{D}(\rho)$ and $\mathcal{N}(\rho)$ we obtain for the total lagrangian density the very simple formula

$$\mathcal{L}_{\text{tot}} = \frac{r_0^2}{\kappa\rho^4}. \tag{34}$$

Since the three-dimensional space (ρ_x, ρ_y, ρ_z) consists of points for which $\rho = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}$ obeys the inequality $\rho \geq \rho_{\text{min}}$, the formula for the total lagrangian reads

$$L_{\text{tot}} = \int\limits_{\rho\geq\rho_{\text{min}}} \mathcal{L}_{\text{tot}}d\rho_x d\rho_y d\rho_z = \frac{4\pi r_0^2}{\kappa\rho_{\text{min}}}. \tag{35}$$

Making use of Eqs. (3)–(5) and (14) we get finally

$$L_{\text{tot}} = c^2\sqrt{\frac{e^2}{k}} + mc^2. \tag{36}$$

This formula shows that the lagrangian in the tetrad representation and the action $S = L_{\text{tot}}t$ are finite in spite of the singular behavior of the electromagnetic and gravitational fields near the point $r = 0$ $(\rho = \rho_{\text{min}})$ due to cancellation of singularities.

If we consider the metric tensor components as the fundamental variables describing the gravitational field, there is no aforementioned cancellation. Indeed, it is well known that the scalar curvature $\mathcal{R}$ and hence the lagrangian density of the gravitational field

$$\tilde{\mathcal{L}}_{\text{g}} = -\frac{\mathcal{R}\sqrt{-g}}{2\kappa} \tag{37}$$

is zero if the matter is represented by the electromagnetic field only [13, 15, 16]. Since the lagrangian density for the electromagnetic field defined by Eqs. (30) has nonintegrable singularity at $r = 0$, the total lagrangian corresponding to Eqs. (22), (30), and (37) is meaningless (infinitely large), while Eq. (36) provides the finite lagrangian in the tetrad representation.

As is shown in [12] the difference of the two lagrangian densities, $\mathcal{L}_{\text{g}}$ and $\tilde{\mathcal{L}}_{\text{g}}$ defined respectively by Eqs. (23) and (37) is the total divergence of a four-vector. This four-vector can be expressed in terms of the tetrad components and their derivatives with respect to ρ^i. Therefore these two lagrangian densities are totally equivalent to each other when solution is regular in all over the four-dimensional spacetime. We conclude from the above consideration that these two lagrangian densities are not equivalent for the singular solutions: the former after adding the lagrangian density of the electromagnetic field, $\mathcal{L}_{\text{em}}$ provides the finite total action, the latter is zero and corresponds to the infinitely large total lagrangian and action. This means that the divergence of the four-vector is infinite for the case under discussion.

The lagrangian $\mathcal{L}_{\text{g}}$ contains the tetrad components and their first derivatives with respect to ρ^i, while the lagrangian $\tilde{\mathcal{L}}_{\text{g}}$ contains not only first derivatives but also second derivatives of the metric tensor components (see for instance [13]) which means that it contains second derivatives of the tetrad components. The absence of the second derivatives is the advantage of the lagrangian $\mathcal{L}_{\rm g}$, that can be constructed only in the tetrad representation, compared to $\tilde{\mathcal{L}}_{\rm g}$ which formula contains g_{ik} and their derivatives. Therefore we assume that it is these sixteen functions $h_i^{(a)}(\rho^k)$ of the coordinates ρ^k which are the fundamental gravitational variables rather than the components of the metric tensor.

The choice of a tetrad obeying Eqs. (19) and (20) is not unique. Tetrad $e^{(a)p}(\rho^i)$ in any point with coordinates ρ^i defined by

$$e^{(a)p}(\rho^i) = L^a{}_c(\rho^i)h^{(c)p}(\rho^i) \tag{38}$$

also gives the same metric tensor as $h^{(a)p}(\rho^i)$ if $L^a{}_c(\rho^i)$ is a matrix of the six-parametric Lorentz group [12, 14]. This means that the lagrangian density $\tilde{\mathcal{L}}_{\rm g}$ in Eq. (37) is invariant under transformation defined by Eq. (38), while $\mathcal{L}_{\rm g}$ containing the covariant derivatives of the tetrad components in Eq. (23) is not invariant. Nevertheless, the difference of two lagrangian densities $\Delta\mathcal{L}_{\rm g}$ expressed with the help of Eq. (23) in terms of $h^{(c)p}(\rho^i)$ and $e^{(a)p}(\rho^i)$ is the divergence of some four-vector. Therefore these two lagrangian densities are equivalent to each other for $h^{(c)p}(\rho^i)$ and $e^{(a)p}(\rho^i)$ regular in all over the four-dimensional spacetime since both of the densities are equivalent to $\tilde{\mathcal{L}}_{\rm g}$ defined in Eq. (37). Nevertheless, if at least one tetrad has singular point, the solutions given by $e^{(a)p}(\rho^i)$ and $h^{(c)p}(\rho^i)$ can be not equivalent to each other and the difference of the corresponding lagrangian densities $\Delta\mathcal{L}_{\rm g}$ can be a singular nonintegrable function.

If we consider the components $h_p^{(c)}(\rho^i)$ of the tetrad as the variables describing the gravitational field, they must obey the Lagrange equations

$$\frac{\partial\mathcal{L}_{\rm tot}}{\partial h_p^{(c)}} = \frac{\partial}{\partial\rho^q}\left[\frac{\partial\mathcal{L}_{\rm tot}}{\partial h_{p,q}^{(c)}}\right]. \tag{39}$$

It was checked in [6] that the tetrad components defined by Eqs. (26)–(28) obey the Lagrange equations.

5. CONSERVATION OF ENERGY AND THREE-MOMENTUM

In the tetrad formalism, let us define the superpotential $U_i{}^{kl}$, proposed by Møller [12, 14]

$$U_i{}^{kl} = -U_i{}^{lk} = h^{(a)k}\frac{\partial\mathcal{L}_{\rm tot}}{\partial h^{(a)i}_{\ ,l}} = h_i^{(a)}\frac{\partial\mathcal{L}_{\rm tot}}{\partial h_{l,k}^{(a)}}. \tag{40}$$

Then the total energy-momentum pseudotensor density, $\mathcal{T}_i{}^k$ is

$$\mathcal{T}_i{}^k = U_i{}^{kl}{}_{,l} \equiv \frac{\partial U_i{}^{kl}}{\partial\rho^l}. \tag{41}$$

It is shown in [12, 14] that

$$U_i{}^{kl} = \frac{|h|}{\kappa} \times \left\{h^k_{(a)}h^{(a)l}_{\ ;i} + \left(\delta_i^k h^{(a)l} - \delta_i^l h^{(a)k}\right)h^s_{(a);s}\right\}. \tag{42}$$

As seen from Eq. (42) Møller's superpotential is the tensor density under arbitrary coordinate transformations since it is expressed in terms of the tetrad vectors $h^{(a)k}$, their covariant derivatives $h^{(a)l}_{\ ;i}$, and the determinant $|h|$. The superpotential $U_i{}^{kl}$ is antisymmetric with respect to the indexes k and l, therefore the divergence of the total energy-momentum pseudotensor density is zero

$$\mathcal{T}_i{}^k{}_{,k} = U_i{}^{kl}{}_{,l,k} \equiv \frac{\partial^2 U_i{}^{kl}}{\partial\rho^k\partial\rho^l} = 0. \tag{43}$$

As is well known [12–14] the conservation of the energy-momentum four-vector is a consequence of Eq. (43).

If the total energy-momentum pseudotensor is localized in the compact three-dimensional region $\mathcal{V}$, such a system will be called the insular one. Then the metric tensor g_{ik} for the insular system goes to its Minkowski limit η_{ik} at $\rho\to\infty$ as

$$g_{ik}(\rho) = \eta_{ik}(\rho) + O_1(\rho). \tag{44}$$

Here, O_n denotes the quantity which main term at $\rho\to\infty$ is proportional to ρ^{-n} for $n = 1, 2, \ldots$. The standard consideration shows that the energy-momentum four-vector P_i defined by

$$P_i = \frac{1}{c}\int\limits_{\mathcal{V}} \mathcal{T}_i{}^0 d^3\rho \tag{45}$$

is conserved if all components $\mathcal{T}_i{}^k$ are zero outside the region $\mathcal{V}$, where $d^3\rho = d\rho^1 d\rho^2 d\rho^3 \equiv d\rho_x d\rho_y d\rho_z$ denotes the volume element. Using Eq. (41) the formula can be rewritten as the surface integral due to the Gauss theorem

$$P_i = \frac{1}{c}\int\limits_{\mathcal{V}} \frac{\partial U_i{}^{0\lambda}}{\partial\rho^\lambda}d^3\rho = \frac{1}{c}\int\limits_{\Sigma} U_i{}^{0\lambda}k_\lambda d\sigma. \tag{46}$$

Here, Σ is the closed surface enclosing the region $\mathcal{V}$, while the three-dimensional vector k_λ is the unit outer normal to the surface. The element of the surface is defined by

$$k_\lambda d\sigma = \epsilon_{\lambda\mu\nu}d\rho^\mu\delta\rho^\nu, \tag{47}$$

where $\epsilon_{\lambda\mu\nu}$ is the totally antisymmetric three-dimensional Levi-Civita symbol, while $d\rho^\mu$ and $\delta\rho^\nu$ are infinitesimal three-vectors on the surface Σ.

When the matter fields are localized mainly in the region $\mathcal{V}$ and $\mathcal{T}_i{}^k$ goes to zero outside $\mathcal{V}$ at $\rho \to \infty$ as a quantity of O_n with $n \geq 4$ the system will be also called the insular system. For this case, the full three-dimensional space ($\mathcal{V}_\infty$) is to be considered and for the surface Σ, we will choose $\Omega_\mathcal{R}$ with $\mathcal{R} \to \infty$. Here $\Omega_\mathcal{R}$ denotes the surface of the sphere with the radius $\mathcal{R}$. The surface integral in Eq. (46) becomes the limit of the integral on $\Omega_\mathcal{R}$ at $\mathcal{R} \to \infty$ if the superpotential $U_i{}^{kl}$ is regular inside the sphere $\Omega_\mathcal{R}$.

Substituting formulas (25) for $|h|$, (27) and (28) for $h^k_{(a)}$, expressions for $h^{(a)l}_{;i}$, calculated with the help of Eq. (21), into Eq. (42) we get for the nonzero components of the superpotential [6] for the RN solution

$$U_0{}^{0\lambda} = -U_0{}^{\lambda 0} = -2\mathcal{N}\frac{n_\lambda \mathcal{D}'}{\kappa\mathcal{D}}, \tag{48}$$

while for $\mu \neq \lambda$, one gets

$$U_\mu{}^{\lambda\mu} = -U_\mu{}^{\mu\lambda} = \frac{n_\lambda}{\kappa}\mathcal{N}'. \tag{49}$$

Note that for the RN solution, the full three-dimensional space for the uniform coordinates ρ_x, ρ_y, ρ_z consists of all points with $\rho = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2} \geq \rho_{\min}$, where $\rho_{\min}$ is given by Eq. (14) and $\mathcal{D}(\rho_{\min}) = 0$. This means that the surface integral in Eq. (46) consists of the integrals over the spheres with the radii $\rho = \rho_{\min}$ and $\rho = \mathcal{R}$ with $\mathcal{R} \to \infty$. As seen from Eq. (48) all the superpotential components $U_0{}^{0\lambda}$ are infinite on the sphere with the radius $\rho_{\min}$ since $\mathcal{D}(\rho_{\min}) = 0$ and $U_0{}^{0\lambda}$ behave near $\rho_{\min}$ as $1/(\rho - \rho_{\min})$. This makes the definition of the energy given by Eq. (46) meaningless for any parameter m obeying inequality in Eq. (6) since the energy is infinitely large. Also, there is no solution with e and m obeying Eq. (6) with a finite total inertial mass which can be used as a model for electrons.

But the total energy of the system becomes finite when the limit $m \to m_{\text{cl}}$ is considered, where

$$m_{\text{cl}} = \sqrt{\frac{e^2}{k}}. \tag{50}$$

Having in mind to discuss elementary particle structure, we put $|e| = 4.80 \times 10^{-10}$ esu for which $r_e = 1.38 \times 10^{-34}$ cm that is calculated with Eq. (4). It is this system which will be referred to as "the classical electron". Its mass is equal to 1.86×10^{-6} g that is much larger than the experimental value of the real electron mass. The solution with the RN parameter $m = m_{\text{cl}}$ corresponds to the black hole with the minimal possible mass for the fixed electrical charge.

If $m = m_{\text{cl}}$, then according to Eqs. (3)–(5), and (14)

$$r_0 = 0, \quad r_e = r_g/2, \quad \rho_{\min} = 0. \tag{51}$$

Using Eqs. (51) we get now instead of Eqs. (9) and (10)

$$\mathcal{D}(\rho) = 1 + \frac{r_g}{2\rho} = 1 + \frac{r_e}{\rho}, \tag{52}$$

$$\mathcal{N}(\rho) = 1. \tag{53}$$

Due to Eq. (53) one gets $\mathcal{N}' = 0$, therefore formula (49) is simplified:

$$U_\mu{}^{\lambda\mu} = -U_\mu{}^{\mu\lambda} \equiv 0. \tag{54}$$

Formula (48) becomes now [6]

$$U_0{}^{0\lambda} = -U_0{}^{\lambda 0} = \frac{m_{\text{cl}}c^2}{4\pi}\frac{n_\lambda}{\rho^2(1 + r_e/\rho)}. \tag{55}$$

Let us calculate the total energy of the classical electron with the help of Eqs. (45), (41), and (55). Since now $\rho_{\min} = 0$, we have

$$\mathcal{E} = \int_0^\infty \mathcal{T}_0{}^0(4\pi\rho^2 d\rho) = \int_0^\infty \frac{e^2(4\pi\rho^2 d\rho)}{4\pi\rho^4(1 + \frac{r_e}{\rho})^2} = \frac{e^2}{r_e}. \tag{56}$$

As seen from this formula the integrand increases with decreasing of ρ as ρ^{-2} when $\rho \gg r_e$ and is divergent if $k = 0$ (gravitation is switched off) and $r_e = 0$. When $k > 0$ and $r_e > 0$, the integrand at $\rho \leq r_e$ goes to a finite constant. As a result, the integral is convergent due to the gravitation contribution and is equal to $e^2/r_e = c^2\sqrt{e^2/k} = m_{\text{cl}}c^2$ if we take into account Eqs. (4) and (50). The mass m_{cl} calculated with the help of Eq. (56) is the inertial mass m_{in} of the electromagnetic and gravitational fields. The total gravitational mass m_{gr} can be obtained from the asymptotic behavior at $\rho \to \infty$ of g_{00} which is to be [13]

$$g_{00} \to 1 - 2km_{\text{gr}}/(c^2\rho). \tag{57}$$

For the parameter m of the RN solution g_{00} behaves at $\rho \to \infty$ according to Eqs. (13), (9), and (10) as

$$g_{00} = \mathcal{N}^2/\mathcal{D}^2 \to 1 - 2km/(c^2\rho). \tag{58}$$

A comparison of Eqs. (57) and (58) shows that the parameter m is always the total gravitational mass: $m = m_{\text{gr}}$. When $m = m_{\text{cl}}$, we have $m_{\text{in}} = m_{\text{gr}}$ which means that the total inertial mass of the system of the electromagnetic and gravitational fields is equal to its total gravitational mass. According to the equivalence principle, this means that there is no need in any point-like particle (usually called the electron with any bare mass) inside the system of the electromagnetic and gravitational fields under consideration.

If Eqs. (51) are fulfilled and $r_g = 2r_e$, then $g_{00} = \Lambda = (1 - r_e/r)^2$ according to Eqs. (1) and (2). The sphere with the radius $r = r_e$ is the horizon surface of the black hole. This means that no information about the region with $r < r_e$ can be obtained by the external observer located in points with $r > r_e$. The integration in Eq. (56) for $\rho \geq 0$ corresponds to the integral for the region $r \geq r_e$. Therefore the integration runs over only those points where the fields can be measured by any exterior observer and the inertial mass is calculated for $r \geq r_e$ only. It is this inertial mass which is compared with the total gravitation mass whose value is established by the study of the asymptotic behavior at $r \to \infty$ of g_{00} in the region where the fields can also be observed. Formally, the formula for the energy-momentum tensor density of the electromagnetic field $t_0^0 = e^2/(8\pi r^4)$ can be considered at $r \to 0$ also but it loses the physical meaning for $r < r_e$ since the electromagnetic field cannot be observed there. Note that $m_{\text{in}} = m_{\text{cl}}$ is not the inertial mass of the point-like particle laying at $r = 0$ but it is the total mass of the fields situated beyond the event horizon.

The space part of the total energy-momentum pseudotensor density is zero ($\mathcal{T}_\mu{}^\nu \equiv 0$), which follows from Eqs. (41) and (54). This assumes the absence of any pressure of any part of the classical electron on other parts. Therefore there is no need in the additional surface-tension (existing in the Lorentz model of the electron [4] considering it as a drop of a charged liquid) which prevents disintegration of the system. Note that only $\mathcal{T}_\mu{}^\nu$ is zero, meanwhile the Maxwell stress tensor for the electromagnetic field t_μ^ν is non-zero.

6. GENERAL DISCUSSION

As is shown above the system of entirely gravitational and electromagnetic fields described by Eqs. (1)–(2) and (32) has a finite total inertial mass if the parameter m of the RN solution is equal to $\sqrt{e^2/k}$. For this case m is both the total inertial and gravitational mass of the black hole, that is in agreement with the equivalence principle. Such a system is called "the classical electron" when e is the experimental electron charge. There is no need in any additional charged point-like particle which is usually called the electron. In the approach of the present paper, the only existing entities are the electromagnetic and gravitational fields, while the solutions, localized in the three-dimensional space regions of the range of about $r_e = 1.38 \times 10^{-34}$ cm, represent the classical electrons. According to Eq. (4) this typical length r_e is expressed through the fundamental constants e, k, and c only.

The important thing is the tetrad representation for the RN solution, which makes the action finite for the parameters e and m obeying Eq. (6). The action is infinite for the same e and m if we consider the metric tensor components as the fundamental variables of the gravitational field. Therefore the tetrad formalism and the approach based on the metric tensor components can be nonequivalent for solutions with singular points.

Up to now, the mass m and the charge e were assumed to be those of an elementary particle. If we denote the mass and charge of a macroscopic object by M and Q, we can define the distance $r_Q = \sqrt{kQ^2}/c^2$ to measure all distances by the dimensionless variable $\zeta = r/r_Q$. The function Λ, that defines the metric tensor by Eqs. (1) and (2), and the only nonzero component of the four-potential of the electric field $A_0 = Q/r$ for the RN solution can be expressed by the functions of ζ

$$\Lambda = 1 - \frac{2|\lambda|}{\zeta} + \frac{1}{\zeta^2}, \tag{59}$$

$$\frac{\sqrt{k}}{c^2} A_0 = \frac{Q}{\sqrt{Q^2}} \frac{1}{\zeta} = \frac{\lambda}{|\lambda|} \frac{1}{\zeta}, \tag{60}$$

where the dimensionless parameter $\lambda = M\sqrt{k}/Q$. Since the parameter $c^2/\sqrt{k}$ is the universal constant, that can be used as a unit for the potential A_0, Eqs. (59) and (60) show that the RN solution has only one dimensionless parameter λ which determines the solution uniquely. The classical electron corresponds to $\lambda = -1$.

Systems of electromagnetic and gravitational fields with parameter sets (M_1, Q_1) and (M_2, Q_2) are described with the same functions Λ and A_0 of ζ if $Q_1/M_1 = Q_2/M_2$ since $\lambda_1 = \lambda_2$. In the classical physics, this relation gives a possibility to study the properties of the elementary particle with the charge e and mass m investigating the macroscopic objects with parameters Q and M if $e/m = Q/M$. The quantum effects violate this scaling and the elementary particle properties with a small scale parameter r_e differ from those of macroscopic bodies characterized by large parameters r_Q ($r_Q \gg r_e$). This reminds the situation with the atom and its classical planetary model in which "electrons" and a "nucleus" are macroscopic bodies. As is well known the quantum effects for atoms are not small corrections and make the lifetime of the atoms infinitely large while the macroscopic model lives for a very short time interval owing to the emission of the electromagnetic waves. The same is probably true for the real electron and its classical model: the ratio of their masses $m_{\rm cl}/m_e$ is about 10^{21} where m_e is the experimental mass of the real electron.

It is well known (see for instance [3]) that the quantum effects become important in QED at distances of about the Compton wavelength of the real electron $\lambda_C = \hbar/(m_e c)$ where $\hbar$ denotes the Plank constant. Since $\lambda_C = 3.86 \times 10^{-11}$ cm, it is much greater than the typical length $r_e = 1.38 \times 10^{-34}$ cm for the classical electron. For the first sight, this means that the above consideration of the classical electron has no physical meaning. Nevertheless, highly likely this argument is not true. Indeed, even in CE, where the integral for the electromagnetic energy is divergent as dr/r^2 at $r \to 0$ (see Eq. (56) for $r_e = 0$ when $r = \rho$), the contribution of gravitation makes the total mass of the point-charge finite. In QED, self-energy Feynman graphs are divergent as dr/r [1–3]. Therefore the fractional contribution of gravitation needed to make the total mass finite can be much less important than in CE. Hence it is not excluded in QED that gravitation can make the electron mass finite.

By changing the sign of the charge in Eq. (32) we get a set of the electromagnetic and gravitational fields corresponding to "a classical positron". It has the same mass as the electron in the approach of the present paper, hence the positron is not an electron with the negative energy as in the Dirac equation [1–3]. It is assumed that in the future version of the quantum field theory, there will be no need in the local electron-positron field having the negative vacuum energy.

In the approach based on the supersymmetry, the negative vacuum energy of the electron-positron field compensates the positive vacuum energy of the photon field. The total vacuum energy of all fundamental fields can be equal to zero if the supersymmetry is not broken. Supersymmetric partners of the existing particles with masses less than about 1 TeV/c^2 are not found up to now. This can mean that the supersymmetry is not a fundamental symmetry of elementary particles. In the absence of the electron-positron field, the vacuum energy can probably be made finite due to the negative contribution of the gravitational field in addition to the positive contribution of the vacuum quantum oscillation of the electromagnetic field. Indeed, let us consider only the modes of the vacuum quantum oscillations with frequencies $\omega < \omega_c$. Let us imagine that the vacuum energy density for these modes with their energies $\epsilon_{\rm vac} = \hbar\omega/2$ is huge but finite in the absence of the gravitational interaction. If the gravitational interaction is switched on, its negative contribution decreases the vacuum energy. In standard QED, the electromagnetic energy density goes to infinity when $\omega_c \to \infty$, the modulus of the gravitation interaction contribution increases to infinity quadratically with respect to the electromagnetic energy density, t_{00}. It is not excluded that the total vacuum energy density becomes finite. This picture is analogous to the considered classical electron case for which the infinite contribution of t_{00} at $\rho \to 0$ is compensated with the contribution of the gravitational attraction of small three-dimensional space regions, filled with the electromagnetic field, with each other. The cancellation of these two contributions leads to the finite value of the total mass.

Real electrons take part in weak interaction and this electron property should be taken into account. Therefore we should try to find solutions of equations for the system of the electromagnetic, gravitational, and weak-boson fields. Another property of the electron, that should be taken into account in its realistic description, is the electron spin s equal to $\hbar/2$. But solutions with $s = \hbar/2$ are not excluded for the nonlinear boson fields. A well known example provides the Skyrme model [17] in which the solutions with the spin $\hbar/2$ (baryons) are constructed, though the fundamental field is the nonlinear field of the pseudoscalar pions. Another way to make the spin is to consider a solution of rotating electromagnetic and gravitational fields described with the Kerr-Newman solution [18, 19].

The localized states of the quantized electromagnetic, gravitational, and weak-boson fields with the $\hbar/2$ spin, the observed values of the electric charge, the weak charge, and the finite masses equal to those of the electron, muon, and tau lepton could exist. In the same way, the localized solutions of the quantized gluon, electromagnetic, weak boson, and gravitational field equations would be quarks and there would not be a need in local bispinor fields corresponding to the point-like massive quarks. This problem cannot probably be solved soon since there is no renormalizable quantum field theory of gravitation, though any estimates of the electron mass could be performed in lattice calculations using the continual integrals for electromagnetic and gravitational fields (see review [20]). Nevertheless, we assume that the idea to construct all observed particles as singular or localized (in a very small three-dimensional space region) solutions of fundamental field equations is constructive.

7. CONCLUSIONS

It is shown, that for the Reissner–Nordström solution, the contribution of gravitation makes the total energy-momentum pseudotensor density integrable function if the parameter m of the solution is equal to $\sqrt{e^2/k}$. Nevertheless, the singular points exist for this case also. The total inertial mass of the system of the electromagnetic and gravitational fields is finite ($m_{\rm in} = \sqrt{e^2/k}$) and equal to its total gravitational mass $m_{\rm gr}$. According to the equivalence principle ($m_{\rm in} = m_{\rm gr}$), this means the absence of an additional contribution to the total mass of any charged point-like particle with a nonzero bare mass. In the approach of the present paper, the classical electron is the system of the electromagnetic and gravitational fields localized in the space region with the typical length of about $r_e = 1.38 \times 10^{-34}$ cm, the electric charge $e = -4.8 \times 10^{-10}$ esu, and the total mass $m = 1.86 \times 10^{-6}$ g.

Since the total stress tensor density $\mathcal{T}_\mu{}^\nu$ is identically zero, there is no need in additional nonelectromagnetic forces preventing the disintegration of the classical electron. Such forces were introduced in the Lorentz model of the electron (surface-tension forces of an unknown nature in a charged liquid drop).

The total lagrangian for the Reissner–Nordström solution is infinitely large if the metric tensor components are considered as fundamental variables of the gravitational field for the parameters e and m obeying the inequality $m < \sqrt{e^2/k}$. The total lagrangian for the Reissner–Nordström solution is shown to be finite in the tetrad representation for the same parameters e and m.

We assume that it is not excluded that in quantum field theory, the only existing fundamental entities are gravitational, electromagnetic, weak-boson, and gluon fields. Leptons and quarks are states of these fields with corresponding quantum numbers localized in very small three-dimensional space regions.

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