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NUCLEI
Theory

On The Contribution of the P and D Partial-Wave States to the Binding Energy of the Triton in the Bethe–Salpeter–Faddeev Approach

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Abstract—The influence of the partial-wave states with nonzero orbital moment of the nucleon pair on the binding energy of the triton $T(nnp)$ in the relativistic case is considered. The relativistic generalization of the Faddeev equation in the Bethe–Salpeter formalism is applied. Two-nucleon t matrix is obtained from the Bethe–Salpeter equation with separable kernel of nucleon–nucleon interaction of the rank one. The kernel form factors are the relativistic type of the Yamaguchi functions. The following two-nucleon partial-wave states are considered: $^1S_0$, $^3S_1$, $^3D_1$, $^3P_0$, $^1P_1$, $^3P_1$. The system of the integral equations are solved by using the iteration method. The binding energy of the triton and three-nucleon amplitudes are found. The contribution of the P and D states to the binding energy of triton is given.

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1. INTRODUCTION

The study of three-nucleon systems has a long history and many works are devoted to the description of such nuclei. One of the common nonrelativistic description is based on the application of the Faddeev equation with various two-particle potentials. Among such potentials, there are realistic [1] and separable [2]. Such studies made it possible to achieve significant progress in the description of static and dynamic properties of the three-nucleon systems.

At the same time planned experiments on the electrons scattering off the ^{3}He and ^{3}H nuclei, for instance, Jefferson Lab Experiment E1210103, with the energies of the initial particles up to 12 GeV, require a relativistic description. There are several ways of relativization of the non-relativistic description, and the methods that follow from the quantum field theory (QFT) first principles. Among them we single out the quasipotential Gross equation with the exchange kernel of a nucleon-nucleon interactions [3], and approaches based on the Bethe–Salpeter formalism with zero range of forces [4] and with a separable kernel of interaction [5, 6]. This work develops the ideas presented in the articles [5], where the triton is considered in the S-state, and [6], where along with the S-state, the contribution of the D-state into the two-particle t matrix is considered. To describe a three-nucleon bound state, the relativistic generalization of Faddeev equations in the Bethe–Salpeter formalism—Bethe–Salpeter–Faddeev equation—is used. For simplicity of calculations, we consider nucleons with the same masses and the scalar propagators instead of the spinor ones. The spin-isospin structure of the system is described by matrices of recoupling coefficient from one partial-wave state to another.

In previous works [7, 8] we considered the case of taking into account the D-wave not only in the two-particle t matrix, but also its amplitudes in the system of integral equations. In the present paper, the equation is generalized to the case of nonzero values of the angular momentum of a pair of nucleons ($L > 0$: P- and D-states). The contributions of the following two-particle partial-wave states with a total angular momentum of the two-nucleon system $j = 0, 1$ are considered: $^1S_0$, $^3S_1$, $^3D_1$, $^3P_0$, $^1P_1$, $^3P_1$. The resulting system of twelve integral equations for real and imaginary parts of amplitudes is solved by the iteration method and the binding energy of the triton, as well as all three-particle amplitudes are calculating.

The work is organized as follows: after a brief description of the solution of Bethe–Salpeter equations for two-nucleon states (sec. 2), the relativistic Bethe–Salpeter–Faddeev equation with scalar propagators is introdused, and the partial-wave decomposition is performed (sec. 3). In sec. 4 the results of solving the system of equations and discussion is presented.

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2. TWO PARTICLES CASE

Since the kernel of the Faddeev equation, written in integral form, contains a two-particle t matrix we consider first the two-body problem.

The system of two relativistic particles can be described using the Bethe–Salpeter equation. The equation for the two-particle t matrix has the following form

$$T(p,p';P) = V(p,p';P) + \frac{i}{(2\pi)^4}\int d^4k V(p,k;P)G(k;P)T(k,p';P), \quad (1)$$

where $p=(p_1-p_2)/2$ [$p'=(p'_1-p'_2)/2$] is the relative 4-momentum of the particles of the system in the initial [final] state, $s=P^2$ is square of the total 4-momentum of the system $P=p_1+p_2=p'_1+p'_2$, $T(p,p';P)$ is two-particle t matrix, $V(p,k;P)$ is kernel (potential) of a nucleon-nucleon (NN) interaction, $G(k;P)$ is the product of two scalar propagators of nucleons,

$$G^{-1}(k;P) = \left[(P/2+k)^2 - m_N^2 + i\epsilon\right] \times \left[(P/2-k)^2 - m_N^2 + i\epsilon\right]. \quad (2)$$

Considering the equation (1) in the center of mass of system of two particles $P=(\sqrt{s},\mathbf{0})$, it is possible to separate the angular dependence and perform the partial-wave decomposition:

$$T_{LL'}(p_0,|\mathbf{p}|,p'_0,|\mathbf{p}'|;s) = V_{LL'}(p_0,|\mathbf{p}|,p'_0,|\mathbf{p}'|;s) + \frac{i}{4\pi^3}\int dk_0|\mathbf{k}|^2 d|\mathbf{k}| \sum_{L''} V_{LL'}(p_0,|\mathbf{p}|,k_0,|\mathbf{k}|;s) \times G(k_0,|\mathbf{k}|;s)T_{L''L'}(k_0,|\mathbf{k}|,p'_0,|\mathbf{p}'|;s). \quad (3)$$

In the present paper, to solve equation we use the kernel of the NN-interaction in the separable form (rank one):

$$V_{LL'}(p_0,|\mathbf{p}|,p'_0,|\mathbf{p}'|;s) = \lambda g^{(L)}(p_0,|\mathbf{p}|)g^{(L')}(p'_0,|\mathbf{p}'|). \quad (4)$$

Substituting the kernel of the NN-interaction in form eq. (4) into the eq. (3), the two-particle t matrix also have a separable form:

$$T_{LL'}(p_0,|\mathbf{p}|,p'_0,|\mathbf{p}'|;s) = \tau(s)g^{(L)}(p_0,|\mathbf{p}|)g^{(L')}(p'_0,|\mathbf{p}'|), \quad (5)$$

where function τ is

$$\tau(s) = 1/(\lambda^{-1}+h(s)) \quad (6)$$

and

$$h(s) = \sum_L h_L(s) = -\frac{i}{4\pi^3}\int dk_0 \int |\mathbf{k}|^2 d|\mathbf{k}| \times \sum_L [g^{[L]}(k_0,|\mathbf{k}|)]^2 S(k_0,|\mathbf{k}|;s). \quad (7)$$

The relativistic generalization of the Yamaguchi-type functions [9,10] is used as form factors $g^{(L)}(p_0,|\mathbf{p}|)$ of NN kernel

$$g^{[S]}(p_0,|\mathbf{p}|) = \frac{1}{p_0^2 - |\mathbf{p}|^2 - \beta_0^2 + i0}, \quad (8)$$

$$g^{[P]}(p_0,|\mathbf{p}|) = \frac{\sqrt{|-p_0^2+|\mathbf{p}|^2|}}{(p_0^2-|\mathbf{p}|^2-\beta_1^2+i0)^2}, \quad (9)$$

$$g^{[D]}(p_0,|\mathbf{p}|) = \frac{C_2(p_0^2-|\mathbf{p}|^2)}{(p_0^2-|\mathbf{p}|^2-\beta_2^2+i0)^2}, \quad (10)$$

where λ, β_0, β_1, β_2 and C_2 are the parameters of the model, which are chosen to describe the two-nucleons observables—the length and phase of the scattering, the effective radius, and in the case when there is a bound state—deuteron ($^3S_1-^3D_1$-state),—binding energy. Numerical values of parameters λ and β can be found in [11].

3. THREE PARTICLES CASE

The system of three relativistic particles can be described by using the Faddeev equations in the Bethe–Salpeter formalism:

$$\begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} - \begin{bmatrix} 0 & T_1G_1 & T_1G_1 \\ T_2G_2 & 0 & T_2G_2 \\ T_3G_3 & T_3G_3 & 0 \end{bmatrix} \begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix}, \quad (11)$$

where full t matrix $T=\sum_{i=1}^3 T^{(i)}$, G_i—two-particle Green's function of particles j and n ((ijn) obeys cyclic permutation):

$$G_i(k_j,k_n) = 1/(k_j^2 - m_N^2 + i\epsilon)/(k_n^2 - m_N^2 + i\epsilon), \quad (12)$$

T_i—two-particle t matrix. For a system of particles with the same masses, Jacobi variables can be introduced:

$$p_i = \frac{1}{2}(k_j - k_n), \quad q_i = \frac{1}{3}K - k_i, \quad K = k_1+k_2+k_3. \quad (13)$$

On the basis of expression (13) the equation (11) can be rewritten in the following way:

$$\begin{aligned} &T^{(i)}(p_i, q_i; p'_i, q'_i; s) \\ &= (2\pi)^4 \delta^{(4)}(q_i - q'_i) T_i(p_i; p'_i; s) \\ &- i \int \frac{dp''_i}{(2\pi)^4} T_i(p_i; p''_i; s) G_i(k''_j, k''_n) \\ &\times \left[T^{(j)}(p''_j, q''_i; p'_i, q'_i; s) + T^{(n)}(p''_i, q''_i; p'_i, q'_i; s) \right]. \end{aligned} \quad (14)$$

We introduce the amplitude $\Psi^{(i)}(p_i, q_i; s)$ for a bound three-particle state:

$$\begin{aligned} &\Psi^{(i)}(p_i, q_i; s) \\ &= \langle p_i, q_i | T^{(i)} | M_B \rangle \equiv \Psi_{LM}(p, q; s), \end{aligned} \quad (15)$$

where $M_B = \sqrt{s} = 3m_N - E_B$—mass of bound state (triton), $s = K^2$—square of the total momentum.

To separate the angular integration and to perform the partial-wave decomposition it is need to take into account, that solution for the two-particle t matrix is found in the system of the center of mass of two nucleons while the solution for the three-particle amplitude—in the center of mass system of three nucleons. Since the radial functions $g^{[L]}(q_0, |\mathbf{q}|)$ depend on the square of the relative 4-momentum the Lorentz transformation must be carried out only for arguments of spherical harmonics. In this paper we assume, that the components of the relative 4-vectors in the two systems coincide i.e we omit the effects of the Lorentz transformation. In this case, the dependence of the three-particle amplitudes on the two 4-vectors p and q can be separated.

We present the total orbital angular momentum of a triton in the following form: $L = l + \lambda$, where l—internal orbital angular momentum of a two-particle subsystem and λ—orbital angular momentum of the third particle relative to the two-particle subsystem.

In order to distinguish the explicit dependence of the amplitude on the angular momentum, we will present it in the following form:

$$\begin{aligned} &\Psi_{LM}(p, q; s) \\ &= \sum_{a\lambda} \Psi^{(a)}_{\lambda L}(p_0, |\mathbf{p}|, q_0, |\mathbf{q}|; s) \mathcal{Y}^{(a)}_{\lambda LM}(\hat{\mathbf{p}}, \hat{\mathbf{q}}), \\ &\mathcal{Y}^{(a)}_{\lambda LM}(\hat{\mathbf{p}}, \hat{\mathbf{q}}) = \sum_{m\mu} C^{LM}_{lm\lambda\mu} Y_{lm}(\hat{\mathbf{p}}) Y_{\lambda\mu}(\hat{\mathbf{q}}), \end{aligned} \quad (16)$$

where two-nucleon states $a \equiv {}^{2s+1}l_j$ are characterized by s is spin, l is angular and j—total angular momentum. In the equation (16) the notetion $\hat{\mathbf{a}} \equiv \Omega_{\mathbf{a}}$ for angular variables of 3-vector $\mathbf{a}$ is introduced, C are the Clebsch-Gordan coefficients, and Y are the spherical functions.

Using the result of the previous section for the two-particle t matrix (5) and partial-wave decomposition the amplitude $\Psi^{(a)}_{\lambda L}$ can be presented in a separable form:

$$\begin{aligned} &\Psi^{(a)}_{\lambda L}(p_0, |\mathbf{p}|, q_0, |\mathbf{q}|; s) = g^{(a)}(p_0, |\mathbf{p}|) \\ &\times \tau^{(a)} \left[\left(\frac{2}{3}\sqrt{s} + q_0 \right)^2 - \mathbf{q}^2 \right] \Phi^{(a)}_{\lambda L}(q_0, |\mathbf{q}|; s). \end{aligned} \quad (17)$$

The functions $\Phi^{(a)}_{\lambda L}$ satisfy the following system of integral equations:

$$\begin{aligned} &\Phi^{(a)}_{\lambda L}(q_0, |\mathbf{q}|; s) \\ &= \frac{i}{4\pi^3} \sum_{a'\lambda'} \int_{-\infty}^{\infty} dq'_0 \int_0^{\infty} \mathbf{q}'^2 d|\mathbf{q}'| \\ &\times Z^{(aa')}_{\lambda\lambda'}(q_0, q; q'_0, |\mathbf{q}'|; s) \\ &\times \frac{\tau^{(a')} \left[\left(\frac{2}{3}\sqrt{s} + q'_0 \right)^2 - \mathbf{q}'^2 \right]}{\left(\frac{1}{3}\sqrt{s} - q'_0 \right)^2 - \mathbf{q}'^2 - m^2 + i\epsilon} \Phi^{(a')}_{\lambda' L}(q'_0, |\mathbf{q}'|; s), \end{aligned} \quad (18)$$

with effective kernels

$$\begin{aligned} Z^{(aa')}_{\lambda\lambda'}(q_0, |\mathbf{q}|; q'_0, |\mathbf{q}'|; s) &= C_{(aa')} \int d\cos\vartheta_{\mathbf{q}\mathbf{q}'} K^{(aa')}_{\lambda\lambda' L}(|\mathbf{q}|, |\mathbf{q}'|, \cos\vartheta_{\mathbf{q}\mathbf{q}'}) \\ &\times \frac{g^{(a)}(-q_0/2 - q'_0, |\mathbf{q}/2 + \mathbf{q}'|) g^{(a')}(q_0 + q'_0/2, |\mathbf{q} + \mathbf{q}'/2|)}{(\frac{1}{3}\sqrt{s} + q_0 + q'_0)^2 - (\mathbf{q} + \mathbf{q}')^2 - m_N^2 + i\epsilon}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} K^{(aa')}_{\lambda\lambda' L}(|\mathbf{q}|, |\mathbf{q}'|, \cos\vartheta_{\mathbf{q}\mathbf{q}'}) &= (4\pi)^{3/2} \frac{\sqrt{2\lambda + 1}}{2L + 1} \\ \times (-1)^{l'} \sum_{mm'} C^{Lm}_{lm\lambda 0} C^{Lm}_{l'm'\lambda' m-m'} Y^*_{lm}(\vartheta, 0) &Y_{l'm'}(\vartheta', 0) Y_{\lambda' m-m'}(\vartheta_{\mathbf{q}\mathbf{q}'}, 0) \end{aligned} \quad (20)$$

and

$$\cos\vartheta = \left(\frac{|\mathbf{q}|}{2} + |\mathbf{q}'|\cos\vartheta_{\mathbf{q}\mathbf{q}'}\right) \Big/ \left|\frac{\mathbf{q}}{2} + \mathbf{q}'\right|,$$

$$\cos\vartheta' = \left(|\mathbf{q}| + \frac{|\mathbf{q}'|}{2}\cos\vartheta_{\mathbf{q}\mathbf{q}'}\right) \Big/ \left|\mathbf{q} + \frac{\mathbf{q}'}{2}\right|.$$

The details of the calculation of the function K can be found in [12].

Since we are considering the ground state of a three-nucleon system $L = 0$ and correspondingly $l = \lambda, l' = \lambda'$, and the function K can be rewritten in the following form:

$$K^{(aa')}_{ll'0} = \sqrt{(4\pi)^3} Y^*_{l0}(\vartheta, 0) A_{l'}(\vartheta', \vartheta_{\mathbf{q}\mathbf{q}'}),$$

$$A_{l'}(\vartheta', \vartheta_{\mathbf{q}\mathbf{q}'}) = \sum_{m'} C^{00}_{lm'l'-m'} Y_{l'm'}(\vartheta', 0) \times Y_{l'-m'}(\vartheta_{\mathbf{q}\mathbf{q}'}, 0),$$

where l, l' correspond to the orbital moments of the partial states $[a, a']$.

To take into account of spin-isospin structure of the equation, kernel can be expressed in terms of matrix of recoupling coefficients from one partial-wave state to another $[(a) = {}^1S_0, {}^3S_1, {}^3D_1, {}^3P_0, {}^1P_1, {}^3P_1]$, which have the following form:

$$C_{(aa')} = \frac{1}{4}\begin{pmatrix} 1 & -3 & -3 & \sqrt{3} & -\sqrt{3} & \sqrt{3} \\ -3 & 1 & 1 & \sqrt{3} & -\sqrt{3} & \sqrt{3} \\ -3 & 1 & 1 & \sqrt{3} & -\sqrt{3} & \sqrt{3} \\ \sqrt{3} & \sqrt{3} & \sqrt{3} & -1 & -3 & -1 \\ -\sqrt{3} & -\sqrt{3} & -\sqrt{3} & -3 & -1 & -3 \\ \sqrt{3} & \sqrt{3} & \sqrt{3} & -1 & -3 & -1 \end{pmatrix}. \tag{21}$$

The system of integral equations (18)–(20) has singularities on q_0, however, in the case of bound three-particles system ($\sqrt{s} < 3m_N$) all these singularities do not cross the path of integration over q_0 and do not affect the implementation of the procedure of Wick rotation $q_0 \to iq_4$.

Table 1. The values of the binding energy of a triton (MeV)

p_D	${}^1S_0 - {}^3S_1$	${}^3D_1$	${}^3P_0$	${}^1P_1$	${}^3P_1$
4	9.221	9.294	9.314	9.287	9.271
5	8.819	8.909	8.928	8.903	8.889
6	8.442	8.545	8.562	8.540	8.527
Experiment			8.48		

System (18)–(20) after Wick rotation can be solved using standard methods for solving integral equations. One of them is discussed in the next section.

4. NUMERICAL CALCULATIONS AND RESULTS

In this paper, a homogeneous system of twelve integral equation with a parameter, which is the binding energy of the triton, are solved using the iteration method.

To determine the binding energy, the following condition is used (more details in [13]):

$$\lim_{n\to\infty} \frac{\Phi_n(s)}{\Phi_{n-1}(s)}\Big|_{s=M_B^2} = 1, \tag{22}$$

where n—iteration number.

The procedure for solving the system of integral equations (18)–(20) by the iteration method has good convergence. In numerical calculations of the binding energy of triton and the amplitudes of its states for the Yamaguchi potential, the ratio of the previous iteration to the next did not change with the growth of the iteration number up to the sixth decimal place starting with the 20th iteration.

For the numerical calculation of the integrals, the Gauss method on two-dimensional grid of nodes by dimension $N_1 \times N_2$ is used with mapping $q_4 = (1 + x)/(1 - x), |\mathbf{q}| = (1 + y)/(1 - y)$. The influence of the number of nodes on the convergence of the result of numerical integration is investigated. For integration on $|\mathbf{q}|$ it is enough $N_2 = 15$ nodes. With further increase quantity of nodes the numerical value of the integral did not change any more. For integration on q_4 it was not enough such number of nodes. For the study of convergence we increased number of nodes to $N_1 = 96$. With further increasing number of nodes the numerical value of the integral changed only in the fourth decimal digit. This accuracy is sufficient and allows us to take into account the contribution of various states to the binding energy.

Table 1 presents the calculated values of the binding energy for different probabilities of the D-state ($p_D = 4, 5, 6$).

The above results show that the main contribution to the binding energy of the triton is from S-states. Contribution of D-state is positive and varies from 0.8 to 1.2 % depending on the probability of D-state in deuteron (p_D = 4-6 %). Contributions of P-states have different signs and partially compensate each other, and their total contribution is -0.2%. So total contribution two-particle P- and D-partial states with a total angular momenta $j = 0, 1$ into the

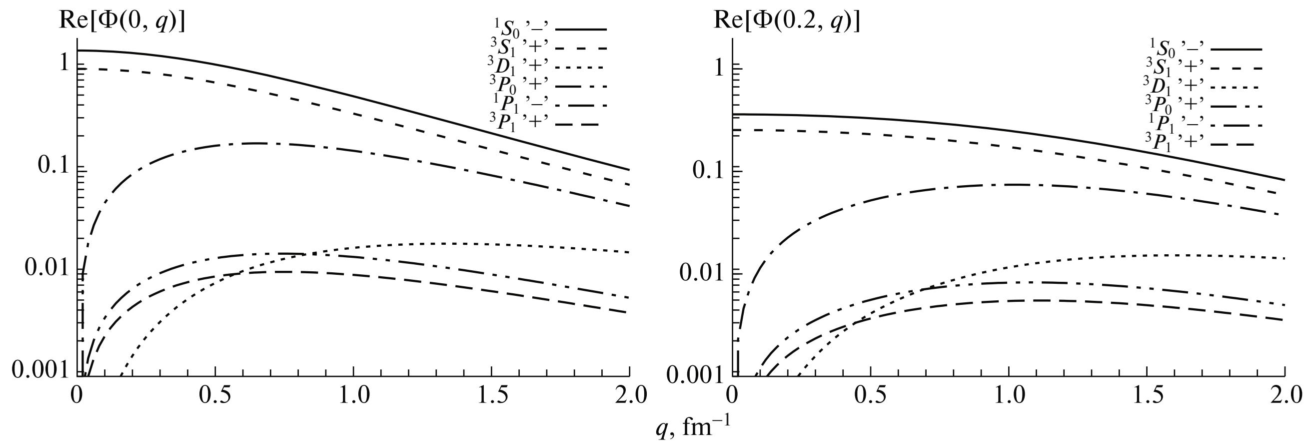

Fig. 1. The real part of the amplitudes for all states considered in the work as a function of $|\mathbf{q}|$ with the value $q_4 = 0$ and $q_4 = 0.2\,\mathrm{Fm}^{-1}$.

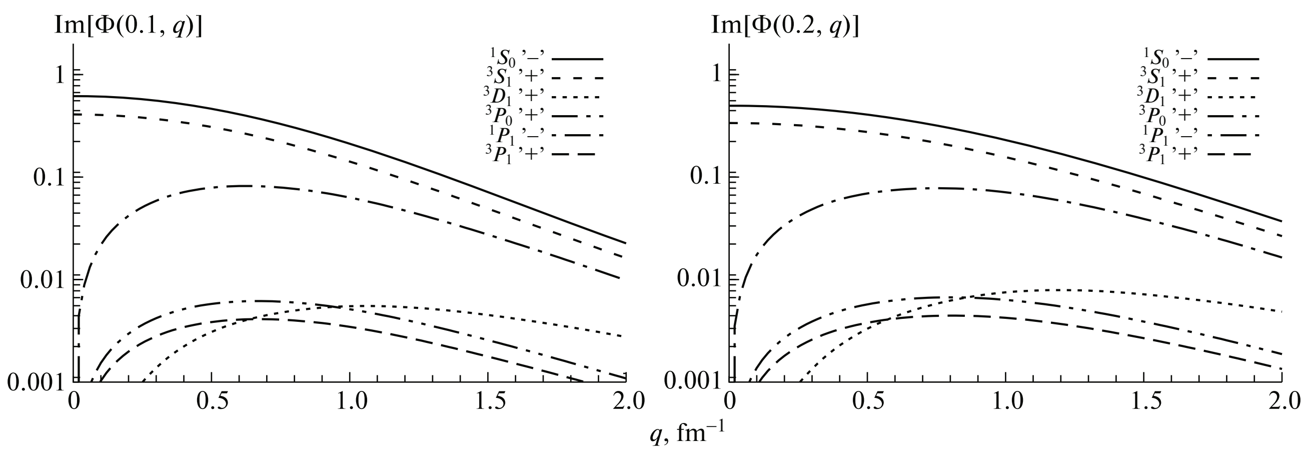

Fig. 2. The imaginary part of the amplitudes for all states considered in the work as a function of $|\mathbf{q}|$ with the value $q_4 = 0.1$ and $q_4 = 0.2\,\mathrm{Fm}^{-1}$.

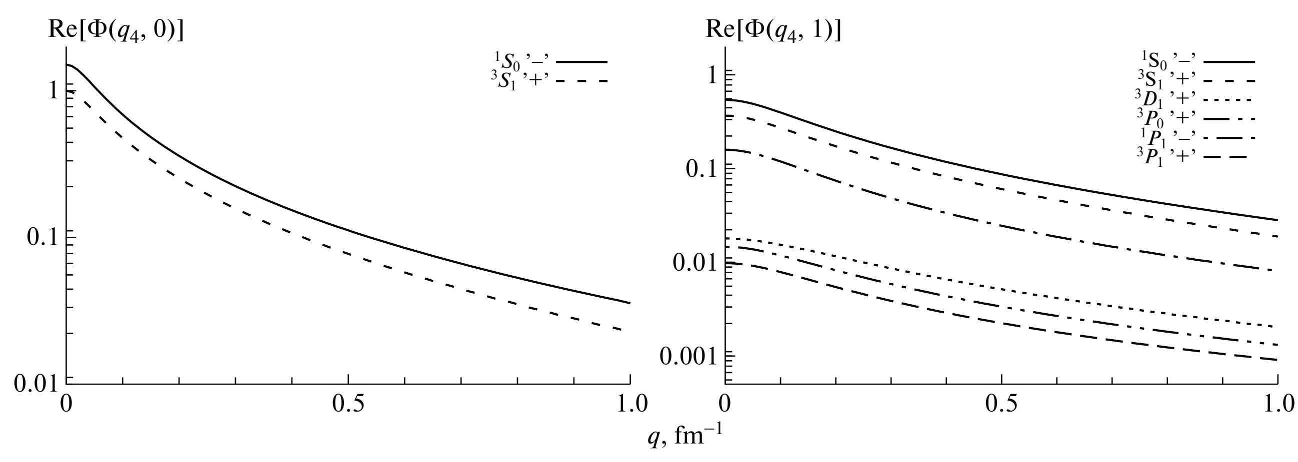

Fig. 3. The real part of the amplitudes for all states considered in the work as a function of q_4 with the value $|\mathbf{q}| = 0$ and $|\mathbf{q}| = 1\,\mathrm{Fm}^{-1}$.

binding energy of a triton is from 0.5 to 1 %. Comparison of nonrelativistic and relativistic calculations of binding energy can be found in [5]. The paper shows that relativistic calculation of binding energy in the case of accounting only S-states larger then nonrelativistic at 0.44 MeV.

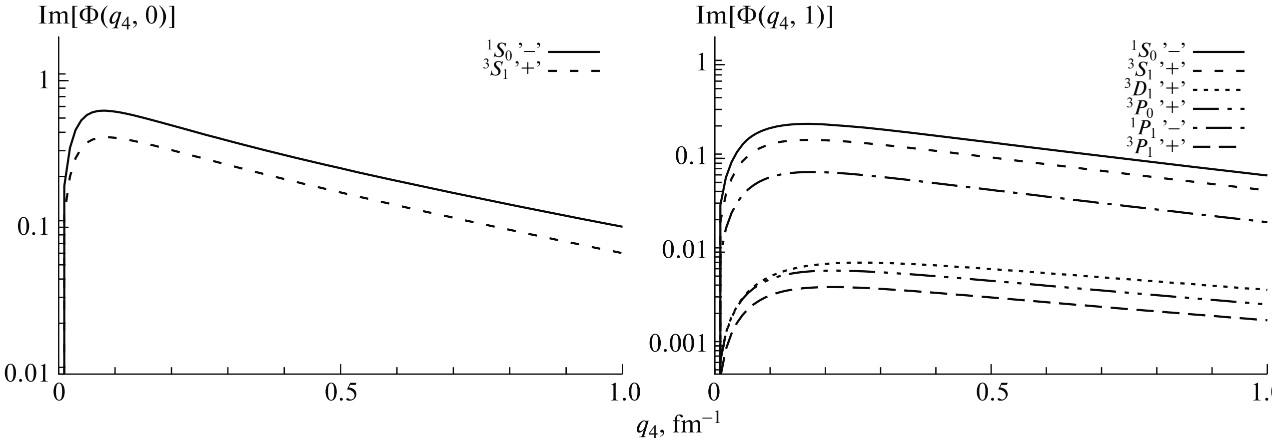

Fig. 4. The imaginary part of the amplitudes for all states considered in the work as a function of q_4 with the value $|\mathbf{q}| = 0$ and $|\mathbf{q}| = 1\ \mathrm{Fm}^{-1}$.

In Fig. 1–4 graphs of real and imaginary parts of partial amplitudes on variables $|\mathbf{q}|$ (at fixed values of q_4) and q_4 (at fixed values of $|\mathbf{q}|$) are presented. As can be seen from the graphs, amplitudes of S-states dominate while other states give a nonzero contribution. However we believe that interference contributions of S-, P- and D-states in form factors of the three-particle system must be taken into account in calculations. Obtained amplitudes will be used to calculate the electromagnetic form factors of the triton using the approximations described in articles [5, 6].

5. CONCLUSION

The solution of the relativistic Bethe–Salpeter–Faddeev equation for a three-nucleon system (triton) are considered in article. A relativistic generalization of the partial-wave decomposition procedure is carried out, which is spread to the nonzero orbital angular momenta of pair of the interacting nucleons. The case of S -, P - and D-partial states of the two-particle subsystems are considered. Using the partial-wave decomposition and potential of NN-interactions in a separable form led to a system of integral equations for the amplitudes of states with different orbital angular moments of particles in the nuclei. The numerical solution of this system using the iteration method allows find the binding energy of a triton and amplitudes of the ${}^1S_0$, ${}^3S_1$, ${}^3D_1$, ${}^3P_0$, ${}^1P_1$, ${}^3P_1$ states as functions of two variables.

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REFERENCES

1. E. van Faassen and J. A. Tjon, Phys. Rev. C **33**, 2105 (1986).
2. G. Rupp, L. Streit and J. A. Tjon, Phys. Rev. C **31**, 2285 (1985).
3. A. Stadler, F. Gross and M. Frank, Phys. Rev. C **56**, 2396 (1997).
4. E. Ydrefors, J. H. Alvarenga Nogueira, V. Gigante, T. Frederico and V. A. Karmanov, Phys. Lett. B **770**, 131 (2017).
5. G. Rupp and J. A. Tjon, Phys. Rev. C **37**, 1729 (1988).
6. G. Rupp and J. A. Tjon, Phys. Rev. C **45**, 2133 (1992).
7. S. G. Bondarenko, V. V. Burov and S. A. Yurev, EPJ Web Conf. **108**, 02015 (2016).
8. S. Bondarenko, V. Burov and S. Yurev, EPJ Web Conf. **138**, 06003 (2017).
9. Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).
10. Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. **95**, 1635 (1954).
11. S. G. Bondarenko, V. V. Burov and S. A. Yurev, Phys. Part. Nucl. Lett. **15**, 442 (2018).
12. A. Ahmadzadeh and J. A. Tjon, Phys. Rev. **139**, B1085 (1965).
13. R. A. Malfliet and J. A. Tjon, Nucl. Phys. A **127**, 161 (1969).