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Theory

Behavior of the Elastic-Scattering Cross Section $\sigma(E)$ in the Vicinity of Two Overlapping Levels with Equal Resonance Energies

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Abstract—The elastic-scattering process proceeding through two resonance levels that have the same spin j and equal resonance energies, $(E_1 = E_2)$, but different widths $(\Gamma_1 \neq \Gamma_2)$ is considered. It is shown that the energy dependence of the total scattering cross section has two equal maxima at the points $E_1 \pm (1/2)\sqrt{\Gamma_1\Gamma_2}$, the cross-section value at the maxima being $4\pi\,(2j+1)\,\lambda\!\!\!^{-2}$, where $\lambda\!\!\!^{-}$ is the wavelength of the incident particle in the c.m. frame, and that, at the energy E_1, the cross section vanishes, $\sigma(E_1) = 0$. The cross section is symmetric with respect to the point E_1.

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Via observing the energy dependence of the cross section for a nuclear reaction, it is possible to unearth the role of long-lived states (resonance levels) in the reaction mechanism. At low excitation energies, we have the well-known Breit–Wigner formula induced by a level of resonance energy E_0 and width $\Gamma_0(E)$, where E is the kinetic energy of the relative motion of colliding particles in the c.m. frame [1–5]. As the excitation energy grows, in which case the level spacing D decreases, while the widths $\Gamma_0(E)$ increase, the levels begin overlapping one another. In the case where $\Gamma \gg D$, the cross sections develop Ericson fluctuations in their energy dependence [6].

The energy dependence of reaction cross sections that was determined by two overlapping levels was explored in a number of studies [7–9]. In particular, the behavior of the cross section for the charge-exchange reaction $\bar{p}p \to n\bar{n}$ was studied in [8] with allowance for the interference between two overlapping resonances (1932 and 2020) and the interference with the background.

The present study is devoted to considering the single-channel reaction (elastic scattering) [7, 9] for the case where the level spacing D is so small that it is legitimate to set it to zero ($D = 0$)—that is, the resonance energies of the levels are equal, $E_1 = E_2$—but where $\Gamma_1 \neq \Gamma_2$. The S-matrix formalism is the most adequate means for describing elastic scattering. Within this formalism, there are no theoretical arguments that would forbid the S matrix to have such a pair of poles. Therefore, there arises the question of how this pair manifests itself in the energy dependence of the cross section for elastic scattering.

In the case of single-channel processes (elastic scattering) of spinless particles, the partial-wave S matrix has the form [1, 4, 5]

$$S_l(k) = e^{i\pi l}\frac{f_l(k)}{f_l(-k)}, \qquad (1)$$

where $f_l(k)$ and $f_l(-k)$ is the Jost function and k is the relative momentum of colliding particles in the c.m. frame. The partial-wave S matrix $S_l(k)$ (1) satisfies the unitarity condition

$$|S_l|^2 = 1. \qquad (2)$$

As was shown in [1, 4, 5], a pole of $S_l(k)$ in the lower half-plane of the complex plane of k at a point $k_j = q_j - i\kappa_j$ (where $\kappa_j > 0$) corresponds to a resonance level (long-lived state). This pole induces yet another pole at the point $-k_j^*$ and two zeros at the points $-k_j$ and k_j^* (requirement of the unitary condition). This arrangement of the singularities in the complex plane of k leads to the appearance of a pole of $S_l(E)$ on the second sheet of the complex plane of E at the point $E_j - (i/2)\Gamma_j$ and a zero on the first sheet at the point $E_j + (i/2)\Gamma_j$, where $2mE_j = q_j^2 + \kappa_j^2$, $\Gamma_j = 2\kappa_j\sqrt{E/m}$, and m is the reduced mass of colliding particles [10].

In the case of two resonance levels, the only possible expression for the partial-wave S matrix $S_l(E)$ such that it satisfies the unitarity condition in (2) has

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the form [9]

$$S_l = e^{2i\delta_l(E)} \prod_{j=1}^{2} \frac{E - E_j - \frac{i}{2}\Gamma_j}{E - E_j + \frac{i}{2}\Gamma_j}, \quad (3)$$

where $\delta_l(E)$ is the phase shift that takes into account nonresonance (so-called potential) scattering. S-matrix theory does not impose any constraint on the parameters E_j and $\Gamma_j(E)$, which are the physical values of the resonance energy of the levels and their levels or on the parameters q_j and κ_j (where $j = 1, 2$) [9].

Expression (3) can be represented in the form of a sum over pole terms [9, 11]; that is,

$$S_l(E) = e^{2i\delta_l(E)} \left(1 - i \sum_{k=1}^{2} \frac{\Gamma_k \gamma_{kj}}{E - z_k}\right), \quad (4)$$

where the complex-valued quantity z_k characterizes the kth resonance level,

$$z_k = E_k - \frac{i}{2}\Gamma_k(E);$$

the quantity γ_{kj} has the form

$$\gamma_{kj} = \frac{z_k - z_j^*}{z_k - z_j};$$

and the index j takes the values of 1 and 2, but $j \neq k$. From these relations, it follows that $|\gamma_{kj}| = |\gamma_{jk}|$. Therefore, the ratio of the residues at the two poles is given by

$$\frac{\Gamma_k \gamma_{kj}}{\Gamma_j \gamma_{jk}} = \frac{\Gamma_k}{\Gamma_j} e^{-2i\Phi}, \quad (5)$$

where

$$\Phi = \arctan \frac{\Gamma_k + \Gamma_j}{2(E_k - E_j)}$$

We can then state that, if the levels do not overlap each other—that is, $\Gamma \ll |E_1 - E_2|$—then $\Phi \approx (\Gamma_k + \Gamma_j)/2(E_k - E_j)$ and tends to zero, so that the ratio of the residues in (5) tends to Γ_k/Γ_j, but, in the case of overlapping levels—Γ_2, $\Gamma_1 \gg |E_1 - E_2|$—$\Phi \approx (\pi/2) - 2(E_k - E_j)/(\Gamma_k + \Gamma_j)$ and tends to $\pi/2$, so that the ratio of the residues tends to $-(\Gamma_k/\Gamma_j)$.

In the following, we consider the case where $E_1 = E_2$ (in the complex plane of k, this condition is equivalent to the equality $q_1^2 + \kappa_1^2 = q_2^2 + \kappa_2^2$), but where $\Gamma_1 \neq \Gamma_2$ (that is, $\kappa_1 \neq \kappa_2$). The existence of such pairs of levels is allowed in S-matrix theory. It is unlikely that such a pair of poles exists within potential theory. It may arise owing to a nonpotential interaction of particles. The present study is aimed at an analysis of the energy dependence that the elastic-scattering cross sections $\sigma(E)$ develops owing to the presence of such pairs of levels. In the following, we assume that $\Gamma_1 > \Gamma_2$.

In the case of $E_1 = E_2$, the partial-wave S matrix $S_l(E)$ in (3) can be represented in the form

$$S_l(E) = e^{2i\delta_l(E)} \left(1 - \frac{i(\Gamma_1 + \Gamma_2)(E - E_1)}{(E - z_1)(E - z_2)}\right). \quad (6)$$

This expression satisfies the unitarity condition in (2).

By employing formulas of scattering theory [3], one can obtain the total cross section for elastic scattering in the form

$$\sigma(E) = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l(E) - \frac{(2j+1)\pi}{k^2} (E - E_1)(\Gamma_1 + \Gamma_2) \quad (7)$$
$$\times \frac{\left\{2\left[(E - E_1)^2 - (1/4)\Gamma_1\Gamma_2\right] \sin 2\delta_j(E) - (E - E_1)(\Gamma_1 + \Gamma_2) \cos 2\delta_j(E)\right\}}{\left[(E - E_1)^2 + (1/4)\Gamma_1^2\right]\left[(E - E_1)^2 + (1/4)\Gamma_2^2\right]}$$

In expression (7), the first term describes nonresonance (potential) scattering, which is completely determined by the phase shifts $\delta_l(E)$, while the second term takes into account purely resonance scattering and the interference between resonance and nonresonance scattering. From expression (7), it follows that, at $E = E_1$, the cross section is completely determined by nonresonance scattering. This feature peculiar to the cross section $\sigma(E)$ in (7) is due exclusively to the unitarity condition in (2) but has nothing to do with the origin of the resonance [see Eq. (6)].

In the following, we consider purely resonance scattering; that is, we set all phase shifts $\delta_l(E)$ to zero ($\delta_l(E) = 0$). The resonance cross section then assumes the form

$$\sigma_{\text{res}}(E) = (2j+1)\pi\lambdabar^2 \quad (8)$$

$$\times \frac{(E-E_1)^2\,(\Gamma_1+\Gamma_2)^2}{\left[(E-E_1)^2+(1/4)\,\Gamma_1^2\right]\left[(E-E_1)^2+(1/4)\,\Gamma_2^2\right]},$$

where $\lambda\!\!\!^{-}$ is the wavelength of the incident particle in the c.m. frame. The cross section $\sigma_{\rm res}(E)$ is symmetric with respect to the point E_1; therefore, we introduce the dimensionless variable $x = 2(E-E_1)/\Gamma_1$ and the dimensionless cross section $\bar{\sigma}(x) = \sigma_{\rm res}(E)/(2j+1)\,4\pi\lambda\!\!\!^{-2}$. Expression (8) then assumes the form

$$\bar{\sigma}(x) = \frac{(1+a)^2 x^2}{(x^2+1)\,(x^2+a^2)}, \qquad (9)$$

where $a = \Gamma_2/\Gamma_1$. Since we assumed that $\Gamma_1 > \Gamma_2$, the parameter a changes within the range of $0 < a < 1$. The cross section $\bar{\sigma}(x)$ has the following characteristic properties: first, it is symmetric with respect to the point $x = 0$ ($E = E_1$); second, it exhibits two maxima at the points $x_\pm = \pm\sqrt{a}$ $\left(E_\pm = E_1 \pm (1/2)\sqrt{\Gamma_1\Gamma_2}\right)$; and, third, the cross section $\bar{\sigma}(x)$ vanishes at the point $x = 0$ ($\bar{\sigma}(x=0) = 0$). All of these special features in the behavior of the cross section $\bar{\sigma}(x)$ stem from the unitarity condition in (2) and are independent of resonance nature. The maximum cross section at the peaks is $\bar{\sigma}(\pm\sqrt{a}) = 1$; that is, $\sigma_{\rm res} = 4\pi(2j+1)\lambda\!\!\!^{-2}$. This value is coincident with the maximum cross section in the case of an isolated resonance [3, 5]. However, the shapes of the curves in the vicinity of the peaks are different. In the case of an isolated resonance, the curve is symmetric with respect to the resonance energy. In the case considered here, the curves are asymmetric with respect to the energy values of $E_\pm$ corresponding to the maxima. In order to characterize this asymmetry, we introduce the quantity $\Gamma_{\rm fwhs}$ defined as the width of the peak of the cross section $\sigma_{\rm res}(E)$ in the vicinity of the points $E_\pm$ at the height equal to half the maximum cross section. These widths satisfy the equation $\bar{\sigma}(x) = 1/2$, whose roots have the form

$$x_{1,2} = \pm\sqrt{a + \frac{1+a}{2}\left(1+a+\sqrt{\Delta}\right)}, \qquad (10)$$
$$x_{3,4} = \pm\sqrt{a + \frac{1+a}{2}\left(1+a-\sqrt{\Delta}\right)},$$

where we have introduced the notation $\Delta \equiv (1+a)^2 + 4a$. The width $\Gamma_{\rm fwhs}$ of the peak in the vicinity of the point $x_+ = \sqrt{a}$ is then given by

$$\Gamma_{\rm fwhs} = E^{(1)} - E^{(3)} \qquad (11)$$
$$= \frac{\Gamma_1}{2}(x_1 - x_3) = \frac{\Gamma_1}{2}(1+a),$$

where $E^{(i)} = E_1 + (\Gamma_1/2)x_i$ ($i = 1, ..., 4$). In the case of an isolated resonance, the quantity $\Gamma_{\rm fwhs}$ is referred to as the resonance width; it is divided by the resonance energy into two equal parts (symmetric curve). In the case being considered, the points $E_\pm = E_1 \pm (1/2)\sqrt{\Gamma_1\Gamma_2}$ at which the cross section peaks divide $\Gamma_{\rm fwhs}$ into two unequal parts; that is,

$$\Gamma^{(1)}_{\rm fwhs} = E^{(1)} - E_+ = \frac{\Gamma_1}{2}(x_1 - \sqrt{a}), \qquad (12)$$
$$\Gamma^{(2)}_{\rm fwhs} = E_+ - E^{(3)} = \frac{\Gamma_1}{2}(\sqrt{a} - x_3).$$

For the difference $\Gamma^{(1)}_{\rm fwhs} - \Gamma^{(2)}_{\rm fwhs}$, which characterizes the asymmetry of the curve with respect to a point of maximum, we then have the relation

$$\Delta\Gamma_{\rm fwhs} = \Gamma^{(1)}_{\rm fwhs} - \Gamma^{(2)}_{\rm fwhs} \qquad (13)$$
$$= \frac{\Gamma_1}{2}\left[\sqrt{(1+a)^2+4a} - 2\sqrt{a}\right].$$

This quantity is maximal at small values of a (that is, $a \to 0$). In the limiting case of $a = 0$ ($\Gamma_2 = 0$), the two roots $x_{3,4}$ of the equation are equal to zero ($x_{3,4} = 0$); accordingly, $\Gamma^{(2)}_{\rm fwhs}$ also vanishes, along with the distance between the maxima, $D_{\max} = \Gamma_1\sqrt{a}$: $\Gamma^{(2)}_{\rm fwhs} = 0$ and $D_{\max} = 0$. This means that the two peaks in the cross sections (8) and (9) merge together into one at the point E_1, whereas expressions (11) and (12) assume the form

$$\Gamma_{\rm fwhs} = \Gamma^{(1)}_{\rm fwhs} = \Gamma_1/2. \qquad (14)$$

Accordingly, expression (8) for the cross section $\sigma_{\rm res}(E)$ goes over in this limiting case to the well-known Breit–Wigner formula [3, 5] with resonance width Γ_1.

The quantity $\Gamma^{(1)}_{\rm fwhs} - \Gamma^{(2)}_{\rm fwhs}$ is minimal for $a \to 1$ and assumes the value of $\Gamma_1(\sqrt{2}-1)$ in the limit of $a = 1$. This asymmetry is the fourth special feature of the cross section under study as a function of energy.

For the ensuing analysis, it is convenient to introduce a dimensional asymmetry, and we denote it by A. Employing Eqs. (11) and (13), we define it as

$$A = \frac{\Delta\Gamma_{\rm fwhs}}{\Gamma_{\rm fwhs}} = \sqrt{1 + \left(\frac{2\sqrt{a}}{1+a}\right)^2} - \frac{2\sqrt{a}}{1+a}. \qquad (15)$$

This quantity is maximal at the point $a = 0$, taking the value of unity ($A = 1$); at the point $a = 1$, the dimensional asymmetry is minimal and has the value of $A = \sqrt{2} - 1$.

Taking into account Eq. (11) and the definition of $D_{\max}$, we arrive at the equation

$$\frac{D_{\max}}{\Gamma_{\rm fwys}} = \frac{2\sqrt{a}}{1+a}, \qquad (16)$$

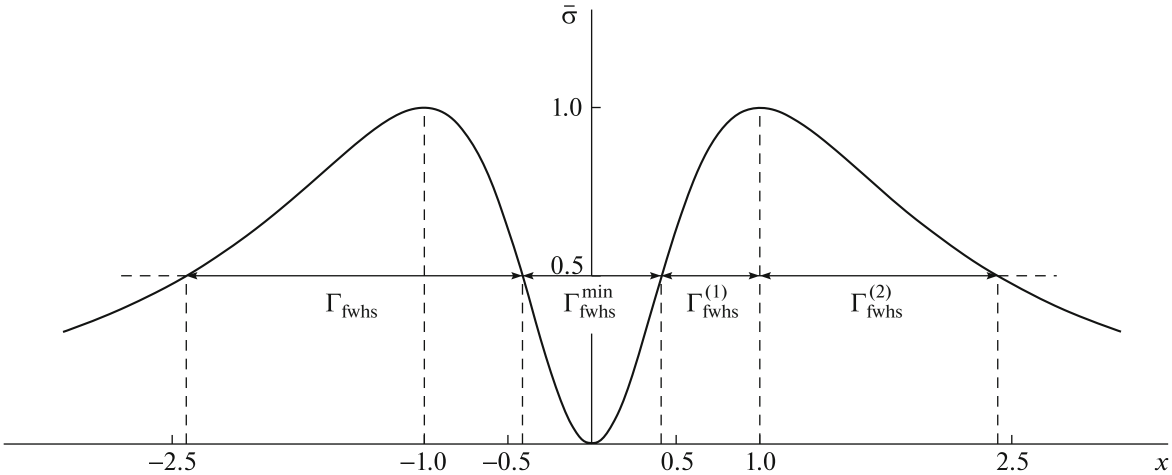

Fig. 1. Cross section $\bar{\sigma}(x)$ as a function of energy at $a = 1$.

which relates a to two experimental observables.

Let us introduce two other experimental observables. The first, $\Gamma_{\rm fwhs}^{(F)}$, is the total width (it involves both maxima and the minimum) of the graph of the scattering cross section as a function of energy at the height equal to half the maximum cross section; that is,

$$\Gamma_{\rm fwhs}^{(F)} = E^{(1)} - E^{(2)} \tag{17}$$
$$= \Gamma_1 \sqrt{a + \frac{1+a}{2}(1 + a + \sqrt{\Delta})}.$$

The second, $\Gamma_{\rm fwhs}^{\rm min}$, is the width of the cross-section minimum at the point E_1 at the height equal to half the maximum cross section; that is,

$$\Gamma_{\rm fwhs}^{\rm min} = E^{(3)} - E^{(4)} \tag{18}$$
$$= \Gamma_1 \sqrt{a + \frac{1+a}{2}(1 + a - \sqrt{\Delta})}.$$

The experimental observables $\Gamma_{\rm fwhs}$, $\Gamma_{\rm fwhs}^{(F)}$, and $\Gamma_{\rm fwhs}^{\rm min}$ satisfy the following trivial relation:

$$\Gamma_{\rm fwhs}^{(F)} = \Gamma_{\rm fwhs}^{\rm min} + 2\Gamma_{\rm fwhs}. \tag{19}$$

Let us consider in more detail the limiting case where $a = 1$ and $\Gamma_1 = \Gamma_2$. It corresponds to the case where the partial-wave S matrix has a second-order pole both in the complex plane of k and in the complex plane of E. In this limiting case, expression (8) for $\sigma_{\rm res}$ reduced to the form

$$\sigma_{\rm res} = 4\pi(2j+1)\lambda\!\!\!^{-2} \frac{\Gamma_1^2 (E - E_1)^2}{\left[(E - E_1)^2 + \frac{1}{4}\Gamma_1^2\right]^2}. \tag{20}$$

From expression (20), it follows that the cross section $\sigma_{\rm res}$ assumes the maximum values of $4\pi(2j+1)\lambda\!\!\!^{-2}$ at the points $E_\pm = E_1 \pm (1/2)\Gamma_1$. In this liming case, the width $\Gamma_{\rm fwhs}$ of the cross-section peaks becomes

$$\Gamma_{\rm fwhs} = \frac{\Gamma_1}{2}\left[\sqrt{3 + 2\sqrt{2}} - \sqrt{3 - 2\sqrt{2}}\right] = \Gamma_1. \tag{21}$$

As was indicated above, the asymmetry of the curve with respect to the points $E_\pm$ is minimal and takes the value of $A_0 = (\sqrt{2} - 1)$. On the contrary, the widths $\Gamma_{\rm fwhs}^{(\rm min)}$ and $\Gamma_{\rm fwhs}^{(F)}$ are maximal in this limiting case, and we have the equalities $\Gamma_{\rm fwhs}^{(\rm min)} = \Gamma_1\sqrt{3 - 2\sqrt{2}}$ and $\Gamma_{\rm fwhs}^{(F)} = \Gamma_1\sqrt{3 + 2\sqrt{2}}$ for them. One can see that Eq. (19), which relates $\Gamma_{\rm fwhs}$, $\Gamma_{\rm fwhs}^{(F)}$, and $\Gamma_{\rm fwhs}^{\rm min}$, holds. Figure 1 shows the scattering cross section $\bar{\sigma}(x)$ as a function of energy at $a = 1$—that is, for the case where the partial-wave S matrix $S_l(E)$ has a second-order pole. This figure also gives the quantities $\Gamma_{\rm fwhs}$, $\Gamma_{\rm fwhs}^{(1)}$, $\Gamma_{\rm fwhs}^{(2)}$, and $\Gamma_{\rm fwhs}^{\rm min}$ introduced in the present study.

It should be noted that the presence of multiple poles in the S matrix was considered earlier in [2, 4, 12]. In particular, the resonance cross section given in [2] for the case where a second-order pole is present is fully coincident with that in expression (20). Also, the scattering cross section as a function of the energy E is given in [2] (Fig. 8.9). However, Fig. 1 of the present article shows the asymmetry of the two-humped curve with respect of the points of maximum, whereas Fig. 8.9 do not exhibit it.

In the present study, it was shown that the appearance of two peaks in the energy dependence of the elastic-scattering cross section $\sigma_{\rm res}(E)$ that are characterized by equal values at the point of maximum may be associated either with the second-order pole of the S matrix or with the presence of two simple poles on the second sheet of the energy surface that correspond to equal resonance energies ($E_1 =$

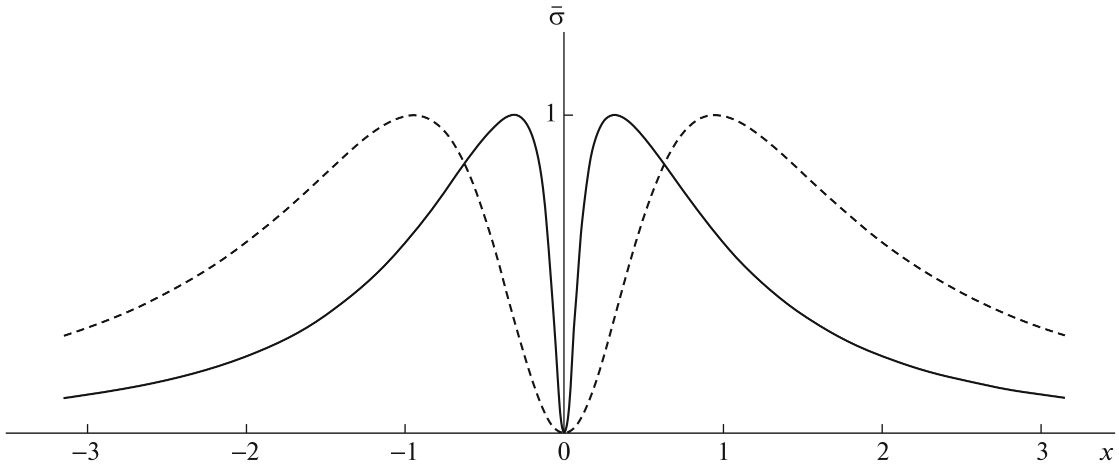

Fig. 2. Cross section $\bar{\sigma}(x)$ as a function of energy at the parameter values of (solid curve) $a = 0.1$ and (dashed curve) $a = 0.9$.

E_2) but different widths ($\Gamma_1 \neq \Gamma_2$). It is possible to discriminate between these two possibilities by studying the asymmetry of the peaks with respect to the point where the cross section has a maximum. In particular, one can determine a by comparing $\Gamma_{\rm fwhs}$ with $D_{\rm max}$ and taking into account Eqs. (16). All of the results of the present study rely on the unitarity condition and are independent of the nature of those long-lived states that are described by resonance levels (poles) in the partial-wave S matrix. It is worth noting that the inclusion of nonresonance scattering [see expression (7)] violates all symmetry properties described above.

Summarizing the foregoing, we will describe the behavior of the curve that represents the cross section as a function of the relative energy E in response to the change in Γ_2 from zero to Γ_1 (that is, in the range of $0 < \Gamma_2 < \Gamma_1$). At $\Gamma_2 = 0$, there is only one level of resonance energy E_1 and width Γ_1. The cross section $\sigma_{\rm res}(E)$ then has one peak with a maximum at the resonance-energy point E_1 and the width Γ_1 (this corresponds to the well-known Breit–Wigner formula). Suppose that the second levels of resonance energy E_1 and width Γ_2 appears and that the value of Γ_2 is very small—that is, $\Gamma_2 \to 0$. The Breit–Wigner peak than splits into two equal halves, and the cross section vanishes at the resonance point—that is, $\sigma_{\rm res}(E_1) = 0$. In this case, the asymmetry of the two emerging peaks is maximal, since $\Gamma^{(2)}_{\rm fwhs} \to 0$, while $\Gamma^{(1)}_{\rm fwhs} \to (1/2)\Gamma_1$. As Γ_2 grows further, the curve representing the cross section as a function of energy develops two peaks in which the cross section reaches the maximum value of $4\pi(2j+1)\lambda\!\!\!^{-2}$ at the points $E_\pm$. In this case, the peak width $\Gamma_{\rm fwhs}$ is defined as $(1/2)(\Gamma_1 + \Gamma_2)$, while the width of the dip with a minimum at the point E_1 is determined by expression (18). As Γ_2 grows, these parameters of the curve increase, while the asymmetry, which is determined by expression (15), decreases. As soon as Γ_2 becomes equal to Γ_1, the two simple poles merge together into a single second-order pole lying on the second sheet of the energy surface. Concurrently, the peak width reaches Γ_1, whereas the dip width becomes $\Gamma_1\sqrt{3 - 2\sqrt{2}}$. The asymmetry is then minimal and takes the value of $A_0 = \sqrt{2} - 1$. As Γ_2 changes from zero to Γ_1, the curve representing the cross section $\sigma_{\rm res}(E)$ as a function of the relative energy E of colliding particles remains symmetric with respect to the resonance energy E_1. At this point, the cross section $\sigma_{\rm res}(E)$ vanishes. All of the aforementioned maximum cross-section values are equal to $4\pi(2j + 1)\lambda\!\!\!^{-2}$; that is, only the shape of the curve changes. Figure 2 illustrates the foregoing. It displays two curves representing the total cross section as a function of energy at two values of the parameter a: (solid curve) $a = 0.1$ and (dashed curve) $a = 0.9$. In the case where the asymmetry A is smaller than A_0, this asymmetry is due to the presence of two overlapping levels in the partial-wave S matrix $S_l(E)$ that have incoincident resonance energies.

Let us now clarify the last statement. Employing either relation (3) or relation (4) and assuming that potential scattering is absent—that is, all of the phase shifts $\delta_l(E)$ are equal to zero—one can obtain the resonance-scattering cross section for the case where the resonance energies are unequal. It has the form

$$\sigma_{\rm res}(E) = \pi(2j+1)\lambda\!\!\!^{-2} \times \frac{(\Gamma_1(E - E_2) + \Gamma_2(E - E_1))^2}{((E - E_1)^2 + \Gamma_1^2/4)((E - E_2)^2 + \Gamma_2^2/4)} \quad (22)$$

Let us introduce the dimensionless parameter $b = 2(E_1 - E_2)/\Gamma_1$ and assume that $E_2 < E_1$. The parameter b than changes from zero at $E_2 = E_1$ to $b_0 =$

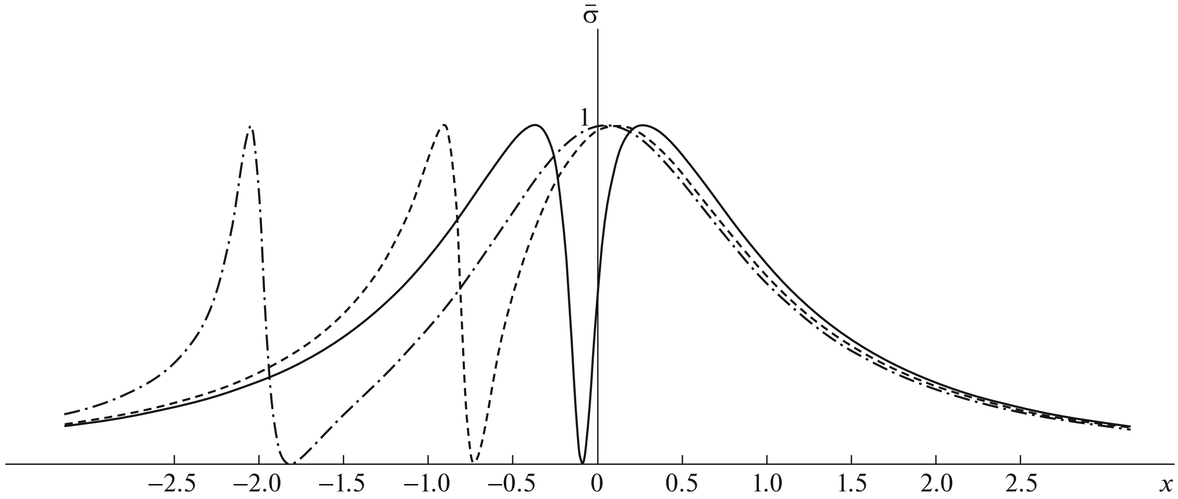

Fig. 3. Cross section $\bar{\sigma}(x)$ as a function of energy at b values of (solid curve) $b = 0.1$, (dashed curve) $b = 1$, and (dash-dotted curve) $b = 2$ for $a = 0.1$.

$2E_1/\Gamma_1$ at $E_2 = 0$ and, accordingly, $\Gamma_2 = 0$. In this limiting case, expression (22) reduces to the well-known Breit—Wigner formula [3, 5]. Using the parameter b, we can recast the cross section in (22) into the form

$$\bar{\sigma}(x) = \frac{(x(a+1)+b)^2}{(x^2+1)((x+b)^2+a^2)}, \qquad (23)$$

where $\bar{\sigma}$, x, and a were defined above. From this equation, it follows that the cross section vanishes at the point $x_0 = -b/(a+1)$ (at $E_1 = E_2$, this point reduces to zero)—that is, at the energy $E^{(0)} = (E_1\Gamma_2 + E_2\Gamma_1)/(\Gamma_1 + \Gamma_2)$. Figure 3 gives three curves. These are the solid curve for $a = 0.1$ and $b = 0.1$, the dashed curve for $a = 0.1$ and $b = 1$ and the dash-dotted curve for $a = 0.1$ and $b = 2$. This figure shows that, as the parameter b grows, the point x_0 and the point where the peak has a maximum move leftward, the asymmetry of the peak decreasing. For $b \to b_0$, $x_0 \to -b_0$, the point at which the right-hand peak has a maximum reaching the point E_1.

Thus, we can state that, in the case of two resonance levels, the unitarity-condition requirement, which couples them, induces an asymmetry for each peak in relation to the symmetric peak of the Breit—Wigner formula. This asymmetry is always present—only its magnitude, which depends on the positions of the poles in the complex plane of E, changes.

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