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# Simulation of Nuclear Reactor Kinetics by the Monte Carlo Method

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**Abstract**—The KIR computer code intended for calculations of nuclear reactor kinetics using the Monte Carlo method is described. The algorithm implemented in the code is described in detail. Some results of test calculations are given.

**Keywords:** calculation, kinetics, nuclear reactor, algorithm, Monte Carlo method, supercomputer

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## INTRODUCTION

A nuclear reactor, if we consider only the process of neutron transport in it, can be defined as an analog multiprocessing mega-supercomputer in which

- each neutron is a computing core;
- the materials of the constructional components of the reactor act as the memory, which "save" the accumulated statistics; the same materials also provide communications between computing cores;
- world constants provide the constant bank;
- laws of the nature play the role of software;
- the reactor core as a whole plays the role of a zero computing core, where all collected statistics are placed.

It is recognized by the world academia that the equation of neutron transport in a reactor–supercomputer is solved by the analog Monte Carlo method.

Generation of 1 W of power requires $3.2 \times 10^{10}$ neutron-induced fissions per second [1]. The BN-600 (thermal power 1470 MW) and the BREST-300-OD (thermal power 700 MW) considered as mega-supercomputers trace histories of $4.7 \times 10^{19}$ and $2.2 \times 10^{19}$ neutrons per second, respectively. Assuming the lifetime of a neutron in the BN-600 is 0.5 μs, and its lifetime in BREST-300-OD is 0.6 μs, it is easy to calculate that one generation of reactor neutrons contains ~$2.4 \times 10^{13}$ and ~$1.3 \times 10^{13}$ neutrons in the BN-600 and BREST-300-OD, respectively.

The number of storage locations in a reactor–mega-supercomputer coincides with the number of cores contained in all reactor materials; i.e., it is practically infinite. Moreover, this is operating storage, and it is located on the hard disk with immediate access.

Note that a nuclear reactor simulates neutron transport solving the problem with a set source since neutrons of each subsequent generation are generated by neutrons of the previous generation. The reactor "knows" the phase coordinates of locations of neutron generation, as well as the spatial distribution, and the time of generation of delayed neutron precursors.

Presently, programs for solution of approximate equations of point or distributed kinetics are used to study kinetic and dynamic processes in nuclear reactors. At the same time, owing to rapid development of multiprocessing computer facilities [2], papers describing algorithms applying the Monte Carlo method for solution of the nonstationary neutron transport equation are more and more frequent [3–10].

## 1. KIR CODE

In 2013, we started to create the codes KIR (KInetics of Reactor) and KIR-P (KInetics of Reactor with approximations) that implement the Monte Carlo method. In the KIR code, the analog Monte Carlo method is applied for simulation of a kinetic process; in the KIR-P code, classical adiabatic, quasi-static, and improved quasi-static approximations are applied. The Monte Carlo method in KIR-P is used to calculate point kinetic parameters and the shape function.

In this paper, the KIR code is presented. The description of KIR-P is under development and will be released in the near future.

## 2. THE PURPOSE OF KIR

KIR is intended for solution of inhomogeneous stationary and nonstationary and homogeneous neutron transport equations by the analog Monte Carlo methods on the basis of evaluated nuclear data in systems with 3D geometry using single-core and multicore computers.

The main sphere of the code application is calculation of kinetics of fast reactors with liquid-metal coolants.

In KIR, the nonstationary neutron transport equation is solved by the Monte Carlo method with time dependences of positions of control rods and densities and temperatures of constructional materials. Delayed neutrons are taken into account, both the ones with precursors accumulated by the start of the simulated temporal process and the ones generated during the process.

## 3. THE EQUATIONS OF REACTOR KINETICS

The kinetics of nuclear reactors, i.e., neutron flux variation depending on time (taking into account delayed neutrons), is described by the following system of equations [11, 12]:

$$\Phi(\mathbf{r},\mathbf{\Omega},E,t)=\int_0^\infty \exp\left[-\int_0^{s'}\Sigma_{\text{tot}}\left(r-s'',\mathbf{\Omega},E,t-\frac{s''}{v}\right)ds''\right] \times q\left(r-s',\mathbf{\Omega},\mathbf{\Omega},E,t-\frac{s'}{v}\right)ds', \quad (1)$$

$$\partial C_i(\mathbf{r},t)/\partial t+\lambda_i C_i(\mathbf{r},t)=\int_E\int_{\mathbf{\Omega}}\beta_i(\mathbf{r},E')\nu(\mathbf{r},E') \times\Sigma_f(\mathbf{r},E')\Phi(\mathbf{r},\mathbf{\Omega}',E',t)d\mathbf{\Omega}dE \quad (2)$$

Here

$$q(\mathbf{r}',\mathbf{\Omega},E,t')=\int_{E'}\int_{\mathbf{\Omega}'}\left(\frac{\chi_p(E)}{4\pi}\nu(1-\beta)\Sigma_f(\mathbf{r}',E',t')\right) \times\Phi(\mathbf{r}',E',\mathbf{\Omega}',t)d\mathbf{\Omega}'dE' + \int_{E'}\int_{\mathbf{\Omega}'}\sum_{x\neq f}\Sigma_x(\mathbf{r}',\mathbf{\Omega}',E',t)f_x(\mathbf{r},\mathbf{\Omega}',E'\to\mathbf{\Omega},E) \times\Phi(\mathbf{r}',E',\mathbf{\Omega}',t)d\mathbf{\Omega}'dE' + \sum_j^{N_d}\frac{\chi_j(E)}{4\pi}\lambda_j C_j(\mathbf{r}',E,t')+Q(\mathbf{r}',\mathbf{\Omega},E,t');$$

($\mathbf{r}$, $\mathbf{\Omega}$, E, $t$) are phase coordinates of a particle: $\mathbf{r}$ is the radius vector of location, $\mathbf{\Omega}$ is the flight direction vector, $E$ is the energy, $t$ is the time; $\nu$ is the neutron velocity; $\mathbf{r}' = \mathbf{r} - s'\mathbf{\Omega}'$; $t' = t - s'/\nu$; $\Phi(\mathbf{r}, \mathbf{\Omega}, E, t)$ is the neutron flux; $\chi_p(E)$ is the normalized spectrum of prompt neutrons; $\nu(\mathbf{r}, E)$ is the expected total number of neutrons emitted per one fission at point $\mathbf{r}$ caused by a neutron with energy $E$; $C_i(\mathbf{r}, t)$ is the density of delayed neutron precursors in group $i$; $N_d$ is the number of groups of delayed neutron predecessors (six or eight groups are usually considered); $\lambda_i$ is the decay constant for group $i$; $\chi_i(E)$ is the energy spectrum of delayed neutrons of group $i$; $\beta_i(\mathbf{r}, E)$ is the fraction of the $\nu(\mathbf{r}, E)$ value assigned to group $i$ of delayed neutron precursors; $\beta_i(\mathbf{r}, E)\nu(\mathbf{r}, E)$ is the expected number of precursors of group $i$ generated as a result of a nuclear fission initiated at point $\mathbf{r}$ by a neutron with energy $E$; $\beta(\mathbf{r}, E)] = \Sigma_i\beta_i(\mathbf{r}, E)$, where $i = 1, ..., N_d$; $[1 - \beta(\mathbf{r}, E)]\nu(\mathbf{r}, E)$ is the number of prompt neutrons emerging as a result of a nuclear fission initiated at point $\mathbf{r}$ by a neutron with energy $E$, $\beta$ and $\beta_i$ are attributed to energy $E'$, and $\tilde{\chi}_p$ and $\tilde{\chi}_j$ are attributed to energy $E$; $\Sigma_{\text{tot}}(\mathbf{r}, E, t)$ is the total macroscopic cross section of neutron interaction; $\Sigma_x(\mathbf{r}; \mathbf{\Omega}', E' \to \mathbf{\Omega}, E; t)$ is the macroscopic cross section of the $x$-type interaction of neutrons with a substance; $f_x(\mathbf{r}; \mathbf{\Omega}', E' - \mathbf{\Omega}, E)$ is the probability that an $x$-type reaction occurs at collision at point $\mathbf{r}$ of a neutron with flight direction $\mathbf{\Omega}'$ and energy $E'$ and a neutron emerges at the same point with flight direction $\mathbf{\Omega}$ and energy $E$ (transition probability); $Q(\mathbf{r}, \mathbf{\Omega}, E, t)$ are external sources.

In each group, precursors decay exponentially with a half-life typical of the group, which determines the rate of emission of delayed fission neutrons. The relative and absolute yields of delayed neutron precursors and their decay constants depend on the type of split nucleus and the energy of the neutron causing the fission [11].

## 4. THE INITIAL CONDITIONS

Each kinetic process starts from a certain initial state. To find this state, the homogeneous transport equation is solved in the KIR code by the Monte Carlo method, and a spatial distribution of the neutron generation reaction rate is obtained over fuel elements or over assemblies with allowance for the height dependence; this distribution is the base for constructing the dynamics simulation.

It is assumed that, before the start of the process, the reactor power was maintained in a steady state long enough so that neutrons and delayed neutron precursors reached the equilibrium corresponding to this power level, i.e., the stationary state. The relations between the neutron flux and the concentrations of precursors in the stationary state are obtained from Eq. (2) under the assumption that the time derivatives are zero:

$$C_i(\mathbf{r},0)=\int_{E'}\int_{\mathbf{\Omega}'}\frac{\beta_i(\mathbf{r},E')}{\lambda_i}\nu(\mathbf{r},E')\Sigma_f(\mathbf{r},E') \times\Phi(\mathbf{r},\mathbf{\Omega}',E',0)d\mathbf{\Omega}'dE'; \quad (3)$$

$$C(\mathbf{r},0)=\sum_{j}^{N_d}\left(\int\limits_{E'}\int\limits_{\Omega'}\frac{\beta_i(\mathbf{r},E')}{\lambda_i}\nu(\mathbf{r},E')\Sigma_f(\mathbf{r},E')\right.$$
$$\left.\times\,\Phi(\mathbf{r},\Omega',E',0)d\Omega'\,dE'\right), \quad (4)$$

where $C(\mathbf{r}, 0)$ is the total number of delayed neutron precursors at $t = 0$, i.e., at the start of the simulation of the kinetic process.

## 5. KIR STRUCTURE

The development of the KIR code was based on the modules of the MCU-5 application package taken as components for assembling the operating programs making it possible to solve only the homogeneous and the stationary inhomogeneous neutron transport equations [13], but not the nonstationary equation.

KIR has a modular structure. The module nomenclature composition coincides with that used in programs assembled of MCU-5 modules.

KIR is a specialized operating code assembled of modules of the following types: controller (CM), track (TM), source (SM), geometrical (GM), physical (PM), registration (RM), and equipment (EM). The purposes of all the modules are presented in [13].

Hence, we can write that KIR = CM + TM + SM + GM + PM + RM + EM.

As in all programs of the MCU family, the track module is engaged in calculation management and consecutively calls the required subroutines of source, geometrical, physical, and registration modules.

For KIR, no corrections are needed in the texts of geometrical module NCG, registration module RGS, and equipment module ENV from the MCU-5 package. Corrections are made in texts of controller module C and SOPHISM, FARION, FIMBROEN, and FIMTOEN submodules of the physical module [13]. The RAPAN and FIMUL programs included in the physical module structure remain without changes.

The chosen arrangement of the physical module provides the possibility of continuous tracing of the energy of a neutron and identification of an isotope with which it interacts. Track module TRACT and source module IST are written specially for KIR. In addition, registration module RGS is supplemented with a user submodule RUGA. Thus, the arrangement of KIR code is defined by the following formula:

KIR = C + TRACT + IST + NCG + (SOPHISM + FARION + FIMBROEN + FIMTOEN + RAPAN + FIMUL) + (RGS + RUGA) + ENV.

## 6. KIR FUNCTIONALITIES

It is assumed that the calculated region is composed of a finite number of geometrical zones confined by planes or surfaces of the second order, whose parameters are set by the user. Each zone is filled with homogeneous material. Each material is defined by the temperature and nuclear concentrations of nuclides contained in it.

Geometrical module NCG allows simulating three-dimensional systems with arbitrary geometry using the combinatorial technique based on description of complex spatial forms by combinations of elemental bodies by means of set-theoretical operations of intersection, addition, and union based on a set of types of primitive bodies. Nets and grids can be obtained by multiplication of some initial elements set by combinatorial methods.

Constants are provided to the program by the MCUDB50 databank composed of a set of sections which can be used for calculations by different submodules of the composite physical module. The MCUDB50 databank contains information on 375 nuclides.

The physical module allows taking into consideration the effects of continuous variation of the particle energy at collisions and both continuous and graduated dependence of cross sections on the energy. The continuous tracing of neutron energy is carried out. At simulation of neutron interactions with a substance, the isotope whose nucleus participated in the collision is identified. The special features of simulation of neutron interactions implemented in the code are presented in [13].

The effective neutron multiplication factor, power flux and fission neutron generation rate distributions over fuel assemblies and individual fuel elements depending on time, and other functionals of neutron flux necessary for calculation of nuclear reactor kinetics are registered.

KIR makes it possible to perform calculations on multiprocessor computers. The code is parallelized on the basis of Message Passing Interface (MPI). It is the most commonly used in parallel programming standard data exchange interface implemented on many computer platforms. In the mode of multiprocessor calculations, the program engages all accessible computing cores.

## 7. KIR ALGORITHM FOR CALCULATION OF NUCLEAR REACTOR KINETICS

Simulation of kinetic processes occurring in nuclear reactors by the Monte Carlo method requires iterative solution of the inhomogeneous neutron transport equation. Moreover, the kinetics calculation by the Monte Carlo method can be defined as a direct simulation of physical processes proceeding in reactors of zero power at a set external source of neutrons.

The following can play the role of external source:

• The initial source of neutrons (ISN) is neutrons located in the reactor at the start of the simulation process.

• The initial source of delayed neutrons (ISDN) is delayed neutrons emerging over time interval [0, $T$] (the time interval of kinetic process simulation) from delayed neutron precursors accumulated in the reactor by the moment $t = 0$.

Note that, to improve the accuracy of evaluation of functionals over time intervals comparable to prompt neutron lifetimes, besides all other things, it is necessary to find $t_{gen}$ and $t_{fis}$ for each neutron from the ISN. The practice of computational investigations has shown that setting $t_{gen} = 0$ results in a certain flux deformation at the initial moment of simulation of the nonstationary process (see below the results of calculations performed in RPIGS test). No deformation is observed at evaluation of functionals over time intervals considerably exceeding the prompt neutron lifetime.

The following algorithm is implemented for evaluation of $t_{gen}$ and $t_{fis}$ in KIR:

Each neutron from ISN is assigned its generation time $t_{gen} = 0$. In the process of simulation of the neutron history up to the moment of its fission, its lifetime $t_1$ is determined, and $t_{fis}$ is defined as $t_{fis} = t_1 \cdot \mathrm{RND}$, where RND is a random number in the interval [0, 1]; i.e., actually, the calculated lifetime of a neutron from ISN decreases in a random manner. This correction is performed only for the starting neutrons from ISN; the lifetimes of their descendants are calculated in the standard way.

The implemented algorithm made it possible to reduce the flux deformation, but not to avoid it completely.

The total flux of neutrons in the reactor is the sum of fluxes formed by ISN and ISDN and their descendants. Thus, the process of kinetics simulation is reduced to calculation of histories of neutrons from the ISN and ISDN and of all their descendants up to the moment of their disappearance from the system (capture, escape or exit over time interval [0, $T$] of the kinetics simulation).

Since the flux and other functionals of neutrons from the ISN and ISDN and their descendants are registered separately, there is no necessity to maintain the set ratio between the numbers of neutrons from ISN and ISDN. In this case, the total flux in the reactor can be defined as a linear combination of fluxes from ISN and ISDN:

$$\Phi(\mathbf{r},\mathbf{\Omega},E,t) = \Phi^{\mathrm{ISN}}(\mathbf{r},\mathbf{\Omega},E,t) + \alpha\Phi^{\mathrm{ISDN}}(\mathbf{r},\mathbf{\Omega},E,t). \quad (5)$$

Coefficient $\alpha$ can be found in various ways, including one based on calculation of the initial critical state of the reactor. However, note that, owing to the statistical nature of the calculation, this relation is usually found with a certain error, which can result in adding a systematic error to the calculation of time-dependent functionals.

The algorithm used in KIR to find coefficient $\alpha$ implies that time interval $[-t_1, 0]$, during which the reactor remains in the critical state, is calculated first. The direct simulation of the kinetics starts at the zero moment of time. Over time interval $[-t_1, 0]$, the neutron flux does not depend on time. Thus, taking into account that the flux is the sum of fluxes from the ISN and ISDN, we find coefficient $\alpha$.

Let us divide time interval $[-t_1, 0]$ into $N$ equal intervals, and register the fluxes from ISN and ISDN in each of them. Obviously,

$$\Phi_{[-t_1,0]} = \sum_{i=1}^{N}\Phi_i^{\mathrm{ISN}} + \alpha\sum_{i=1}^{N}\Phi_i^{\mathrm{ISDN}},$$

where $\Phi_{[-t_1,0]}$ is the total flux over interval $[-t_1, 0]$, and $\Phi_i^{\mathrm{ISN}}$ and $\Phi_i^{\mathrm{ISDN}}$ are the fluxes from the ISN and ISDN, respectively, in the $i$th time interval. Since the flux in interval $[-t_1, 0]$ is constant, the following relation is true in each interval $i$:

$$\Phi_{[t_1,0]}/N = \Phi_i^{\mathrm{ISN}} + \alpha\Phi_i^{\mathrm{ISDN}}.$$

Thus, the problem is reduced to finding the minimum of the functional

$$\sum_{k=1}^{N}\left(\Phi_k^{\mathrm{ISN}} + \alpha\Phi_k^{\mathrm{ISDN}} - \frac{\sum_{i=1}^{N}\Phi_i^{\mathrm{ISN}} + \alpha\sum_{i=1}^{N}\Phi_i^{\mathrm{ISDN}}}{N}\right)^2.$$

After equating the derivative with respect to $\alpha$ to zero, we obtain

$$\alpha = \frac{\sum_{k=1}^{N}\Phi_k^{\mathrm{ISN}}\sum_{k=1}^{N}\Phi_k^{\mathrm{ISDN}}N - \sum_{k=1}^{N}\Phi_k^{\mathrm{ISN}}\Phi_k^{\mathrm{ISDN}}}{\sum_{k=1}^{N}\left(\Phi_k^{\mathrm{ISDN}}\right)^2 N - \left(\sum_{k=1}^{N}\Phi_k^{\mathrm{ISDN}}\right)^2}. \quad (6)$$

This approach makes it possible to use any number of neutrons both in ISN and in ISDN, depending on properties of the simulated kinetic process. Obviously, the larger is $T$, the larger the delayed neutron source contribution to the integrated (both spatial and time) flux.

The simulation accuracy and allowable duration of the considered time interval of the process are determined only by the accuracy of the evaluated nuclear data and the available computing resources for collecting statistics from the source neutrons and their descendants.

The algorithm of solution of the nonstationary transport equation is implemented in KIR in three stages.

(1) Calculation of the reactor stationary critical state (criticality problem) to find the spatial distribution of the neutron generation reaction rate.

(2) Shaping of ISN and ISDN allowing for the spatial distribution of the neutron generation reaction rate and calculation of the nonstationary process for the reactor critical state over interval $[-t_1, 0]$ to find the relation between ISN and ISDN (coefficient $\alpha$).

(3) Calculation of the nonstationary kinetic process over interval $[0, T]$.

The reactor stationary critical state is calculated by the standard Monte Carlo method over generations and does not need explanations.

In simulation of the kinetic process, for reasons of analysis of its results, time interval $[-t_1, T]$ is divided into intervals $i_{t\max}$, and functionals set by the user are registered in each interval. The reactor properties are considered constant over each time interval.

Let us introduce the following designations: the bank of the first turn is a packet of neutrons in the current calculated time interval; the bank of the second turn is a packet of descendants from the ISN outside the current time interval; the bank of the third turn is a packet of descendants from the ISDN outside the current time interval.

The following values are set for calculation of the nonstationary kinetic process over time interval $[-t_1, T]$:

NHISTP is the number of neutrons in the ISN.

NHISTD is the number of neutrons in the ISDN.

NPROC is the number of available processors (cores).

IPROMPT is a parameter taking on two values: if the neutron is prompt, then IPROMPT = 1; if the neutron is delayed, then IPROMPT = 0.

$i_{\text{cur}} = 1$ is the current time interval.

$i_{t\max}$ is the total number of calculated intervals.

$\alpha = 0$.

Thus, the total number of neutrons in ISN and ISDN used for simulation of the nonstationary process is NHISTP · NPROC and NHISTD · NPROC, respectively.

Proceeding from the data obtained from calculation of the stationary state, the source module of KIR generates the following parameters for each neutron of the source:

• the coordinates of the point of generation obtained from the power flux spatial distribution;

• the energy obtained from the neutron generation rate distribution over split nuclides and the corresponding spectrum: prompt neutron spectrum, if the neutron is prompt, and delayed neutron spectrum, if the neutron is delayed;

• the guiding cosines of flight obtained from the isotropic distribution;

• the time of generation of a delayed neutron obtained proceeding from equilibrium concentrations of delayed neutron precursors in the registration zone corresponding to the coordinates of point of generation and the nuclide in time interval $[-t_1, T]$. The time of generation of a prompt neutron is set equal to $-t_1$. This time is assumed to be the initial age of the prompt neutron from ISN.

The algorithm of calculation of the reactor kinetics by KIR is described by the following steps:

(1) NHISTP of prompt neutrons is generated, and the neutrons are placed in the bank of the first turn. IPROMPT is set equal to 1; go to (10).

(2) NHISTD of delayed neutrons is generated. If the generation time of a delayed neutron exceeds the first time interval, the neutron is placed in the bank of the third turn, otherwise it is placed in the bank of the first turn. IPROMPT is set equal to zero; go to (10).

(3) The neutron free path is played.

(4) Using the neutron free path and velocity, the neutron lifetime up to the point of collision is calculated and added to its total age.

(5) If the neutron total age exceeds the upper limit of time interval T of the kinetic process simulation, go to (10).

(6) Contributions of collisions to evaluations are made over the registration space and time grid. If IPROMPT = 1, contributions are made for functionals from ISN, otherwise they are made from ISDN.

(7) The type of collision is simulated in the standard way.

(8) In the case of scattering, tracing of the track continues—go to (3); if the neutron escapes or capture occurs without fission, go to (10), otherwise go to (9).

(9) Number of neutrons $N_f$ generated as a result of fission is played; the numbers of prompt and delayed neutrons are found. Prompt neutrons are assigned the total age of the neutron causing the fission. If IPROMPT = 1 and the neutron causing the fission is the starting neutron, i.e., is taken directly from ISN, the lifetime of the neutron is multiplied by random number RND; i.e., the total age of the starting neutron causing the fission and, hence, the time of generation of secondary neutrons is defined as $-t_1 + t_1 \times \text{RND}$.

For delayed neutrons, the time of generation (emergence from a precursor) is played. The delayed neutrons are assigned the total age of the neutron causing the fission plus the time of generation of a delayed neutron. Neutrons whose total age exceeds the upper limit of time interval $i_{\text{cur}}$ are placed in the bank of the second turn at IPROMPT = 1 and in the bank of the third turn at IPROMPT = 0.

Neutrons with total age exceeding $T$ are ignored, and remaining neutrons are placed in the bank of the first turn; go to (10).

**Table 1.** One-group cross sections of the calculated area

| Cross section | $cm^{-1}$ |
|---|---|
| $\Sigma_{tot}$ | 1.0 |
| $\Sigma_{fis}$ | 0.25 |
| $\Sigma_s$ | 0.4118 |
| $\Sigma_{abs} = \Sigma_{tot} - \Sigma_s$ | 0.5882 |

**Table 2.** Properties of groups of delayed neutrons

| Group number | Decay constant, $s^{-1}$ | Relative yield |
|---|---|---|
| 1 | 0.0127 | 0.00026 |
| 2 | 0.0317 | 0.001459 |
| 3 | 0.1156 | 0.001288 |
| 4 | 0.311 | 0.002788 |
| 5 | 1.4 | 0.000877 |
| 6 | 3.87 | 0.00178 |

(10) Check whether the bank of the first turn is empty. If not, a neutron is chosen from the bank, and go to (3), otherwise go to (11).

(11) If IPROMPT = 1 and $i_{cur} = 1$, go to (2), otherwise go to (12).

(12) If IPROMPT = 0, then $i_{cur} = i_{cur} + 1$ is assigned. If the lower limit of interval $i_{cur}$ is 0, go to (16), otherwise go to (13).

(13) If $\alpha \neq 0$ and IPROMPT = 0, functionals for time interval $i_{cur} - 1$ are obtained by formulas similar to (5) and placed in the output file. Contributions of ISN and ISDN to the functionals are also placed in the output file. If $i_{cur} > i_{max}$, go to (17).

(14) If IPROMPT = 1, then IPROMPT = 0 is assigned, and on the contrary, if IPROMPT = 0, then IPROMPT = 1 is assigned; go to (15).

(15) If IPROMPT = 0, neutrons from the bank of the third turn with total lifetime within the time interval $i_{cur}$ are sent to the bank of the first turn. If IPROMPT = 1, then the corresponding neutrons from the bank of the second turn are sent to the bank of the first turn; go to (10).

(16) Calculation of coefficient $\alpha$ by formula (6). Go to (14).

(17) End of calculation.

Thus, the proposed algorithm makes it possible to calculate the nonstationary process without entering weighting functions for prompt and delayed neutrons, which significantly simplifies the algorithm of registration of the calculated functionals and calculation of the statistical error.

Note that the algorithm implemented in KIR allows using several temporal grids, in as much detailed as possible, with a time step comparable with the average lifetime of fission neutrons. This makes possible to relocate, for example, control units or redefine temperatures and densities of materials not at particular initially preset instants, but directly within the simulation of the kinetic process, using such criteria as the reactor power variation limits and allowable maximum variations of power flux in fuel elements or their parts.

Upon availability of sufficient computing resources, parallel calculation of the ISN and ISDN can be arranged, having set NHISTD equal to C (see formula (4)); i.e., all precursors decay with probability of 1, tracing these delayed neutrons on the processors. In this case, provided that the speed of the computer used for calculation of the nonstationary process allows tracing the number of neutrons specified in the Introduction, it is possible to precisely simulate the operation of the reactor-supercomputer, in which precursors decay by their own laws independently of neutrons located in the reactor. However, the computing resources available today make it possible to simulate only reactor operation.

## 8. KIR TESTING

### *8.1. Test RP1GC: Critical Rectangular Prism*

In [4], the description of a kinetic test with set one-group constants is presented. For reasons of further presentation, we shall designate this test as RP1GC (Rectangular Prism with 1-Group Constants Critical). The calculated region is a rectangular prism with size of $10 \times 20 \times 24$ $cm^3$ placed in vacuum. The one-group cross sections are displayed in Table 1; the delayed neutron properties are given in Table 2. The neutron velocity is $\nu = 22\,000$ cm/s. The number of secondary neutrons per one fission event is $\nu = 2.5$. The delayed neutron fraction is $\beta = 0.00685$.

Over the time interval of 0–10 s, the calculated region is in a critical state. After 10 s, the absorption cross section ($\Sigma_{abs}$) instantaneously decreases from 0.5882 to 0.5870 $cm^{-1}$ with the respective decrease in $\Sigma_{tot}$. After 40 s of the kinetic process, it becomes instantaneously equal to the initial value. This problem actually simulates the process of a reactor switching from one power level to another.

The results of calculation by KIR of this process are presented in Fig. 1. The flux is registered over time intervals of 0.1 s. For comparison, the results of calculation by the TRIPOLI 4.7 code from [4] are presented.

As follows from Fig. 1, data obtained by KIR show good consistency with the results of [4].

The power surges registered after 10 s and 40 s of the kinetic process are related to the prompt neutron flux increase and decrease, respectively. Further power variations are connected with the increase (after 10 s) and decrease (after 40 s), respectively, in concen-

trations of delayed neutron precursors. In Figs. 2 and 3, the power variations are presented close to the moment of variation of neutronic properties of the calculated region.

### *8.2. Test RP1GS: Subcritical Rectangular Prism*

In this test proposed in [5], a subcritical rectangular prism placed in vacuum and filled with the same material as in the RP1GC test is considered. For reasons of further presentation, we designate this test as RP1GS (Rectangular Prism with 1-Group Constants Subcritical).

The prism dimensions are $8 \times 18 \times 22$ cm$^3$. In [5], the value of the effective neutron multiplication factor $K_{\text{eff}} = 0.97823 \pm 0.00001$ is given. Within the calculation by KIR, the effective multiplication factor is $K_{\text{eff}} = 0.9790 \pm 0.0005$.

The simulation starts from solution of the conditionally critical problem, and the process is monitored for 1000 s. Under the conditions of the test problem, it is assumed that, at the start of the kinetic process simulation, delayed neutron sources have equilibrium concentrations.

In [5], within the kinetic process simulation, the neutron flux evaluation was carried out over exponentially increasing time intervals starting from $10^{-4}$ s, which was comparable to the neutron lifetime in the system. In calculation by KIR, the neutron flux was registered over two temporal grids. The first detailed grid with step of $10^{-4}$ s was used in the interval [0, 2] s; the second with the step of 0.1 s was used over the whole period of the process.

The calculations were carried out on a PC. Histories of 1 million prompt neutrons and 500 000 delayed neutrons, whose precursors were accumulated by the start of the kinetic process, were traced. The statistical error of the neutron integral density increases from 0.1% to 10% depending on the duration of the kinetic process if no account is taken of correlation of sequential generations of neutrons. If the correlation is taken into account, it causes a slight increase in the statistical error.

The calculation results are presented in Fig. 4 in comparison with the data obtained using TRIPOLI 4.7 [5].

The results obtained using KIR and TRIPOLI 4.7 codes show good consistency both at the start of the process, when prompt neutrons make the predominant contribution to the flux, and at the end of the time interval, when the delayed neutron contribution is determining.

Nevertheless, it is necessary to note that the neutron flux density calculated by KIR increases at the start of the kinetic process (up to about 0.001 s), which is apparently inconsistent with the physics of the problem to be solved (see Fig. 5).

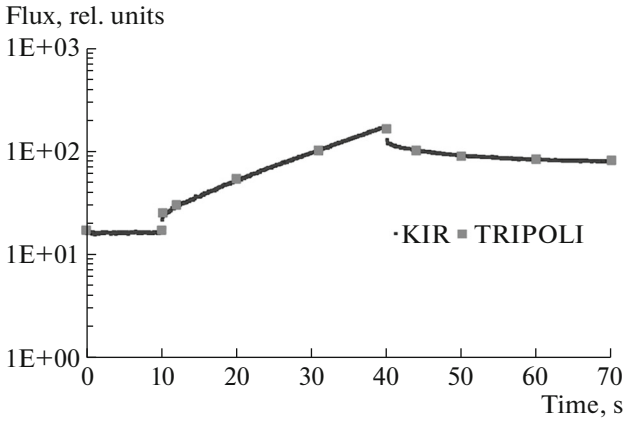


**Fig. 1.** Dependence of power on duration of the kinetic process for RP1GC one-group test.

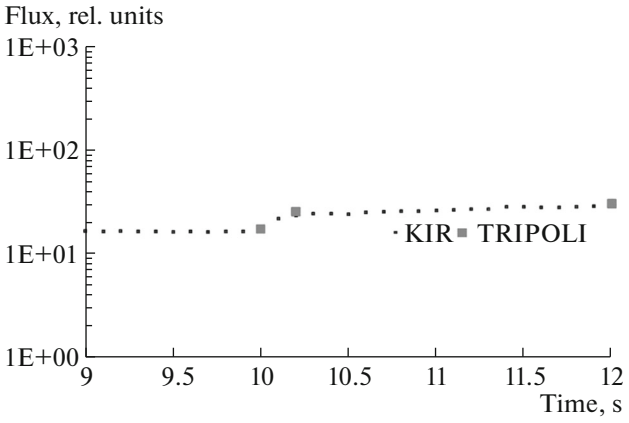


**Fig. 2.** Dependence of power on duration of the kinetic process upon decrease in $\Sigma_{\text{abs}}$.

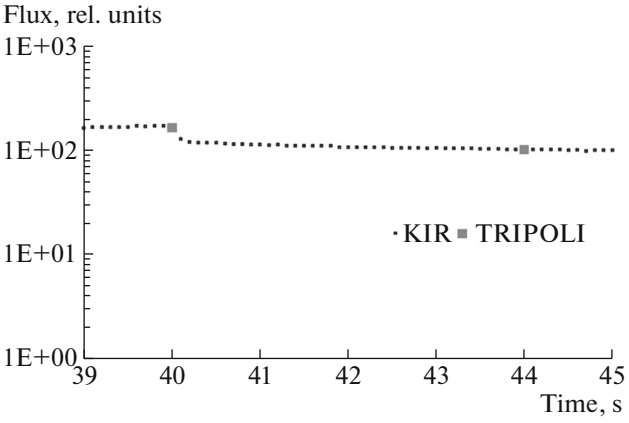


**Fig. 3.** Dependence of power on duration of the kinetic process upon increase in $\Sigma_{\text{abs}}$.

This behavior of the neutron flux density is explained by the fact that, at formation of ISN within KIR calculations, all neutrons were assigned zero

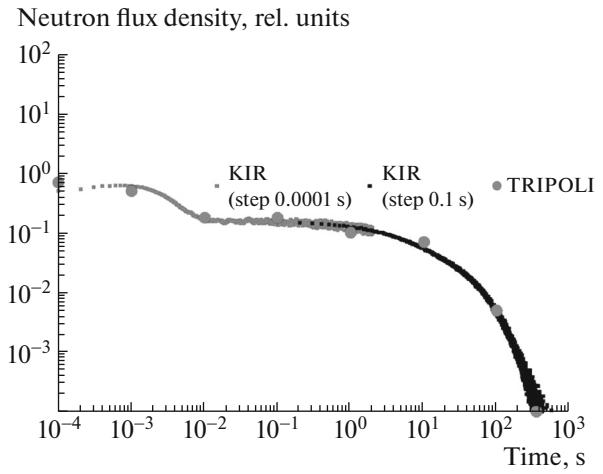


**Fig. 4.** Neutron flux density in subcritical prism as a function of duration of the kinetic process.

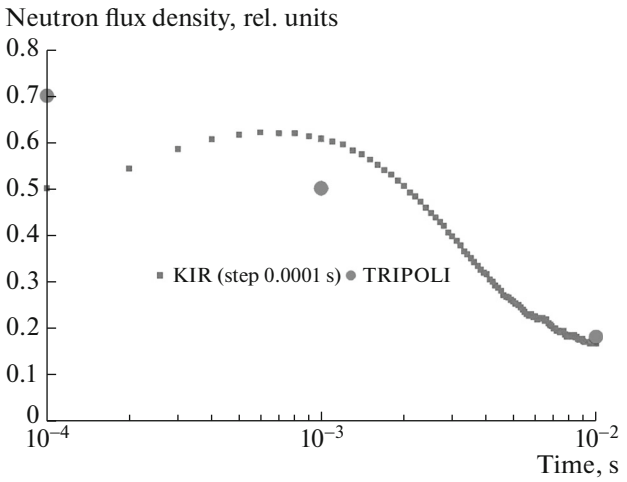


**Fig. 5.** Neutron flux density in subcritical prism over time interval of 0–0.01 s of the kinetic process.

starting time. Actually, the neutrons show a certain distribution in time. During simulation of motion of prompt neutrons and their descendants, their redistribution over the time scale occurs, so that in about 0.005 s they obtain their real time parameters consistent with the physical properties of the calculated region. The time interval in which the redistribution occurs depends on the neutron lifetime in the calculated region. This time interval is about tens of lifetimes. In this test, the interval is about 0–0.005 s.

Apparently, this inconsistency of the initial neutron distribution over the time scale does not affect in any way the neutron flux at simulation of long-term kinetic processes. Upon considering registration time intervals of 0.1 s, it is impossible to detect the initial flux surge (see Fig. 4).

However, in simulation of kinetic processes whose physical properties noticeably vary over time intervals comparable with neutron lifetimes, the inconsistency in definition of the initial temporal distribution of prompt neutrons can add considerable errors to the end result. As mentioned above, the algorithm of starting prompt neutron lifetime correction is implemented in KIR; nevertheless, it has still been impossible to completely avoid the initial deformation of the flux.

For correct solution of this problem, additional studies are needed, which are beyond the scope of this paper. In particular, it is necessary to develop and carry out calculations of special mathematical benchmarks with the precise monitoring of prompt neutron lifetimes at the start of the kinetic process.

### *8.3. Test RPCEU235: A Rectangular Prism with $^{235}$U Isotope*

In [4], the description of a kinetic test with a preset $^{235}$U concentration is given. For reasons of further presentation, we designate this test as RPCEU235 (Rectangular Prism with U235). The calculated region is a rectangular prism placed in vacuum with the dimensions of $10 \times 20 \times 24$ cm$^3$ filled with isotope $^{235}$U.

In [4], the critical $^{235}$U concentration of $0.044925 \pm 10^{24}$ 1/cm$^3$ is given. The critical concentration is obtained by TRIPOLI 4.7 using constants prepared from JEFF-3.1.1 files of evaluated nuclear data [4]. The calculation by KIR with this concentration gives the multiplication factor of $1.003 \pm 0.001$. To bring the calculated region to the critical state, the $^{235}$U concentration is reduced to $0.044630 \pm 10^{24}$ 1/cm$^3$. The multiplication factor is $1.0001 \pm 0.0001$.

Under the test problem conditions, after 10 s, the $^{235}$U concentration increases by $0.000075 \pm 10^{24}$ 1/cm$^3$ and remains constant, and after 40 s, it returns to its initial value. In simulation of the kinetic process by KIR, the increase in $^{235}$U concentration is corrected proportionally to the $^{235}$U concentration decrease to the critical state; i.e., in KIR calculations, the $^{235}$U concentration increases over the interval [10, 40] s by $0.00007451 \pm 10^{24}$ 1/cm$^3$.

The calculations were performed on a 240-core supercomputer. The statistical error without taking into account the neutron number correlation between generations is less than 0.01%. This approximately corresponds to 1 billion fissions over a time interval 0.1 s.

In Fig. 6, the dependence of the flux on the kinetic process duration is presented. For comparison, the results obtained by using TRIPOLI 4.7 are given. Registration of the flux in KIR calculations was carried out over time intervals of 0.1 s.

Calculations were carried out by KIR and TRIPOLI 4/7 using libraries assembled from different files of evaluated nuclear data, which, in particular, differ in properties of delayed neutrons. In addition, the

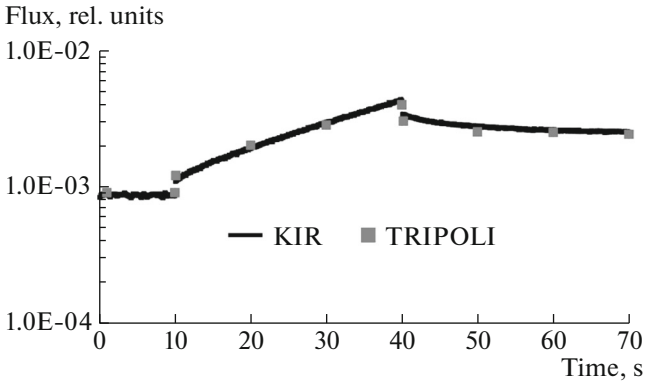


**Fig. 6.** Dependence of power on duration of the kinetic process for RPCEU235 test—rectangular prism with $^{235}$U isotope.

accuracy of the gained critical state is not specified in [4]; namely, no value of the multiplication factor is given.

The neutron lifetime in the considered problem is about $10^{-8}$ s. Therefore, about $10^{10}$ generations are replaced during 70 s of the kinetic process. Even a small inaccuracy in the definition of the $^{235}$U critical concentration can result in significant variations of power over the observed time interval.

Hence, the results obtained with the help of the KIR and TRIPOLI 4.7 codes, notwithstanding the generally good consistency, can be compared only qualitatively, since there are some uncertainties in interpretation of the initial data.

## CONCLUSIONS

A description of the KIR code intended for analog simulation of kinetic processes in nuclear reactors by the Monte Carlo method is presented.

The algorithm of solution of the nonstationary transport equation by the Monte Carlo method implemented in KIR taking into account delayed neutrons method is detailed. The peculiarity of the algorithm is the exclusion of weight coefficients from the simulation of neutron motion both in space and in time.

Some tests are given as examples of comparative analysis of the results obtained by the KIR and TRIPOLI 4.7 codes, in which the nonstationary neutron transport equation is solved by the Monte Carlo method.

The results are quite consistent: for the one-group tests, the results differ within the limits of statistical errors, and for the rectangular prism containing only the $^{235}$U isotope, the difference between integral neutron fluxes does not exceed 5%. This discrepancy can be explained by the use of different libraries of nuclear data in these programs.

There is no doubt that, in the near future, with development of computing facilities and their increasing accessibility to specialists in nuclear reactor safety, the calculations of kinetic processes by the Monte Carlo method will become an everyday reality.

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