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ELEMENTARY PARTICLES AND FIELDS
Theory

Quark-Flavor Symmetries and their Violation in Quantum Chromodynamics*

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Abstract—In Quantum Chromodynamics, the hadrons consisting of light (u, d, s) and heavy (c, b) quarks are subject to approximate flavor symmetries, providing the basis for powerful effective theories. I will briefly overview the origin of these symmetries and the scale of their violation. The current precision tests of Standard Model in the electroweak decays of hadrons demand an accurate quantitative account of flavor-symmetry violation effects. I will discuss the continuum (non-lattice) QCD calculation of these effects in hadronic matrix elements, taking as an example the decay constants of heavy–light hadrons.

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1. QUARK MASSES IN QCD

Quantum chromodynamics (QCD) is a part of the Standard Model (SM), and is described by the well-known Lagrangian:

$$\mathcal{L}_{\rm QCD}(x) = -\frac{1}{4}G^a_{\mu\nu}(x)G^{a\mu\nu}(x) + \sum_{f=u,d,s,\ldots}\bar\psi_f(x)(iD_\mu\gamma^\mu - m_f)\psi_f(x), \quad (1)$$

where $G^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g_s f^{abc} \times A^b_\mu(x)A^c_\nu(x)$ is the gluon field-strength tensor, and $D_\mu = \partial_\mu - ig_s\frac{\lambda^a}{2}A^a_\mu(x)$ is the covariant derivative. Within QCD, the quark fields $\psi_f(x)$ with different flavors $f = \{u, d, s, c, b, t\}$ differ only by their masses. The primordial values of the mass parameters m_f originate beyond QCD. In Standard Model, the quark masses are generated due to the Yukawa couplings of quark fields with the scalar Higgs field. The three-generation hierarchy of quark flavors, and the relation between quarks and leptons constitute the fundamental "flavor problem" of the SM. In what follows, we consider m_f as an external input in Eq. (1). The quark–gluon interactions in QCD via quantum-loop effects lead to the renormalization-scale dependence ("running") of the quark masses,

$$\overline{m}_q(\mu) = \overline{m}_q(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_0/\beta_0} + \text{NLLO}, \quad (2)$$

similar to the scale dependence of the effective quark–gluon coupling

$$\alpha_s \equiv g_s^2/(4\pi) \to \alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\alpha_s(\mu_0)}{4\pi}\beta_0\log\frac{\mu^2}{\mu_0^2}} + \text{NLLO}, \quad (3)$$

where $\beta_0 = 11 - 2/3N_f > 0$, $\gamma_0 = 4$ and N_f is the number of "active" flavors in the loops. The logarithmic decrease of the coupling (3) at large scales reflects the color-charge screening, and leads to the "asymptotic freedom" of QCD at $\mu \to \infty$, allowing one to establish the perturbation theory in α_s valid at the scales $\mu \geq 1$ GeV. The world average [1] is $\alpha_s(m_Z) = 0.1185 \pm 0.0006$, so that $\alpha_s(1\ \text{GeV}) \simeq 0.467$.

At low scales, around $\Lambda_{\rm QCD} \sim 200$–$300$ MeV QCD undergoes a transition to the nonperturbative regime where the perturbation theory in the α_s coupling is not applicable. QCD dynamics at these scales is characterized by vacuum fields (condensates), confinement of color charges and hadronization.

For our discussion the actual values of the quark masses are crucial. One subdivides the quark flavors into two distinct groups with respect to the QCD scale $\Lambda_{\rm QCD}$: the light quarks $q = u, d, s$ with $m_{u,d} \ll \Lambda_{\rm QCD}$, $m_s \lesssim \Lambda_{\rm QCD}$ and the heavy quarks $Q = c, b$ with $m_Q \gg \Lambda_{\rm QCD}$. The superheavy top quark will not play role in the following.

The world averages of quark-mass determination obtained combining the results of lattice QCD and the continuum method of QCD sum rules, can be found in the Review of Particle Properties [1]. The

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light quark masses normalized at a default scale $\mu_0 = 2$ GeV, are:

$$m_u = 2.3^{+0.7}_{-0.5}\,\text{MeV}, \quad m_d = 4.8^{+0.5}_{-0.3}\,\text{MeV}, \quad m_s = 95 \pm 5\,\text{MeV}, \tag{4}$$

and the heavy–quark masses normalized at the same mass scale are:

$$m_c(\mu = m_c) = 1.275 \pm 0.025\ \text{GeV}, \quad m_b(\mu = m_b) = 4.18 \pm 0.03\ \text{GeV}. \tag{5}$$

Hereafter, we only use the (renormalization scheme dependent) quark masses in the $\overline{MS}$ scheme, their relations to the other quark-mass definitions can be found in the literature. Note that the ratios of the quark masses remain scale independent. The errors of the quark mass determination quoted above are dominated by the lattice QCD results, whereas QCD sum rules yield quark masses with somewhat larger theoretical uncertainties.

2. FLAVOR SYMMETRIES IN QCD

Certain flavor symmetries of the hadronic (strong) interactions were known long before the formulation of QCD. The isospin symmetry for nuclei was introduced by W. Heisenberg and studied by E. Wigner in 1930's. The $SU(3)_{fl}$ symmetry relating strange and nonstrange hadrons was formulated by Gell-Mann and Neeman in 1961 [2]. The most spectacular prediction of this symmetry was the Ω^- hyperon, the spin-3/2 hadronic state with three valence s quarks. Its existence is at the same time one of the strongest implications of the color quantum number in QCD, providing the necessary total asymmetry of the constituent fermions.

In QCD the above-mentioned and other flavor symmetries originate due to accidental relations between various quark masses with respect to Λ_{QCD} and/or with respect to each other. E.g., the origin of the isospin symmetry lies in a small mass difference $m_u - m_d \ll \Lambda_{\text{QCD}}$. This symmetry is described by the $SU(2)$ global gauge transformation, rotating the doublet formed by u and d quark fields in the Lagrangian (1). The isospin invariance of hadrons is the most accurate flavor symmetry. Its characteristic violation at the percent level is caused by the smallness of the ratio $(m_u - m_d)/\Lambda_{\text{QCD}} \sim 1\%$ and in addition, by the electromagnetic effects $\sim O(\alpha_{em})$. The latter evidently violate the isospin symmetry since u and d quarks have different electric charges. At the same time, both m_u and m_d are also small, hence the chiral-symmetry limit $m_q = 0$ seems to be a reasonable one for $q = u, d$ allowing one to develop the QCD-based chiral perturbation theory (ChiPT). The chiral symmetry in QCD is however nontrivially implemented being spontaneously violated by the nonperturbative vacuum condensate, $\langle 0|\bar{q}q\,0\rangle \neq 0$. For a detailed description of the chiral symmetry and related dynamics see, e.g. the reviews [3, 4].

Adding s quark to the u, d quarks allows one to formulate the approximate $SU(3)_{fl}$ symmetry. Its accuracy is reasonably good for hadronic systems where the mass difference $m_s - m_{u,d} \sim 100$ MeV is still smaller than the total binding energy due to quark–gluon interactions. For example, baryons or vector mesons (with $J^P = 1^-$) consisting of light quarks fall under this category. In the heavy–light hadrons, such as B or D mesons, the typical interaction energy of quarks, defined as the difference between the meson mass and the heavy–quark mass, is also substantially larger than m_s. Hence, the mass differences of $m_{B_s} - m_B \sim m_{D_s} - m_D$ nicely reproduce the m_s value. The $SU(3)_{fl}$ symmetry can however be noticeably violated, e.g. in the ratio of the leptonic decay constants of kaon and pion, $f_K/f_\pi \simeq 1.2$. This is not surprising at all, because the mass difference $m_s - m_{u,d}$ is in fact not much smaller than Λ_{QCD}. The $SU(3)_{fl}$ symmetry has three $SU(2)$ subgroups, one of which is the isospin which we already mentioned. The two others are U spin and V spin, based on the (d, s) and (u, s) quark doublets, respectively. Although the U-spin invariance is protected with respect to the electromagnetic interaction of d, s quarks having the same electric charge, the violation of this symmetry is at the same level as the general $SU(3)_{fl}$-symmetry violation. Approximate chiral symmetry and ChiPT can also be extended to the $SU(3)_{fl}$ level, however, with the same reservations as the symmetry itself, e.g., involving a large symmetry-violation effects when transforming from pions to kaons.

The light–quark flavor symmetries have their most important applications in the relations between various hadronic parameters of nonperturbative origin (decay constants, form factors and other hadronic matrix elements) which are not calculable in perturbative QCD and demand nonperturbative methods. In all these applications the isospin symmetry relations are usually taken as granted, anticipating a large degree of precision, whereas the $SU(3)_{fl}$ violation needs case-by-case quantitative estimates of its violation.

Turning to the heavy–quark sector, one encounters the heavy–quark symmetries manifesting themselves in the limit $m_Q \to \infty$, justified by the fact that $m_c, m_b \gg \Lambda_{\text{QCD}}$. These symmetries classify/relate the heavy–light mesons $Q\bar{q}$ and baryons Qqq', ($Q = c, b$; $q, q' = u, d, s$) and their hadronic matrtix elements (decay constants and form factors). In the heavy–quark limit, the mass scale m_Q and the Q-quark spin effectively decouple from the dynamics of the heavy–light state. In particular, replacing the b quark by c quark does not change the hadronic matrix elements of $b \to c$ weak transitions at certain kinematical regions. The heavy–quark symmetry triggered the development of the QCD-based heavy–quark effective theory (HQET) [5–11]. Hadronic matrix elements relevant for heavy-hadron decays are very important for revealing the quark-flavor structure of the SM and for the search for new physics effects. The currently achieved level of accuracy in these studies needs calculating and taking into account the symmetry-violating $O(1/m_Q)$ corrections.

3. QUARK MASS DETERMINATION IN NON-LATTICE QCD

Since flavor symmetries are closely related to the quark masses, the first task is to determine these masses with the highest possible accuracy, using nonperturbative method in QCD and the data on hadron masses. There is a continuous progress in solving this problem in the framework of the lattice QCD using a numerical simulation of QCD functional integrals on the space–time lattice. Here I will briefly discuss the application of the alternative, continuum (non-lattice) method of QCD sum rules [12]. For the heavy–quark mass determination, the "cleanest" hadronic system is the heavy quarkonium $\bar{Q}Q$, with $Q = c$ and b, observed as a series of $J/\psi, \psi(2S), \ldots$ and $\Upsilon, \Upsilon(2S), \ldots$ resonances, respectively [1]. To obtain the QCD sum rules for quarkonium [12, 13], e.g., for the b-quark case, one introduces a correlation function of the two quark–antiquark vector (electromagnetic) currents:

$$\Pi_{\mu\nu}(q) = \int d^4x \times e^{iqx}\langle 0|T\{\bar{b}(x)\gamma_\mu b(x)\bar{b}(0)\gamma_\nu b(0)\}|0\rangle. \qquad (6)$$

A perturbative expansion in $\alpha_s(m_b) \ll 1$ is applicable at $q^2 \ll m_b^2$, where this correlation function is described by highly virtual b-quark loops with gluon radiative corrections (see Fig. 1). In addition, the nonperturbative contributions of the vacuum-gluon condensate $\langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle$ have to be included; they are power suppressed with respect to the perturbative part. Note that due to the absence of light quarks in the currents, the quark-condensate effects are absent at the level of leading contributions. In the most recent determinations [14, 15], gluon radiative corrections up to $O(\alpha_s^3)$ are taken into account.

Employing dispersion relation (analyticity and unitarity) in the variable q^2, the correlation function (6) calculated in QCD is matched to the sum of

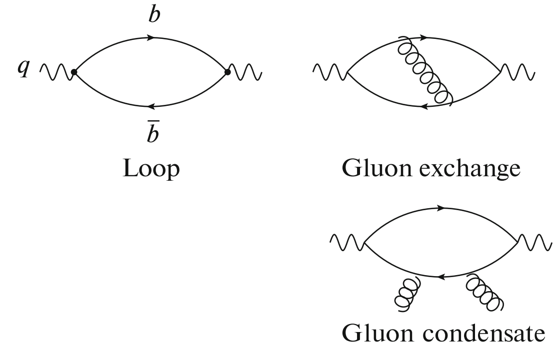

Fig. 1. Sample of quark–gluon diagrams for the correlation function (6).

intermediate bottomonium-state (Υ) contributions, schematically:

$$\Pi^{(\mathrm{QCD})}_{\mu\nu}(q^2, \alpha_s, m_b, \langle 0|GG|0\rangle) = \sum_{\Upsilon} \frac{\langle 0|\bar{b}\gamma_\mu b|\Upsilon\rangle\langle\Upsilon|\bar{b}\gamma_\nu b|0\rangle}{m_\Upsilon^2 - q^2}, \qquad (7)$$

where the hadronic matrix elements $\langle 0|\bar{b}\gamma_\mu b|\Upsilon\rangle$ on r.h.s. (decay constants of the Υ states) are known from the measured $\Upsilon \to e^+e^-$ widths [1]. The hadronic continuum states with $\bar{b}b$ quantum numbers located above $\Upsilon(3S)$ and not shown in the above sum explicitly, are taken into account using the quark–hadron duality assumption [12] based on the asymptotically free limit of the correlation function at $|q^2| \to \infty$. The interval of m_b obtained from the QCD sum rule analyses, is currently in a very good agreement with the one obtained from lattice QCD. Quite analogously, m_c is determined from the correlation function of the product of $\bar{c}c$ currents, matched to the hadronic sum over charmonium levels.

The light-quark masses can also be accurately determined using the QCD sum rule method. Let us consider as an example the s-quark mass determination. One possibility to access this mass parameter of QCD is to introduce the underlying correlation function with pseudoscalar strangeness currents $j_{5s} = \partial^\mu \bar{s}\gamma_\mu\gamma_5 q = (m_s + m_q)\bar{s}\gamma_5 q$, $(q = u, d)$:

$$\Pi_{5s}(q^2) = i\int d^4x \times e^{iqx}\left\langle 0\left|T\left\{j_{5s}(x)j^\dagger_{5s}(0)\right\}\right|0\right\rangle. \qquad (8)$$

The dispersion relation (doubly differentiated) for $\Pi_{5s}(q^2)$ is employed at spacelike $Q^2 = -q^2 \gg \Lambda^2_{\mathrm{QCD}}$:

$$\frac{1}{2}\frac{d^2}{d(q^2)^2}\Pi^{(\mathrm{QCD})}_{5s}(q^2, \alpha_s, m_s, \langle\bar{q}q\rangle, \ldots) = \int\limits_0^\infty ds \frac{\rho^{(\mathrm{hadr})}_{5s}(s)}{(s - q^2)^3}, \qquad (9)$$

where the hadronic sum

$$\rho_{5s}^{(\mathrm{hadr})}(s) = \sum_K \langle 0|j_{5s}|K\rangle\langle K|j_{5s}|0\rangle\delta(m_K^2 - s) \quad (10)$$

is saturated by the kaon and its radial excitations. Their contributions to Eq. (10) are fixed by the meson masses and decay constants, the latter defined as $\langle 0|j_{5s}|K\rangle = f_K m_K^2$. The value $f_K = 159.8$ MeV for the kaon is measured in $K \to \mu\nu_\mu$ decays. Note that according to ChiPT, the excited $K(1460)$, $K(1830)$ resonances have very small decay constants $f_{K(1460)}$, $f_{K(1830)} \ll f_K$, hence the accuracy of these hadronic parameters has a small impact on the sum in (10). The remaining contribution of hadronic states at $s > m_{K(1830)}^2$ is estimated using the quark–hadron duality approximation.

The diagrams contributing to the correlation function (8) are shown in Fig. 2. Their sum forms the operator-product expansion (OPE) consisting of the perturbative part (the loop diagram and gluon radiative corrections) and nonperturbative, power-suppressed part (condensate contributions). The latter are strongly suppressed in this particular case.

The radiative gluon corrections to the loop diagrams have been calculated up to $O(\alpha_s^4)$ in [16]. The OPE result for the correlation function can be cast in a form of expansion in the powers $1/(Q)^{d+2}$, $d = 0, 2, 4, 6$, where the dominant m_s dependence is factorized:

$$\frac{d^2}{d(Q^2)^2}\left[\Pi_5^{(\mathrm{QCD})}(Q^2)\right] = \frac{3(m_s+m_u)^2}{8\pi^2 Q^2} \times \left\{1 + \sum_{i=1}^{4} C_{0,i}\left(\frac{\alpha_s}{\pi}\right)^i - 2\frac{m_s^2}{Q^2}\left(1 + \sum_{i=1,2} C_{2,i}\left(\frac{\alpha_s}{\pi}\right)^i\right) + \frac{\{d=4\}}{Q^4} + \frac{\{d=6\}}{Q^6}\right\}. \quad (11)$$

In the above, the abbreviation $\{d = 4\}$ for the coefficient denotes a combination of vacuum-condensate densities with dimension four (the terms $\sim m_s\langle 0|\bar{q}q|0\rangle$, $\langle 0|GG|0\rangle$) and some minor $O(m_s^4)$ corrections from the perturbative part, whereas $\{d = 6\}$ contains dimension six coefficients $\sim m_s\langle 0|\bar{q}Gq|0\rangle, \langle 0|\bar{q}q|0\rangle^2$. One of the analyses of this sum rule [17] contributing to the PDG average yields

$$\overline{m}_s(2\ \mathrm{GeV}) = \left(105 \pm 6\Big|_{\mathrm{OPE}} \pm 7\Big|_{\mathrm{hadr}}\right)\ [\mathrm{MeV}], \quad (12)$$

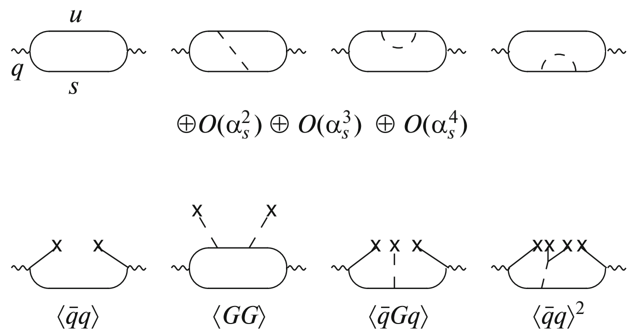

Fig. 2. Diagrams for the correlation function (8) corresponding to the perturbative (upper row) and nonperturbative (lower raw) contributions.

consistent with the other s-quark mass determinations. The average of u-, d-quark masses can be obtained repeating the same analysis for the correlation functions of the $j_{5ud} = \bar{u}\gamma_5 d$ currents; the hadronic input in this case provided by the pion states.

Note that the m_s determination is sufficient if one calculates then $m_{u,d}$, employing very accurate ChiPT relations relating π and K masses squared to the ratios of light-quark masses [18]. Importantly, the key nonperturbative parameter of the QCD vacuum, the quark condensate density is determined from the relation [19] between the pion characteristics and the u, d-quark masses:

$$\langle 0|\bar{q}q|0\rangle = \frac{m_\pi^2 f_\pi}{m_u + m_d}, \quad (13)$$

with negligibly small corrections. The current interval of the quark condensate density normalized at the default scale $\mu = 2$ GeV is: $\langle 0|\bar{q}q|0\rangle(\mu = 2\ \mathrm{GeV}) = (-277^{+12}_{-10}\ \mathrm{MeV})^3$. Another alternative method to determine the light-quark masses is to use the correlation functions with scalar ($J^P = 0^+$) quark–antiquark currents saturating the hadronic sum with the parameters of the light scalar ($J^P = 0^+$) mesons (see e.g., [20]).

Replacing nonstrange quarks with s quarks in the condensate density we arrive at a qualitatively new situation in nonperturbative QCD: the $SU(3)_{fl}$ symmetry is effectively violated in the vacuum by the ratio:

$$\frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{q}q|0\rangle} = 0.8 \pm 0.2, \quad (14)$$

where $q = u, d$ and isospin symmetry is assumed. A direct relation of this ratio to $m_s/m_{u,d}$ cannot be simply derived. The above estimate is taken from the dedicated analysis of the QCD sum rules for strange baryons (hyperons) versus nucleon sum rules [21]. A numerically very similar relation is obtained for the quark–gluon condensate density ratios. Importantly, the s-quark condensate density in the kaon sum rule (11), being multiplied by a very small $m_{u,d}$, does not influence the result for the s-quark mass. On the other hand, the relation similar to (13) for the strange-quark condensate density receives large corrections and cannot be directly used. A further improvement of the precision in the ratio (14) represents a topical problem.

Comparing QCD sum rules for correlation functions constructed from similar quark currents of different flavors, it is possible to assess the effects of flavor-symmetry violation in a quantitative way. One has to keep in mind that the $SU(3)_{fl}$-symmetry violation extends beyond the simple quark-mass difference.

4. CALCULATING SYMMETRY VIOLATING RATIOS OF HADRONIC MATRIX ELEMENTS

Having at hand a reliable method of quark mass determination, on can extend the technique of QCD sum rules with universal nonperturbative input parameters (condensate densities) to calculate various hadronic matrix elements involving heavy–light mesons. Here one "inverts" the sum rule setup: knowing the OPE for a dedicated correlation function, one predicts the unknown hadronic parameters entering the spectral density in the dispersion relation. Since the calculation is done at finite mass and in full QCD, quantitative estimates of the flavor-symmetry-violation effects can be obtained straightforwardly. Importantly, the results for the ratios of hadronic parameters are more accurate than the individual parameters, due to correlation of the input.

As an example, let me illustrate how this approach works for the decay constants of heavy–light $D_{(s)}$ and $B_{(s)}$ mesons and their vector meson counterparts $D^*_{(s)}$ and $B^*_{(s)}$ mesons. The B-meson decay constant (see Fig. 3) is defined via the hadronic matrix element $\langle 0|j_5|B\rangle = f_B m_B^2$, where the heavy–light quark current is $j_5 = \bar{q}\gamma_5 b\,(q = u, d)$. All other decay constants are obtained by replacing quark flavors ($q \to s$, $b \to c$) in the currents. For the vector meson the corresponding definition reads: $\langle 0|j_\mu|B^*\rangle \equiv f_{B^*} m_{B^*}\epsilon_\mu$, where $j_\mu \equiv \bar{q}\gamma_\mu b$ and ϵ_μ is the polarization vector of the meson.

Calculating the $SU(3)_{fl}$ and heavy–quark symmetry-violating ratios represents a topical problem for quark-flavor physics. The heavy–light meson decay constants enter the widths of weak leptonic decays. E.g., the $B_u \to \tau\nu_\tau$ decay width depends on the single hadronic parameter f_B, whereas the rare flavor-changing neutral-current decay $B_s \to \mu^+\mu^-$ recently observed by LHCb and CMS Collaborations [22] is determined by f_{B_s}. The vector meson decay constants are not directly measured but enter the calculation of more complicated hadronic matrix elements such as the strong coupling $B^*B\pi$.

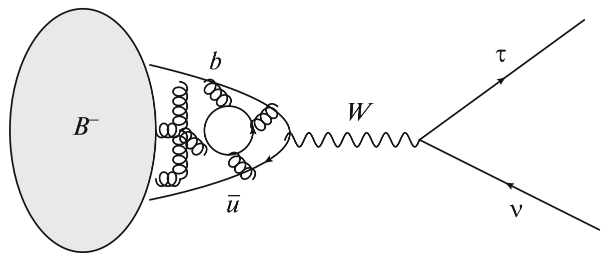

Fig. 3. Schematic view of the leptonic decay of B meson.

The question, how much the ratio f_{B_s}/f_B deviates from its $SU(3)_{fl}$ limit equal to one, is nontrivial because in QCD the heavy–light mesons involve several mass/energy characteristic scales from the largest (m_b), to smallest ($\Lambda_{\rm QCD}$) one. Hence, a "guesstimate" for the symmetry violation can span from $O(m_s/m_b)$ to $O(m_s/\Lambda_{\rm QCD})$.

To obtain the QCD sum rule, we consider f_B as a study case and "design" an appropriate correlation function of two currents $j_5 = \bar{q}\gamma_5 b$ and its conjugate ($q = u, d$):

$$\Pi_5^{(B)}(q^2) = \int d^4x e^{iqx}\langle 0|T\{j_5(x)j_5^\dagger(0)\}|0\rangle. \quad (15)$$

At $q^2 \ll m_b^2$ the expansion of this correlation functions in perturbative loops and vacuum-condensate contribution is possible (see Fig. 4). The currently achieved accuracy includes $O(\alpha_s^2)$ gluon radiative corrections to the leading-order quark loop and $O(\alpha_s)$ corrections to the quark-condensate contribution. The latter is enhanced by the heavy–quark mass factor. The hadronic sum (spectral density in the dispersion relation) contains the ground-state B-meson contribution and the sum of the contributions of its excitations. The latter is estimated with the help of quark–hadron duality. Calculating m_B from the sum rule, one fixes the effective threshold parameter. The resulting QCD sum rule has the form, schematically:

$$\Pi_{\rm QCD}(q^2; \alpha_s, m_b, m_u, \langle 0|\bar{q}q|0\rangle, ...)$$
$$= \frac{\langle 0|j_5|B\rangle\langle B|j_5|0\rangle}{m_B^2 - q^2} + \left\{\sum_{B_{\rm exc}}\right\}_{\text{duality approx.}}, \quad (16)$$

from which f_B is extracted. It is clear that this sum rule allows one the access to flavor symmetry violation replacing $\{u, d\} \to s$ and $b \to c$ and forming the ratios of the corresponding sum rules.

The most recent results on QCD sum rules for $B_{(s)}$, $B^*_{(s)}$, $D_{(s)}$, $D^*_{(s)}$ decay constants are obtained

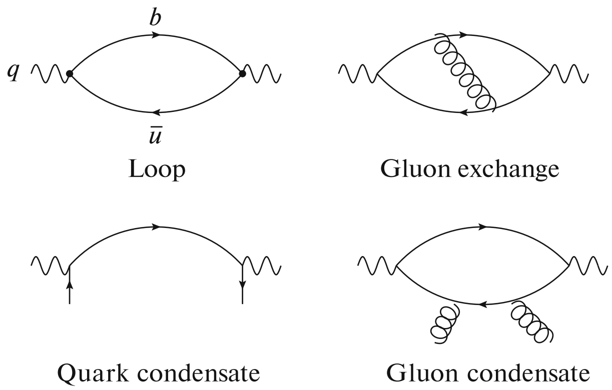

Fig. 4. Sample of diagrams for the correlation function (15).

with NNLO accuracy in [23]. The prediction of this method $f_B = 207^{+17}_{-9}$ MeV, is in agreement with the lattice QCD result $f_B = 197 \pm 9$ MeV [24].

Let me quote the $SU(3)_{fl}$-violating ratios predicted in [23] by dividing to each other the corresponding sum rules and varying the common input parameters concertedly, so that the correlations are taken into account:

$$f_{B_s}/f_B = 1.17^{+0.03}_{-0.04}, \quad f_{D_s}/f_D = 1.18^{+0.04}_{-0.05}. \quad (17)$$

Also calculated are the heavy–quark spin-symmetry violating ratio:

$$f_{B^*}/f_B = 1.02^{+0.07}_{-0.09}, \quad f_{B_s^*}/f_{B_s} = 1.04^{+0.01}_{-0.08},$$
$$f_{D^*}/f_D = 1.20^{+0.13}_{-0.07}, \quad f_{D_s^*}/f_{D_s} = 1.24^{+0.13}_{-0.05}, \quad (18)$$

and the heavy–quark flavor symmetry violating ratio:

$$f_B/f_D = 0.93\text{–}1.19. \quad (19)$$

Comparison of the latter ratio with the HQET result [25], including radiative corrections

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{6/25}$$
$$\times\left(1 + 0.894\frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right) \simeq 0.69, \quad (20)$$

clearly indicates that the power corrections $O(1/m_{c,b})$ not accounted in HQET but implicitly included in the sum rule calculation are essential.

5. CONCLUSION

The flavor symmetries of hadrons: the isospin, $SU(3)_{fl}$, and heavy–quark symmetries, emerge in QCD due to the interplay of quark masses and the $\Lambda_{\rm QCD}$ scale. These symmetries form a basis for classification of hadrons, provide many useful relations between hadronic amplitudes, and serve as a starting point for powerful effective theories such as ChiPT and HQET. Employing the dedicated correlation functions of quark currents, hadronic dispersion relations and quark–hadron duality, one obtains QCD sum rules, providing the determination of quark masses and a possibility for an accurate estimate of flavor-symmetry violating effects. In the case of $SU(3)_{fl}$ symmetry a simple quark mass difference is not sufficient, one needs to take into account a nontrivial deviation between strange- and nonstrange-quark condensates. I have briefly discussed the possibility to extend these methods to hadronic matrix elements for heavy–light hadrons, where $SU(3)_{fl}$-violation effects and heavy–quark symmetry-violating effects are calculated. The same methods are useful for other topical problems where $SU(3)_{fl}$ violation is important such as the relation of hadronic matrix elements in B_d–$\overline{B}_d$ and B_s–$\overline{B_s}$ mixing. Concluding, let me mention a possibility to access the flavor-symmetry violation also in the heavy–light hadronic transition form factors (e.g., $B \to \pi$ vs $B_s \to K$ or $D \to K$ vs $D \to \pi$ transitions) calculated with the method of QCD light-cone sum rules, e.g., in [26–28].

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