

The Interacting Quasiparticle–Phonon Picture and Odd–Even Nuclei. Overview and Perspectives*

S. Mishev^{1),2)}** and V. V. Voronov¹⁾***

Received February 5, 2016

Abstract—The role of the nucleon correlations in the ground states of even–even nuclei on the properties of low-lying states in odd–even spherical and transitional nuclei is studied. We reason about this subject using the language of the quasiparticle–phonon model which we extend to take account of the existence of quasiparticle⊗phonon configurations in the wave functions of the ground states of the even–even cores. Of paramount importance to the structure of the low-lying states happens to be the quasiparticle–phonon interaction in the ground states which we evaluated using both the standard and the extended random phase approximations. Numerical calculations for nuclei in the barium and cadmium regions are performed using pairing and quadrupole–quadrupole interaction modes which have the dominant impact on the lowest-lying states’ structure. It is found that states with same angular momentum and parity become closer in energy as compared to the predictions of models disregarding the backward amplitudes, which turns out to be in accord with the experimental data. In addition we found that the interaction between the last quasiparticle and the ground-state phonon admixtures produces configurations which contribute significantly to the magnetic dipole moment of odd-*A* nuclei. It also reveals a potential for reproducing their experimental values which proves impossible if this interaction is neglected.

DOI: 10.1134/S1063778816060193

1. INTRODUCTION

The perception of a nucleon orbiting around a doubly even inert core lead to numerous valuable conclusions about the validity of the nuclear single-particle shell model. One of its undoubted validations is the reproduction of the experimentally measured nuclear spins and parities [1]. This picture needs amendments if continuously varying quantities such as the energy of levels and the transitions between them as well as the electric and magnetic moments are to be described. Departing from the magic configurations—the landmarks of the extreme shell model—parts of the nucleon–nucleon interaction which cannot be incorporated in the mean field start to play a significant role in determining the spectroscopic properties of the nuclear states. This residual interaction imposes a many-body problem whose solution is often based on physical insights. For odd-*A* nuclei this problem can be approached by the interaction between the last

nucleon with the different modes of the even–even core. In this work we present the most important results based on the particle–core coupling concept for the lowest-lying states which have been obtained from the authors during the last decade. The outcome from these research facilitates the process of describing the nuclear properties far away from the valley of stability which is one of the major directions in the development of nuclear physics.

A model which consistently incorporates the interaction between the different vibrational modes of the even–even core with the last nucleon is the well established quasiparticle–phonon nuclear model [2]. Originally [3] it treated the influence of the excited states of the core on the last nucleon—its shift in energy, fragmentation as well as its decay properties. The achievements of this approach for medium excitation energies are reviewed in [4, 5]. In a series of papers [6–9] other authors and we applied efforts to tackle the fluctuations of the ground state, the so-called zero-point motion, and their effects on the lowest-lying states in odd–even isotopes. In physical terms, we suggested the existence of (quasi)nucleons and phonons in the ground state of the even–even cores. Due to the limited applicability of the standard perturbation theory to this problem [10], we applied

*The text was submitted by the authors in English.

¹⁾Joint Institute for Nuclear Research, Dubna, Russia.

²⁾Institute for Advanced Physical Studies, New Bulgarian University, Sofia.

**E-mail: mishev@theor.jinr.ru

***E-mail: voronov@theor.jinr.ru

the Random Phase Approximation (RPA) [7] as well as a state-of-the-art realization of the more consistent but complicated extended RPA (ERPA) [8]. These approximate methods yield progressively improving results for even–even nuclei when applied to calculating both energies and transitions.

The main focus of our research is the effects of the particle–vibration coupling on the various measured quantities related to states close to the ground state. In this respect the generation of the single-particle basis is performed using a standard Woods–Saxon mean field. On top of it we added pairing correlations as well as different channels of the long-range part of the residual interaction. In QPM the respective Hamiltonian is diagonalized approximately in a step-by-step procedure by introducing fictitious particles—the quasinucleons and the phonons. This approach gives the model the advantage of being flexible and versatile enough to be tailored to focus on the physical phenomena of interest. The latter happens to be quite helpful not only in terms of reducing the amount of performed numerical operations but also to keep a steadfast eye on the physical outcome of the calculations.

In this review paper we briefly outline in Section 2 the main elements of the theory with respect to the studied quantities. Results from calculations for the energies of the low-lying states, the transitions between them as well as their magnetic moments are presented in Section 3. Perspective directions for future developments in this area as well as the potential improvements of the current scheme are discussed in the concluding Section 4.

2. MODEL OVERVIEW

The QPM approach to the nuclear many-body problem involves assumptions related to the Hamiltonian of the system, the choice of model wave functions and finally the set of approximation methods for solving the Schrödinger equation. A key feature of this model is the introduction of two types of virtual particles which represent the basic modes of nuclear excitations—the single-particle and vibrational motions. These particles are the Bogoliubov quasiparticles and the nuclear phonons which are formulated as

$$\alpha_{jm} = u_j a_{jm} - (-)^{j-m} v_j a_{j-m}^\dagger \quad (1)$$

and

$$Q_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} \left[\psi_{jj'}^{\lambda i} A(jj'; \lambda\mu) - (-1)^{\lambda-\mu} \varphi_{jj'}^{\lambda i} A^\dagger(jj'; \lambda - \mu) \right], \quad (2)$$

respectively. The spherical single-nucleon basis states a_{jm} are used throughout this paper. Details on the quasiparticle–phonon nomenclature, which we follow in this paper, can be found in [2].

In order to diagonalize a complex Hamiltonian like the one involving short-range pairing and long-range interactions on top of the mean field

$$H = H_{\text{MF}} + H_{\text{RAIR}} + H_{\text{RES}}, \quad (3)$$

one transforms it to obtain a picture of freely propagating and interacting quasiparticles and phonons. Initially, the first two terms of the Hamiltonian are diagonalized by using the BCS ansatz. From the long-range part of the residual interaction we consider in our research the multipole and spin-multipole terms:

$$H_{\text{RES}} = H_M + H_{\text{SM}}, \quad (4)$$

which are expressed as

$$H_M = -\frac{1}{2} \sum_{\substack{\lambda \\ \rho=\pm 1}} \left(\kappa_0^{(\lambda)} + \rho \kappa_1^{(\lambda)} \right) \times \sum_{\substack{\mu \\ \tau=n,p}} M_{\lambda\mu}^\dagger(\tau) M_{\lambda\mu}(\rho\tau), \quad (5)$$

$$H_{\text{SM}} = -\frac{1}{2} \sum_{\substack{L=\lambda, \lambda\pm 1 \\ \rho=\pm 1}} \left(\kappa_0^{(\lambda L)} + \rho \kappa_1^{(\lambda L)} \right) \times \sum_{\substack{M \\ \tau=n,p}} (S_{LM}^\lambda)^\dagger(\tau) S_{LM}^\lambda(\rho\tau), \quad (6)$$

where the index τ enumerates the neutron (n) and proton (p) subsystems.

$$M_{\lambda\mu}^\dagger = \sum_{jj'mm'} \langle jm | i^\lambda R_\lambda(r) Y_{\lambda\mu} | j'm' \rangle a_{j'm'}^\dagger a_{jm} \quad (7)$$

and

$$(S_{LM}^\lambda)^\dagger = \sum_{jj'mm'} \langle jm | i^\lambda R_\lambda(r) [\sigma Y_\lambda]_{LM} | j'm' \rangle a_{j'm'}^\dagger a_{jm} \quad (8)$$

are the single-particle multipole and spin-multipole operators [2]. From the sum over L in Eq. (6) we include only the terms with $L = \lambda - 1$. In the phonon space, the eigenstates of this part of the interaction are of unnatural parity $(-1)^{L-1}$. The reduced matrix elements related to equations (7) and (8) are denoted by $f_{jj'}^{(\lambda)}$ and $f_{jj'}^{(\lambda L)}$, respectively.

The transformed Hamiltonian (4) in terms of quasiparticle and phonon operators, which we skip here for brevity, embodies explicitly the interaction between these two types of objects. The effect of this

interaction on the physical system under consideration hinges on the elemental content introduced in the wave function. We elaborated on multi-particle–multi-hole configurations of the ground states of even–even nuclei since their properties determine to a large extent the structure of the lowest-lying states of odd- A nuclei. For that reason in addition to configurations built on top of the ground state we allowed excitation channels through annihilation of quasiparticle⊗phonon configurations existing in this ground state. A superposition of such elementary excitations forms the wave function which we have explored in our studies:

$$\Psi_\nu(JM) = \left[C_{J\nu} \alpha_{JM}^\dagger + \sum_{j\lambda i} D_{j\lambda i}(J\nu) P_{j\lambda i}^\dagger(JM) \right]$$

$$- E_{J\nu} \tilde{\alpha}_{JM} - \sum_{j\lambda i} F_{j\lambda i}(J\nu) \tilde{P}_{j\lambda i}(JM) \Big] | \rangle, \quad (9)$$

with $| \rangle$ denoting the ground state of the even–even core.

The coefficients from Eq. (9) are subject to the equation of motion [11]

$$\begin{aligned} & \langle \{ \delta O_{JM\nu}, H, O_{JM\nu}^\dagger \} \rangle \quad (10) \\ & = \eta_{J\nu} \langle \{ \delta O_{JM}, O_{JM}^\dagger \} \rangle, \end{aligned}$$

which yields a generalized eigenvalue problem owing to the non-orthogonality of the basis states induced by the Pauli principle

$$\begin{aligned} & \begin{pmatrix} \varepsilon_J & V(Jj'\lambda'i') & 0 & -W(Jj'\lambda'i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda'i') & -\varepsilon_J & -V(Jj'\lambda'i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda i') \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix} \quad (11) \\ & = \eta_{J\nu} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - \mathcal{L}(Jj\lambda i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 - \mathcal{L}(Jj\lambda i) \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}, \end{aligned}$$

where

$$V(Jj\lambda i) = -\frac{1}{\sqrt{2}}(1 + \mathcal{L}(Jj\lambda i))\Gamma(Jj\lambda i), \quad (12)$$

$$\begin{aligned} W(Jj\lambda i) &= -\frac{1}{4} \frac{\pi_\lambda}{\pi_J} (1 + \mathcal{L}(Jj\lambda i)) \quad (13) \\ &- \mathcal{L}(jJ\lambda i) \sum_{i'\tau_0} \mathcal{A}_{\tau_0}(\lambda i i') \varphi_{j_j}^{\lambda i'} \end{aligned}$$

and

$$\begin{aligned} K_J(j\lambda i j'\lambda' i') &= \delta_{j j'} \delta_{\lambda \lambda'} \delta_{i i'} (1 + \mathcal{L}(Jj\lambda i)) \quad (14) \\ &\times (\omega_{\lambda i} + \varepsilon_j - \mathcal{R}(Jj\lambda i)). \end{aligned}$$

The emerging off-diagonal quantities $V(Jj\lambda i) \sim \langle | \alpha_{JM} H \alpha_{jm}^\dagger Q_{\lambda\mu i}^\dagger | \rangle$ and $W(Jj\lambda i) \sim \langle | \alpha_{JM} H \alpha_{jm} Q_{\lambda\mu i} | \rangle$ represent the interaction between quasiparticles and phonon in both the excited and the ground states, respectively. The exclusion of certain three-quasiparticle states inhibited by the

Pauli principle, as well as other effects, related to this principle are taken into account by using the exact commutation relations between the quasiparticle and phonon operator. These blocking effects are quantified by $[1 - \mathcal{L}(Jj\lambda i)]$ which reduces the weight of certain quasiparticle⊗phonon configurations as well as by $\mathcal{R}(Jj\lambda i)$ inducing a shift in the energy of such configurations. Hereafter $\pi_\lambda = \sqrt{2\lambda + 1}$.

Having determined the structural composition of the odd–even nucleus, estimates for the observable quantities of interest are obtained by evaluating the average values of the corresponding operators. For the magnetic dipole and electric quadrupole moments they are defined as

$$\mu_1(J\nu) = \sqrt{\frac{4\pi}{3}} \langle J J \nu | \mathcal{M}(M; 10) | J J \nu \rangle, \quad (15)$$

$$Q_2(J\nu) = \sqrt{\frac{16\pi}{5}} \langle J J \nu | \mathcal{M}(E; 20) | J J \nu \rangle, \quad (16)$$

Values of the matrix elements $V^2(Jj\lambda i)$ and $W^2(Jj\lambda i)$ as well as other Pauli induced factors calculated for $J^\pi = 1/2^+$ at the lowest poles

State	Nuclide	Pole's structure	V^2	W^2	\mathcal{R}	$1 + \mathcal{L}$
$1/2^+$	^{131}Ba	$2d_{3/2} \otimes 2_1$	0.0361	1.302	-0.36	0.93
	^{133}Ba	$2d_{3/2} \otimes 2_1$	0.1225	2.1	-0.43	0.932
	^{135}Ba	$2d_{3/2} \otimes 2_1$	0.4	0.837	-0.11	0.99
	^{137}Ba	$2d_{3/2} \otimes 2_1$	0.56	0.49	-0.053	0.9948

where the electric and magnetic multipole operators are expressed as

$$\mathcal{M}(X; \lambda\mu) = \frac{1}{\pi\lambda} \sum_{\substack{j_1 m_1 \\ j_2 m_2}} (-1)^{j_2 - m_2} \mathcal{F}_{j_1 j_2}^{(\lambda)}$$

$$\times \langle j_1 m_1, j_2 - m_2 | \lambda\mu \rangle a_{j_1 m_1}^+ a_{j_2 m_2}.$$

$\mathcal{F}_{j_1 j_2}^{(\lambda)}$ are the reduced single particle matrix elements:

$$\mathcal{F}_{j_1 j_2}^{(\lambda)} = \begin{cases} e \langle j_2 || r^\lambda i^\lambda Y_{\lambda\mu} || j_1 \rangle, & \text{for electric transitions,} \\ \mu_0 \left(g_s \langle j_2 || s \nabla (r^\lambda Y_{\lambda\mu}) || j_1 \rangle + g_l \frac{2}{\lambda+1} \langle j_2 || l \nabla (r^\lambda Y_{\lambda\mu}) || j_1 \rangle \right), & \text{for magnetic transitions.} \end{cases} \quad (17)$$

Here e and μ_0 are the electron charge and nuclear magneton, respectively.

3. RESULTS

From the various manifestations of the correlated motion of nucleons in odd- A isotopes we present below characteristic shifts in the calculated energies of nuclear levels and their magnetic moments which bring the obtained results closer to the experimental values. Importantly, the described effects cannot be reproduced if the even-even core is understood as composed of particles moving independently one from the other. The calculations have been performed systematically in a series of barium and cadmium isotopes with shapes close to spherical and as well for such belonging to the transitional region with $E(4_1^+)/E(2_1^+) < 2.5$.

Prior to analyzing the results obtained for measurables the magnitudes of the quantities introduced in the previous section for the case of ^{131}Ba are presented in the table. As shown in [7], the correlations in the ground state tend to increase together with the quantities $W(Jj\lambda i)$ with decreasing number of neutrons with respect to the magic number 82. In the table the squares of the interaction vertices $W^2(Jj\lambda i)$ and $V^2(Jj\lambda i)$ are evaluated at the states $|j\lambda i\rangle$ with lowest energy. Following the Green's function representation of Eq. (11) (conf. [7]) it can be concluded that the contributions from quasiparticle \otimes phonon configurations, lying at higher energies, peter out

because of the large values of the corresponding denominators and the weakened quasiparticle \otimes phonon interaction in the ground state. We therefore examine only levels in the vicinity of the Fermi level, since they interact with the ground state stronger than for those lying at higher energies. Having the lowest quasiparticle energies, the quasiparticle states $\nu 1h11/2$, $\nu 3s1/2$, and $\nu 2d3/2$ experience the greatest part of the interaction with the remaining quasiparticles in the ground state. It is also argued that the values of $\mathcal{R}(Jj\lambda i)$ tend to grow as the isotopes move away from the magic number 82 of the neutron subsystem. This is explained by the strong dependence of $\mathcal{R}(Jj\lambda i)$ on the degree of collectivity of the vibrational states in the even-even nuclei [12].

3.1. Shifts in the Energies

In this section we discuss the effect of increasing the density of low-energy states which we attribute to the correlations in the ground states. We direct our attention to the third and fourth columns of Fig. 1. In the fourth column in this figure we can see higher densities of the states with energies of up to 1 MeV as compared to the results in the corresponding third column which disregard the backward-going terms in the odd-even nuclear wave function. The correctness of this result is borne out by the experimental data which are presented in the first column. The reason for that is simple: the quasiparticle-phonon interaction $W(Jj\lambda i)$ pushes the energy of the lowest solution up from its unperturbed position, which

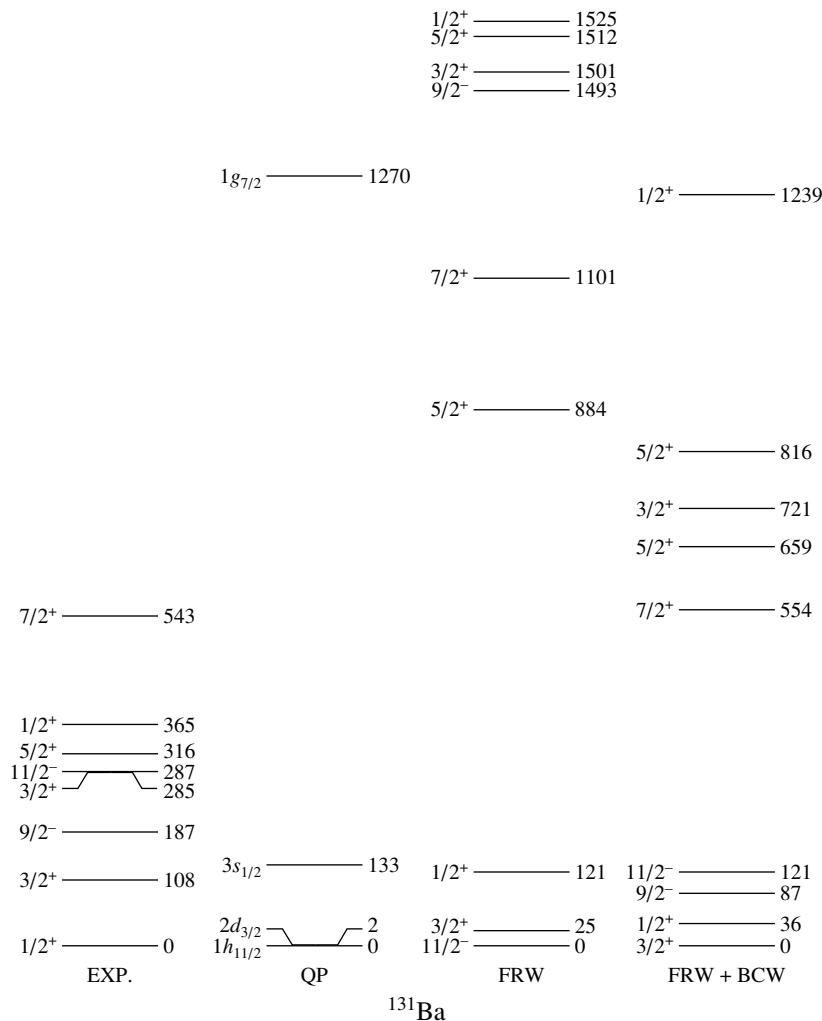


Fig. 1. The lowest part of the energy spectrum of ^{131}Ba (in keV). The calculations are performed within a model version including backward and forward amplitudes, denoted by BCW + FRW, as well as by using a model version with only forward amplitudes, denoted by FRW.

thereby approaches the energy of the lowest pure quasiparticle \otimes phonon state. As a result, it becomes closer to the second solution with the same quantum numbers, thus reducing the energy gap between the two lowest eigenvalues. It is worthwhile to notice that the forward $V(Jj\lambda i)$ and backward vertices $W(Jj\lambda i)$ work in opposite directions as the former shifts the first solution to lower energies as compared to the quasiparticle energy [2].

This same important effect is visualized in Fig. 2, where in addition the strengths of the individual levels with signature $5/2^+$ are plotted. As seen from this figure, the fragmentation of the quasiparticle strengths due to the backward amplitudes increases substantially. We found a similar behavior for the rest of the states from the valence shell in all nuclei from the considered region.

3.2. Magnetic Moments

The ground state admixtures modify considerably the results of calculations for the magnetic moments as will be argued below for the case of the isotopes $^{117-127}\text{Cd}$ [9]. Although the contribution to the wave function coming from configurations arising to the coupling of the last nucleon with the magnetic giant dipole resonance of the core are small, their influence on the magnetic moments are significant because of the strong $M1$ transition to the ground state. The effects of the $M1$ giant resonance on the magnetic moments (the Arima–Horie effect) were the main focus of many research studies, while the contribution from other modes seems to be a less explored territory. In [13] a systematic theoretical analysis of experimental data on magnetic moments in different nuclei is performed utilizing the theory of finite Fermi systems. The influence on the magnetic moments coming from

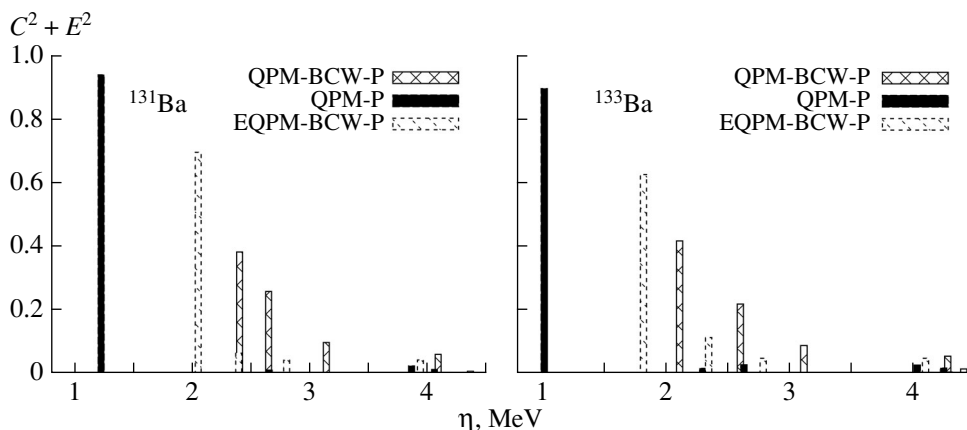


Fig. 2. Quasiparticle strength distribution ($C^2 + E^2$) of the state $\nu 2d_{5/2}$ in ^{131}Ba and ^{133}Ba . The quadrupole–quadrupole interaction strength $\kappa^{(2)}$ is kept constant in the calculations within the three model versions.

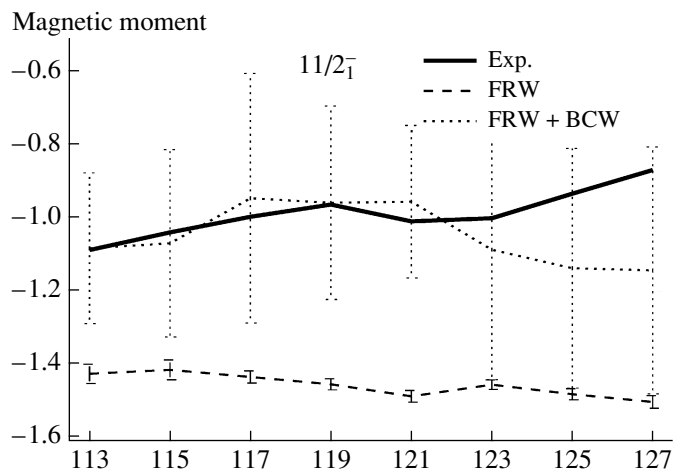


Fig. 3. Magnetic moments in units μ_0 of the first $11/2^-$ state in the chain $^{111-127}\text{Cd}$. The solid line represents the experimental values [15, 16]. The calculations are performed within BCW+FRW (dotted line) and FRW (dashed line) versions of the model. The error bars give the uncertainty in evaluating the magnetic moments when varying the strength of the quadrupole–quadrupole interaction in a wide range of values.

the coupling with the low-lying collective quadrupole and octupole core excitations is studied in [14].

The results from the calculations performed using the featured QPM versions are plotted in Fig. 3 and are compared to the experimental values. The dotted and dashed lines in this figure depict the fact that the interaction between the last quasiparticle and the ground-state phonon admixtures produces configurations which contribute significantly to the magnetic moment of odd- A nuclei and reveal a potential for reproducing their experimental values which proves impossible if this interaction is neglected. The importance of the contributions to the magnetic dipole moment coming from different components of the wave function (conf. [9]) is shown in Fig. 4. The enhanced fragmentation due to the quasiparticle–phonon interaction in the ground state leads systematically to

shrunk values of the single quasiparticle contribution μ_{qp-qp} and to an increase in the quasiparticle–phonon contribution μ_{qp-ph} leading to an overall decrease in the magnitude of the magnetic moment. The enhancement of the magnetic transitions between different quasiparticle \otimes phonon configurations, given by μ_{ph-ph} , is due to configurations involving a quadrupole phonon, of which $\nu 1h_{11/2} \otimes 2_1^+$ plays the most important role. It is worth noting that because of the weakened coupling between the quasiparticles and the quadrupole phonons in the core’s ground state near the neutron shell closure, the quantity μ_{ph-ph} tends to diminish while μ_{qp-qp} remains almost unchanged along the isotope chain. This interaction, however, leaves the first order core polarization term μ_{qp-ph} , describing the magnetic transitions

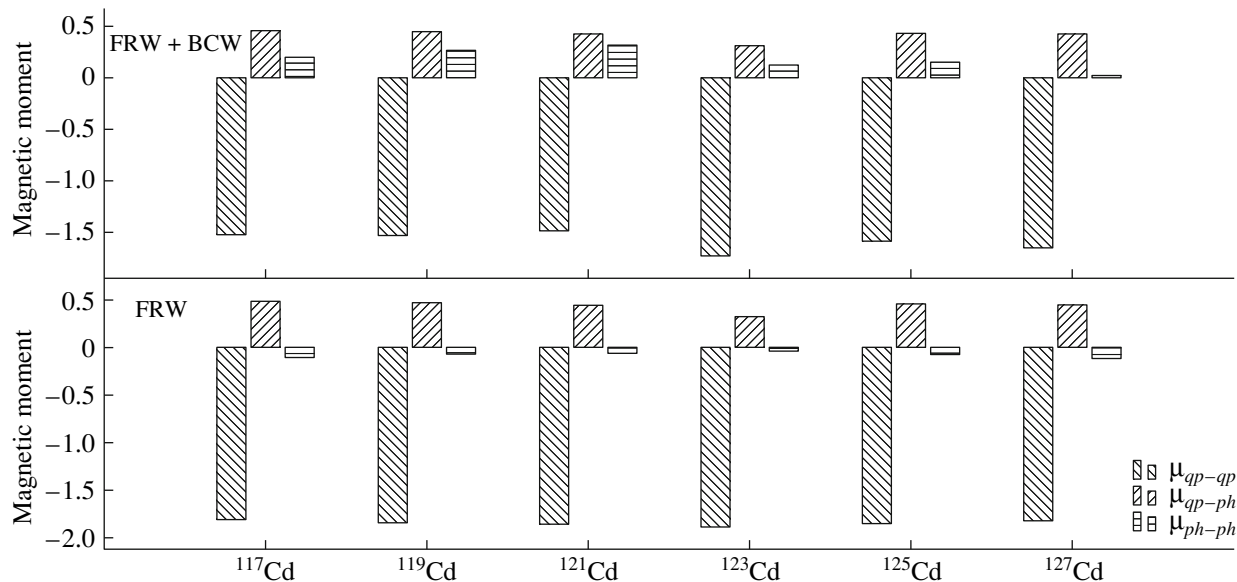


Fig. 4. Contributions from different components of the wave function to the magnetic dipole moment of the $11/2$ states in the series of isotopes $^{117-127}\text{Cd}$.

between quasiparticle and quasiparticle $\otimes 1^+$ -phonon states virtually unaffected because the latter configurations represent a negligible part in the $11/2_1^-$ state mixture.

4. OUTLINE AND CONCLUSIONS

The energies and the magnetic dipole moments of the low-lying states in odd- A barium and cadmium nuclei are found to be significantly affected by the correlations in the ground state. The obtained corrections allow one to reproduce the experimental values in open-shell nuclei which proves impossible if the existence of the quasiparticle \otimes phonon configurations in the ground states of even–even nuclei is ignored. However, despite its capacity of reaching the experimental values, the described theoretical development suffers from the shortcoming (conf. [6, 7]) that the residual interaction strength which yields results that are of sound agreement with the odd- A experimental data, generate substantially less collective 2_1^+ states in the even–even cores than the ones implied from the data for the neighboring even–even nuclei. The origin of this inconsistency is the set of approximation techniques embedded in the considered QPM versions, namely the BCS and RPA, which tend to overestimate the degree of correlations in the ground state for open-shell nuclei. One path to overcome this problem is to apply more consistent RPA-like approaches, or to use tractable methods based on the variational principle for the ground state as in [17].

REFERENCES

1. M. G. Mayer, Phys. Rev. **78**, 16 (1950).
2. V. G. Soloviev, *Theory of Complex Nuclei* (Pergamon, Oxford, 1976); *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Institute of Physics, Bristol and Philadelphia, 1992).
3. A. I. Vdovin and V. G. Soloviev, Teor. Mat. Fiz. **19**, 275 (1974) [Theor. Math. Phys. **19**(2), 275 (1974)].
4. A. I. Vdovin, V. V. Voronov, V. G. Soloviev, and Ch. Stoyanov, Fiz. Elem. Chastits At. Yadra **16**, 245 (1985) [Sov. J. Part. Nucl. **16**, 246 (1985)].
5. S. Galès, Ch. Stoyanov and A. I. Vdovin, Phys. Rep. **166**, 125 (1988).
6. V. Van der Sluys, D. Van Neck, M. Waroquier, and J. Ryckebusch, Nucl. Phys. A **551**, 210 (1993).
7. S. Mishev and V. V. Voronov, Phys. Rev. C **78**, 024310 (2008).
8. S. Mishev and V. V. Voronov, Phys. Rev. C **82**, 64312 (2010).
9. S. Mishev and V. V. Voronov, Phys. Rev. C **92**, 044329 (2015).
10. J. Li, J. X. Wei, J. N. Hu, et al., Phys. Rev. C **88**, 064307 (2013).
11. D. J. Rowe, *Nuclear Collective Motion* (Menthuen, London, 1970).
12. C. Z. Khuong, V. G. Soloviev, and V. V. Voronov, J. Phys. G **7**, 151 (1981).
13. I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Yad. Fiz. **71**, 493 (2008) [Phys. Atom. Nucl. **71**, 469 (2008)].
14. A. I. Levon, S. N. Fedotkin, and A. I. Vdovin, Yad. Fiz. **43**, 1416 (1986) [Sov. J. Nucl. Phys. **43**, 912 (1986)].
15. N. J. Stone, At. Data Nucl. Data Tables **90**, 75 (2005).
16. D. T. Yordanov et al., Phys. Rev. Lett. **110**, 192501 (2013).
17. S. Mishev, Phys. Rev. C **87**, 064310 (2013).