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NUCLEI
Theory

$E2$ Transitions between Excited Single-Phonon States: Role of Ground-State Correlations

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Abstract—The probabilities for $E2$ transitions between low-lying excited 3^- and 5^- single-phonon states in the ^{208}Pb and ^{132}Sn magic nuclei are estimated on the basis of the theory of finite Fermi systems. The approach used involves a new type of ground-state correlations, that which originates from integration of three (rather than two, as in the random-phase approximation) single-particle Green's functions. These correlations are shown to make a significant contribution to the probabilities for the aforementioned transitions.

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"Each nucleus requires an individual approach."
V. G. Soloviev

1. INTRODUCTION

Theoretical methods for taking into account quasiparticle–phonon interaction that were implemented within the quasiparticle–phonon model developed by V.G. Soloviev and his coauthors [1, 2] are quite seminal and have withstood the tests of time. Not only has the quasiparticle–phonon model been widely applied in low-energy nuclear physics, but it is still undergoing a vigorous development—for example, in what is concerned with taking into account the consistency of the mean field and effective interaction on the basis of the Skyrme energy density functional [3]. It is also necessary to mention the important results obtained earlier within the quasiparticle–phonon model that concern structures in photoabsorption cross sections for many nuclei. It was shown (see, for example, [2, 4, 5]) that a Lorentzian extrapolation is overly rough in the region extending up to the neutron-separation energy, so that, sometimes, the existence of substructures (irregularities) in cross sections is not consistent with a Lorentzian extrapolation of giant dipole resonances. This issues have become ever more important in the course of the past decade in connection with the development of experimental techniques (for an overview, see [6]) and the growing demand for nuclear data [9, 10]—first of all, for calculating radiative strength functions, which are needed for describing all nuclear reactions involving photons. There are two definitions of the radiative strength function—one involves transitions between the ground state and excited states, while the other, which is used, as a rule, contains transitions between excited states. The radiative strength function of the first kind was calculated on the basis of the quasiparticle–phonon model [5] in the region around the neutron-separation energy, where reasonably good agreement with experimental data was obtained. However, it is of interest to consider the whole region below the neutron-separation energy—that is, the region of the pygmy dipole resonance [6]. In order to calculate the radiative strength function of the second kind, it is necessary to know here the nature of the nuclear levels being considered. Therefore, use is made, as a rule, of the Brink–Axel hypothesis [7], which states that, on the basis of each excited state, it is possible to construct a giant dipole resonance (in the present-day formulation, any giant resonance) of the same form as that based on the ground state, its features being independent of the nature of this state (for more details, see [8]).

In textbooks on nuclear data [9, 10], use is extensively made of the assumption that the cross section for dipole photoabsorption at energies below the neutron-separation energy can be described in terms of various present-day forms of a smooth Lorentzian curve. In [9], for example, there are six versions of this curve. As was indicated above, it was shown within the quasiparticle–phonon model that this is at odds with the existence of structures in the photoabsorption cross section. It seems that the community of nuclear-data specialists perceived this circumstance in 2006 (even though the authors of [11, 12] emphasized earlier the importance of taking into

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account microscopic effects in considering nuclear data), when microscopic calculations of radiative strength functions on the basis of the Hartree–Fock–Bogolyubov method and the quasiparticle random-phase approximation (QRPA) were included in [9]. The approach in question has a substantially higher predictive power, and this is of crucial importance for unstable nuclei and astrophysics and permits describing ground and excited states of nuclei on equal footing. As yet another example of intensively employing methods of the quasiparticle–phonon model, we can indicate studies (see, for instance, [13]) devoted to combinatorial calculations of the level density. The combinatorial self-consistent level-density models developed by Goriely and other authors [10] (for a brief overview, see [14]) appeared as a natural continuation of those studies.

However, it was shown at a phenomenological level in the studies of a group from Oslo (see, for example, [15]) that, in order to explain experimental data on radiative strength functions, the QRPA method should be supplemented with taking into account coupling to phonons. A self-consistent approach to calculating radiative strength functions over the whole energy range was developed in [8, 14, 16–19]. This approach employed the Green's function method and took into account QRPA effects, a discretized single-particle spectrum, and coupling to phonons. These calculations of radiative strength functions and respective radiative features revealed that the contribution of coupling to phonons is quite sizable and is mandatory for explaining experimental data (provided that the Brink–Axel hypothesis is valid). Thus, we see that, in accordance with the early results based on the quasiparticle–phonon model and the recent results of applying the self-consistent approach, there arises the possibility of dispensing with a Lorenzian extrapolation of the photoabsorption cross section. But if one does not invoke the Brink–Axel hypothesis (in general, its validity is not obvious—see [8]), then, according to the definition of the radiative strength function, a microscopic description of this quantity requires calculating electromagnetic transitions at least between excited single-phonon states, for which an extensive body experimental data is already available. An attempt at solving this problem by the Green's function method is among the purposes pursued in the present study. It should be noted that a similar problem was tackled in [20] and [21] within non-self-consistent methods of the quasiparticle–phonon model and Green's functions, respectively.

In the present study, we also aim at an analysis of ground-state correlations belonging to a new type. Earlier, Voronov and his coauthors [22] worked on evolving procedures for an improved inclusion of ground-state correlations, implying ground-state correlations known within the QRPA framework. Such correlations may be called two-quasiparticle correlations, since, in terms of Green's functions, they are associated with integration of two Green's functions. Those studies relied on the fact that, within the QRPA framework, ground-state correlations are small. Improved QRPA phonons were used to describe effects of quasiparticle–phonon interaction, with the result that the equations of the quasiparticle–phonon model for even–even nuclei were generalized in such a way as to include phonons of the extended QRPA scheme [22]. For odd nuclei, ground-state correlations were analyzed in [23, 24] on the basis of the quasiparticle–phonon model, as well as in [25] (without allowance for the Pauli exclusion principle). Such correlations may be called quasiparticle–phonon ground-state correlations. The calculations performed in [22–25] showed that the inclusion of such ground-state correlations improved agreement with experimental data but was unlikely to make a sizable contribution to the features being studied. In [26] and [27], two-phonon ground-state correlations were also considered on the basis of, respectively, the quasiparticle–phonon model and the Green's function technique. Thus, the problem of ground-state correlations has a rich and long history, which deserves a continuation.

Calculations of ground-state correlations belonging to a new type were recently performed in the problem of quadrupole moments of the first excited 2^+ levels in magic and semimagic nuclei, and a large contribution of these ground-state correlations (50% to 60% of observables) was likely to be observed for the first time [28]. A triangle that contains an integral of three Green's functions is the main quantity that determines the effect. As any other Feynman diagram, it involves diagrams going backward—that is, ground-state correlations. These ground-state correlations may be called three-quasiparticle ground-state correlations, in contrast to two-quasiparticle ground-state correlations appearing in the QRPA scheme. The quadrupole-moment problem is that where the phonons in the triangle are identical and corresponds to the diagonal case of the triangle.

In the present study, we consider the nondiagonal case that corresponds to a transition between two single-phonon states. This problem is more complicated algebraically than the problem of calculating the quadrupole moment in an excited state. We study probabilities for $E2$ transitions between single-phonon states, describing, in just the same way as in [28], the problem within the QRPA scheme. It will be seen below that ground-state correlations of the new type were disregarded in [20]. In order to estimate the probabilities for $E2$ transitions, we make several simple approximations in our calculations. Specifically, effects involving ground-state correlations of the new type are estimated in the same approximations as those that are used in dealing without these correlations.

2. DESCRIPTION OF THE METHOD

We make use of the method intended for analyzing anharmonic effects and developed by Khodel for magic nuclei [29]. The method takes consistently into account all effects proportional to the small parameter g^2, where g is the dimensionless phonon-production amplitude. The amplitude $M_{ss'}$ for the transition between the excited single-phonon states s and s' has the form (in the representation of single-particle wave functions) [28, 30]

$$M_{ss'} = M^{(1)}_{ss'} + M^{(2)}_{ss'} \tag{1}$$
$$= \sum_{123} [V_{12}(g^s_{31})^* g^{s'}_{23} A^{(1)}_{123} + V_{12} g^{s'}_{31} (g^s_{23})^* A^{(2)}_{123}],$$

where the index 1 runs through the following set of values: (n_1, j_1, l_1, m_1). This corresponds to the two Feynman (triangle) diagrams in the figure. Here, V is the vertex that describes the medium effect (nucleon–nucleon interaction) in the theory of finite Fermi systems [31] (in our case, $E2$ transitions, which generate the effect of medium quadrupole polarization); g is the phonon-production amplitude described within the QRPA scheme; and $A^{(1)}$ is an integral of three Green's functions,

$$A^{(1)}_{123}(\omega_s, \omega_{s'}) \tag{2}$$
$$= \int G_1(\varepsilon) G_2(\varepsilon + \omega) G_3(\varepsilon + \omega_s) d\varepsilon$$
$$= \frac{[(1-n_1)(1-n_2)n_3 - n_1 n_2 (1-n_3)]}{(\varepsilon_{31} - \omega_s)(\varepsilon_{32} - \omega_{s'})}$$
$$+ \frac{1}{\varepsilon_{12} + \omega} \left[\frac{n_1(1-n_2)(1-n_3) - (1-n_1)n_2 n_3}{\varepsilon_{13} + \omega_s} - \frac{(1-n_1)(1-n_3)n_2 - (1-n_2)n_1 n_3}{\varepsilon_{23} + \omega_{s'}} \right].$$

Further, we have

$$A^{(2)}_{123}(\omega_s, \omega_{s'}) = A^{(1)}_{123}(-\omega_{s'}, -\omega_s). \tag{3}$$

In expression (1), we retained only those terms that correspond to the triangles in the figure, which involve three Green's functions, and omitted other terms that contain $\delta\mathcal{F}$ and which take into account the change in the effective interaction $\mathcal{F}$ in the phonon field. Since they made a small contribution in calculating the quadrupole moments of the first 2^+ states [28], there is every reason to believe that their quantitative role is insignificant in our nondiagonal problem as well.

After summation over the projections m_1, m_2, and m_3, we obtain

$$M_{ss'} = (-1)^{M_s+1} \begin{pmatrix} I_s & I_{s'} & L \\ -M_s & M_{s'} & M \end{pmatrix} \tag{4}$$
$$\times \sum_{123,p,n} \left[\begin{Bmatrix} I_s & I_{s'} & L \\ j_2 & j_1 & j_3 \end{Bmatrix} \langle 1||V \parallel 2\rangle \times \langle 3||g^s||1\rangle \langle 2||g^{s'} \parallel 3\rangle A^{(1)}_{123} \right.$$
$$+ (-1)^{(I_{s'}+I_s+L)} \begin{Bmatrix} I_{s'} & I_s & L \\ j_2 & j_1 & j_3 \end{Bmatrix} \langle 1||V \parallel 2\rangle$$
$$\left. \times \langle 3||g^{s'}||1\rangle \langle 2||g^s \parallel 3\rangle A^{(2)}_{123} \right],$$

where $1 = (n_1, j_1, l_1)$. In the second term, we interchange the indices 1 and 2 and make use of the relation $\langle 1||V||2\rangle = (-1)^{j_1-j_2}\langle 2||V||1\rangle$. After that, we finally find that the reduced probability for the $s \to s'$ transition of energy $\omega = \omega_s - \omega_{s'}$ has the form

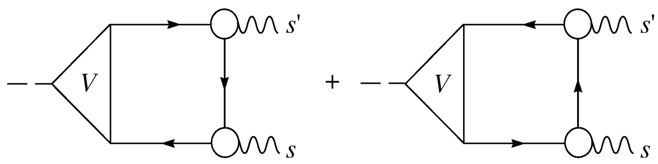
$$B(E(M)L) = \frac{1}{2I_s+1} |\langle \parallel M_{ss'} \parallel \rangle|^2, \tag{5}$$

where

$$M_{ss'} = \sum_{123} \begin{Bmatrix} I_s & I_{s'} & L \\ j_2 & j_1 & j_3 \end{Bmatrix} V_{12} g^s_{31} g^{s'}_{23} \tag{6}$$
$$\times \left[A^{(1)}_{123}(\omega_s, \omega_{s'}) + (-1)^{(I_s+I_{s'}+L)} A^{(1)}_{213}(-\omega_{s'}, -\omega_s) \right].$$

Here, we have introduced the notation $\langle 1||V||2\rangle = V_{12}$, etc. For the bracketed expression on the right-hand side of Eq. (6), we obtain

$$\left[A^{(1)}_{123}(\omega_s, \omega_{s'}) + (-1)^{(I_s+I_{s'}+L)} \right. \tag{7}$$
$$\left. \times A^{(1)}_{213}(-\omega_{s'}, -\omega_s) \right] = \left[(1-n_1)(1-n_2)n_3 \right.$$
$$\left. - n_1 n_2 (1-n_3) \right] \left(\frac{1}{(\varepsilon_{31} - \omega_s)(\varepsilon_{32} - \omega_{s'})} + \frac{1}{(\varepsilon_{31} + \omega_s)(\varepsilon_{32} + \omega_{s'})} \right)$$
$$+ \left[n_1(1-n_2)(1-n_3) - (1-n_1)n_2 n_3 \right]$$
$$\times \frac{-2\omega\omega_s - 2\varepsilon_{12}\varepsilon_{13}}{(\omega^2 - \varepsilon^2_{12})(\varepsilon^2_{13} - \omega^2_s)} + \left[n_2(1-n_1)(1-n_3) \right.$$
$$\left. - (1-n_2)n_1 n_3 \right] \frac{2\omega\omega_{s'} - 2\varepsilon_{12}\varepsilon_{23}}{(\omega^2 - \varepsilon^2_{12})(\varepsilon^2_{23} - \omega^2_{s'})}.$$



Transition amplitude $M_{ss'}$ (1).

The parenthetical terms in the second row on the right-hand side of Eq. (7) appear in the respective formula for the reduced transition probability in the problem that involves pairing [20] (for more details, see [28]). The remaining terms in (7), which, as we will see, make a significant quantitative contribution, were disregarded in [20]. Following [21], we call these remaining terms ground-state correlations (GSC). In the limiting case where there is no pairing and where $\omega = 0$, expression (7) reduces to the respective formula in [28].

3. ESTIMATING $B(E2)$ IN DOUBLY MAGIC NUCLEI

By means of Eqs. (5)–(7), we have calculated the probabilities for $E2$ transitions between the 3^- and 5^- excited low-lying single-phonon states in the ^{208}Pb and ^{132}Sn doubly magic nuclei. We have employed the following approximations:

(i) The phonon-production amplitude g was calculated on the basis of the Bohr–Mottelson model; that is,

$$g(\mathbf{r}) = \frac{\beta}{2L+1} r \frac{dU}{dr} Y_{LM}, \tag{8}$$

where β was determined on the basis of the experimental (for ^{208}Pb from [32]) and calculated (for ^{132}Sn from [33]) values of $B(EL)$ (see Table 1).

Table 1. Energies and total amplitudes of vibrations in ^{208}Pb and ^{132}Sn

Nucleus	Level	E, MeV	β
^{208}Pb	3_1^-	2.61	0.12
[32]	5_1^-	3.20	0.072
	5_2^-	3.71	0.034
^{132}Sn	3_1^-	4.352	0.082
[33]	5_1^-	4.943	0.018

(ii) It is rather difficult to calculate the vertex $V(\omega)$, but, in our case of doubly magic nuclei, the energy ω is small in relation to the energy of the first 3^- level, which corresponds to the pole of the vertex V; therefore, we can disregard the energy dependence of the vertex and to express it approximately in terms of the effective quadrupole charge [34, 35] as $V = e_{\rm eff} V^0$, where $e_{\rm pol} = e^n_{\rm eff} = 0.6$ and $e^p_{\rm eff} = 1.6$. This circumstance simplifies our task substantially.

In our calculations, we relied on the self-consistent single-particle scheme described in [28]. With allowance for the aforementioned approximations, our calculation is not fully self-consistent. However, there are reasons to believe that a comparative analysis of terms that involve ground-state correlations and terms that does not involve such correlations, which is the main objective of our present study, will be quite reliable, since we perform it on the basis of the same procedure.

The results of the calculations are given in Table 2. A comparison with the only reliable experiment for ^{208}Pb [$B(E2)_{\rm expt}$ =$28e^2$ fm^4] shows that, in view of the roughness of the approximations used, our estimate is quite reasonable. Our main objective was to clarify the role of ground-state correlations, and we found that, for the first transition in ^{208}Pb, the contribution of ground-state correlations increased $B(E2)$ by a factor of 1.5. The results for other transitions show that the contributions of ground-state correlations are quite sizable for them as well. In view of the incoherence of the terms in the transition amplitudes and a significant degree of inconsistency in our calculations, all this means that a quantitative contribution of ground-state correlations belonging to the new type is substantial and calls for a more thorough analysis.

It is of interest to examine the contribution of quadrupole polarizability of nuclei—that is, the difference between the vertex V and the bare vertex $e_q V^0$. For this purpose, we have calculated $B(E2)$ for the first transition in ^{208}Pb at $e_{\rm pol} = 0$ and obtained the value of $B(E2)$= $9.64e^2$ fm^4, which is nearly one-fourth as large as that in the case of $e_{\rm pol} = 0.6$. Thus, we have seen that, in calculating probabilities for

Table 2. Amplitudes and reduced probabilities for the $5^- \to 3^-$ $E2$ transitions in ^{208}Pb and ^{132}Sn (in e^2 fm^4) units

Nucleus	GSC ≠ 0				GSC = 0			Exp.
	transition	$M^n_{ss'}$	$M^p_{ss'}$	$B(E2)$	$M^n_{ss'}$	$M^p_{ss'}$	$B(E2)$	
^{208}Pb	$5^-_1 \to 3^-_1$	−3.42	−16.48	36.00	−2.16	−13.87	23.36	28 ± 2
	$5^-_2 \to 3^-_1$	−5.86	0.24	2.87	−4.82	1.20	1.19	
^{132}Sn	$5^-_1 \to 3^-_1$	−0.25	−0.08	0.01	−0.20	0.05	0.002	

transitions between single-phonon states, it is necessary to take simultaneously into account, in just the same way as in dealing with the static case [28], effects of nuclear polarizability and effects of ground-state correlations.

4. CONCLUSIONS

In the present study, we have estimated the probabilities for $E2$ transitions between the 3^- and 5^- excited states in the ^{208}Pb and ^{132}Sn magic nuclei. We have performed our calculations on the basis of quantum many-body theory—more precisely, within the approach that was developed in [30] and where effects of order g^2 are taken consistently into account. In contrast to the usual quantum-mechanical method implemented within the quasiparticle–phonon model, our approach contains new effects associated with ground-state correlations and caused by integration of three Green's functions. It turns out that, in the problem being considered, these effects are sizable, making a significant contribution to $B(E2)$—for example, they increase $B(E2)$ for the first transition in ^{208}Pb by a factor of 1.5. This is the main result of our present study. We have also shown that, in calculating probabilities for transitions between for single-phonon states, it is necessary to take simultaneously into account nuclear-polarizability effects and effects associated with ground-state correlations. It is of great interest to perform further calculations for transitions between single-phonon states in nuclei that involve pairing. The use of a more consistent approach to the problem being considered is also highly desirable.

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