ISSN 1063-7788, Physics of Atomic Nuclei, 2016, Vol. 79, No. 1, pp. 113–117. © Pleiades Publishing, Ltd., 2016.
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Total Cross Sections for Hadron Collisions on the Basis of the HPR1R2 Model

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Received February 20, 2015; in final form, August 24, 2015

Abstract—The results of a quantitative, statistically complete, description of the total-cross-section data obtained worldwide for hadron—hadron (photon—hadron) collisions and compiled in the Particle Data Group surveys are presented for several versions of a universal analytic parametrization of amplitudes for forward hadron—hadron (photon-hadron) scattering.

DOI: 10.1134/S1063778816010075

INTRODUCTION

New experimental data on total cross sections for nucleon—nucleon collisions at the c.m. collision energy of 7 TeV from experiments of the ATLAS Collaboration at the Large Hadron Collider (LHC, CERN) [1] required additionally tuning parameters of the HPR1R2 model. In this article, we present a parameter-dependent description of an extended set of experimental data from [2] on two observables that are determined by the elastic-scattering amplitudes $T^{ab}(s,t)$ for particles a and b at zero scattering angle $[(\theta = 0) \sim (t = 0)]$. These are the total collision cross sections $\sigma^{ab}_{\mathrm{tot}}(s) \sim \mathrm{Im}\, T^{ab}(s,0)$ and the parameter $\rho^{ab}(s) = \dfrac{\mathrm{Re}\, T^{ab}(s,0)}{\mathrm{Im}\, T^{ab}(s,0)}$.

The variables s and t are Mandelstam variables in GeV^2 units (see Kinematics, page 7, in http://pdg.lbl.gov/2012/reviews/contents_sports.html).

In the following, we will sometimes employ the generic symbol model(s_i) instead of $\sigma^{ab}_{\mathrm{tot}}(s_i)$ or $\rho^{ab}(s_i)$, thereby going over to a shorthand notation without loss of meaning of respective expressions.

EXPERIMENTAL DATA

An integrated compilation of data obtained over the period extending up to 1988 and saved on computer data carriers was composed by the CERN-HERA group (see [3–5]). Extended data are available from the website http://pdg.lbl.gov/2014/html/computer_read.html. In Figs. 1 and 2, the points with total-error bars stand for experimental data, while the curves represent a parameter-dependent description of data for various collisions, the parameter values being fitted to experimental data at $\sqrt{s} \geqslant 5$ GeV.

In order to avoid encumbering Fig. 1, we do not show there data on reactions involving neutrons. The arrows above the curves that represent the total cross sections indicate the lower threshold in energy for the data sample used in the fitting procedure. The parametrizations of the amplitudes are constructed as linear forms in the function $\ln^2(s/s_{\mathrm{M}})$, which specifies the asymptotic behavior according to Heisenberg [6–8], and the functions $(s/s_{\mathrm{M}})^{-\eta_1}$ and $(s/s_{\mathrm{M}})^{-\eta_2}$, which stand for the contributions of the C^+ and C^- Reggeons, respectively.

The table gives parameter values for $\sqrt{s} \geqslant 5$ GeV.

A data analysis involves a simultaneous description of 20 sets of data on the total cross sections for

$$(\bar{p})p(p,n); \quad \Sigma^- p; \quad \pi^{\mp} p; \quad K^{\mp}(p,n);$$
$$(\bar{p})pd; \quad \pi^{\mp} d; \quad K^{\mp} d; \quad \gamma p; \quad \gamma d; \quad \gamma\gamma,$$

collisions and six sets of data on the parameters ρ. The total statistical data sample includes $N_{pt} = 1048$ data points. The model used involves 35 adjustable parameters; of these, three are common to the parametrizations of all observables in all of the collision types considered in the present data analysis.

The Particle Data Group (PDG) archive http://pdg.lbl.gov/2012/html/ rpp_archives.html gives an idea of the evolution of descriptions of total cross sections in the Review of Particle Properties (RPP) and

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of the analytic data parametrizations adopted there in different years.[1)]

ANALYTIC PARAMETRIZATIONS

Here, we will consider modifications to the HPR1R2 parametrization:

$$\sigma^{ab} = \begin{cases} \mathrm{H}^{ab}\ln^2(s/s_{\mathrm{M}}) & \text{Heisenberg term,} \\ +\mathrm{P}^{ab} & \text{Pomeranchuk term,} \\ +R_1^{ab}(s_{\mathrm{M}}/s)^{\eta_1} & C^{+}\text{-Reggeon term,} \\ +R_2^{ab}(s_{\mathrm{M}}/s)^{\eta_2} & C^{-}\text{-Reggeon term,} \end{cases}$$

$$\sigma^{\bar{a}b} = \begin{cases} \mathrm{H}^{ab}\ln^2(s/s_{\mathrm{M}}) & \text{Heisenberg term,} \\ +\mathrm{P}^{ab} & \text{Pomeranchuk term,} \\ +R_1^{ab}(s_{\mathrm{M}}/s)^{\eta_1} & C^{+}\text{-Reggeon term,} \\ -R_2^{ab}(s_{\mathrm{M}}/s)^{\eta_2} & C^{-}\text{-Reggeon term,} \end{cases}$$

In specifying these formulas for various collisions, use is made of the colliding-particle masses m_a and m_b in GeV units. In the parametrizations involving photons, use is made of the constant $m_{\gamma *} = m_{\rho(770)}$.

The numerical values of the masses and fundamental constants $\hbar$ and c were taken from RPP-2012 (see also [10]).

The adjustable parameters include (i) M, which has dimensions of energy in GeV units and which specifies the rate $\mathrm{H}^{ab} = \pi\,(\hbar c/\mathrm{M})^2$ (in millibarns) and the beginning of growth, $s_{\mathrm{M}} = (m_a + m_b + \mathrm{M})^2$, of Heisenberg's contribution; (ii) P^{ab}, R_1^{ab}, and R_2^{ab}, which have dimensions of squared length in millibarn units; and (iii) η_1, η_2, δ, and λ, which are dimensionless.

The parameters η_1 and η_2 determine the asymptotic rate of decrease in contributions of secondary Reggeons as the collision energy grows, and δ is a parameter that is necessary for the inclusion of data on γp, γd, and $\gamma\gamma$ collisions in a simultaneous analysis together with hadron—hadron collisions, the following substitutions being mandatory:

$$\mathrm{P}^{\gamma p} + \pi\,(\hbar c/\mathrm{M})^2 \ln^2(s/s_{\mathrm{M}}) \to \delta\left(P^{pp} + \pi\,(\hbar c/\mathrm{M})^2 \ln^2(s/s_{\mathrm{M}})\right),$$

$$\mathrm{P}^{\gamma\gamma} + \pi\,(\hbar c/\,\mathrm{M})^2 \ln^2(s/s_{\mathrm{M}}) \to \delta^2\left(P^{pp} + \pi\,(\hbar c/\mathrm{M})^2 \ln^2(s/s_{\mathrm{M}})\right).$$

The parameter λ controls the hypothesis that the growth of Heisenberg's contribution is universal for hadron—nucleus and photon—nucleus collisions inclusive [11], the substitution $\mathrm{H}^{ad} \to \lambda\cdot\pi\,(\hbar c/\mathrm{M})^2$ being necessary.

The parametrizations of $\rho^{ab} = \mathrm{Re}T^{ab}(s,0)/\mathrm{Im}T^{ab}(s,0)$ do not involve additional adjustable parameters [12] and have the form

$$\rho^{ab} = \frac{1}{\sigma^{ab}}\begin{cases} \pi\mathrm{H}^{ab}\ln(s/s_{\mathrm{M}}) & \text{Heisenberg,} \\ -R_1^{ab}(s_{\mathrm{M}}/s)^{\eta_1}\tan\left(\frac{\eta_1\pi}{2}\right) & C^{+}\text{Reggeon,} \\ -R_2^{ab}(s_{\mathrm{M}}/s)^{\eta_2}\cot\left(\frac{\eta_2\pi}{2}\right) & C^{-}\text{Reggeon,} \end{cases}$$

$$\rho^{\bar{a}b} = \frac{1}{\sigma^{ab}}\begin{cases} \pi\mathrm{H}^{ab}\ln(s/s_{\mathrm{M}}) & \text{Heisenberg,} \\ -R_1^{ab}(s_{\mathrm{M}}/s)^{\eta_1}\tan\left(\frac{\eta_1\pi}{2}\right) & C^{+}\text{Reggeon,} \\ +R_2^{ab}(s_{\mathrm{M}}/s)^{\eta_2}\cot\left(\frac{\eta_2\pi}{2}\right) & C^{-}\text{Reggeon.} \end{cases}$$

Specific parametrizations of observables for each collision type are presented in the *.nb files of the *Mathematica*^{TM} system.

SIMULTANEOUS ADJUSTMENT OF PARAMETERS

The fitted parameters were evaluated according to the standard NonlinearModelFit procedure of the *Mathematica*^{TM} system. Specifically, the procedure consists in seeking a set of parameters that minimize the quadratic form

$$\chi^2_{\min} = \min_{\text{params}} \sum_{i=1}^{N_{pt}} \left[\frac{\text{data}(s_i) - \text{model}(s_i;\text{params})}{w_i}\right]^2,$$

where data(s_i) is the experimental value of the observable model$(s_i$; params) at the value of s_i and w_i is the total error in the respective value of data(s_i). The minimum in question is chosen in the process of iterative activations of the minimization procedure. The calculations are stopped as soon as the value of the fit-quality indicator (Fit Quality) $\mathrm{FQ} = \chi^2_{\min}/(N_{pt} - N_{\mathrm{par}})$ is stabilized to a precision of $\Delta_k(\mathrm{FQ}) = \mathrm{FQ}_i - \mathrm{FQ}_{i+1} \leqslant 10^{-8}$. We assume the adjustment to be reliable if (i) $\mathrm{FQ} \approx 1$, (ii) the distributions of $\chi^2_{\min}/n_{pt}$ values with respect to observables and with respect to corresponding data sets differ only modestly (uniformity of description), and (iii) the minimum eigenvalue of the correlation matrix for the uncertainties in the parameters is strictly positive. We discriminate between two fit-quality values: "intrinsic" ones, $\mathrm{FQ}_{\mathrm{int}}$, calculated with computer-accuracy parameters and "extrinsic" ones, $\mathrm{FQ}_{\mathrm{ext}}$, calculated with parameter values rounded (see table) in accordance with the thresholds for safe uniform rounding [13].

[1)] It is noteworthy that the use of analytic functions of several complex variables in simulating the amplitudes in question halves the number of fitted parameters. For an exhaustive (up to 1988) survey of the approaches to constructing analytic expressions for observables in axiomatic versions of quantum field theory, we refer the interested reader to [9].

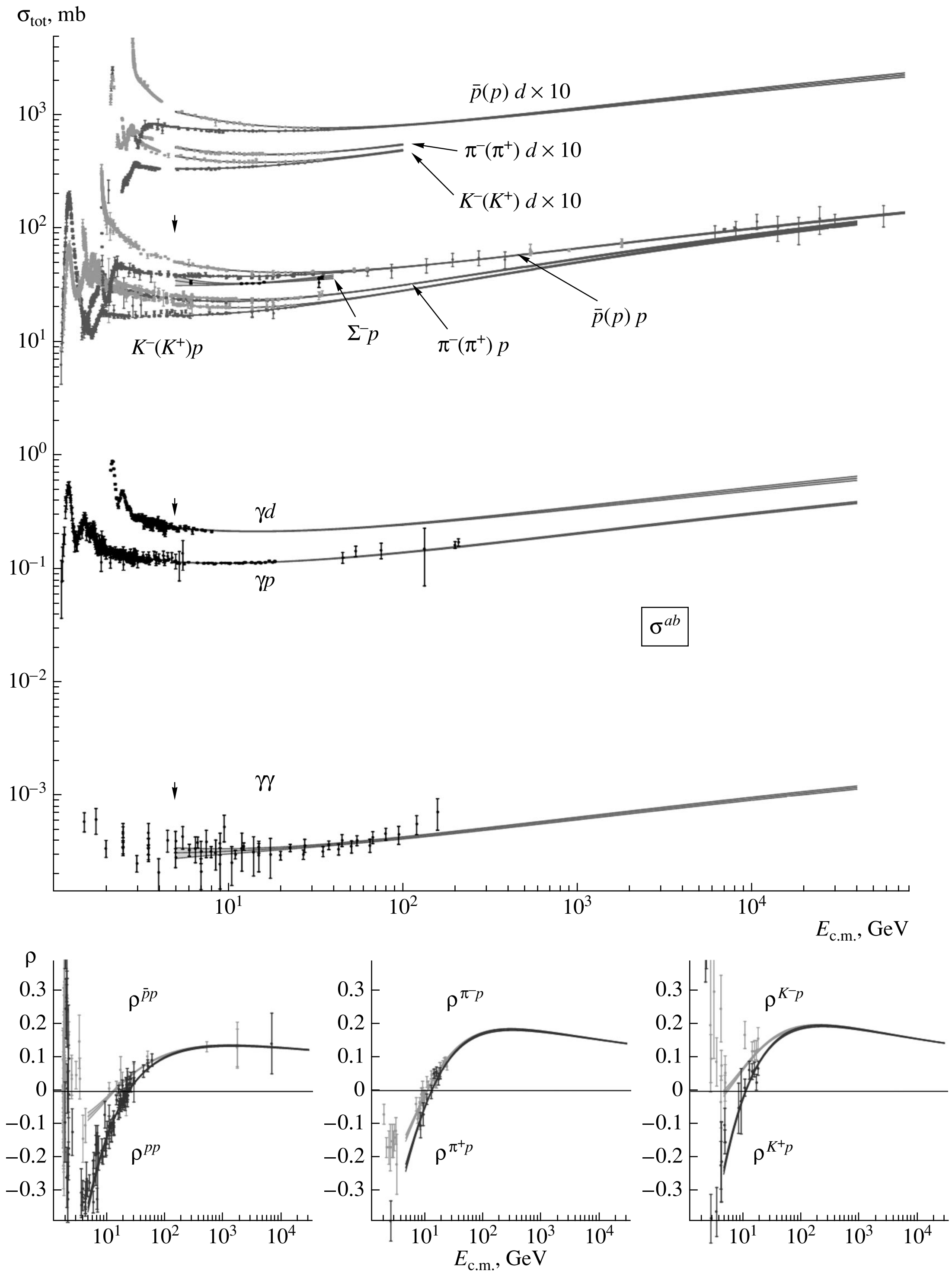

Fig. 1. Theoretical results and experimental data in the energy region of $\sqrt{s} \geqslant 5$ GeV.

The spread region is constructed by the method of a direct transfer of the uncertainties in experimental data to the uncertainties in the tuned parameters.

The direct-transfer procedure is implemented according to the JCGM/WG 1 (Joint Committee for Guides in Metrology Working Group 1) recommendations [14] by the Monte Carlo method:

(i) Each experimental value of an observable is replaced by a value chosen at random according to its normal distribution:

$$\text{data}\ (s_i) \Rightarrow \text{data}_R(s_i) \in N(\text{data}\ (s_i), w_i);$$

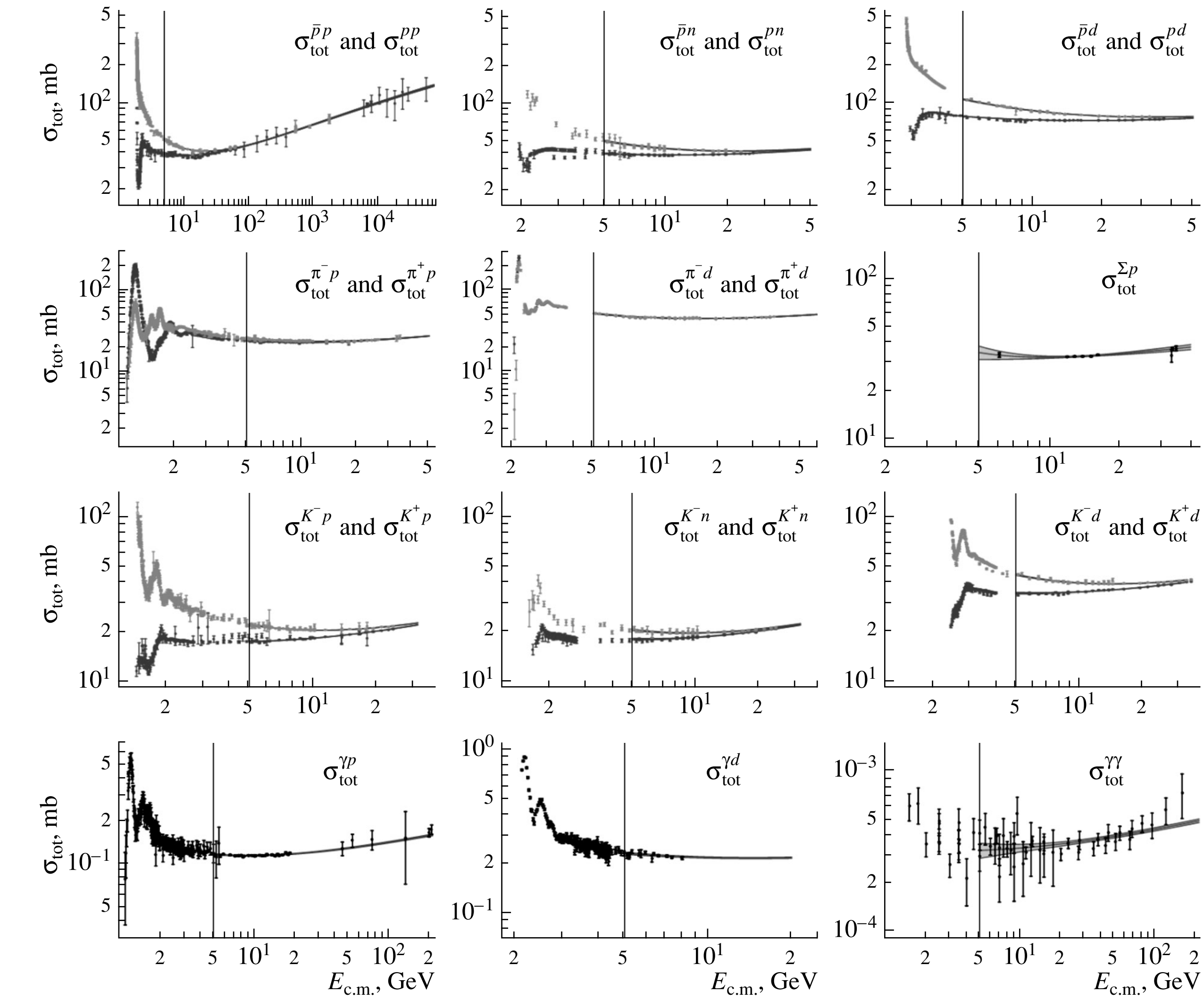

Fig. 2. Behavior of the cross sections in the energy region of $\sqrt{s} \geqslant 5$ GeV.

(ii) A set of vectors params$_R$ obtained according to a "biased" procedure specified as

$$\chi_R^2 = \min_{\text{params}} \sum_{i=1}^{N_{pt}} \left[\frac{\text{data}(s_i) - \text{model}(s_i;\text{params})}{w_i} \right]^2$$

is accumulated, whereby, a representative Monte Carlo set (of power MC$_i$) of vectors from the spread region is formed upon a sufficient number of Monte Carlo iterations.

In subsequently deriving the required estimates, we assume that all experimental estimates of the uncertainties in the input data used come from normal distributions of the total uncertainties (combined statistical and systematic ones) and are statistically independent. This enables us to avoid complicating evaluation procedures and to test the efficiency of the JCGM/WG 1 recommendations for performing a global evaluation of the components of the 35-dimensional vector of tuned parameters in the nonlinear evaluation problem.

The resulting Monte Carlo set of vectors contains the most comprehensive body of information about the region of spread of tuned parameters around their best least squares estimates and permits obtaining more correct estimates of the second moment of simultaneous distributions, since the procedure of error transfer takes fully into account the nonlinearity of the model. A numerical expression for the best estimates of the tuned parameters, the Monte Carlo sets, and the estimates of their second moments for all data analyses described in the present article are given with a computer accuracy of 16 decimal places. This is the representation that we call a statistically complete representation of the results of "indirect measurements" of tuned model parameters.

Parameter values in the energy region of $\sqrt{s} \geqslant 5$ GeV

HPR1R2 in the region of $\sqrt{s} \geq 5$ GeV	$M = 2.121 \pm 0.013$ GeV $H = 0.272 \pm 0.0033$ mb $\eta_1 = 0.447 \pm 0.013$ $\eta_2 = 0.5486 \pm 0.007$ $\delta = (3.063 \pm 0.021) \times 10^{-3}$ $\lambda = 1.624 \pm 0.048$				$FQ_{\mathrm{int}} = 0.96$ $FQ_{\mathrm{ext}} = 0.96$
P, mb	R_1, mb	R_2, mb	beam/target	$N_{pt} = 1048$ n_{pt}	χ^2/n_{pt} in groups
34.41 ± 0.19	13.07 ± 0.23	7.39 ± 0.11	$\bar{p}(p)/p$	258	1.14
34.71 ± 0.24	12.52 ± 0.47	6.66 ± 0.22	$\bar{p}(p)/n$	67	0.48
34.7 ± 2.0	-46.0 ± 29.0	-48.0 ± 29.0	Σ^-/p	9	0.37
18.75 ± 0.16	9.56 ± 0.20	1.767 ± 0.042	π/p	183	1.02
16.36 ± 0.12	4.29 ± 0.18	3.408 ± 0.059	K/p	121	0.82
16.31 ± 0.13	3.70 ± 0.26	1.83 ± 0.10	K/n	64	0.58
	0.0139 ± 0.0017		γ/p	41	0.62
	$(-4.0 \pm 25.0) \times 10^{-6}$		γ/γ	37	0.75
	0.0370 ± 0.0028		γ/d	13	0.9
64.45 ± 0.51	29.66 ± 0.59	14.94 ± 0.24	$\bar{p}(p)/d$	85	1.52
36.65 ± 0.42	18.75 ± 0.57	0.34 ± 0.12	π/d	92	0.72
32.06 ± 0.31	7.70 ± 0.48	5.62 ± 0.12	K/d	78	0.79

CONCLUSIONS

The above analysis of an extended set of experimental data has revealed that, starting from the energy of 5 GeV, the HPR1R2 version of the RPP-2012 model describes well a complete set of data on cross sections for the collisions considered in our present study. In order to perform a more detailed test of the universality hypothesis and to analyze collisions involving nuclear targets, one needs data on (anti)proton–nucleus collisions from still operating accelerators such as the RHIC, FNAL, and LHC. Within the *Mathematica*^{TM} system, there is a set of procedures for constructing a statistically complete description of experimental data on the basis of parameter-dependent models. These include searching for values of adjusted parameters, evaluating the region of spread of parameters via a direct transfer of the uncertainties in experimental data to uncertainties in adjusted parameters, and forming a complete numerical account of the results of the evaluation session and placing it at the disposal of potential users.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project no. 14-07-000950).

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