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Study of the Charge Dependence of the Pion–Nucleon Coupling Constant on the Basis of Data on Low-Energy Nucleon–Nucleon Interactions

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Abstract—The relation between quantities that characterize the pion–nucleon and nucleon–nucleon interactions is studied with allowance for the fact that, at low energies, nuclear forces in nucleon–nucleon systems are mediated predominantly by one-pion exchange. On the basis of the values currently recommended for the low-energy parameters of the proton–proton interaction, the charged pion–nucleon coupling constant is evaluated at $g^2_{\pi^\pm}/4\pi = 14.55(13)$. This value is in perfect agreement with the experimental value of $g^2_{\pi^\pm}/4\pi = 14.52(26)$ found by the Uppsala Neutron Research Group. At the same time, the value obtained for the charged pion–nucleon coupling constant differs sizably from the value of the pion–nucleon coupling constant for neutral pions, which is $g^2_{\pi^0}/4\pi = 13.55(13)$. This is indicative of a substantial charge dependence of the coupling constant.

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1. The pion–nucleon coupling constants are fundamental physical features of strong nuclear interaction. These quantities play an important role in studying the nucleon–nucleon and pion–nucleon interactions [1–5]. In view of this, much attention has been given for many decades to studying and refining their values. A detailed history of the development of the situation around the pion–nucleon coupling constant can be found in [5–7].

At the present time, there is no substantial controversy about the neutral pion–nucleon coupling constant $g^2_{\pi^0}/4\pi$. The value of $g^2_{\pi^0}/4\pi = 13.52(23)$ from [8], which is among the values determined experimentally in recent years, is in perfect agreement with the earlier results from [9], $g^2_{\pi^0}/4\pi = 13.55(13)$, and [10], $g^2_{\pi^0}/4\pi = 13.61(9)$, as well as with the averaged value of $g^2_{\pi^0}/4\pi = 13.6(3)$ quoted in [5, 11].

However, there is no unanimous consensus on the value of the charged pion–nucleon coupling constant $g^2_{\pi^\pm}/4\pi$. The well-known compilation of Dumbrajs and his coauthors [12] gives the value of $g^2_{\pi^\pm}/4\pi = 14.28(18)$ obtained in [13, 14] from data on $\pi^\pm p$ scattering. On the basis of an energy-dependent partial-wave analysis (PWA) of data on nucleon–nucleon scattering, the Nijmegen group found the value of $g^2_{\pi^\pm}/4\pi = 13.54(5)$ [6, 15] for the charged pion–nucleon coupling constant. This result was nearly coincident with coupling constant $g^2_{\pi^0}/4\pi = 13.55(13)$ determined for neutral pions by the same group in [9]. The values $g^2_{\pi^\pm}/4\pi \sim 13.7–13.8$ of the charged pion–nucleon coupling constant, which are close to the constant $g^2_{\pi^0}/4\pi \sim 13.6$ for neutral pions, were recently obtained on the basis of data on $\pi^\pm p$ interaction in some other studies [16–19]. At the same time, the Uppsala Neutron Research Group obtained much greater values for the charged pion–nucleon coupling constant, $g^2_{\pi^\pm}/4\pi = 14.62(35)$ [20], $g^2_{\pi^\pm}/4\pi = 14.52(26)$ [21], and $g^2_{\pi^\pm}/4\pi = 14.74(33)$ [22], which exceed substantially the average value of the coupling constant for neutral pions, $g^2_{\pi^0}/4\pi = 13.6(3)$ [5, 11]. Thus, it is of paramount importance to address presently the problem of the possible charge dependence of the pion–nucleon coupling constant—in other words, the question of whether the pion–nucleon coupling constants for neutral and charged pions differ from each other.

In the present study, we consider the pion–nucleon coupling constants for neutral and charged pions on the basis of data on low-energy nucleon–nucleon (NN) scattering. At the present time,

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semiphenomenological one-boson-exchange potential models, which include the exchange of various mesons, are frequently used to describe the NN interaction. In this approach, the exchange of pions, which are light, determines primarily the long-range part of the NN potential, while the exchange of rho and omega mesons, which are heavier, determines the interaction at intermediate and short distances, which is substantial at higher energies. At extremely low energies, which effectively correspond to long distances, the use of the simplest one-pion-exchange potentials is quite appropriate for describing the NN interaction. In view of this circumstance and under the assumption that nuclear forces in the NN system at low energies are due primarily to the exchange of virtual pions, we use here the well-known Yukawa potential, which directly follows from meson field theory [1–3], to describe the NN interaction, matching the parameters of this potential (that is, its depth V_0 and range R) with the low-energy parameters of NN scattering in the ${}^1S_0$ spin-singlet state. By employing the parameters of the Yukawa potential for proton–proton (pp) and neutron–proton (np) interactions, we obtain here an equation that relates the charged ($f^2_{\pi^\pm}$) and neutral ($f^2_{\pi^0}$) pseudovector pion–nucleon coupling constants to each other.

2. According to meson field theory, strong nuclear interaction between two nucleons is due largely to the exchange of virtual pions, which determine primarily the long-range part of the NN interaction and, accordingly, NN scattering at extremely low energies. The nucleon–nucleon potential that follows from meson field theory and which one calls the Yukawa potential has the form [1–3]

$$V(r) = -V_0 \frac{e^{-\mu r}}{\mu r}, \tag{1}$$

where r is the distance between two nucleons and μ is related to the pion mass m_π by the equation

$$\mu = \frac{m_\pi c}{\hbar}. \tag{2}$$

Here, c is the speed of light and $\hbar$ is the reduced Planck constant. The nuclear-force range R is in inverse proportion to the pion mass and is small:

$$R \equiv \frac{1}{\mu} = \frac{\hbar}{m_\pi c} \sim 1.4 \text{ fm}. \tag{3}$$

As a matter of fact, the nuclear-force range R is coincident with the Compton wavelength of the pion.

The depth V_0 of the potential in (1) is related to the dimensionless pseudovector pion–nucleon coupling constant f_π by the simple equation [1–3, 5, 23]

$$V_0 = m_\pi c^2 f_\pi^2. \tag{4}$$

Thus, the pion mass m_π and the pion–nucleon coupling constant f_π are basic features of the pion–nucleon interaction, which play a significant role in studying the nucleon–nucleon and pion–nucleus interactions [1–5]. The pseudovector coupling constants for neutral ($f^2_{\pi^0}$) and charged ($f^2_{\pi^\pm}$) pions are related to the pseudoscalar coupling constants $g^2_{\pi^0}$ and $g^2_{\pi^\pm}$ by the equations [5, 11]

$$\frac{g^2_{\pi^0}}{4\pi} = \left(\frac{2M_p}{m_{\pi^\pm}}\right)^2 f^2_{\pi^0}, \tag{5}$$

$$\frac{g^2_{\pi^\pm}}{4\pi} = \left(\frac{M_p + M_n}{m_{\pi^\pm}}\right)^2 f^2_{\pi^\pm}, \tag{6}$$

where M_p and M_n are, respectively, the proton and neutron masses and $m_{\pi^\pm}$ is the charged-pion mass.

We now present those experimental values of the nucleon and meson masses and the $\hbar c$ value that we will use in the ensuing calculations. We have

$$M_p = 938.272\,046 \text{ MeV}/c^2, \quad M_n = 939.565\,379 \text{ MeV}/c^2, \tag{7}$$

$$m_{\pi^0} = 134.976\,6 \text{ MeV}/c^2, \quad m_{\pi^\pm} = 139.570\,18 \text{ MeV}/c^2, \tag{8}$$

$$\hbar c = 197.326\,971\,8 \text{ MeV fm}, \tag{9}$$

where m_{π^0} is the neutral-pion mass. All of the values in Eqs. (7)–(9) were taken from the last compilation of the Particle Data Group [24].

Two charged protons interact via the exchange of a neutral pion. According to Eqs. (2) and (4), the parameters μ_{pp} and V_0^{pp} of the Yukawa potential (1) are then determined by the neutral-pion mass m_{π^0} and the coupling constant f_{π^0}. But in the case of neutron–proton interaction, both neutral and charged pions are exchanged. In the latter case, one should determine the parameters μ_{np} and V_0^{np} in the potential (1) by employing [25] the averaged values of the pion mass,

$$\overline{m}_\pi = \frac{1}{3}\left(m_{\pi^0} + 2m_{\pi^\pm}\right) \tag{10}$$

and of the pion–nucleon coupling constant,

$$\overline{f^2_\pi} = \frac{1}{3}\left(f^2_{\pi^0} + 2f^2_{\pi^\pm}\right). \tag{11}$$

In order to determine the parameters of the potential in (1), we will use data on the interaction of two nucleons at low energies in the ${}^1S_0$ spin-singlet state. Further, we evaluate the proton–proton parameters μ_{pp} and V_0^{pp} on the basis of the scattering length

a_{pp} and the effective range r_{pp}. In doing this, corrections associated with electromagnetic interaction should be removed from the experimental values of the nuclear Coulomb low-energy parameters of pp scattering. After the removal of these corrections, the values of the purely nuclear scattering length a_{pp} and effective range r_{pp} for proton–proton scattering become [4]

$$a_{pp} = -17.3(4) \text{ fm}, \quad (12)$$

$$r_{pp} = 2.85(4) \text{ fm}. \quad (13)$$

By employing the variable-phase approach [26] and the values of the pp-scattering parameters in (12) and (13), we obtain the following results for the parameters of the Yukawa potential (1) in the case of proton–proton scattering:

$$\mu_{pp} = 0.8393 \text{ fm}^{-1}, \quad (14)$$

$$V_0^{pp} = 44.8295 \text{ MeV}. \quad (15)$$

In accordance with Eqs. (2), (4), (14), and (15), the mass and the pion–nucleon coupling constant for the neutral pion in the case of the Yukawa potential for proton–proton interaction proved to be

$$m_{\pi^0}^{\mathrm{Y}} = 165.6108 \text{ MeV}/c^2, \quad (16)$$

$$\left(f_{\pi^0}^{\mathrm{Y}}\right)^2 = 0.2707. \quad (17)$$

They are much greater than the experimental values

$$m_{\pi^0} = 134.9766 \text{ MeV}/c^2 \text{ [24]}, \quad (18)$$

$$f_{\pi^0}^2 = 0.0749(7) \text{ [9]}. \quad (19)$$

Thus, we have

$$m_{\pi^0}^{\mathrm{Y}} = b m_{\pi^0}, \quad (20)$$

$$\left(f_{\pi^0}^{\mathrm{Y}}\right)^2 = d f_{\pi^0}^2, \quad (21)$$

where b and d are constants, which, in general, depend on the form of nucleon–nucleon interaction. From Eqs. (16)–(19), it follows that, for the Yukawa potential, these constants are

$$b = 1.2270, \quad (22)$$

$$d = 3.6142. \quad (23)$$

It is natural to assume that relations similar to Eqs. (20) and (21) hold for the charged-pion mass and charged pion–nucleon coupling constants and, hence, for the averaged pion mass and the averaged pion–nucleon coupling constant. In view of this, it can readily be shown that the neutron–proton parameters μ_{np} and V_0^{np} of the potential in (1) are related to the analogous proton–proton interaction parameters μ_{pp} and V_0^{pp} by the equations

$$\mu_{np} = \frac{\overline{m}_\pi}{m_{\pi^0}} \mu_{pp}, \quad (24)$$

$$V_0^{np} = \frac{\overline{m}_\pi}{m_{\pi^0}} \frac{\overline{f_\pi^2}}{f_{\pi^0}^2} V_0^{pp}. \quad (25)$$

In accordance with Eqs. (8) and (10), the ratio of the average pion mass $\overline{m}_\pi$ to the neutral pion mass m_{π^0} is

$$\frac{\overline{m}_\pi}{m_{\pi^0}} = 1.0227. \quad (26)$$

By using Eqs. (14), (24), and (26) and the experimental value of the singlet np scattering length for the $^1S_0$ state [27, 28],

$$a_{np} = -23.71(2) \text{ fm}, \quad (27)$$

we obtain the following values for the parameters μ_{np} and V_0^{np} in the case of np interaction in the form of the Yukawa potential:

$$\mu_{np} = 0.8584 \text{ fm}^{-1}, \quad (28)$$

$$V_0^{np} = 48.0742 \text{ MeV}. \quad (29)$$

As before, we use the variable-phase approach [26] in our calculations.

The effective range r_{np} found for np scattering by using the Yukawa potential with the parameter values in (28) and (29),

$$r_{np} = 2.70(4) \text{ fm}, \quad (30)$$

is in perfect agreement with the experimental value [27, 28]

$$r_{np} = 2.70(9) \text{ fm}. \quad (31)$$

The values of the singlet np scattering length in (27) and effective range in (30) are in good agreement with the values of $a_{np} = -23.7154(80)$ fm and $r_{np} = 2.706(67)$ fm that we obtained in [29, 30] by using the experimental values of the np-scattering cross section and the experimental values of the deuteron features. Thus, the use of the experimental values of the neutral- and charged-pion masses in (8) leads to a consistent description of experimental data on proton–proton and neutron–proton scattering in the region of low energies.

From Eqs. (11) and (25), one can obtain an important equation that relates the pseudovector charged and neutral pion–nucleon coupling constants. Specifically, we have

$$f_{\pi^\pm}^2 = \frac{1}{2}\left(3 \frac{V_0^{np}}{V_0^{pp}} \frac{\mu_{pp}}{\mu_{np}} - 1\right) f_{\pi^0}^2. \quad (32)$$

Employing Eq. (32) and taking into account the neutral pion–nucleon coupling constant in (19), along with the parameters of proton–proton scattering in (14) and (15) and the parameters of neutron–proton scattering in (28) and (29), we obtain the following value for the pseudovector charged pion–nucleon coupling constant $f^2_{\pi^\pm}$:

$$f^2_{\pi^\pm} = 0.0804(7). \quad (33)$$

Employing Eqs. (5) and (6) and taking into account the pseudovector coupling constants for the neutral pion in (19) and for the charged pions in (33), along with the proton and neutron masses in (7) and the charged-pion mass in (8), we obtain the following values for the pseudoscalar pion–nucleon coupling constants:

$$\frac{g^2_{\pi^0}}{4\pi} = 13.55(13), \quad (34)$$

$$\frac{g^2_{\pi^\pm}}{4\pi} = 14.55(13). \quad (35)$$

The value in (35) that we found for the pseudoscalar charged pion–nucleon coupling constant is in perfect agreement with the coupling constant

$$\frac{g^2_{\pi^\pm}}{4\pi} = 14.52(26), \quad (36)$$

determined by the Uppsala Neutron Research Group [21].

Thus, the value in (35) obtained in the present study for the charged pion–nucleon coupling constant differs sizably from the value of the neutral pion–nucleon coupling constant in (34). In relative units, this difference is about 7%, which is indicative of a substantial charge dependence of the pion–nucleon coupling constants.

3. Considering that nuclear forces in nucleon–nucleon systems are due primarily to the exchange of virtual pions, we have studied relations between quantities that characterize the pion–nucleon and nucleon–nucleon interactions. We have derived an equation that relates pseudovector charged and neutral pion–nucleon coupling constants to the depths of the potentials that describe the np and pp interactions. By employing the values of $a_{pp} = -17.3(4)$ fm and $r_{pp} = 2.85(4)$ fm currently recommended for the purely nuclear pp scattering length and effective range, respectively, along with the experimental value of $a_{np} = -23.71(2)$ fm for the singlet np scattering length and the value of $g^2_{\pi^0}/4\pi = 13.55(13)$ for the pseudoscalar neutral pion–nucleon coupling constant [9], we have obtained the value of $g^2_{\pi^\pm}/4\pi = 14.55(13)$ for the charged pion–nucleon coupling constant. This value is in good agreement with the experimental values of $g^2_{\pi^\pm}/4\pi = 14.52(26)$ [21] and $g^2_{\pi^\pm}/4\pi = 14.74(33)$ [22] and with the value of $g^2_{\pi^\pm}/4\pi = 14.28(18)$ found previously in [13, 14]. At the same time, the value that we obtained is at odds with the value of $g^2_{\pi^\pm}/4\pi = 13.54(5)$ [6, 15] found by using data on NN scattering and the value of $g^2_{\pi^\pm}/4\pi = 13.76(1)$ [16, 17] determined from data on πN scattering. It is worth noting, however, that values of the charged pion–nucleon coupling constant that are smaller than $g^2_{\pi^\pm}/4\pi = 14.55(13)$ lead to the weakening of the neutron–proton potential and to the singlet np scattering length underestimated in magnitude with respect to its experimental value of $|a_{np}| = 23.71(2)$ fm.

The results obtained in the present study are indicative of a substantial charge dependence of the pion–nucleon coupling constants. A simultaneous analysis of low-energy pion–nucleon and nucleon–nucleon parameters leads to the conclusion that the violation of the charge independence of nuclear forces is due primarily to the difference in mass and in pion–nucleon coupling constants between the neutral and charged pions. Indications of effects stemming from the mass difference between the neutral and charged pions and leading to a violation of the charge independence of nuclear forces can be found in [1, 23]. Our results show that a difference of 3.4% in mass and a difference of 7% in pion–nucleon coupling constant between the neutral and charged pions lead to the difference of pp and np scattering lengths that is equal to $\Delta a_{\rm CIB} \equiv a_{pp} - a_{np} = 6.41$ fm, which is about 30% in relative units. In that case, the difference of the effective ranges for proton–proton and neutron–proton scattering is $\Delta r_{\rm CIB} \equiv r_{pp} - r_{np} = 0.15$ fm.

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