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Light Scattering by a One-Dimensional Absorbing Layer with a Continuous Refractive Index between Two Dielectrics

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Abstract—We consider the passage of a plane electromagnetic wave through a 1D absorbing layer sandwiched between two generally different semi-infinite dielectrics. The transmission, reflection, and absorption spectra of the incident wave, as well as the electric field intensity distribution in the absorbing layer, are obtained using the transfer matrix method and the method based on the solution of the Cauchy problem for a system of two first-order differential equations.

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1. INTRODUCTION

The problem of propagation of electromagnetic waves in heterogeneous layered media is often encountered in many applications. This problem plays an especially important role in the development of various layered photonic structures including dielectric mirrors [1, 2], optical sensors [3, 4], metamaterials [5, 6], devices based on liquid crystals [7, 8], and others [9, 10]. For this reason, a large number of methods for solving this problem have been developed, which include the direct solution of the wave equation [11], classical methods of the transfer-matrix [12, 13] and the Green's function [14, 15], the invariant embedding method [16], etc.

The transfer-matrix method deserves special attention because it is one of the most popular techniques, which is also used in deriving many other methods. This method is especially effective for layered media since the transfer matrix relating the field amplitudes at different interfaces can easily be obtained for each homogeneous layer. In this case, the transfer-matrix method is very convenient for numerical calculations as well as for analytic approach. In addition, this method can be extended to media with heterogeneous layers (e.g., with a linear or a parabolic profile).

The Green function method is also frequently used, especially in solving problems for 2D and 3D structures. It also makes it possible to calculate the photonic density of states within the layer under investigation, which is a useful tool in analysis of various photonic structures. It should be noted that each of the aforementioned methods has its own advantages and limitations; for this reason, new methods are being developed [17–21], which are better optimized for numerical calculations or make it possible to simplify analytic expressions for problems with specific conditions.

A new method has been proposed earlier [22–24] for determining the transmission and reflection coefficients for a heterogeneous layer with a continuous dependence of the refractive index, which is based on solving a system of two first-order ordinary differential equations. The advantages of this method include its applicability to a medium with an arbitrary continuous coordinate dependence of the refractive index. In particular, in the method proposed in [24], it is presumed that a heterogeneous layer is in contact with two dielectrics, the refractive indices of which coincide with the values in the layer at the corresponding interfaces, which ensures the continuity of refractive index n at both interfaces and their zero contribution to the reflectance of the transmitted wave. This simplifies the solution of the scattering problem and the determination of the field distribution in the layer in the above conditions; however, the continuity condition for the refractive index at the layer boundaries is often not satisfied in actual practice. For this reason, scattering of waves at the interfaces must be taken into account additionally.

An analogous problem has already been considered by other authors (e.g., in [25]); however, they have presumed, in particular, the absence of absorption and amplification of radiation in the medium, which requires separate account in the derivation of expressions. In this study, we derive expressions for the reflection and transmission coefficients, as well as the electric field intensity distribution within a heterogeneous layer with absorption (amplification), after which the derived expression are used in the solution of various problems.

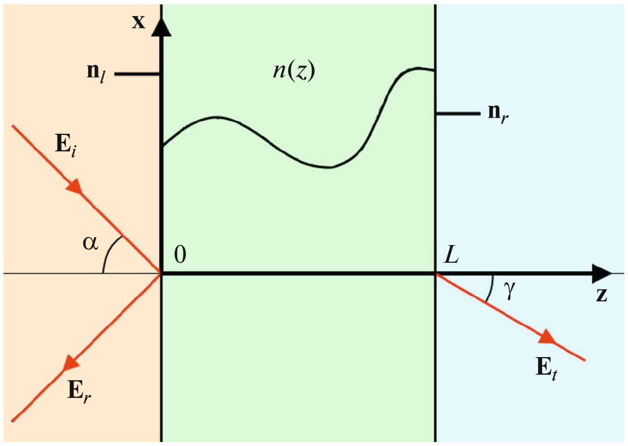

Fig. 1. Transmission of a plane electromagnetic wave through an isotropic 1D layer with an arbitrary dependence of the refractive index.

2. MODEL AND METHODOLOGY

Let us first consider the problem of scattering of a plane wave from a heterogeneous 1D layer (Fig. 1), which has an infinitely large length along the x and y axes, is located between planes $z = 0$ and $z = L$, and its refractive index n is an arbitrary continuous function of z only. We assume that the plane of incidence coincides with the xy plane, and the wave is incident at angle α to the normal to the layer boundary, which coincides with the xy plane. We also assume that the layer of the medium is isotropic and nonmagnetic ($\mu = 1$), and the medium is nonabsorbing ($\mathrm{Im}\, n \equiv 0$). Domains $z < 0$ and $z > L$ are filled with homogeneous dielectrics with refractive indices n_l and n_r, respectively.

The electric field s of the incident, reflected, and transmitted waves will be denoted by $\mathbf{E}_i$, $\mathbf{E}_r$, and $\mathbf{E}_t$. We represent these fields in the form

$$\mathbf{E}_{i,r,t} = E^s_{i,r,t}\mathbf{n}_s + E^p_{i,r,t}\mathbf{n}_p = \begin{pmatrix} E^s_{i,r,t} \\ E^p_{i,r,t} \end{pmatrix}, \qquad (1)$$

where $\mathbf{n}_s$ and $\mathbf{n}_p$ are unit vectors of the s- and p-polarizations and $E^s_{i,r,t}$ and $E^p_{i,r,t}$ are the corresponding amplitudes of the incident, reflected, and transmitted waves. Complex transmission and reflection coefficients for the s and p waves can be written as

$$r^{s,p} = \frac{E^{s,p}_r}{E^{s,p}_i}, \quad t^{s,p} = \frac{E^{s,p}_t}{E^{s,p}_i}. \qquad (2)$$

2.1. Heterogeneous Layer with a Continuous Refractive Index at the Boundaries

In accordance with the approach developed earlier in [24], complex amplitude coefficients $t^{s,p}$ and $r^{s,p}$ for a layer with refractive index $n(z)$, which is in contact on both sides with isotropic dielectrics with refractive indices $n_l = n(0)$ and $n_r = n(L)$, can be written in form

$$t^{s,p} = \frac{2\exp(-ikL)}{Q^{s,p}(L) + F^{s,p}(L)}, \quad r^{s,p} = \frac{(Q^{s,p}(L) - F^{s,p}(L))^*}{Q^{s,p}(L) + F^{s,p}(L)}, \qquad (3)$$

where the asterisk indicates complex conjugation, $k(z) = (2\pi/\lambda)n(z)\cos\beta(z)$, $\beta(z)$ is the angle of refraction, and λ is the wavelength of incident radiation. Functions $Q^{s,p}$ and $F^{s,p}$ are the solutions to the system of differential equations

$$\frac{dF^{s,p}}{dz} = -ikQ^{s,p} + A^{s,p}\frac{1}{k}\frac{dk}{dz}F^{s,p}, \qquad (4)$$

$$\frac{dQ^{s,p}}{dz} = -ikF^{s,p} + B^{s,p}\frac{1}{k}\frac{dk}{dz}Q^{s,p} \qquad (5)$$

with initial conditions

$$F^{s,p}(0) = 1, \quad Q^{s,p}(0) = 1.$$

In system of equations (4), (5), we have

$$\frac{dk}{dz} = \frac{2\pi}{\lambda}\frac{1}{\cos\beta}\frac{dn}{dz}, \quad A^s = 1, \quad B^s = 0, \quad A^p = \cos^2\beta, \quad B^p = \sin^2\beta. \qquad (6)$$

The total field in each of the media can be written as follows:

$$\mathbf{E}_0 = \mathbf{E}_i + \mathbf{E}_r, \quad \mathbf{E}_1 = \mathbf{E}_{\mathrm{in}}, \quad \mathbf{E}_2 = \mathbf{E}_t, \qquad (7)$$

where subscripts 0, 1, and 2 indicate the fields corresponding to the media on the left of the 1D layer of the photonic crystal, in this layer, and on the right of it. Total field E_{in} within the 1D layer is related with remaining fields (1) via boundary conditions. For E_{in}, we have

$$E^{s,p}_{\mathrm{in}}(z) = \frac{k(0)}{k(L)}[(F^{s,p}(z))^* + r^{s,p}F^{s,p}(z)]E^{s,p}_i. \qquad (8)$$

Finally, the electric field intensity in the layer is defined as

$$I^{s,p}_{\mathrm{in}}(z) = |E^{s,p}_{\mathrm{in}}(z)|^2. \qquad (9)$$

Let us now assume that there is absorption (amplification) of radiation within the layer ($\mathrm{Im}\, n \neq 0$). According to the results obtained in [26], to correctly account for radiation absorption (or amplification), it is necessary to replace complex conjugation in all expressions by the inversion of the wavevector ($\mathbf{k} \to -\mathbf{k}$). Then expressions (3) and (8) take form

$$t^{s,p}_k = \frac{2\exp(-ikL)}{Q^{s,p}_k(L) + F^{s,p}_k(L)}, \quad r^{s,p}_k = \frac{Q^{s,p}_{-k}(L) - F^{s,p}_{-k}(L)}{Q^{s,p}_k(L) + F^{s,p}_k(L)}, \qquad (10)$$

$$E^{s,p}(z) = \frac{k(0)}{k(L)}[F_{-k}^{s,p}(z) + r_k^{s,p} F_k^{s,p}(z)]E_i^{s,p}. \quad (11)$$

Here, functions $Q_k^{s,p}$ and $F_k^{s,p}$ are solutions to system (4), (5), while functions $Q_{-k}^{s,p}$ and $F_{-k}^{s,p}$ are solutions to the system of equations, which is obtained from (4), (5) after the substitution $k \to -k$.

2.2. Reflection at the Layer Boundaries

It should be recalled that we have considered until now the situation in which the refractive index is continuous at the layer boundaries, i.e., $n_l = n(0)$ and $n_r = n(L)$. This condition often does not hold in practice, for example, when the layer borders air on both sides ($n_l = n_r = 1$). For this reason, it is important to generalize this method to the case of arbitrary media on the left and right of a heterogeneous layer. This can be done, for example, using the Ambartsumyan method for the summation of layers [27, 28] or the transfer-matrix method [12, 13]. Let us consider here the transfer-matrix method.

Transfer matrix $M^{s,p}$ of the entire structure, which connects the amplitudes of the incident, reflected, and transmitted wave, can be written in form

$$M^{s,p} = \frac{k_l}{k_r}\begin{pmatrix} 1/t_r^{s,p} & -r_r^{s,p}/t_r^{s,p} \\ -r_r^{s,p}/t_r^{s,p} & 1/t_r^{s,p} \end{pmatrix} \times \begin{pmatrix} 1/t_{-k}^{s,p} & -r_{-k}^{s,p}/t_{-k}^{s,p} \\ -r_k^{s,p}/t_k^{s,p} & 1/t_k^{s,p} \end{pmatrix} \times \begin{pmatrix} 1/t_l^{s,p} & -r_l^{s,p}/t_l^{s,p} \\ -r_l^{s,p}/t_l^{s,p} & 1/t_l^{s,p} \end{pmatrix}. \quad (12)$$

The second matrix determines the variation of the field amplitude during the propagation within the heterogeneous layer, while the first and third matrices determine the variation of the field amplitude during reflection from the right and left boundaries, respectively. The coefficients in the first and third matrices can easily be obtained using the Fresnel formulas:

$$\frac{1}{t_l^s} = \frac{k_l + k(0)}{2k_l}, \quad \frac{r_l^s}{t_l^s} = \frac{k_l - k(0)}{k_l + k(0)},$$
$$\frac{1}{t_r^s} = \frac{k(L) + k_r}{2k(L)}, \quad \frac{r_r^s}{t_r^s} = \frac{k(L) - k_r}{k(L) + k_r}. \quad (13)$$

Here, $k_l = (2\pi/\lambda)n_l\cos\alpha$ and $k_r = (2\pi/\lambda)n_r\cos\gamma$. The expressions for the p polarization can be obtained analogously. On the other hand, matrix $M^{s,p}$ can be expressed in terms of the reflection and transmission coefficients of the entire structure:

$$M^{s,p} = \frac{k_l}{k_r}\begin{pmatrix} 1/T_{-k}^{s,p} & -R_{-k}^{s,p}/T_{-k}^{s,p} \\ -R_{-k}^{s,p}/T_k^{s,p} & 1/T_k^{s,p} \end{pmatrix}. \quad (14)$$

Therefore, evaluating matrix (12) and equating it to matrix (14), we can obtain sought quantities $T_k^{s,p}$ and $R_k^{s,p}$. We write here the explicit expressions for the s polarization:

$$\frac{1}{T_k^s} = \frac{(k_l - k(0))Q_{-k}^s(L) + (k_l + k(0))Q_k^s(L)}{2k_l} + \frac{k_r(k(0) - k_l)F_{-k}^s(L) + k_r(k(0) + k_l)F_k^s(L)}{2k(L)k_l}, \quad (15)$$

$$R_k^s = 1 - \left[\frac{k(0)k_rF_k^s(L) - k_lk_rF_{-k}^s(L)}{k(L)k_l} + \frac{k(0)}{k_l}(Q_k^s(L) - Q_{-k}^s(L))\right]T_k^s. \quad (16)$$

Analogously, we can obtain the expression for the field distribution within the heterogeneous layer. For this, we express the amplitudes of the wave at point z within the layer ($0 < z < L$) in terms of the field at point $z = 0$:

$$\begin{pmatrix} E_+^{s,p}(z) \\ E_-^{s,p}(z) \end{pmatrix} = \frac{k_l}{k(z)}\begin{pmatrix} 1/t_{-k}^{s,p}(z) & -r_{-k}^{s,p}(z)/t_{-k}^{s,p}(z) \\ -r_k^{s,p}(z)/t_k^{s,p}(z) & 1/t_k^{s,p}(z) \end{pmatrix} \times \begin{pmatrix} 1/t_l^{s,p} & -r_l^{s,p}/t_l^{s,p} \\ -r_l^{s,p}/t_l^{s,p} & 1/t_l^{s,p} \end{pmatrix}\begin{pmatrix} 1 \\ R_k^{s,p} \end{pmatrix}E_i^{s,p}. \quad (17)$$

Here, $t_k^{s,p}(z)$ and $r_k^{s,p}(z)$ are the transmission and reflection coefficients for a part of the layer of thickness z, $E_i^{s,p}$ is the amplitude of the incident wave at point $z = 0$, and $E_+^{s,p}(z)$ and $E_-^{s,p}(z)$ are the amplitudes of the waves propagating in the layer to the right and to the left, respectively. The total field amplitude at point z is defined as the sum of the waves, $E^{s,p}(z) = E_+^{s,p}(z) + E_-^{s,p}(z)$:

$$E^s(z) = \frac{(k(0) + k_l)F_{-k}^s(z) + (k(0) - k_l)F_k^s(z)}{2k(z)} + \frac{(k(0) - k_l)F_{-k}^s(z) + (k(0) + k_l)F_k^s(z)}{2k(z)}R_k^s. \quad (18)$$

3. NUMERICAL RESULTS AND DISCUSSION

In this section, we apply our modified method to two problems of scattering of a plane wave by various structures. Let us first consider an absorbing photonic crystal (PC) with a refractive index varying over the amplitude modulation length (apodized lattice):

$$n(z) = n_1 + n_2(z)\sin^2\left(\frac{\pi}{\Lambda}z\right) + i\kappa, \quad (19)$$

where κ is the absorption coefficient and

$$n_2(z) = n_{02} + \delta n\frac{z}{L}. \quad (20)$$

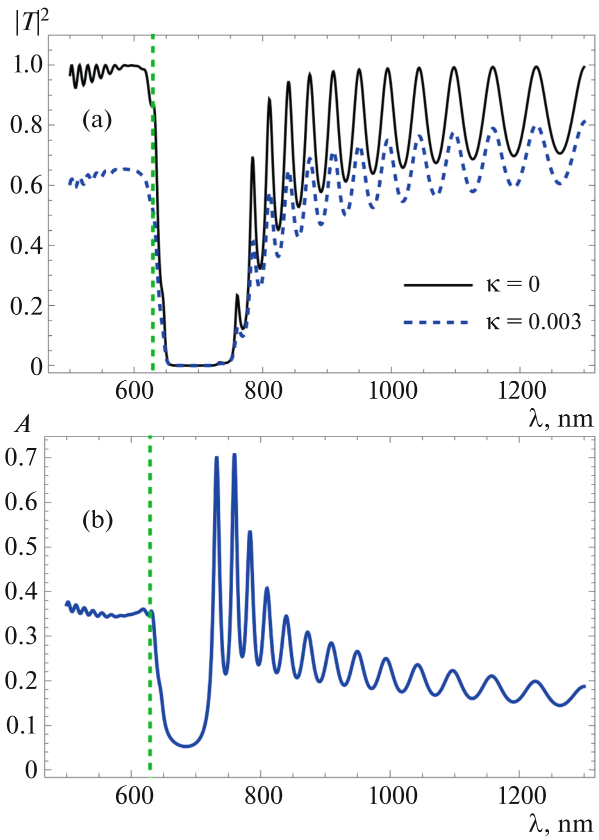

Fig. 2. (a) Transmission and (b) absorption spectra of an apodized PC with absorption coefficient $\kappa = 0.003$ (blue curve) and without absorption (black curve). Structure parameters: $n_1 = 1.5$, $n_{02} = 0.3$, $\delta n = 0.3$, $L = 6000$ nm, $\Lambda = 200$ nm, and $n_l = n_r = 1$.

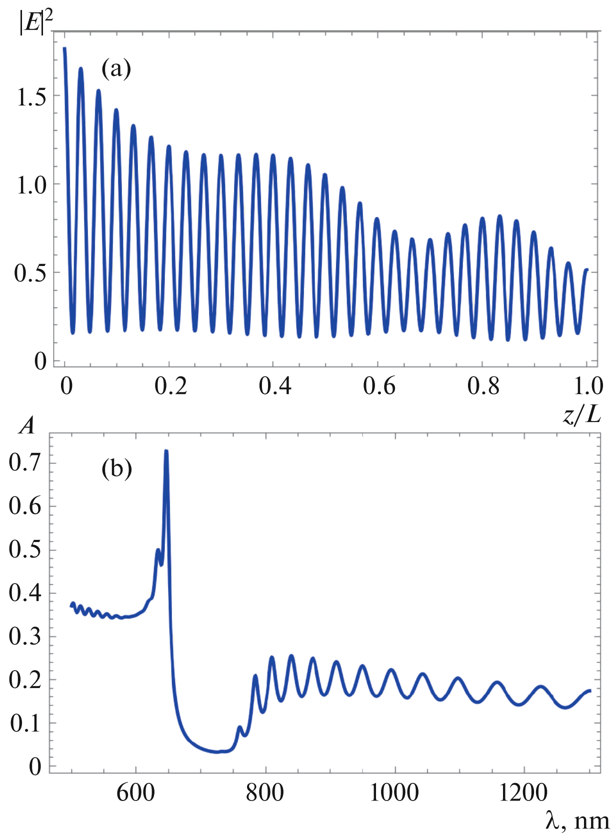

Fig. 3. (a) Field distribution within an apodized PC with absorption coefficient $\kappa = 0.003$ at wavelength $\lambda = 629$ nm (green dashed lines in Fig. 2). (b) Absorption spectrum of an apodized PC like in Fig. 2, but for the radiation incidence in the opposite direction (from the medium on the right of the PC).

Figure 2 shows the transmission spectrum $|T|^2$ and absorption spectrum $A = 1 - |T|^2 - |R|^2$ for the normal incidence in the cases without absorption ($\kappa = 0$) and with absorption ($\kappa = 0.003$). The transmission spectrum clearly shows the characteristic range 650–800 nm with a transmission minimum known as the photonic bandgap (PBG). The addition of absorption reduces transmission through the structure in the entire spectral range except PBG. Because of apodization, the long-wavelength edge of the PBG has a more complex structure as compared to an ideal PC. This is due to the fact that at these wavelengths, the field has an increased localization within the PC, but low transmission. This also explains the absorption peaks at the long-wavelength PBG boundary.

Figure 3a shows the field distribution within an active PC at wavelength $\lambda = 629$ nm. It can be seen that the envelope of the distribution gradually decreases during the propagation along the PC. Figure 3b shows the absorption spectrum for an electromagnetic wave incident from the medium to the right of the same PC. The transmission spectra for the wave incident from the left and from the right fully coincide; however, the reflection and absorption spectra can exhibit considerable asymmetry. In our case, this is manifested in the position of modes with a high absorption: for the incidence from the right, these modes are located at the PBG short-wavelength boundary, while for incidence from the left, they are located at the long-wavelength boundary.

Let us now consider an active PC with the refractive index modulation period varying over the length (chirped lattice):

$$n(z) = n_1 + n_2 \sin^2\left(\frac{\pi}{\Lambda(z)} z\right) - ig, \qquad (21)$$

where g is the gain and

$$\Lambda(z) = \Lambda_0 + \delta\Lambda \frac{z}{L}. \qquad (22)$$

Figure 4 shows transmission spectrum $|T|^2$ and reflection spectrum $|R|^2$ in the cases without amplification ($g = 0$) and with gain $g = 0.0045$. As can be seen from these spectra, the active PC increases the ampli-

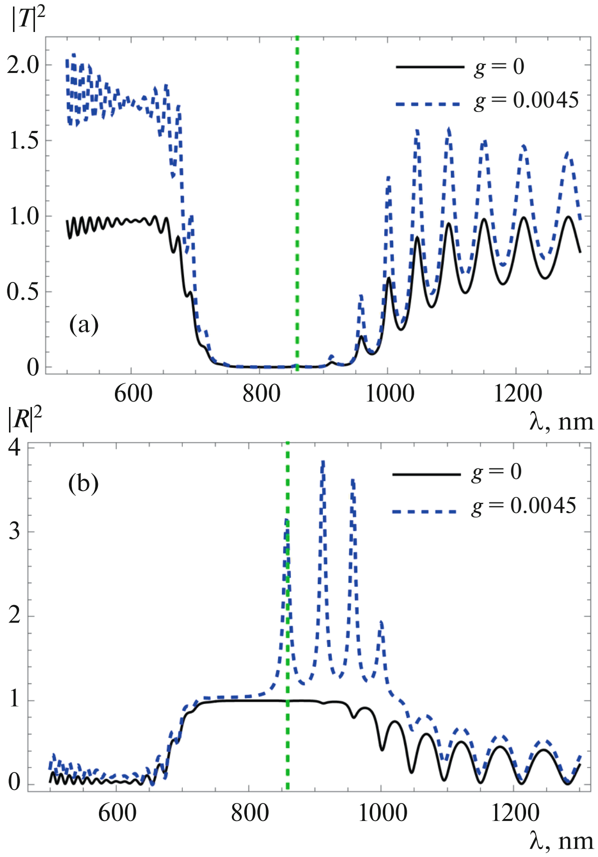

Fig. 4. (a) Transmission and (b) reflection spectra of a chirped PC for the normal incidence with gain $g = 0.0045$ (blue curve) and without gain (black curve). Structure parameters: $n_1 = 1.5$, $n_2 = 0.6$, $L = 5980$ nm, $\Lambda_0 = 200$ nm, $\delta\Lambda = 30$ nm, and $n_l = n_r = 1$.

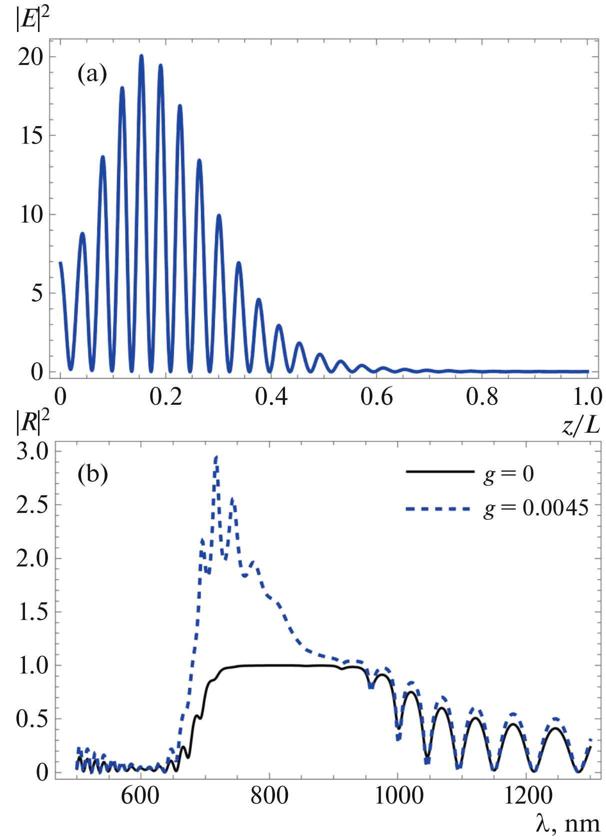

Fig. 5. (a) Field distribution within a chirped PC with gain $g = 0.0045$ at wavelength $\lambda = 859$ nm (shown by green dashed lines in Fig. 4). (b) Reflection spectra of a chirped PC as in Fig. 4, but for radiation incidence in the opposite direction (from the medium on the right of the PC).

tudes of the reflected and transmitted waves. An especially large gain is observed for the reflected wave at several wavelength at the PBG long-wavelengths edge. At these wavelengths, the given structure behaves as a reflective optical amplifier.

Figure 5a shows the field distribution within an active PC for one of the modes with wavelength $\lambda = 859$ nm. Figure 5b shows the reflection spectrum in the case of incidence of an electromagnetic wave from the medium on the right of the same PC. Analogously to the previous case, we again observe asymmetry for the reflection spectra for the "left" and "right" problems. It should also be noted that the modes with a high gain at the short-wavelength boundary in the right problem merge into a relatively broad band.

4. CONCLUSIONS

In this study, we have modified and generalized the method for solving the problem of scattering of a plane wave from a heterogeneous absorbing (amplifying) layer with an arbitrary refractive index, which has been proposed earlier; namely, we have derived the expressions for the transmission and reflection coefficients for the heterogeneous layer pressed between two different dielectrics, as well as the expression for the electric field distribution within such a layer. These coefficients can be expressed in terms of a pair of complex functions, which are solutions to the Cauchy problem for a system of two ordinary differential equations, as well as parameters of the layer and of the external medium. This method has an advantage as compared to the direct solution of the wave equation from the standpoint of the numerical solution of the problem since it is easier to solve a system of two first-order equations than one second-order equation. Finally, we have demonstrated the application of this method to the problem of scattering of a plane wave from two photonic crystals with gradient parameters as an example.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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