ATOMS, MOLECULES, OPTICS

Can Isotropic Negative Permittivity ε and Permeability μ Metamaterials Exist?

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Abstract—An attempt is made to answer the question posed in the title. Periodic metamaterials, namely, cubic photonic crystals with dielectric inclusions (including those with metallic properties), methods for introducing material equations (homogenization), and the possibility of describing metamaterials by scalar permeability and permittivity (or two scalar parameters) including negative values of their real parts, are considered. For photonic crystals, the Landau conclusion about the absence of high-frequency (optical) magnetic properties is confirmed. In particular, negative values of magnetic permeability or any of its components are impossible. Configurations of metamaterials with almost isotropic properties are presented without regard for spatial dispersion.

INTRODUCTION

In a broad sense, metamaterials are considered to be any artificial media (AM), which have been intensively created and widely theoretically studied in the last two decades owing to the development of modern technologies, including nanotechnologies. Their investigation began more than a hundred years ago, and by the middle of the last century, important results were obtained on artificial dielectrics and structures (see, e.g., review [1]). Created AM can be periodic and chaotic, i.e., with a periodic and chaotic arrangement of particles or metaatoms (MAs) in a base, which is usually a homogeneous and isotropic dielectric medium. Another type of AM is created and studied using the theory of mixtures, mixing formulas, the percolation theory, the compact group method, and some other approaches [2, 3]. Here, such metamaterials are not considered: we study only periodic AM, which are also called photonic crystals. Correspondingly, there is an analogy between them and crystals in optics, and amorphous dielectrics serve as an analogy for mixtures. Metamaterials are often considered in a narrow sense as periodic AM containing metallic MAs or particles or, even in a narrower sense, as AM with "simultaneously negative dielectric permittivity ε and magnetic permeability μ" [2, 4]. They are also associated with AM in which bulk back waves (BWs) propagate [4]. Interest in the metasurfaces along which BWs are possible has recently been quickened, and the requirement of simultaneous negativity of permittivity and permeability (or any of their components) is not mandatory [5]. Let us immediately make a reservation

that neither ε nor μ can be negative, since these are complex quantities, all the more so that the losses in metallic MA are significant and the structures with BWs under study are resonant. In addition, BWs exist in any photonic crystal which is strictly described only by a tensor permittivity with allowance for spatial dispersion [6–8]. Therefore, many work have been devoted to the fabrication and study of fully dielectric AM with simultaneous electric and magnetic responses. The answer to the question of whether it is possible to achieve negative effective values of ε and μ (and even more so their simultaneously negative values) in fully dielectric AM is one of the purposes of this work and will be given below. Usually, "simultaneity" is understood as values at a single fixed frequency. This idea is tempting, since the loss tangent of good dielectrics over a wide range from microwaves to optics can be on the order of or less than 10^{-4}, and cryogenic temperatures are not required here. However, these parameters also depend on the wavevector **k**; i.e., it is necessary to understand simultaneity as belonging to a fixed point in an isofrequency surface (dispersion surface in **k** space). Accordingly, the transition from one region or branch of the dispersion surface to another can be accompanied by a transition from direct waves to back waves. The losses can be decreased by two orders of magnitude using helium temperatures. Such periodic AM with metallic MAs can be considered as photonic crystals along with dielectric AM. In such electromagnetic crystals, the absorption of photons may be neglected, and then their behavior is similar to the behavior of electrons in ordinary crystals. Just like electrons and holes in a solid crystal, these are quasi-

particles, namely, quasi-photons or polaritons, which have dispersion and allow efficient control of light; therefore, such nanophotonic structures are of particular interest. Quasi-photons have dispersion and almost forbidden zones are possible for them. The word "almost" can be omitted if we neglect dissipation and consider an infinite periodic photonic crystal.

As is known in the electrodynamics of continua, permeability μ loses its meaning with increasingly frequency and is $\mu \approx 1$ in the optical frequency range; therefore, to consider a magnetic response is meaningless [9, §79, §103]. This statement has been repeatedly questioned and refuted over the past twenty years (see, e.g., [10]). However, the conclusion about the inexpediency of a high-frequency description of natural materials and metamaterials using permeability is correct. In this paper, this conclusion, which was made in [9], is confirmed in the general case specifically for photonic crystals using classical electrodynamics and homogenization. It can be extended to the interaction of radiation with periodically located MAs as quantum dots. The problem of creating isotropic metamaterials is also considered. In this case, the crystal size or the number of periods is important. This number should be so large that the radiation losses can be neglected. Using these approximations, we will solve the problem. For example, a thin 100-nm-thick film with five periods and two different layers of 10 nm per period is not an 1D crystal. It is more consistent with an optical filter [11, 12]. For a band gap to appear, a film must have a thickness of more than 800 nm, where the number of periods is larger than or equal to 40 [12]. A photonic crystal (like an ordinary crystal) is described by the permittivity tensor $\hat{\varepsilon}$ rather than a negative scalar. However, there are many works in which periodic AM are described using permittivity $\varepsilon < 0$ and permeability $\mu < 0$. Such parameters are often entered to analyze and some properties, such as the behavior of rays. Then, the wave vector electrodynamics is replaced by the beam optics. At best, such permeabilities are introduced into Maxwell's equations. This is due to the fact that strict solutions are very complex and have not been used in the works known to us (except for the use of commercial software packages). Below, we present approaches to a rigorous numerical–analytical solution of such problems.

It is known that the internal field plays a key role in the description of permittivity and permeability in natural substances. Usually, permittivity and permeability are introduced by averaging the fields, inductions, and polarizations of microscopic Maxwell–Lorentz equations over a physically infinitesimal volume [6–8, 13]. This process is called homogenization for crystals and mixtures. For mixtures, averaging is carried out over small volume with the linear size significantly smaller than the wavelength and with many MAs. For a photonic crystal, averaging is performed over a periodicity cell. It is not necessary to require its smallness in comparison with the wavelength, but it is necessary for the

locality of material equations. Low-frequency homogenization means $\mathbf{k} = 0$, i.e., neglected spatial dispersion (SD). For a calculated band structure, the homogenization depends on a point in a dispersion surface. The averaging also depends on a medium model [1, 4–8, 14]. For example, it is possible not to introduce permeability at all, i.e., not to use a symmetric approach and to determine only $\hat{\varepsilon}$ tensor. To determine its six components (in general, complex ones), six scalar Maxwell's equations are quite sufficient, which determines its advantages. The symmetric approach based on $\hat{\varepsilon}$ and $\hat{\mu}$ requires the introduction of additional conditions usually associated with symmetry [15, 16].

BACKGROUND

In the first works on homogenization [17–19], magnetic properties were obtained using only dielectric MA when setting a medium model in the form of ε and μ scalars. For example, Lewin [17, 18] considered the reflection of a normally incident wave from the cubic lattice of spherical magnetodielectric MAs. Wave polarization was fixed along one of the axes. A number of approximations were used. Diamagnetism was obtained for metallic MAs. For dielectric MAs with a high permittivity, a wide variation of the values of μ was obtained with the statement that this quantity can lie almost at any point in the complex plane. This is not entirely true. Figure 1 shows the results of calculations using the formulas from [17, 18]; they show that $\mu'_e > 0$ and $\mu''_e > 0$ at $\varepsilon'_2 < 5000$. At higher values of permittivity and sufficiently large balls, resonances with a small negative permittivity zone are possible. However, there are no substances with such high linear isotropic permittivities, and the resonances are caused by the function $F(k_0 a \sqrt{\varepsilon_2 \mu_2})$ introduced in [18] and are nonphysical. At a high permittivity of the balls, permittivity ε'_e is saturated, which also indicates the limitation of the model. The permittivity of the base in the calculations was $3 - i0.0003$, and $\mu''_e / \mu'_e \sim 10^{-4}$ was obtained. The permittivity of the balls was changed from 1 to 10000 and the dielectric loss tangent was taken to be 10^{-4}. Lewin noted that the results are only suitable for the macroscopic reflection coefficient at normal incidence. That is, the parameters can change for incidence at a certain angle. The results are valid under the assumption $k_0 a \ll 1$, where a is the ball radius, and the more rigid condition $k_0 d \ll 1$, where $a \ll d$ and d is the period, should be used for homogenization. If this rigid condition is met, the effective permittivity differs weakly from the permittivity of the base and the permeability decreases by an order of magnitude or more. Thus, the condition $\mu'_e \gg 1$ is the result of a strong mutual influence of MAs (influence of the internal field), and the model becomes less strict because of spatial dispersion and other factors. The
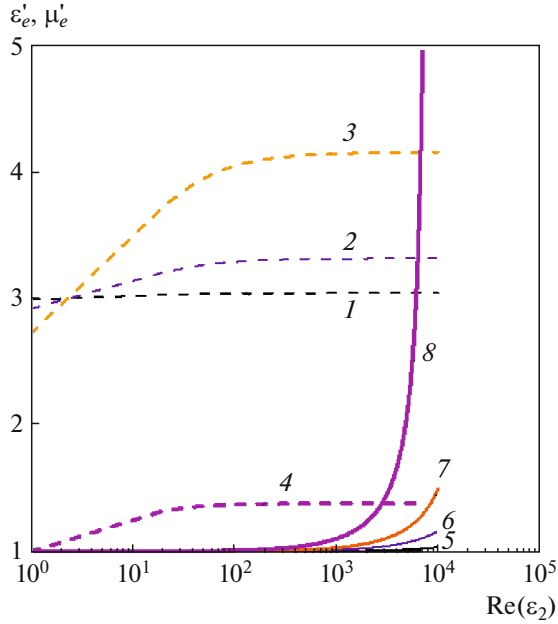
Fig. 1. (*1–4*) Re(ε_e) and (*5–8*) Re(μ_e) vs. Re(ε_2) at $\mu_1 = \mu_2 = 1$ and $k_0 d\sqrt{\varepsilon_1} = 0.1$ [12, 13]: $\varepsilon_1 = $ (*1–3, 5–7*) 3 and (*4, 8*) 1. $a/d = $ (*1, 5*) 0.1, (*2, 3*) 0.2, and (*4, 7, 8*) 0.3.

main limitation of the model induced by an increase in the size is the disappearance of spherical symmetry used to derive formulas.

Rytov [19] considered a plane-layered periodic medium. Scalar permittivity and permeability describing waves of two transverse directions were found for it. The conditions were obtained by equating the wave impedance and the deceleration factor to their effective values, $\sqrt{\mu_e/\varepsilon_e}$ and $n = \sqrt{\varepsilon_e\mu_e}$. Two conditions determine the two effective quantities ε_e and μ_e. These quantities are different in different directions. Even if the layers are dielectric, permeability appears. On the other hand, the following material equations for such a uniaxial dielectric AM were obtained in the long-wavelength limit: $\varepsilon_{e\perp} = (\varepsilon_1 d_1 + \varepsilon_2 d_2)/d$ an dependences $\varepsilon_{e\parallel}^{-1} = (\varepsilon_1^{-1} d_1 + \varepsilon_2^{-1} d_2)/d$. In addition, a similar formula for permeability was derived in the case of AM with magnetic layers. Such 1D metamaterials are described by diagonal tensors, which include the permeabilities of the layers and their thicknesses. If the layers have no magnetic properties, we have $\mu_1 = \mu_2 = 1$ and $\mu_e = 1$; i.e., AM also has no them. However, such a metamaterial can be described by a model taking into account magnetic properties by introducing two scalar parameters, ε_e and μ_e, instead of $\hat{\varepsilon}_e$ tensor. For example, we can require that the coefficient of reflection R from such a medium be equal to the coefficient of reflection from an isotropic effective medium. The reflection coefficient depends on the angle of incidence, i.e., vector **k** in the medium; therefore, this approach allows us to take into account SD. It can be extended to a small-scale period structure in a large-scale period [20]. If a structure has a large number N of periods of two dielectric layers per period, its reflection and transmission coefficients are exactly calculated. It is convenient to use a transmission matrix. By describing the structure as homogeneous with two parameters ε_e and μ_e, we can relate them to the parameters of the layers. Such a model will correctly describe diffraction by plane-layered structures at other large values of N, i.e., at other thicknesses of the entire structure. However, it is not applicable to describe the waves inside. A model based on the permittivity tensor of a uniaxial crystal is more suitable. The above model describing $\varepsilon_{e\perp}$ and $\varepsilon_{e\parallel}^{-1}$ is approximate. It is possible to take into account SD by joining fields and calculating the polarization under the action of a plane wave of direction **k** [1],

$$\varepsilon_{xx} = k_0^{-1}\left[k_x^2 + k_{1z}^2 d^{-2}(d_1^2 + d_1 d_2) + k_{2z}^2 d^{-2}(d_2^2 + d_1 d_2)\right],$$

$$\varepsilon_{zz} = k_0^{-2} k_x^2 \left\{ 1 - \frac{k_{1z}^2 d^{-2}(d_1^2 + d_1 d_2 \varepsilon_2/\varepsilon_1) + k_{2z}^2 d^{-2}(d_2^2 + d_1 d_2 \varepsilon_1/\varepsilon_2)}{k_x^2 + k_{1z}^2 d^{-2}(d_1^2 + d_1 d_2) + k_{2z}^2 d^{-2}(d_2^2 + d_1 d_2)}\right\}^{-1}.$$

Here, ε_n is the permittivity, d_n is the layer thickness, $k_{nz}^2 = k_0^2 \varepsilon_n - k_x^2$, and $d = d_1 + d_2$. It is interesting to note that the result of these formulas in a number of particular cases coincides with the result given above. The wave impedance of an extraordinary wave in such an 1D photonic crystal normalized to the impedance $Z_0 = \mu\sqrt{\mu_0/\varepsilon_0}$ is $\sqrt{\varepsilon_{xx}^{-1} - (k_x^2/k_0^2)/(\varepsilon_{xx}\varepsilon_{zz})}$. In vacuum, we have $\tilde{Z}_0 = E_x/H_y = Z_0\sqrt{1 - k_x^2/k_0^2}$. The reflection coefficient $R = (Z/Z_0 - 1)/(Z/Z_0 + 1)$ is related to permittivity tensor, which gives another possibility of theoretical and experimental determination of its compo-

nents. On the other hand, using the Floquet−Bloch equation $\cos(\tilde{k}d) = (a_{11} + a_{22})/2$, we can determine Bloch wavenumber $\tilde{k}$ and the impedance of an E wave in the z direction, $Z = \sqrt{\mu_0/\varepsilon_0}k_0\tilde{k}/(\tilde{k}^2 + k_x^2)$. Here, a_{nn} are the elements of the transmission matrix of one period of two layers. The difference between the two impedances is that the first is based on homogenization in the form of a Fresnel equation, and the second corresponds to a rigorous solution of the Floquet−Bloch equation. The second formula for the reflection coefficient does not include effective permeability and

permittivity. Equating both reflection coefficients, we obtain the following formula for homogenization:

$$\sqrt{1 - \frac{k_x^2/k_0^2}{\varepsilon_{zz}}} = \frac{k_0\sqrt{\varepsilon_{xx}}\tilde{k}}{\tilde{k}^2 + k_x^2}.$$

To study the properties of metamaterials, researchers used commercial software packages to calculate configuration-complex periodic microstructures. Such packages can demonstrate the solutions that illustrate negative refraction (NR). However, NR does not mean that $\varepsilon < 0$ and $\mu < 0$ and does not even say anything about the fact that some tensor components are negative. Usually, NR is related to bulk BWs. However, BW and NR are different phenomena and can exist independently [1, 4]. Such AMs are called left-handed media or metamaterials with a negative group velocity or a negative refractive index. The refractive index cannot be negative [21]. This concept was introduced in optics at the dawn of its development and is applicable for isotropic media, where the wavelength is larger than the molecule size by 10^5 times or more. For AM, this ratio is at least three orders of magnitude lower, and for them permittivity $\hat{\varepsilon}(\omega, \mathbf{k})$ is a tensor dependent of wavevector $\mathbf{k}$ [4, 6–8]. The bulk waves in such a crystal satisfy the Fresnel equation, in the general case, of the fourth order in $\mathbf{k}$ and the sixth order in wavenumber k_0. The maximum number of such waves is four, and the question of a direct or back volume wave in a nondissipative photonic crystal is solved by determining the angle between the vectors $\mathbf{k}$ and $\mathbf{v}_g = \nabla_k\omega(\hat{\varepsilon}, \mathbf{k})$. The type of wave depends on the direction of the group velocity (normal to the isofrequency surface) with respect to vector $\mathbf{k}$. In a dissipative crystal, the Poynting vector $\mathbf{S} = \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*)/2$ should be used instead of $\mathbf{v}_g$. In the general case without bianisotropy, the isofrequency surface is determined by the equation $\omega(\hat{\varepsilon}(\omega, \mathbf{k}), \hat{\mu}(\omega, \mathbf{k}), \mathbf{k}) = \mathrm{const}$, which takes into account SD. For a hyperbolic metamaterial, this leads to a limited isofrequency surface [1]. A bianisotropic metamaterial with a magnetic response is described by two more cross-polarization tensors [14–16]. It is characterized by a strong SD. However, it can also be described by only one permittivity tensor, but such a tensor in the general case cannot be reduced to a diagonal form. To obtain negative refraction, it is important at what angle a wave falls and what is the optical density of the incidence medium [4]. For incidence from a medium with permittivity $\tilde{\varepsilon}$, the vector $\tilde{\mathbf{k}}$ component that is tangent to the surface is preserved and the vector fulfills the Fresnel equation $\tilde{\mathbf{k}} = k_0^2\tilde{\varepsilon}$. Inside the crystal, vector $\mathbf{k}$ satisfies a more complex Fresnel equation. By changing the angle of incidence, we change the position of the end of the vector in the isofrequency surface. It is impossible to introduce a single refractive index for such crystals, even if we take it as a tensor. Moreover, it is impractical to introduce it: vector $\mathbf{k}$ and wave matrix impedances are introduced in

electrodynamics, since homogenized (macroscopic) Maxwell's equations take the form

$$\mathbf{k} \times \mathbf{H} = -Z_0 k_0 \hat{\varepsilon}_e(\omega, \mathbf{k})\mathbf{E},$$

$$\mathbf{k} \times \mathbf{E} = -Z_0^{-1} k_0 \hat{\mu}_e(\omega, \mathbf{k})\mathbf{H},$$

and the impedances are determined from them when dependence $\mathbf{k}(\omega)$ is determined for a particular dispersion branch and effective parameters $\hat{\varepsilon}_e(\omega, \mathbf{k})$ and $\hat{\mu}_e(\omega, \mathbf{k})$ are obtained. In dissipative AM, a negative group velocity and bulk BWs are not identical concepts. In the case of dissipation, the $\mathbf{k}$-space is no longer three-dimensional: it is six-dimensional complex space. This fact does not allow one to determine the group velocity as the gradient of a scalar. Therefore, the of energy motion velocity and the group velocity are also different concepts. In metallic photonic crystals, we have $|\varepsilon'| \sim \varepsilon''$ in the plasmon resonance region, and it is almost impossible to make $\varepsilon''/|\varepsilon'| < 10^{-3}$ at room temperature. Surface waves can also be back [22], and this does not require the presence of magnetic properties in a medium, although there are also back magnetostatic waves [23]. In contrast to induced magnetism in AM, the anisotropic magnetic response in ferrites manifests itself in the microwave range in the presence of an external magnetic field, which is due to limited saturation magnetization. Surface BWs can experience negative refraction at metasurfaces.

The concepts discussed above require correct use. Often a number of them are devoid of physical meaning. For example, the formal substitution of homogeneous and isotropic quantities $\varepsilon_e = \varepsilon' - i\varepsilon''$ and $\mu_e = \mu' - i\mu''$ into Maxwell's equations at $\varepsilon'' > 0$ and $\mu'' > 0$ gives $n' < 0$ for refractive index $n_e = n' - in''$ if we take $\varepsilon' < 0$ and $\mu' < 0$. The purpose of this work is to consider whether it is possible to form an isotropic medium with $\varepsilon' < 0$ and $\mu' < 0$. Below, we will show that this is impossible, which in some way echoes the conclusions of [24]. The waves in such a metamaterial in the low-frequency limit obey the Fresnel equation $\mathbf{k}^2 = k_0^2\varepsilon_e\mu_e$. Constructed as a crystal, it should have the following properties. The lattice should be cubic with a period $a \ll \Lambda$, where Λ is the minimum internal wavelength and $\lambda = \Lambda\sqrt{\varepsilon_e\mu_e}$ is the wavelength in vacuum. The particles included in lattice sites should be symmetric, i.e., have the same dipole moments (polarizability coefficients) along each axis. The dissipation should be small: $|\mathbf{k}''|/|\mathbf{k}'| \ll 1$ and $|n''/n'| \ll 1$, where $\mathbf{k}' - i\mathbf{k}''$ is the wavevector and $n = \sqrt{\varepsilon_e\mu_e} = n' - in''$ is the homogenization-induced refractive index (note that vector $\mathbf{n}' = \mathbf{k}'/k_0$, the modulus of which determines deceleration in the $\mathbf{k}'$ direction, may be introduced). The SD and biisotropy should be negligible. The latter corresponds to the fact that a metamaterial should be considered far from resonances and band gaps at small $|\mathbf{k}|$ and large wavelengths compared to the particle size Λ. The neglect of biisotropy means

that the secondary electric fields induced by the polarization of each particle do not contribute to its magnetic polarization and the magnetic polarization of its neighbors. The same applies to magnetic fields. Once again, we emphasize that the use of scalar quantities $\varepsilon < 0$ and $\mu < 0$ in Maxwell's equations gives a solution in the form of a plane wave, for which $\mathbf{k} \cdot \mathbf{S} < 0$, which does not require the introduction of either a negative refractive index or a negative group velocity. Since a monochromatic wave has no wave group, there is no reason to introduce it.

ALMOST ISOTROPIC METAMATERIALS

An electric dipole of length l is well modeled by a small thin metallic cylinder of small radius $r \ll l$. An isotropic dipole consists of three such mutually perpendicular cylinders (Fig. 2a). Its radiation is isotropic. A magnetic dipole is a wire loop. An isotropic magnetic dipole is represented as three crossed loops (Fig. 2). We place two (magnetic and electric) isotropic dipoles in each cubic lattice site (Fig. 2a). To eliminate the mutual influence of the dipoles of the neighboring sites, we assume $R \ll a$. In addition, we assume $l < R$. Even in this case, there is a near-field mutual influence of dipoles at a site: the excitation of an electric dipole leads to radiation, which excites currents in the loops, and the excitation of the loops, in turn, excites electric dipoles. Therefore, such a crystal incompletely meets our requirements. It is more likely to be biisotropic. Each emitter at a site in it belongs to eight cubic cells. Since a cube has eight cubic cells, one emitter belongs to one cube. It can be placed in the center of the cube. Then, it is necessary to set the decomposition of currents in the wires, to calculate their fields using a periodic Green's function (GF) of a given photonic crystal, and to impose boundary conditions at the wires. This allows us to formulate a dispersion equation. We will call it microscopic. It allows us to construct an exact band structure and a microscopic isofrequency surface. To construct it, we need a dispersion equation, which includes only fields, frequency, and wavevector $\mathbf{k}$. Now, we can perform homogenization and determine the average fields, the inductions, and the polarizations; as a result, we can determine the average (effective) homogeneous (in the general case) tensor permittivity and permeability. They correspond to a Fresnel equation for a photonic crystal, which includes $\hat{\varepsilon}_e$, $\hat{\mu}_e$, $\mathbf{k}$, and k_0. When k_0 is fixed, it gives a macroscopic isofrequency surface and can be considered a macroscopic dispersion equation.

Figure 2b shows the AM that is more consistent with the isotropy requirements. Obviously, the three ring loops can be replaced with a metallic ball. The ring currents on the ball surface induce magnetic polarization. These are low-frequency currents. The ball can also be an electric dipole, quadrupole, octupole, etc. [25]. Multipole resonance currents correspond to higher frequencies, although they exist at all
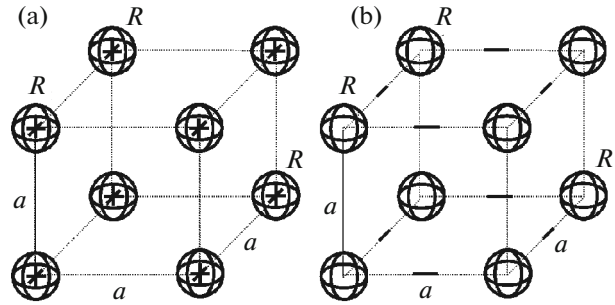

Fig. 2. Photonic crystal with magnetic and electric dipoles in the cubic lattice.

frequencies due to expansion of the total current in terms of multipoles, i.e., the derivatives of Legendre polynomials. However, the z axis orientation can be chosen arbitrarily; therefore, these are isotropic emitters. The currents are often considered to be surface currents. Therefore, the penetration depth must be significantly smaller than the MA size. When we talk further about the currents, we always mean their densities, except in certain specified cases. Too low frequencies or too small sizes can lead to complete field penetration into the balls or wires. For nanoscale metallic structures, this situation is possible in wide frequency ranges, including optical and UV regions. Figure 3 shows the wavelength-normalized depth of penetration into silver as a function of the reciprocal wavelength in meters. The Drude−Lorentz formula for a bulk sample was used. For nanoscale films and quantum filaments, these results are approximate, since the computation requires quantum approaches.

Various resonant emitters, such as open rings, double open rings, etc., are not isotropic. The photonic crystals based on them are not isotropic media. In the general case, metamaterials with such MAs are bianisotropic: an electric field induces both electric and magnetic polarizations in them, and a magnetic field induces magnetic and electric polarizations, respectively. Such AM can exhibit BWs, but they are not structures with $\varepsilon < 0$ and $\mu < 0$ or with $n < 0$. Their rigorous analysis is very complex. Other almost isotropic structures are shown in Fig. 4. They are easier for an analysis and homogenization. The metallic balls in Fig. 4a are characterized by diamagnetism and a small effective permittivity [26]. The metallic cylinders in Fig. 4b exhibit isotropic electric and magnetic polarizations. The dielectric balls and metallic rings in Fig. 4c demonstrate isotropic polarization of both types. The structure in of two shifted rectangular lattices with different dielectric balls in the given low-frequency approximation can only create an isotropic electric polarization (Fig. 4d). Such a metamaterial consists of two identical cubic lattices shifted by half the period with different spherical MAs in them. Replacement of metal cylinders with dielectric ones is possible. Replacing the balls with symmetrically
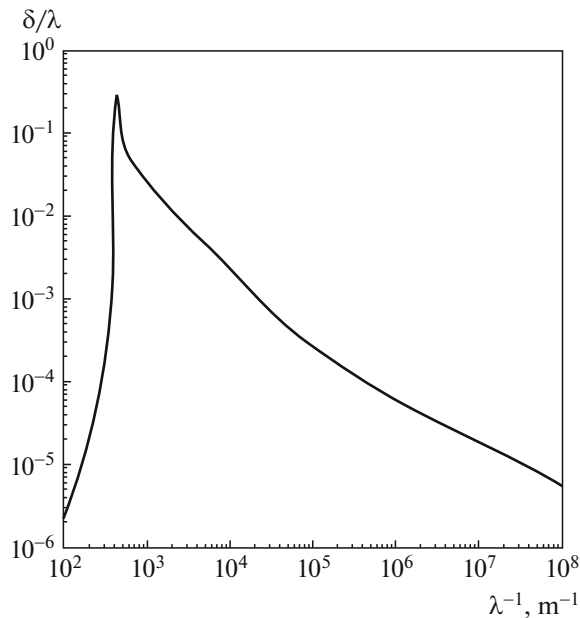
Fig. 3. Normalized penetration depth in silver vs. the reciprocal wavelength.
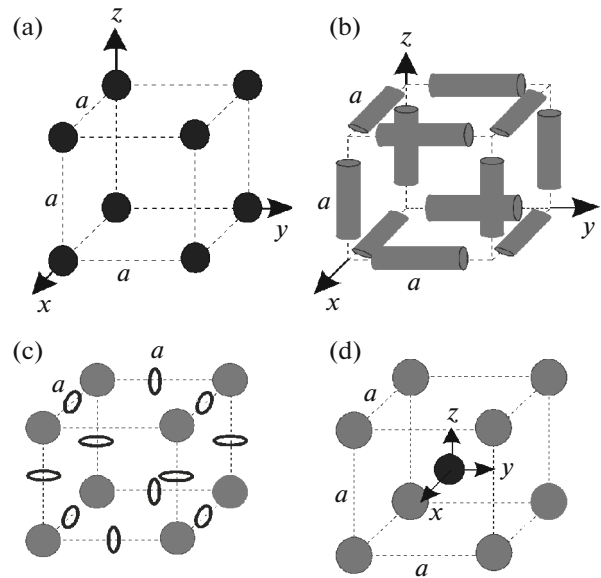

Fig. 4. Elementary cells of cubic photonic crystals with (a, d) spherical, (b) cylindrical, (c) spherical and ring MA inclusions in a dielectric base (matrix).

arranged cubes is also possible. Cubes are much more convenient for modeling. But technologically, it is impossible to place cubes at lattice sites without random turns. The authors of [27−31] proposed to use the AM in Fig. 4d as a metamaterial with both negative ε and μ. This explanation was as follows. Let the resonance frequencies of any E and H modes coincide for two different balls. Then, they contribute to both electric and magnetic polarizations. Slightly shifting the frequency up, we should obtain the polarizations shifted in phase relative to the fields. Here, you can argue like this. First, all resonances are high-frequency. The well-known isotropic dielectric materials with low losses do not have a very high permittivity. MAs should be included in a dielectric base, which weakens the effect. Second, the mode of an individual resonator and the resonator field in a photonic crystal are completely different when a wave moves in it. A wave of given direction **k** in a crystal made of such MAs corresponds to a certain point on a dispersion surface, and a change in the frequency leads to a change in **k**. The polarization direction depends on this fact. Due to the openness of a spherical resonator, its modes are quasi-eigenmodes with complex resonance frequencies. They do not form a complete system of functions, and MAs exchange energy with all neighbors, including distant ones. The wave-excited field inside MA consists of an infinite set of such modes and continuous spectrum functions; i.e., it is impossible to distinguish one mode. Third, the crystal under study has a strong SD (i.e., it is not isotropic), since the excitation of a ball at high resonance frequencies depends on direction **k** and the response is

not local. Fourth, magnetic properties can be not introduced for dielectric structures, and they can be described only by a permittivity tensor. Fifth, even if magnetic polarization is introduced for some reason, it is necessary to prove that some of its components are in antiphase to a magnetic field in order to state that the permeability components are negative.

INTEGRAL AND DISPERSION EQUATIONS

Consider a photonic crystal with cubic cells of face size a, which are filled with a dielectric medium with permittivity $\tilde{\varepsilon}$, and dielectric bodies or MAs in it of volume V with surface S. The area can be multiply connected: the volume can consist of the sum of the volumes bounded by noncontacting surfaces. The complex permittivity $\varepsilon(\omega) = \varepsilon'(\omega) - i\varepsilon'(\omega)$ of MA is considered independent of coordinate **r** inside a body, the MA boundaries are considered sharp; that is, the permittivity undergoes a jump when crossing the boundary, $\varepsilon^-(\omega, \mathbf{r}) \neq \varepsilon^+(\omega, \mathbf{r}) = \tilde{\varepsilon}$, $\mathbf{r} \in S$. Here, the normal component of the electric field also undergoes a jump on surface S, $E_n^-\tilde{\varepsilon} = E_n^+\tilde{\varepsilon}$, and sources, specifically, an induced bound-charge surface density, exists at the boundary. It is obvious that its integral over the entire surface, i.e., the total bound charge, is zero. In this formulation, it is convenient to solve the problem using volume integral equations and a periodic scalar GF [1],

$$\tilde{G}(\mathbf{r}) = \frac{1}{a^3} \sum_{k,l,m=-\infty}^{\infty} \frac{\exp(-i\lfloor \tilde{k}_{xk}x + \tilde{k}_{yl}y + \tilde{k}_{zm}z \rfloor)}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2\tilde{\varepsilon}}. \quad (1)$$

Here, $\tilde{k}_{xk} = k_x + 2k\pi/a$, $\tilde{k}_{yl} = k_y + 2l\pi/a$, and $\tilde{k}_{zm} = k_z + 2m\pi/a$, which determines the periodicity in $\mathbf{k'}$ to within space harmonics. When losses are taken into account, we have $\mathbf{k} = \mathbf{k'} - i\mathbf{k''}$. GF (1) satisfies the equation

$$(\nabla^2 + k_0^2\tilde{\varepsilon})\tilde{G}(\mathbf{r} - \mathbf{r'}) = -\delta(\mathbf{r} - \mathbf{r'})\exp(-i\mathbf{k}\cdot\mathbf{r'}).$$

A dielectric body c permittivity ε in a base with permittivity $\tilde{\varepsilon}$ creates a polarization current density $\mathbf{J}_p(\mathbf{r}) = i\omega\varepsilon_0(\varepsilon - \tilde{\varepsilon})\mathbf{E}(\mathbf{r})$. The polarization current is additional to the bias current in the base. Hence, it is easy to see that a phase shift by $-\pi/2$ between the polarization current and the field takes place in a negative permittivity medium and that this shift is $\pi/2$ in a conventional dielectric. This approach is applicable for all frequencies and any MAs. For ultralow frequencies, we have $\mathbf{J}_p(\mathbf{r}) = (\varepsilon_0\omega_p^2/\omega_c)\mathbf{E}(\mathbf{r}) = \sigma_0\mathbf{E}(\mathbf{r})$, where dc conductivity is indicated. At low frequencies, this is the Drude conductivity. In optics, the Lorentz term is essential. For thin and long wire structures, only the longitudinal component of the current is important, which simplifies the calculation. The integral of GF (1) with density $\mathbf{J}_p(\mathbf{r})$ gives vector potential $\mathbf{A}(\mathbf{r})$; differentiating this potential, we arrive at the volume integral equation $\mathbf{E}(\mathbf{r}) = (i\omega\varepsilon_0\tilde{\varepsilon})^{-1}(\nabla\nabla \cdot \mathbf{A}(\mathbf{r}) + k_0^2\tilde{\varepsilon}\mathbf{A}(\mathbf{r}))$. From here on, we designate $k = k_0\sqrt{\tilde{\varepsilon}}$. In addition, we have magnetic field $\mathbf{H}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$. We can write several equations for the field $\mathbf{E}$ and field $\mathbf{H}$ and for their combination and equations loaded by surface integrals along boundary S and without them [1]. In the general case, they have the form of integro-differential equations, since the desired quantities are under the signs of both integral and derivatives. As is seen from Figs. 2 and 4, the use of surface integrals is inconvenient for algorithmization. We use the volume integral equation

$$\mathbf{E}(\mathbf{r})(2 + \varepsilon(\mathbf{r})/\tilde{\varepsilon})/3$$
$$= \text{p.v.}\int_V \hat{L}\hat{G}(\mathbf{r} - \mathbf{r'})(\varepsilon(\mathbf{r'})/\tilde{\varepsilon} - 1)\mathbf{E}(\mathbf{r'})d^3r'. \qquad (2)$$

Here the operator $\hat{L} \equiv k^2\hat{I} + \nabla\otimes\nabla$ is denoted. The integral in Eq. (2) is taken as the Cauchy principal value. The dependence of GF on k and $\mathbf{k}$ is omitted despite the fact that it determines the dispersion equation. In the right-hand side of Eq. (2), the multiplier $\kappa = \varepsilon/\varepsilon - 1$, which does not depend on the volume of homogeneous particles, is taken out from the integral sign. Due to GF (1), it is sufficient to solve this integral equation only in one (zero) periodicity cell and only inside particles, where $\varepsilon(\mathbf{r}) \neq \tilde{\varepsilon}$. Therefore, the multiplier of the field in the left-hand side is $1 + \kappa/3$. GF ensures the correct interaction of the entire infinite ensemble of MAs. In Eq. (2), we can use the ratio $\hat{L}\tilde{G}(\mathbf{r} - \mathbf{r'}) = \hat{L}'\tilde{G}(\mathbf{r} - \mathbf{r'})$, where the prime means differentiation with respect to the dashed coordinates (point of origin). Field $\mathbf{E}$ is approximated by three-dimensional cubic piecewise constant finite elements, numbering them with one-dimensional index m. Let the number of such elements be M. The Galerkin method for Eq. (2) leads to a homogeneous system of linear algebraic equations (SLAE) $(1 + \kappa/3)\breve{I}E = \kappa\breve{G}E$ of dimension $3M$, where $\breve{I}$ is a unit matrix of dimension $3M$ and E is a vector column of dimension $3M$ consisting of vector columns E^x, E^y, and E^z of dimension M. The values of E_m^x, E_m^y, and E_m^z correspond to the corresponding electric field components in an element with number m. Matrix $\breve{G}$ of dimension $3M$ has a block structure $\breve{G}_{mm'}^{\alpha\beta}$, where α and β take the values of x, y, and z. The dispersion equation has the form of equality of the determinant of the SLAE to zero. Finding the roots of such an equation is very difficult. It is more convenient to use the stationary quadratic functional obtained by scalar multiplication of Eq. (2) by $\mathbf{E}^*(\mathbf{r})$ and by volume integration. It is convenient to solve it for k^2,

$$k^2 = \frac{(1 + k/3)\int_V |\mathbf{E}(\mathbf{r})|^2 d^3r - \kappa \cdot \text{p.v.}\int_V\int_V \mathbf{E}(\mathbf{r})\nabla\otimes\nabla\tilde{G}(\mathbf{r} - \mathbf{r'})d^3r'd^3r}{\int_V\int_V \mathbf{E}^*\tilde{G}(\mathbf{r} - \mathbf{r'})\mathbf{E}(\mathbf{r'})d^3r'd^3r}. \qquad (3)$$

The right-hand side of Eq. (3) depends nonlinearly on k; therefore, Eq. (3) should be solved iteratively together with Eq. (2). It is convenient to find an approximate solution to the dispersion equation, to set one of the amplitudes (e.g., E_1^x, which specifies the wave amplitude), and to solve the SLAE using direct methods. This allows us to refine the solution of the dispersion equation and to use iterations. If we divide a particle into 10 finite elements for each coordinate, we form 10^3 elements and the dimension of the problem is 3×10^3. For the SLAE matrix and functional (3), quadrature formulas can be written. The diagonal ele-

ments in calculating integrals in the sense of a principal value can be equated to zero, or other approximate estimates can be used.

In principle, the dispersion equation allows us to determine ω at a given $\mathbf{k}$. Without taking into account the dissipation, $\mathbf{k}$ can be set arbitrarily. By fixing k_0, we can construct an isofrequency surface in the $\mathbf{k}$ space. Quantity $\mathbf{v}_g = \nabla\omega(\mathbf{k})$ is the group velocity indicating the energy motion direction. In the case of dissipation, the group velocity cannot be used: the cell-averaged Poynting vector $\langle\mathbf{S}\rangle$ should be calculated. It can not match the direction of $\mathbf{v}_g$. In this case, the vector $\mathbf{k} = \mathbf{k}' - i\mathbf{k}''$ is complex. The scalar components to be determined become larger (six) and one complex dispersion equation is no longer sufficient. The $\mathbf{k}$ space also becomes six-dimensional. It is necessary to calculate the Poynting vector $\mathbf{S} = \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*)/2$ and to use the condition $\mathbf{S}/|\mathbf{S}| = \mathbf{k}''/|\mathbf{k}''|$. Its meaning is that the wave attenuation direction $\mathbf{k}''$ coincides with the energy motion direction.

Since the dispersion equation in the form of a high-order determinant equal to zero is very complex, a simpler approximate approach can be based on an equation in the form of a functional based on Eq. (2). To this end, we multiply Eq. (2) by $\mathbf{E}^*$ and integrate in volume or MAs. An approximate equation is obtained by choosing an electric field (or current) for physical reasons. For wire MAs, where the penetration depth is smaller than the wire radius, it is convenient to use an axial current instead of a field and to impose boundary conditions at the wires surfaces. This gives rise to simple dispersion equations [26]. For MAs in the form of a homogeneous dielectric cylinder or ball, it is convenient to use explicit low-frequency solutions of the Helmholtz equation inside and to determine unknown coefficients from functional extremum conditions. Often, e.g., for thin and long cylinders, some field components (radial and/or azimuthal ones) may be neglected.

HOMOGENIZATION

Homogenization is a procedure (generally ambiguous) for achieving effective parameters of homogeneous media, which are electrodynamically equivalent in some sense to the original AMs; it is based on solving inverse problems. In the general case, it requires some averaging procedures. The dependence of averaging methods is one of the ambiguity causes. Another cause is the ambiguity of a homogeneous medium model. The third cause consists in different homogenization methods. One of the first methods used in the theory of artificial dielectrics is the use of diffraction parameters [17]. To meet the extinction theorem conditions, AM must have many periods in each dimension. This approach is electrodynamically complex, especially for 3D problems. The second method is based on comparing the dispersion calculations based on electrodynamic models (e.g., equations of type (2), (3)) with the Fresnel-equation-based dispersion. It is relatively simple if a microscopic dispersion equation is obtained. The third method, which will be used here, is the homogenization based on calculating the cell-averaged polarization periodicity for a wave specified by wavenumber k_0 and wavevector $\mathbf{k}$. Conceptually, it corresponds to the determination of the macroscopic electrodynamic parameters by averaging over physically infinitesimal volume. However, periodicity cell size a is not necessarily small as compared to the wavelength. In any case, for a rigorous solution of Maxwell's equations, averaging is possible for any relations, unless the condition $a \ll \lambda$ is imposed. This condition is important for introducing local permeabilities. We introduce cell-averaged fields, polarizations, and inductions, e.g.,

$$\langle E_x \rangle = \frac{1}{a^3} \int_V E_x(\mathbf{r}) d^3 r.$$

The electric polarization is written as

$$\langle \mathbf{P}^e \rangle = \varepsilon_0(\hat{\varepsilon}_e - \tilde{\varepsilon}\hat{I})\langle\mathbf{E}\rangle + c^{-1}\hat{\xi}\langle\mathbf{H}\rangle.$$

Here, we have considered a bianisotropic model. Similarly, for magnetic polarization we have

$$\langle \mathbf{P}^m \rangle = \mu_0(\hat{\mu}_e - \hat{I})\langle\mathbf{H}\rangle + c^{-1}\hat{\zeta}\langle\mathbf{E}\rangle.$$

We introduce magnetization $\mathbf{M} = (\hat{\mu}_e - \hat{I})\mathbf{H}$. Writing other averaged components, we obtain SLAE for determining the effective parameters. The approach is suitable for anisotropic and bianisotropic medium models with allowance for symmetry relations for material parameters. At a large number of unknowns, we can write an overdetermined SLAE and solve it by regularization. In the case of an isotropic model, we have $\hat{\xi} = \hat{\zeta} = 0$. In the case of a biisotropic model, these parameters are nonzero pseudoscalars. In the case of a dielectric anisotropic medium, we have three equations,

$$\langle P_x^e \rangle = \varepsilon_0((\varepsilon_{exx} - \tilde{\varepsilon})\langle E_x \rangle + \varepsilon_{exy}\langle E_y \rangle + \varepsilon_{exz}\langle E_z \rangle)$$

and two similar ones for other components. If dissipation is neglected, the Onsager–Casimir symmetry conditions $\varepsilon_{e\alpha\beta} = \varepsilon_{e\beta\alpha}$ take place; i.e., the tensor is symmetric. In the case of dissipation, the tensor is not Hermitian but the condition $\varepsilon'_{e\alpha\beta} = \varepsilon'_{e\beta\alpha}$ is met. From the three equations, three quantities are determined if symmetry conditions are taken into account. The equations are simplified by reducing the permittivity tensor to principal axes. Thus, three permittivity tensor components can be determined from three polarizations. If there are only two tensors ($\hat{\varepsilon}_e$, $\hat{\mu}_e$) and they can be simultaneously reduced to a diagonal form, in the general case six complex components can be determined from six independent complex scalar Maxwell's equations. However, the possibility of this reduction

depends on the structure of AM. For example, suppose we have an AM with magnetic and electric dipoles at the sites of two identical interpenetrating cubic lattices. We can make AM by moving the lattices relative to each other by an arbitrary distance. Formally, we can also rotate one lattice relative to the other through three arbitrary angles relative to different axes. It is unlikely that such a thought experiment should be implemented, but this distinguishes AM from natural environments, where symmetry is essential. In the case of only a dielectric AM model without dissipation, the Onsager–Casimir conditions are met and the tensor is reduced to a diagonal form in the general case. It may turn out that the coordinate system in which it is sufficient to determine six rather than three components from six Maxwell's equations is more convenient. For the isotropic AM of interest, it is sufficient to write relations for only one component. We have

$$\varepsilon_e = \tilde{\varepsilon} + \varepsilon_0^{-1}\langle P_x^e\rangle/\langle E_x\rangle,$$
$$\mu_e = 1 + \mu_0^{-1}\langle P_x^m\rangle/\langle H_x\rangle. \tag{4}$$

Having obtained the solution of the problem for $\mathbf{E}$, field $\mathbf{h(r)} = \nabla \times \mathbf{A(r)}$ is found as $H = \breve{A}E$, where the introduced matrix is determined by the integro-differential operator for obtaining a magnetic field. Consider the polarization calculation

$$\langle \mathbf{P}^e\rangle = \frac{1}{a^3}\int_V \rho(\mathbf{r})\mathbf{r}d^3r,$$
$$\langle \mathbf{M}\rangle = \frac{1}{a^3}\int_V \mathbf{r}\times \mathbf{J}_p(\mathbf{r})d^3r. \tag{5}$$

Here, there the charge density is $\rho(\mathbf{r}) = i\nabla\cdot\mathbf{J}_p/\omega = -\varepsilon_0\nabla\cdot[(\varepsilon - \tilde{\varepsilon})\mathbf{E}]$. Since we consider MAs of a homogeneous dielectric, we have $\nabla\cdot\mathbf{E}$ inside it. Therefore, the surface divergence associated with the induced surface charge density $\rho_S = \varepsilon_0 E_\nu(1 - \tilde{\varepsilon}/\varepsilon)$ should be calculated. Here, E_ν is the external normal coordinate. Since $\nabla\cdot(\varepsilon\mathbf{E}) = 0$, we have $\tilde{\varepsilon}\nabla\cdot\mathbf{E} = -(\tilde{\varepsilon}/\varepsilon)\mathbf{E}\cdot\nabla\varepsilon$ and $\nabla\varepsilon = (\tilde{\varepsilon} - \varepsilon)$ and the charge density is proportional to the delta function of the normal coordinate. Therefore, the first integral in Eq. (5) becomes a surface integral,

$$\langle \mathbf{P}^e\rangle = \frac{1}{a^3}\oint_S \rho_S(\mathbf{r})\mathbf{r}d^2r.$$

Denote $\chi = \varepsilon/\tilde{\varepsilon} - 1$. Equation (2) can be written in the following form: $\mathbf{E} = \hat{G}(\chi\mathbf{E})$ and $\hat{G} = \hat{L}\hat{\tilde{G}}()$, where $\hat{\tilde{G}}$ is an integral operator with kernel $\tilde{G}$ and $\hat{L} = \nabla\otimes\nabla + k^2$ is a differential operator. Since $\nabla\cdot\nabla\otimes\nabla = \nabla\cdot\nabla^2$ and $(\nabla^2 + k^2)\tilde{G} = -\delta(\mathbf{r})$, we can easily see that the action of the divergence operator on the left on the integral equation gives identity. When solving this equation, we have to determine the normal component of the electric field and the surface charge density.

Consider a number of simple cases. Let us have a photonic crystal of wire rings oriented along the z axis. For this crystal, we have $\varepsilon_e = 1$, $\mu_{ezz} = \mu$, and $\mu_{exx} = \mu_{eyy} = 1$. We have one electric current component $J_\varphi = \sigma(\omega)E_\varphi$. For simplicity, let the wire radius be smaller than the skin layer thickness and the current is uniformly distributed. The solenoidal current creates two vector-potential components [25],

$$A_\rho = \int_V J_\varphi \sin(\varphi - \varphi')\tilde{G}(\mathbf{r},\mathbf{r}')d^3r',$$

$$A_\varphi = \int_V J_\varphi \cos(\varphi - \varphi')\tilde{G}(\mathbf{r},\mathbf{r}')d^3r'.$$

Integration is carried out over the ring volume, and the current can be taken out of the integral, since it is constant. The wires are taken to be thin and a field is taken to penetrate into them completely. It is more convenient for us to use the conventional GF

$$G_0(\mathbf{r},\mathbf{r}') = |4\pi(\mathbf{r} - \mathbf{r}')|^{-1}\exp(-ik_0|\mathbf{r} - \mathbf{r}'|),$$

and to continue it periodically. We have

$$\nabla\cdot\mathbf{A} = \rho^{-1}\partial_\rho(\rho A_\rho) + \rho^{-1}\partial_\varphi A_\varphi,$$

and for GF we write the expressions

$$G_{xlm} = \exp(-ilk_xa - imk_za)G_{lm},$$
$$G_{ylm} = \exp(-ilk_ya - imk_za)G_{lm}.$$

GF G_{xlm} corresponds to a source located at point $x = la$, $y = 0$, and $z = ma$, and GF G_{ylm} corresponds to a source located at point $x = 0$, $y = la$, and $z = ma$. However, fields from all sources are considered in a zero cell. Therefore, using the summation

$$\tilde{G} = \sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\exp(-ia[mk_z + l(k_x + k_y)])G_{lm}$$

we obtain a periodic GF acting in the zero cell associated with the origin of coordinates. Denote $\psi_{lm}(k_x, k_y, k_z) = a\lfloor mk_z + l(k_x + k_y)\rfloor$. For GF, we have [25]

$$G_{lm} = \sum_{n=-\infty}^{\infty}\exp(-in(\phi - \phi'))$$

$$\times\int_0^\infty \frac{\exp(-\sqrt{\kappa^2 - k_0^2}|z - z' - ma|)J_n(\kappa\rho)J_n(\kappa(\rho' + la))}{4\pi\sqrt{\kappa^2 - k_0^2}}\kappa d\kappa. \tag{6}$$

Obviously, $G_0(\mathbf{r},\mathbf{r}') = G_{00}(\mathbf{r},\mathbf{r}')$. Calculating the integral equation in the zero cell, we obtain the solution of the problem. In order not to move to a toroidal coordinate system, we are still in the cylindrical system and replace the circular cross section of the wire of radius r_0 with a rectangular cross section of area $4r_0^2$, almost without changing the result. Assuming the field is constant, we have

$$\frac{1}{\pi}\int_0^{2\pi}\sin(\varphi-\varphi')\exp(-in(\varphi-\varphi'))d\varphi' = i(\delta_{n,-1}-\delta_{n,1}),$$

$$\frac{1}{\pi}\int_0^{2\pi}\cos(\varphi-\varphi')\exp(-in(\varphi-\varphi'))d\varphi' = \delta_{n,-1}+\delta_{n,1}.$$

Denoting the integral in Eq. (6) as $g_{\text{nim}}(\rho,z|\rho',z')$, we obtain

$$A_\rho = i\int(g_{-1lm}(\rho,z|\rho',z')$$
$$-g_{1lm}(\rho,z|\rho',z'))\rho'd\rho'dz' = 0,$$

$$A_\varphi = 2\int g_{1lm}(\rho,z|\rho',z')\rho'd\rho'dz'.$$

We used the equality $J_{-1}(x) = -J_1(x)$ for the Bessel functions. The vector potential has one component independent of φ. The electric field has three components, $E_\rho = \partial_\rho A_\varphi/(i\omega\varepsilon_0)$, $E_\varphi = k_0^2 A_\varphi/(i\omega\varepsilon_0)$, and $E_z = \partial_z A_\varphi/(i\omega\varepsilon_0)$. The magnetic field has two components, $H_\rho = -\partial_z A_\varphi$ and $H_z = \rho^{-1}\partial_\rho(\rho A_\varphi)$. The Cartesian system has all six components. Components E_ρ and E_z inside a wire may be neglected in comparison with the constant azimuthal component of the field. The electric field inside the wire (E_φ) is the sum of all the periodic-GF-induced responses,

$$E_\varphi = \sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\exp(-ia[mk_z + l(k_x + k_y)])E_{\varphi lm}, \tag{7}$$

$$E_{\varphi lm} = \frac{\pi k_0^2\sigma(\omega)}{i\omega\varepsilon_0}\int\rho'd\rho'dz'\int_0^\infty\exp\frac{\exp(-\sqrt{\kappa^2-k_0^2}|z-z'-ma|)J_1(\kappa\rho)J_1(\kappa(\rho'+la))}{\sqrt{\kappa^2-k_0^2}}\kappa d\kappa. \tag{8}$$

In Eq. (7), we performed integration over angle; therefore, the integration in Eq. (8) is performed over the cross section of the wire. Assuming the field is constant, we multiply Eq. (7) by ρ, integrate again over the cross section, and cancel the field. $4Rr_0^2$ is retained in the left-hand side of Eq. (7). The right-hand side of Eq. (8) is considered separately for zero and nonzero values of subscript m, denoting $\alpha = \sqrt{\kappa^2-k_0^2}$. In the first case, the dashed coordinate area of integration is divided into two. The result has the form

$$f_0(\kappa,k_0,r_0) = \int_{-r_0}^{r_0}\int_{-r_0}^{r_0}\exp(-\sqrt{\kappa^2-k_0^2}|z-z'|)dzdz'$$
$$= \frac{4r_0}{\alpha}\left(1 - \frac{1-\exp(-2\alpha r_0)}{2\alpha r_0}\right).$$

If α is small, we have $\kappa^2 \approx k_0^2$ and $f = 4r_0^2$. In the second case, integrals with positive subscripts and the negative subscripts that are equal in absolute value are

$$f_m(\kappa,k_0,r_0) = f_{-|m|}(\kappa,k_0,r_0)$$
$$= -4\exp(-\alpha|m|a)\left(\frac{\sinh(\alpha r_0)}{\alpha}\right)^2.$$

Passing to summation over positive indices, we obtain $2f_m(\kappa,k_0,r_0)$. Calculating integrals over ρ', we perform integration over regions $la - R - r_0 < \rho' < la - R + r_0$ and $la + R - r_0 < \rho' < la + R + r_0$ and obtain

$$I_l(\kappa,r_0,R) = \int_{la-R-r_0}^{la-R+r_0}J_1(\kappa\rho)\rho d\rho$$
$$+ \int_{la+R-r_0}^{la+R+r_0}J_1(\kappa\rho)\rho d\rho \approx 2r_0[(la-R)J_1(\kappa(la-R))$$
$$+ (la+R)J_1(\kappa(la+r))].$$

Here, R is the ring radius. We used the smallness of the wire radius and the mean-value theorem. Since $I_{-l} = I_l$, we perform summation over positive indices by doubling I_l. An analogous result is obtained for a complex-conjugate dispersion equation. Taking into account $\sigma(\omega) = i\omega\varepsilon_0(\varepsilon_L - \omega_P^2/(\omega^2 - i\omega\omega_c))$ and multiplying both results, we derive the equation

$$1 = \left(\frac{\pi k_0^2|\sigma(\omega)|}{\omega\varepsilon_0}\right)^2$$
$$\times\left|\sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\exp(-i\psi_{ml}(k_x,k_y,k_z))\right.$$
$$\times(2-\delta_{m0})(2-\delta_{l0})$$
$$\times\left.\int_0^\infty\frac{f_m(\kappa,k_0,r_0)I_l(\kappa,k_0,r_0)J_1(\kappa R)}{r_0\sqrt{\kappa^2-k_0^2}}\kappa d\kappa\right|^2. \tag{9}$$

It connects the frequency and the wavevector. This result shows that the solutions in noncoordinate problems take a complex form even for strong simplifications. In the low-frequency limit, we have $\alpha = \kappa$ and can explicitly express k_0^2 from Eq. (9). The magnetic moment for the problem takes the form $\langle M_z\rangle = J_\varphi(\pi r_0 R)^2/a^3$. To solve the question of the sign of per-

meability, it is necessary to calculate the average component of the field,

$$\langle H_z \rangle = \frac{1}{a^3} \int\limits_{-a/2}^{a/2} dx \int\limits_{-a/2}^{a/2} dy \int\limits_{-a/2}^{a/2} dz H_z(\mathbf{r}, \mathbf{k}, k_0).$$

Such calculations are more convenient to perform in the Cartesian coordinate system. The sign of permeability is determined by the formula $\mu = 1 + \langle M_z \rangle / \langle H_z \rangle$. If there is no dissipation, the magnetization should be in antiphase to the field, i.e., negative, and exceed unity in absolute value.

We show how this problem is solved in the Cartesian system. From here on, instead of Eq. (9), we derive a new dispersion equation assuming that the loop is rectangular. A ring of radius R is replaced by a rectangular loop with an arm $b = R\sqrt{\pi}$ and cross section $4\delta^2 = \pi r_0^2$. We take density J directed clockwise along the x and y axes out of the integral and obtain the components of the vector potential and the field,

$$A_x(\mathbf{r}) = \frac{-16iJ}{a^3} \sum_{k,l,m=-\infty}^{\infty} \frac{\exp(-i\lfloor \tilde{k}_{xk}x + \tilde{k}_{yl}y + \tilde{k}_{zm}z \rfloor)}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2}$$
$$\times \frac{\sin(\tilde{k}_{zm}\delta)}{\tilde{k}_{zm}} \frac{\sin(\tilde{k}_{xk}b/2)}{\tilde{k}_{xk}} \frac{\sin(\tilde{k}_{yl}\delta)}{\tilde{k}_{yl}} \sin(\tilde{k}_{yl}b/2), \quad (10)$$

$$A_y(\mathbf{r}) = \frac{16iJ}{a^3} \sum_{k,l,m=-\infty}^{\infty} \frac{\exp(-i\lfloor \tilde{k}_{xk}x + \tilde{k}_{yl}y + \tilde{k}_{zm}z \rfloor)}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2}$$
$$\times \frac{\sin(\tilde{k}_{zm}\delta)}{\tilde{k}_{zm}} \frac{\sin(\tilde{k}_{xk}b/2)}{\tilde{k}_{xk}} \frac{\sin(\tilde{k}_{yl}\delta)}{\tilde{k}_{yl}} \sin(\tilde{k}_{xk}b/2), \quad (11)$$

$$H_z = \partial_x A_y(\mathbf{r}) - \partial_y A_x(\mathbf{r}) = -\frac{4Jb^2\delta^2}{a^3}$$
$$\times \sum_{k,l,m=-\infty}^{\infty} \frac{\exp(-i\lfloor \tilde{k}_{xk}x + \tilde{k}_{yl}y + \tilde{k}_{zm}z \rfloor)}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2} F_{mlk}(\mathbf{k}), \quad (12)$$

$$F_{mlk}(\mathbf{k}) = \mathrm{sinc}(\tilde{k}_{zm}\delta)\mathrm{sinc}(\tilde{k}_{xk}b/2)$$
$$\times \mathrm{sinc}(\tilde{k}_{yl}b/2)[(\tilde{k}_{yl})^2\mathrm{sinc}(\tilde{k}_{yl}\delta) + (\tilde{k}_{xk})^2\mathrm{sinc}(\tilde{k}_{xk}\delta)]. \quad (13)$$

Now, we average field (12) as follows:

$$f(\tilde{k}_{xk}) = \frac{1}{a^3} \int\limits_{-a/2}^{a/2} \exp(-i\tilde{k}_{xk}x)dx$$
$$= \frac{2\sin(\tilde{k}_{xk}a/2)}{\tilde{k}_{xk}a} = \mathrm{sinc}(\tilde{k}_{xk}a/2).$$

As a result, we have

$$\langle H_z \rangle = -\frac{4Jb^2\delta^2}{a^3}$$
$$\times \sum_{k,l,m=-\infty}^{\infty} \frac{\mathrm{sinc}(\tilde{k}_{xk}a/2)\mathrm{sinc}(\tilde{k}_{yl}a/2)\mathrm{sinc}(\tilde{k}_{zm}a/2)}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2} F_{mlk}(\mathbf{k}). \quad (14)$$

The functions $\mathrm{sinc} = \sin(x)/x$ are even and rapidly decreasing. Although the zero term of the sum in Eq. (14) makes a significant contribution, a strict result requires taking into account a large number of terms. We now write

$$\chi_{ezz} = \mu - 1 = -\frac{1}{\displaystyle\sum_{k,l,m=-\infty}^{\infty} \frac{\mathrm{sinc}(\tilde{k}_{xk}a/2)\mathrm{sinc}(\tilde{k}_{yl}a/2)\mathrm{sinc}(\tilde{k}_{zm}a/2)}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2} F_{mlk}(\mathbf{k})}. \quad (15)$$

In Eq. (15), the solution to the microscopic dispersion equation should be substituted. If we substitute the wavenumber squared from the macroscopic Fresnel equation $k_0^2 = (k_x^2 + k_y^2)/\mu_{ezz} + k_z^2$, we have an implicit equation by setting $\mathbf{k}$. Its solution is symmetric in wavevector, $\mu(\mathbf{k}) = \mu(-\mathbf{k})$. Simplification can be achieved by choosing the wave propagation direction. For example, choosing $k_x = k_y$ and $k_z = 0$, we find the frequency from Eq. (9) and substitute it into Eq. (15). We have $F_{000}(\mathbf{k}_\perp) = 2\mathrm{sinc}(k_x\delta)[k_x\mathrm{sinc}(k_xb/2)]^2$ and the following zeroth approximation:

$$\mu_{(0)} = 1 - \frac{2k_x^2 - k_0^2}{2[k_x\mathrm{sinc}(k_xa/2)\mathrm{sinc}(k_xb/2)]^2}.$$

It is very rough, which is determined by the slow convergence of series (15). For a slow wave, the sign of

susceptibility $\chi_{ezz}^{(0)}$ is negative. As follows from the Fresnel equation, $2k_x^2 = \mu k_0^2$; therefore, the solution $\mu_{(0)} = 1$ takes place in the low-frequency limit. If $\mu < 0$, the wavenumbers are imaginary. This is possible for the Bragg resonance in the band gap. For a slow wave, the deceleration is maximal at $k_x = k_y = \pi/a$; hence, the maximum value is $\mu = 2\pi^2/(k_0a)^2 > 1$. The maximum deceleration frequency can be found from the dispersion equation. Above the resonance frequency, we have $\mu < 0$ in the band gap. However, this is true on the assumption that the Fresnel equation is still valid. This approach for a crystal made of metal loops was used in [26]. Here, there is an analogy with crystal optics, for which strong SD and Bragg resonances occur in the X-ray range. Using an approximate formula with allowance for only one term of the series, we obtain

$$\mu_{(0)} = 1 - \frac{2\pi^2 - k_0^2}{8\mathrm{sinc}^2(\pi b/(2a))} \approx 1 - \frac{2\pi^2 - k_0 a}{8}.$$

We took into account that $b \ll a$. In the general case, we denote the sum in the denominator in Eq. (15) as

$$S = S_0(\mathbf{k}) + \tilde{S}(\mathbf{k}) + \tilde{S}(-\mathbf{k}),$$

$$\tilde{S} = S_{00m}(\mathbf{k}) + S_{k00}(\mathbf{k}) + S_{0l0}(\mathbf{k}) + S_{0lm}(\mathbf{k})$$
$$+ S_{k0m}(\mathbf{k}) + S_{kl0}(\mathbf{k}) + S_{klm}(\mathbf{k}),$$

where

$$S_0(\mathbf{k}, k_0) = \frac{\mathrm{sinc}(k_x a/2)\mathrm{sinc}(k_y a/2)\mathrm{sinc}(k_z a/2)}{k_x^2 + k_y^2 + k_z^2 - k_0^2} F_{000}(\mathbf{k}),$$

$$S_{00m}(\mathbf{k}, k_0)$$
$$= \sum_{m=1}^{\infty} \frac{\mathrm{sinc}(\tilde{k}_{xk} a/2)\mathrm{sinc}(\tilde{k}_{yl} a/2)\mathrm{sinc}(\tilde{k}_{zm} a/2)}{k_x^2 + k_y^2 + k_{zm}^2 - k_0^2} F_{00m}(\mathbf{k}),$$

$$S_{0lm}(\mathbf{k}, k_0)$$
$$= \sum_{l,m=1}^{\infty} \frac{\mathrm{sinc}(k_x a/2)\mathrm{sinc}(\tilde{k}_{yl} a/2)\mathrm{sinc}(\tilde{k}_{zm} a/2)}{k_x^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2} F_{0lm}(\mathbf{k}),$$

$$S_{klm}(\mathbf{k}, k_0)$$
$$= \sum_{k,l,m=1}^{\infty} \frac{\mathrm{sinc}(k_x a/2)\mathrm{sinc}(\tilde{k}_{yl} a/2)\mathrm{sinc}(\tilde{k}_{zm} a/2)}{k_x^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2} F_{klm}(\mathbf{k}).$$

All other sums, namely, triple, double, and single ones taken at positive subscripts, are calculated in the same way. Then, we have

$$\mu(\mathbf{k}, k_0) = 1 - \frac{1}{S_0(\mathbf{k}) - \tilde{S}(\mathbf{k}) + \tilde{S}(-\mathbf{k})}.$$

Since $\tilde{S}(\mathbf{k}) = \tilde{S}(-\mathbf{k})$, we have $\mu(-\mathbf{k}, k_0) = \mu(\mathbf{k}, k_0)$. For $\mathbf{k} = 0$, the solution to the dispersion equation is $k_0 = 0$; therefore, $\mu(0, 0) = 1$. For small sizes δ and b, we have $\mu(\mathbf{k}, k_0) \approx 1$. A strong SD means that it is impossible to obtain a $\mu(\omega)$ dependence: the result depends significantly on $\mathbf{k}$. The resulting material equation is nonlocal: it refers to the entire cell area, and locality occurs at $\lambda \gg a$, i.e., at $\mathbf{k}^2 \approx k_0$, when $\mu(\mathbf{k}, k_0) \approx 1$. By increasing the loop sizes, we can obtain the permeabilities that are different from unity and even negative; however, it is not a fact that a negative permeability is actually achieved in the usual sense: in this case, the model becomes very approximate and the result depends on $\mathbf{k}$.

We now replace a metallic ring with a thin dielectric one and neglect the transverse fields in comparison with the toroidal field. A polarization current flows through the ring, and it excites the same field as the conduction current. For a metallic ring, there is no difference: a high-frequency current is also a polarization current to some extent. For a dielectric ring, there is no current at zero frequency. In the above formulas,

it is sufficient to put $\omega_P^2 = 0$ for conductivity. It is very tempting to form magnetic properties of AM on structures such as dielectric rings, dielectric spirals, and in general on MAs with solenoidal polarization currents. However, the currents should be strictly solenoidal. For a dielectric ring of radius R, the condition $k_0\sqrt{\varepsilon}R \ll 1$ must be met; for a high permittivity, it leads to very small radii and small effective permeabilities. The permittivity of a dielectric ring located in a dielectric base should differ significantly from its permittivity. The dielectric ring should be very thin so that transverse fields and resonances can be neglected. This leads to small effective parameters. Unlike a metal, the polarization current at low frequencies is low, and the approximation works poorly at high frequencies. Nevertheless, the results obtained can be formally used for dielectric rings. In vacuum, such a ring is a resonator with the $H_{01\delta}$ type of oscillation (see [29]). In a photonic crystal, such a resonator is the source of a wave and, at the same time, is excited by it. The polarization and the ring current should be in antiphase to the H_z component. When the MP component is negative, the photonic crystal becomes a hyperbolic metamaterial. Considering a square dielectric loop, we can use components (10) and (11) to obtain a dispersion equation. The electric field is determined in terms of the vector potential as $\mathbf{E} = (k_0^2 + \nabla\nabla\cdot)\hat{\tilde{G}}((\varepsilon - 1)\mathbf{E})$, where $\mathbf{A} = i\omega\varepsilon_0\hat{\tilde{G}}((\varepsilon - 1)\mathbf{E})$ and $\hat{\tilde{G}}$ is an integral operator with kernel (1). To use an algorithm with the kernel $\nabla \otimes \nabla\tilde{G}$ is problematic due to its non-integrable feature. Assuming the field is solenoidal inside a ring and transferring operator $\nabla\cdot$ to $(\varepsilon - 1)\mathbf{E}$, we obtain surface integrals of the surface charge density. Since we neglected the normal components of the field at the ring boundary, we also neglected the surface charges, and a good approximation is the integral equation $\mathbf{E} = k_0^2\hat{\tilde{G}}((\varepsilon - 1)\mathbf{E}) = k_0^2(\varepsilon - 1)\hat{\tilde{G}}(\mathbf{E})$. A dispersion equation based on it is written in the form

$$\int_V |\mathbf{E}|2d^3r = k_0^2(\varepsilon - 1)$$
$$\times \int \mathbf{E}^*(\mathbf{r})\tilde{G}(\mathbf{r},\mathbf{r}')\mathbf{E}(\mathbf{r}')d^3r d^3r'. \tag{16}$$

In the Cartesian system, it is more convenient to consider a rectangular loop. Considering now the field constant and having one constant component, E_x or E_y, in each arm, $E_x = (x, -b/2, z) = -E_x (x, b/2, z) = E_y(b/2, y, z) = -E_y(-b/2, y, z) = E$, we obtain the equation

$$1 = \frac{4k_0^2\delta^2 b(\varepsilon - 1)}{a^3} \sum_{k,l,m=-\infty}^{\infty} \frac{|G_{klm}(\mathbf{k})|^2}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2},$$

where

$$|G_{klm}(\mathbf{k})|^2 = \text{sinc}^2(\tilde{k}_{zm}\delta)\text{sinc}^2(\tilde{k}_{xk}b/2)$$
$$\times \text{sinc}^2(\tilde{k}_{yl}b/2)[\sin^2(\tilde{k}_{xk}b/2) + \sin^2(\tilde{k}_{yl}b/2)].$$

Setting vector $\mathbf{k}$, we can determine the wavenumber using this equation,

$$k_0^2 = \cfrac{1}{\cfrac{4\delta^2 b(\varepsilon - 1)}{a^3} \sum_{k,l,m=-\infty}^{\infty} \cfrac{|G_{klm}(\mathbf{k})|^2}{\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 - k_0^2}}. \quad (17)$$

Equation (17) can be solved iteratively. It is also suitable for a metallic loop, the permittivity of a metal is taken as ε. If $\tilde{\varepsilon}$ is the permittivity of the base, it is necessary to make the substitutions $k_0^2 \to k_0^2\tilde{\varepsilon}$ and $\varepsilon \to \varepsilon/\tilde{\varepsilon}$. Assuming $\mathbf{k} = \mathbf{x}_0 k_x$ and $k_x \approx k_0 \ll \pi/a$ and taking into account only the large zero term of the sum, we obtain $k_0^2 \approx k_x^2/\lfloor 1 + 4\delta^2 b(\varepsilon - 1)/a^3 \rfloor$. The approximation is valid if $4\delta^2 b(\varepsilon - 1)/a^3 \ll 1$. In this case, we have $k_x \approx k_0$. Of course, Eq. (17) is very approximate. Specifying the field as a cyclic polarization current, we imposed the following symmetry conditions: $E_x(x, y, z)$ is an even function in x, z and odd function in y, and $E_y(x, y, z)$ is an even function in y, z and odd in x. Due to symmetry, such a solution can exist, but solutions with other even−odd relations can exist. Solving the problem in a general form, e.g., by the finite element method, we can find waves of all possible symmetries and their corresponding dispersions. Equation (17) for a completely even field is even. In the case of wave diffraction by a metamaterial of finite sizes, the given symmetry of the fields, generally speaking, does not hold. It means the presence of two electrical walls normal to the loop plane, and it can be implemented using phase shift π per cell along both coordinates, which can be performed in a band gap, which corresponds to the attenuation band of the filter made of a finite sample. For diffraction by a metamaterial in the low-frequency limit, a near-zero phase shift takes place. The use of constant components of the electric field inside a loop is the most severe limitation of the model. However, it is this limitation that allowed us to obtain a simple explicit result.

NEGATIVE ε'_e AND μ'_e IN THE LORENTZ MODEL

The classical Lorentz model for a rarefied oscillator gas (electric dipoles or oscillators) with frequencies ω_{0el} gives the following permittivity equation [13]:

$$\varepsilon_L(\omega) = 1 + \sum_{l=1}^{N} \frac{\omega_{el}^2}{\omega_{0el}^2 - (\omega^2 - i\omega\omega_{cl})}. \quad (18)$$

Here, the frequency squared $\omega_{el}^2 = e^2 N_l/(\varepsilon_0 m_l)$ is determined by the concentration (N_l) and the mass of charged particles of kind l, and the pulse relaxation frequency ω_{cl} (collision frequency) determines the broadening of a spectral line. The quantum description based on the semi-classical approximation gives the same form of permittivity [6, 13, 32],

$$\varepsilon_L(\omega) = 1 + \frac{e^2}{\varepsilon_0 m_e} \sum_{\nu} \sum_{k} N_k^{\nu} \sum_{l \neq k} \frac{f_{lk}^{\nu}}{(\omega_{0lk}^{\nu})^2 - (\omega^2 - i\omega\omega_{lk}^{\nu})}. \quad (19)$$

Here, the summation over subscripts ν is performed for all kinds of atoms, over subscripts k, for all energy levels, and over l, for all allowed transitions from level k. Accordingly, N_k^{ν} is the number of atoms of kind ν in state k, $\omega_{0\nu} = \omega_{lk}^{\nu} = (E_k - E_l)\hbar$, and ω_{lk}^{ν} is the width of the transition $k \to l$ line associated with the lifetime. If the number of atoms in excited states is larger than that in the ground state, we have $\varepsilon'' < 0$ and the medium is active. The oscillator strength $f_{lk}^{\nu} = 2m_e|\omega_{0lk}^{\nu}\|d_{lk}^{\nu}|^2/(3e_2\hbar g_l)$ of a dipole transition for an atom with dipole moment $\mathbf{d}^{\nu}$ is determined by the squared modulus of the matrix element $d_{lk}^{\nu} = -e\langle\psi_l|\mathbf{r}|\psi_k\rangle$ [6, 32]. In the general case, to calculate (19), it is necessary to perform summation over all electric multipole transitions that satisfy the selection rules. It is easy to see that the condition $\varepsilon'_L(\omega) < 0$ can be met. However, Eq. (19) does not give the correct value of permittivity, since it was derived when the field of a plane wave in vacuum is applied to a single atom by summing the contributions of atoms to polarization. It is necessary to take into account the mean field [6]. Researchers often write a formula of type (18) for μ_e to achieve $\mu'_e < 0$ from solutions to diffraction problems using computer simulation packages. However, a formula of type (18) has no relation to μ_e. In quantum analysis, a perturbed Hamiltonian is known to be written as $\hat{H} = \hat{H}_0 + \hat{H}'$, where $\hat{H}' = -(e/m)\hat{\mathbf{p}} \cdot \mathbf{A}^e + (e\mathbf{A}^e)^2/(2m) + \hat{H}_s$ the perturbation associated with field $\mathbf{B} = \nabla \times \mathbf{A}$ [32, 33]. Here, $\hat{H}_s = \mu_B g_s \mathbf{B} \cdot \hat{\mathbf{s}}/\mu_0$, $\mathbf{A}^e = \mu_0\mathbf{A}$, $\mathbf{A}$ is the vector potential introduced earlier, $g_s \approx 2$, and one electron in an atom with spin $\hat{\mathbf{s}}$ is considered for simplicity. Neglecting the second term and taking into account the commutation properties, we can write $\hat{H}' = \mu_B(\hat{\mathbf{L}} + 2\hat{\mathbf{S}})/\mu_0$, where μ_B is the Bohr magneton and $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$ are the complete operators of the spin and orbital moments of the atom (including nucleus) [33]. The magnetic susceptibility depends on the total magnetic moment of an atom; if it is nonzero, the susceptibility consists of polarization paramagnetism, precession diamagnetism, and orientation paramagnetism [34]. In weak fields, to determine the contribution of an atom to magnetic polarization, it is necessary to calculate matrix elements $\mathbf{m}_{lk} = -\mu_B\langle\psi_l|2\hat{\mathbf{S}} + \hat{\mathbf{L}}|\psi_k\rangle$. The rotational spectra of polar molecules in a gas and liquid can also form a magnetic moment. However, this is usually a microwave range

with strong line broadening, and no contribution to permeability occurs. In the optical range, natural substances do not exhibit a significant difference between the permeability and unity [9, 34]. AM made of metallic nanoparticles, which are quantum dots or 3D quantum boxes, can be modeled using quantum methods. By determining the energy levels and wavefunctions for them or using the density functional theory, one can calculate the current density induced by field $\mathbf{B}(\omega)$. This field also acts on a classic loop with a current; therefore, $\mathbf{P}^m(\omega)$ is proportional to $\mathbf{B}(\omega)$ rather than $\mathbf{H}(\omega)$ in weak fields.

Considering classic contours with currents excited by magnetic induction $\mathbf{B}(\omega) = \mu_0 \mu_e \mathbf{H}(\omega)$, we have to take into account that the current in the circuit in the absence of a field is attenuated and has no resonance frequencies. At room temperature, the free path of electrons in a metal is several tens of nanometers. In addition, unlike a spin, the magnetic moment of the circuit in an atom cannot change the orientation direction depending on the angle of a magnetic field. This induced magnetism at low frequencies is diamagnetism [26]. It is possible to derive a formula of type (18) without resonances for μ_e^{-1}, since field $\mathbf{B}(\omega)$ act on moving charges. The MA-induced contribution to polarization is proportional to $\mathbf{B}(\omega)$ rather than $\mathbf{H}(\omega)$. The classical approach can easily be applied to the integral equation for wire microrings (Fig. 4c). It is convenient to place them on the edges of a cubic crystal. Resonance structures such as open microrings with a capacitance gap are often considered. Such an RCL circuit has a resonance frequency $(LC)^{-1/2}$ of forced resonance, but it is very high because the edge capacitance is low. To decrease ω_0, it is necessary to increase the capacitance. A ring with a capacitance create magnetic and electric dipoles oriented normal and tangentially to the plane of the ring, respectively. The electric dipole is normal to the gap, which makes the ring an asymmetric emitter in its plane. Therefore, a crystal with open rings is not isotropic (Fig. 4c). A double open ring does not save the situation. We need quadruple open rings with gaps rotated through 45° and placed on the edges of a cubic photonic crystal. Due to the Lenz rule, such an AM has diamagnetism at low frequencies. It is important to synchronize the rotation angles of all rings. In reality, such a 3D photonic crystal is possible in the radio range, where concentrated capacitances can also be used. Creating such an AM in the microwave range is already problematic.

Near the resonance frequency, we have $\mu_e^{-1}(\omega) = 1 + \omega_m^2/\lfloor \omega_0^2 - (\omega^2 - i\omega\omega_c) \rfloor$. Consider μ_e'. We have

$$\mu_e' = 1 - \frac{\omega_m^2(\omega_0^2 - \omega^2 + \omega_m^2)}{(\omega_0^2 - \omega^2 + \omega_m^2)^2 + (\omega\omega_c)^2}.$$

The condition $\mu_e' < 0$ has the form $(\omega_m^2 - \omega_c^2)^2 > 4(\omega_m\omega_c)^2$. Denote $\Delta = (\omega_m^2 - \omega_c^2)^2 > 4(\omega_0\omega_c)^2$. For weak dissipation, we have $\omega_m^2 - \omega_c^2 > 2\omega_0\omega_c$ or $\omega_m > \sqrt{2\omega_0\omega_c}$. In this case, there is a region of negative $\mu_e' < 0$; it is- located at

$$\omega_0^2 + \frac{\omega_m^2 - \omega_c^2}{2} - \Delta < \omega^2 < \omega_0^2 + \frac{\omega_m^2 - \omega_c^2}{2} + \Delta.$$

Denoting $\Omega = \omega^2 - \omega_0^2$, we have the equation $\Omega^2 - \Omega(\omega_m^2 - \omega_c^2) + (\omega_0\omega_c)^2 = 0$ to determine the boundary frequencies at which $\mu_e' = 0$. Such frequencies can be absent if we take into account dissipation. At the resonance frequency, we have $\mu_e' = 1 - 1/\lfloor 1 + (\omega_0\omega_c/\omega_m^2)^2 \rfloor > 0$. In this case, we obtain $\mu_e' = 1/2$ at $\omega_0\omega_c = \omega_m^2$. When dissipation increases, μ_e' at resonance increases and tends toward 1; when dissipation tends toward zero, it also tends toward zero. At low frequencies, we have $0 < \mu_e' < 1$, i.e., diamagnetism, and $\mu_e' \to 1$ at $\omega \to 0$. At very high frequencies ($\omega^2 \gg \omega_0^2 + \omega_m^2$), we have $\mu_e' = 1 + (\omega_m^2 + 2\omega_c^2)/\omega^2 \to 1$, i.e., paramagnetism. Note that we introduced the resonance frequency artificially using the circuit theory and the formula to be derived in terms of electrodynamics. That is, we have to solve the problem for open rings rigorously.

Thus, permeability can be negative in resonant AM but only in a resonant region, where SD is significant, and the description by two scalar quantities, ε_e and μ_e, is incompletely correct. In the low-frequency region (where these parameters can be introduced), AM behaves like a diamagnet, $0 < \mu_e < 1$. Since the particle size is substantially smaller than a, these resonances usually lie above the first Bragg resonance, where the isotropic approximation is not applicable. Induced magnetism, however, takes place over a wide frequency range.

FRESNEL EQUATION AND NUMERICAL RESULTS

Periodic photonic crystals are described by the Fresnel equation. This is a dispersion equation based on the results of homogenization. In the classical case, there are two approaches to describing effective AM parameters: symmetric in terms of fields $\mathbf{E}$ and $\mathbf{H}$ and inductions $\mathbf{D}$ and $\mathbf{B}$ and asymmetric using three vectors $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{B}$ [8, 9]. In this section, we follow the first approach. In this approach, the following four tensor parameters are introduced in the general case of a bianisotropic AM: permittivity tensor $\hat{\varepsilon}(\omega, \mathbf{k})$, permeability tensor $\hat{\mu}(\omega, \mathbf{k})$, and cross-polarization tensors $\hat{\xi}(\omega, \mathbf{k})$ and $\hat{\zeta}(\omega, \mathbf{k})$. During the propagation of plane waves $\mathbf{E} = \mathbf{E}_0\exp(i\omega t - i\mathbf{k} \cdot \mathbf{r})$ (or for a combination of the fields of such waves) for symmetric consideration, the relations

$$\mathbf{D}(\omega, \mathbf{k}) = \varepsilon_0 \hat{\varepsilon}(\omega, \mathbf{k}) \mathbf{E}(\omega, \mathbf{k}) + c^{-1} \hat{\xi}(\omega, \mathbf{k}) \mathbf{H}(\omega, \mathbf{k}),$$

$$\mathbf{B}(\omega, \mathbf{k}) = \mu_0 \hat{\mu}(\omega, \mathbf{k}) \mathbf{H}(\omega, \mathbf{k}) + c^{-1} \hat{\zeta}(\omega, \mathbf{k}) \mathbf{E}(\omega, \mathbf{k}),$$

and the Fresnel equations [1, 9]

$$\det \left\lfloor (k_0^{-1} \hat{k} + \hat{\xi}) \hat{\mu}^{-1} (k_0^{-1} \hat{k} - \hat{\zeta}) + \hat{\varepsilon} \right\rfloor = 0,$$
$$\det \left\lfloor (k_0^{-1} \hat{k} - \hat{\zeta}) \hat{\varepsilon}^{-1} (k_0^{-1} \hat{k} + \hat{\xi}) + \hat{\mu} \right\rfloor = 0. \qquad (20)$$

take place. Here $\nabla \times \mathbf{E} = -i\mathbf{k} \times \mathbf{E} = \hat{k}\mathbf{E}$ and matrix $\hat{k}$ determines curl operator

$$\hat{k} = -i \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix}.$$

If we consider isotropic metamaterials in the sense of [4], i.e., consider all effective material parameters as scalars, then, in the general case, AM becomes biisotropic with scalar parameters ε, μ, ξ, and ζ and the Fresnel equation $\det \lfloor \hat{k}^2 + \hat{k}(\xi - \zeta) + \hat{I}k_0^2 \varepsilon\mu \rfloor = 0$. If polarization can be neglected and anisotropy cannot be neglected, the Fresnel equation takes the form

$$\det \left\lfloor \hat{\varepsilon}^{-1} \hat{k} \hat{\mu}^{-1} \hat{k} - \hat{I}k_0^2 \right\rfloor = 0,$$
$$\det \left\lfloor \hat{\mu}^{-1} \hat{k} \hat{\varepsilon}^{-1} - \hat{I}k_0^2 \right\rfloor = 0. \qquad (21)$$

In the completely isotropic case ($\hat{\varepsilon} = \varepsilon_{\text{eff}} \hat{I}$, $\hat{\mu} = \mu_{\text{eff}} \hat{I}$), the Fresnel equation $\det \lfloor \hat{k}^2 - \hat{I}k_0^2 \varepsilon\mu \rfloor = 0$ is equivalent to the simplest dispersion equation for an ordinary wave $\mathbf{k}^2 = k_0^2 \varepsilon\mu$.

For AM made of balls of two types, we estimate the resonance frequencies, which, of course, does not mean that a homogeneous metamaterial described by permittivity and permeability is achievable (Fig. 4d). To justify this statement, we set field $E_\varphi = \text{const}$ in small balls and place these balls on the edges. As noted above, this excitation cannot appear during wave diffraction by a finite sample. Since the orientations of the axes are different, the balls contribute to the magnetic moment in three directions. Such balls are similar to the considered rings and create a magnetic moment. Of course, this is an approximation: the field in the balls should be expanded in all spherical harmonics [18, 25]. Let balls of a larger size be located in the corners. These balls create an electric moment. The permittivity of the balls is taken to be the same. To expand the field, we can take several spherical harmonics with respect to three axes. We artificially introduced symmetry into such an AM. Then, we have to calculate the field excited by the balls taking into account periodic GF (1) and to solve an integral equation. This solution takes into account the mutual influence of all balls. Then, we should calculate the electric and magnetic moments and the averaged fields. In natural media, the atomic size is $r \approx 0.05$ nm; at a light wavelength $\lambda \approx 500$ nm and base permittivity

$\tilde{\varepsilon} \approx 10$ or lower, we arrive at the ratio of the size to the wavelength $r/\Lambda \approx 3 \times 10^{-4}$ in the medium. For crystals, this ratio has the same order. Atoms are affected by a local field, and the precession of spins gives a magnetic moment, which is proportional to the component of the local magnetic field in a linear approximation. To excite a current loop or ball, their size, which is comparable with the wavelength, is important. The magnetic moment is not proportional to the local field, and the magnetization depends on $\mathbf{k}$. For a photonic crystal with $a \approx 50$ nm, we have $a/\Lambda \approx 0.1$, and particles with $r < 5$ nm should be used for sparseness. We estimate minimum Λ at 100 GHz, a wavelength of 3 mm, $\tilde{\varepsilon} = 10$, and $\varepsilon = 400$. Since isotropy involves operation far from Bragg resonances, a good and accurate approach for this is the use of methods for chaotic metamaterials, e.g., mixtures: mixing formulas for effective medium models, compact group method, and the percolation theory methods [7, 35]. Using Garnett's formula $(\tilde{\varepsilon}_e - \tilde{\varepsilon})/(\tilde{\varepsilon}_e + 2\tilde{\varepsilon}) = C(\varepsilon - \tilde{\varepsilon})/(\varepsilon + 2\tilde{\varepsilon})$, we can estimate for dielectric ball concentration C for $\tilde{\varepsilon}_e = \tilde{\varepsilon} + 1 = 11$, $C = 1/30$. A similar result is given by the Bruggeman effective field formula. Since $C = (4\pi/3)(r_1^3 + r_2^3)/a^3$, we have $r_1 \sim a \times 10^{-4}$ at $r_2 = 2r_1$. At $\tilde{\varepsilon}_e = 11$, we should have $\Lambda < 1$ mm, i.e., at least $a < 0.1$ mm, which gives an estimate $r_1 \sim 10^{-5}$ mm. The minimum distance between emitters is $a\sqrt{3}/2$, therefore, it is impossible to take r_1 larger than 0.01 mm without a significant violation of sparseness. However, this increase violates the condition $\tilde{\varepsilon}_e \approx \tilde{\varepsilon}$ and decreases Λ. It is easy to see using the exact formulas from [36] that the resonance frequencies of such isolated nanoballs do not lie in the range stated by the authors of the idea. In free space, $H_{01\delta}$ is the fundamental (low-frequency) magnetic mode, and the indices (except for azimuthal one) cannot be considered as integers (due to radiation losses, they even need to be considered complex numbers). A photonic crystal has no radiation losses, but it exhibits dipole−dipole interaction of scatterers, which is to be strictly taken into account.

The low-frequency dispersion branch corresponding to Eq. (17) is shown in Fig. 5. Figure 6 (curve 1) shows the results of calculating permeability by Eq. (15). The sums of Eqs. (15) and (17) have poles determined by the condition $\tilde{k}_{xk}^2 + \tilde{k}_{yl}^2 + \tilde{k}_{zm}^2 = k_0^2$. They result in zeros in the right-hand sides of Eqs. (15) and (17). In addition, the functions $F_{mlk}(\mathbf{k})$ and $|G_{mlk}(\mathbf{k})|^2$ have zeros. They take place at sufficiently high $|\mathbf{k}|$ and cause poles. Such resonances in permeability are nonphysical and occur when a band gap is reached, i.e., where the model is not applicable. Poles can be avoided by taking into account losses or building a more rigorous model. In the low-frequency region $|\mathbf{k}| \approx k_0$, a zero compensates for a pole, the permeability is almost unity, and the model demonstrates
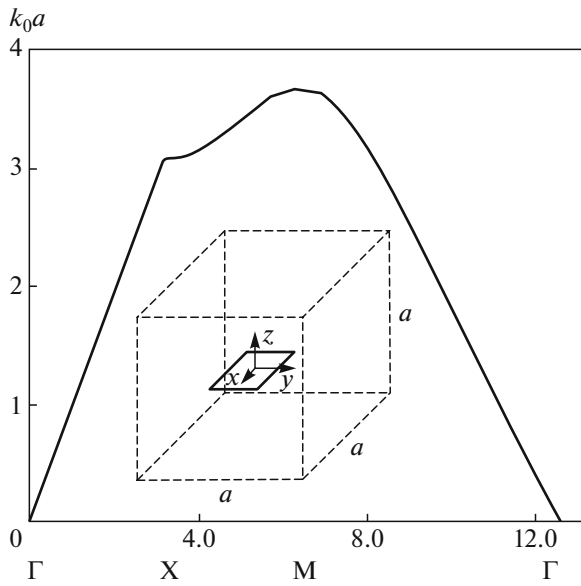
Fig. 5. Dispersion in a metamaterial made of rectangular dielectric loops.

infinitesimal diamagnetism. Negative values are not reached. The calculations in this region were performed by introducing an infinitesimal dielectric loss tangent of 10^{-10}. To calculate permeability at high $|\mathbf{k}|$, we used the relations $k_x = k_x(k_0)$ and $k = k_x + k_y = k(k_0)$ determined from Eq. (17) and took a loss tangent of 10^{-2}. Note the dispersion of permeability can be constructed using the homogenization results and the Fresnel equation $\mu = (k_x/k_0)^2$. This model at $|\mathbf{k}| \approx k_0$ gives weak paramagnetism. At point X, we have $\mu =$

1.061. When moving to the reverse branch $k_x \to \pi/a - k_x$, the deceleration and the permeability grow strongly. In the band gap, the k_x component is imaginary and the permeability is negative. The permeability becomes first negative and then positive and smaller than unity during passage through band gap to the high-frequency (optical) dispersion branch when only k_x changes. Like in the resonance fields in Fig. 6, the resonance values of permeability that differ from unity here have no physical meaning, although these values describe the dispersion by the Fresnel model. When this model is used for diffraction by a photonic crystal sample, it is important at which point of the dispersion surface at a given frequency vector $\mathbf{k}$ is located with allowance for all waves, including reflected ones, which is very difficult.

At low frequencies, other deceleration dependences can be constructed using an effective dielectric medium in the form of a mixture. For it, we have a filling factor $C = 16b\delta^2/a^3$ and depolarization coefficients $L_x = L_y = 1/2 - \delta/b$ and $L_z = 2\delta/b$ (see [7, 37]). Using Garnett's formula, for a thin dielectric loop we obtain [1]

$$\varepsilon_{exx} = \varepsilon_{eyy} = 1 + \frac{C(\varepsilon - 1)}{1 + L_x(\varepsilon - 1 - C)}$$

$$\approx 1 + \frac{16b\delta^2(\varepsilon - 1)/a^3}{1 + (\varepsilon - 1 - 16b\delta^2/a^3)/2},$$
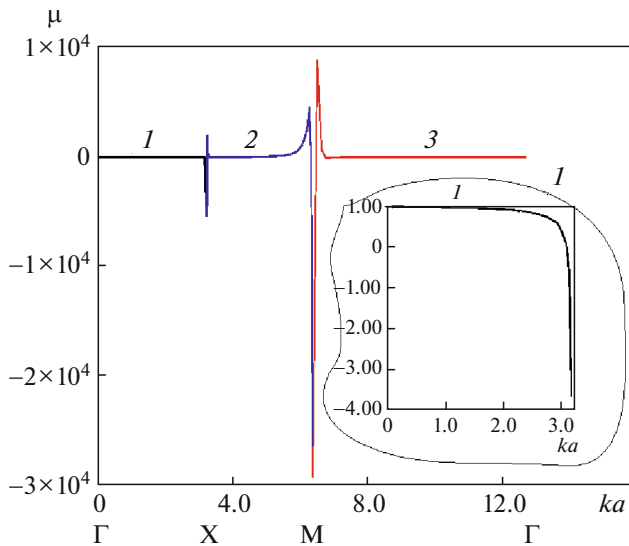

Fig. 6. Effective permeability of a metamaterial made of rectangular dielectric loops according to Eq. (15), which corresponds to the dispersion of Fig. 7.
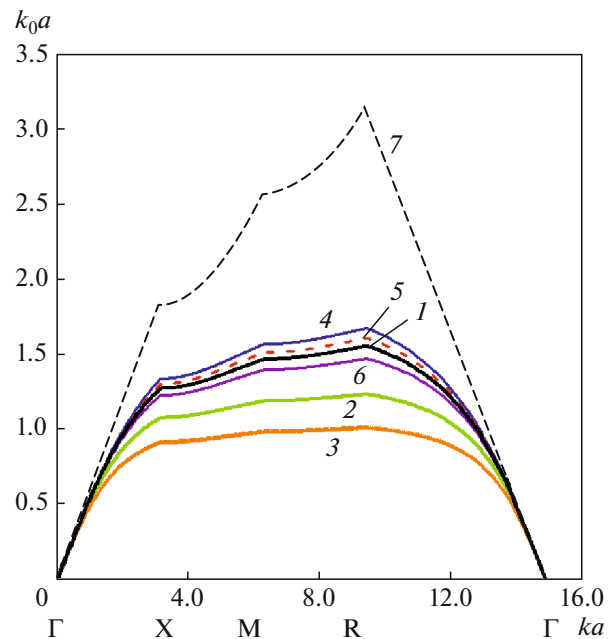

Fig. 7. Dispersion (lower frequency branches) in a cubic photonic crystal with cubic $a_0 \times a_0 \times a_0$ MAs in a dielectric base with $\tilde{\varepsilon} = 3.0$. The permittivity of inclusions is $\varepsilon = (1-4)\ 6.0$, $(5)\ 0.5$, $(6)\ -1.0$, and $(7)\ 3.1$. $a_0 = (1, 4-7)\ 0.1a$, $(2)\ 0.2a$, and $(3)\ 0.5a$.
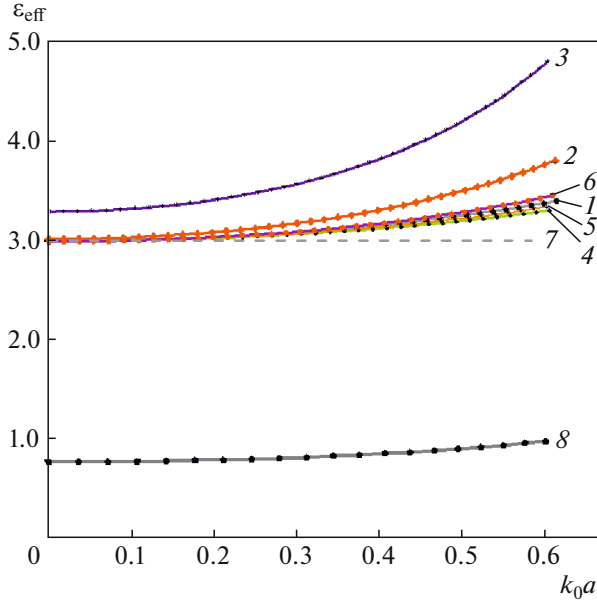
ε_{eff}

Fig. 8. Homogenized effective permittivity ε_{eff} for configurations (1–7) corresponding to Fig. 7. (8) $\varepsilon = -1.0$, $a_0 = 0.3a$, and $\tilde{\varepsilon} = 1.0$.

$$\varepsilon_{ezz} = 1 + \frac{16b\delta^2(\varepsilon-1)/a^3}{1 + 2\delta(\varepsilon-1-16b\delta^2/a^3)/b}$$
$$\approx 1 + 16b\delta^2(\varepsilon-1)/a^3.$$

Thus, at low frequencies, the AM is described by dielectric properties and a wave with $k_z = 0$ moves almost at the velocity of light. Now the Fresnel equation at $k_y = k_z = 0$ has the form $k_x = k_0\sqrt{\varepsilon_{ezz}}$.

The results of calculating the low-frequency dispersion branches for (17) with an effective permittivity for a cubic crystal with cubic dielectric MAs are shown in Figs. 7 and 8. Since an approximate model in the form of approximation of the E_x, E_y, and E_z components by piecewise constant elements with a symmetric field distribution was used, the equation was solved in the first octant of the cube. Two or three approximations per component were used; therefore, the dimension of the problem was either 8 or 27. Homogenization was performed by a formula similar to Eq. (15). Note that a simple algorithm can be created by taking field approximations of the type $E_x(x, y, z) = E_{0x}\cos(\alpha_x x)\cos(\beta_x y)\cos(\gamma_x z)$. Then a dispersion equation is derived in the form of a functional, which depends on 12 parameters in addition to k_0 and $\mathbf{k}$. They can be reduced to 8 by imposing a solenoidity condition and the condition that each wave component satisfies a wave equation. However, such a problem is nonlinear.

CONCLUSIONS

The structures of periodic metamaterials with isotropic effective material parameters, including those with permittivity ε_e and permeability μ_e, were considered. Since all MAs (excluding ferromagnetic particles in an external magnetic field) can be described by permittivity, a symmetric cubic crystal is also modeled only by a diagonal tensor with the same components $\varepsilon_e(k_0, \mathbf{k})$ [1, 6]. Therefore, the introduction of μ_e is an artificial technique. An exception may be magnetic MAs made of ferrites and magnetic metals, the magnetism of which is due to the precession of the uncompensated magnetic moments of atoms or domains in an external magnetic field. Such AM can exhibit resonant properties and a change in the sign of the permeability components, but they are anisotropic and usually exhibit such properties in the microwave range or below. Gyrotropy takes place there.

The so-called induced magnetism demonstrates the dependence of magnetization on $\mathbf{k}$ even in the low-frequency limit. In this case, we have $\mu_e(0, 0) = 1$. Weak diamagnetism was shown to occur in ring AM in the low-frequency limit $|\mathbf{k}| \approx k_0(0, 0) \ll \pi/a$ (see [26]). We now consider this situation qualitatively for a single wire loop of area S. We have $\mathbf{A} = \hat{G}(\mathbf{J})$, $\mathbf{H} = \nabla \times \mathbf{A} = -j\mathbf{k} \times \hat{G}(\mathbf{J})$, and $d\mathbf{M} = \mathbf{r} \times \mathbf{J}dl$. For a flat round loop with current $I = |\mathbf{J}|\delta^2$, we obtain $\mathbf{M} = \mathbf{n}_0 SI$. Diamagnetism provides the Lenz rule. Since the averaged magnetic field is not local, local material parameters cannot be introduced. The high values of permeability are due to the fact that $\langle H_z(k_0, \mathbf{k}) \rangle \approx 0$ for some points in a dispersion surface. A nonlocal magnetic field means that a high permeability does not have a usual physical meaning. Permeability takes on a local meaning in the low-frequency limit at a small phase shift per cell.

For a wave-supporting polarization current $\mathbf{J}_p$ (even a closed one), we may not to introduce magnetization and to introduce only electric polarization $\langle \mathbf{P}^e \rangle = \varepsilon_0(\varepsilon - 1)\langle \mathbf{E} \rangle_V$. Averaging here is performed over MAs. Such a dipole moment differs from (5) and is more approximate. In the low-frequency limit, the field is constant and $\langle \mathbf{E} \rangle_V = V\langle \mathbf{E} \rangle/a^3$. Formally, we can introduce various AM models using tensor or scalar material parameters and then determine them under homogenization conditions. These parameters can be represented by the properties of waves in a metamaterial, the scattering parameters, and the polarizability parameters. The ambiguity of the inverse homogenization problem requires the solution of an overdetermined system of equations. In particular cases, the number of equations and unknowns may coincide. Determining permeability by such methods does not guarantee that there is a local relation between magnetic induction and a magnetic field. Formally, the introduction of magnetic properties is based on the fact that the scattering of a plane wave by a small par-

ticle in the low-frequency limit can be described as the appearance of electric and magnetic dipoles in it [7, 17, 37]. However, to meet boundary conditions, we can introduce an additional orthogonal electric dipole rather than a magnetic dipole. We have not considered a number of important issues: the influence of quadrupole and higher multipole moments on polarization, close-packed (not sparse) photonic crystals, strongly nonlocal AM, and some others.

Thus, homogeneous periodic AM described by permittivity can exist in the low-frequency limit if SD is neglected. When their permittivity and permeability are described, the latter is close to unity, especially in the optical range. Homogeneous metamaterials with negative permittivity can also exist. These are metals at ultralow temperatures and the frequencies up to plasma ones. Mixtures of metallic and dielectric MAs in the low-frequency limit can also exhibit a negative permittivity. Induced magnetism is possible; however, the corresponding metamaterials are not isotropic and can be described by other material parameters. In optics, the permeability of metamaterials is almost unity, which is consistent with the analogous conclusion drawn for natural substances [7, 30]. For periodic AM, the question posed in the title should be answered in the negative.

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