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Generation of Short Electron Bunches by a Laser Pulse Crossing a Sharp Boundary of Inhomogeneous Plasma

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Abstract—The formation of short electron bunches during the passage of a laser pulse of relativistic intensity through a sharp boundary of semi-bounded plasma has been analytically studied. It is shown in one-dimensional geometry that one physical mechanism that is responsible for the generation of electron bunches is their self-injection into the wake field of a laser pulse, which occurs due to the mixing of electrons during the action of the laser pulse on plasma. Simple analytic relationships are obtained that can be used for estimating the length and charge of an electron bunch and the spread of electron energies in the bunch. The results of the analytical investigation are confirmed by data from numerical simulations.

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1. INTRODUCTION

Laser-plasma-induced acceleration of electrons in rarefied plasma has been extensively studied in the past decade. This research interest is related to the fact that, according to theoretical estimations, the field strength in an accelerating plasma wake wave can exceed 300 GV m^{-1}, which is several orders of magnitude greater than the strength of the accelerating electric field (~0.01 GV m^{-1}) in modern accelerators of traditional types [1]. Investigations of the laser-plasma acceleration of electron bunches in various laboratories (see review [2] summarizing the experimental results) confirmed the validity of the premises underlying the idea of laser-plasma acceleration and, despite some technical difficulties and theoretical problems, demonstrated a gradual increase in the average energy of electrons in a bunch up to approximately 1 GeV. The best result was obtained at Berkeley [3], where a laser pulse of 300 TW peak power (40 fs duration, 0.815 μm wavelength) in a gas-filled capillary discharge accelerated bunched electrons up to an energy of 4.2 GeV in a 9-cm long waveguide. Electron bunches of this energy are of interest in many practical applications.

At the same time, questions related to the quality of accelerated electron bunches, including their monoenergeticity, duration, emittance, and charge, are still not completely clear. As an example, approximately 6% of the relative energy spread of electrons in accelerated bunches in the aforementioned experiment [3] was also among the best achieved results. However, the degree of electron beam nonmonoenergeticity that is desirable for practical applications must not exceed 1% and in some cases should even fall within tenths of a percent [4, 5].

Evidently, the quality characteristics (monoenergeticity and emittance) of accelerated electron bunches are determined to a considerable degree by the method that is used to inject electrons into the accelerating wake field and by the initial parameters of the injected bunch. In particular, the shorter the initial length of injected electron bunch is compared to the wake field wavelength, the more a monoenergetic bunch is obtained upon acceleration [6, 7]. Theoretical estimations show that at a characteristic wake wavelength of 10–100 μm, high-energy electron bunches with a small energy spread require accurate injection of very short (~1–10 μm) initial bunches into the appropriate phase of an accelerating electric field. Obtaining electron bunches of such short lengths is a difficult task.

There are several possible methods of injecting electrons into the accelerating wake field, each of which possesses its own disadvantages. The usual photocathode based high-frequency injector can, in principle, generate high-quality electron bunches but with durations of 100 fs [8] and above, which are too long. The best modern injectors [9] can generate much shorter bunches of approximately 5 fs, but only with a charge of ~1 pC, which is too small. In addition, the use of external injection in laser-plasma accelerators encounters the need to solve the quite difficult task of bunch synchronization in both time and space with a certain optimum phase of the wake field. Of course, the external injector requires additional equipment. Thus, it is evident that an external photocathode based high-frequency injector and the general idea of using external injection of electrons into the wake wave cannot help in solving the task of creating a compact accelerator.

Various methods have also been proposed for the optical injection of electrons into the wake wave. The essence of these methods is to act upon plasma, in which a laser pulse that generate the wake field wave propagates, at an appropriate moment of time. This can be achieved, e.g., by using one or several auxiliary laser pulses of lower energy, which act at this special moment (consistently with the main pulse that generates the wake wave) on background plasma electrons in order to favor their trapping in the wake field [10, 11]. Alternatively, a rather powerful auxiliary laser pulse can also be used to additionally ionize plasma in a desired phase of a wake wave that propagates in incompletely ionized plasma. The additional electrons generated by this auxiliary laser pulse via photoionization of incompletely ionized background gas can also be trapped by the wake wave [10]. However, these optical methods of electron injection also require extremely accurate matching of several laser pulses in both space and time, which presents considerable technical difficulties.

The above problems can be eliminated using a scheme with self-injection [12] of electrons into the wake field, which employs a single laser pulse passing through inhomogeneous plasma. According to this scheme, background plasma electrons are trapped due to the wake-wave breakage in a region of smoothly decreasing plasma density, whose spatial scale is much greater than the wake wavelength. When a laser pulse propagates along a descending plasma-density gradient, the phase front velocity of the wake potential gradually decreases and at some moment of time becomes equal to the oscillatory velocity of the electron component of plasma, which leads to the wake-wave breakage, the trapping of background plasma electrons, and their subsequent acceleration. The scheme with self-injection of electrons into the wake wave can be implemented in cases of both a negative [12, 13] or a positive gradient of inhomogeneous plasma density [14, 15] with respect to the direction of laser-pulse propagation.

The method for electron injection into a wake field wave proposed in [12] was much simpler compared to other injection schemes, but electrons were trapped into the wake wave from a relatively large phase volume, which did not improve the quality characteristics of the trapped bunch. Evidently, the process of electron trapping can be either fast or slow depending on the parameters of the plasma and laser pulse, with the corresponding change in characteristics of the trapped bunch quality, in particular, its length and monoenergeticity. In order to make the injection faster, it is necessary to use the vacuum–plasma density transition or the density jump inside inhomogeneous plasma with a steeper gradient between regions.

Recently, Li et al. [16] proposed a promising method for injecting background electrons into the wake field wave from plasma with an up-ramp density profile and performed numerical simulations that demonstrated that under certain conditions of laser pulse–plasma interaction electrons are injected from a narrow region where the up-ramp density profile attains a plateau. This method is performed essentially under conditions of one-dimensional (1D) breakage of the wake field wave. The 1D character of the wake-wave breakage can be provided by the size of the laser focal spot being large such that the transverse breakage of the wake wave is excluded. The resulting length of electron bunches injected under these conditions can be very small, on the order of several dozen attoseconds at a bunch charge on a level of nanoCoulombs.

It has been pointed out [16] that bunches with these parameters are appropriate objects for subsequent acceleration in a multistage laser-plasma accelerator with the first stage acting as an electron injector that operates on the proposed principle. However, the theoretical analysis of the method for generating short electron bunches with large charges [16] was not exhaustive. As an example, the concept proposed for explaining this phenomenon was rather general and could not describe the essential details of the process of background electron trapping by the wake wave. The physical interpretation was based on the assumption that the phase velocity of the laser-pulse-generated wake wave in the transition layer (where the plasma density increases with the pulse propagation) is greater than the oscillatory velocity of plasma electrons and then (upon attaining the plateau of the density profile) sharply drops to a level below the electron velocity, which results in the self-injection of electrons. It was also stated [16] that this change in the wake-wave velocity occurs not strictly at the point where the plateau begins, but somewhat later, although no convincing explanation was provided.

The present work was aimed at elucidating and justifying a particular mechanism of electron bunch self-injection into the wake wave, the properties of which would explain (under conditions of the phenomenon considered in [16]) where and when the self-injection of background electrons begins, as well as when and why this mechanism ceases to operate. These details of the self-injection process are quite important, since they determine the charge of a bunch trapped in the wake wave. In this work, the problem is considered in 1D geometry, but the domain of physical parameters that characterizes the generation of electron bunches by laser pulse passing through a boundary of inhomogeneous plasma corresponds to conditions for which the 2D modeling [16] proved the adequacy of the 1D setting.

2. TRAJECTORIES OF BACKGROUND ELECTRONS INITIATED BY A LASER PULSE PASSING THROUGH INHOMOGENEOUS PLASMA

Consider semi-bounded plasma in the absence of external static fields. Let us describe it in the framework of a model in which only the electron component is mobile (with neglect of the intrinsic thermal motion of electrons) while ions constitute the immobile, positively charged homogeneous background. For the sake of simplicity, the problem is considered in the 1D geometry and the plasma–boundary interface is assumed to be sharp.

Let a short laser pulse be normally incident on the plasma boundary, as a 1D packet of circularly polarized electromagnetic waves with frequencies much greater than the plasma frequency (i.e., plasma can be treated as rarefied). For certainty, the laser pulse propagates at group velocity V_{gr} left to right in the positive direction of the z axis with origin at the plasma boundary. Each electron in the plasma that interacts with the laser pulse rapidly oscillates in the transverse direction to the z axis and moves along this axis under the action of a ponderomotive Miller force caused by the pressure of the electromagnetic field of the laser pulse.

In the 1D geometry with the circularly polarized electromagnetic waves of the laser pulse that acts on the plasma electrons, their longitudinal motion along the z axis has no high-frequency component and is described by the following equations:

$$\frac{dP}{dt} = |e|\frac{\partial\varphi}{\partial z} - mc^2\frac{\frac{\partial}{\partial z}\left(\frac{eA}{mc^2}\right)^2}{2\sqrt{1+\frac{P^2}{m^2c^2}+\left(\frac{eA}{mc^2}\right)^2}}, \tag{1}$$

$$\frac{dz}{dt} = u = \frac{P/m}{\sqrt{1+\frac{P^2}{m^2c^2}+\left(\frac{eA}{mc^2}\right)^2}}, \tag{2}$$

where $A(z, t)$ is the amplitude of the envelope of the laser-pulse vector potential; $\varphi(z, t)$ is the scalar potential of the charge-separation field; P and u are the electron momentum and velocity, respectively; and $-|e|$ and m are the electron charge and mass, respectively. The charge-separation field occurs due to the laser-pulse action on electrons, which leads to their shift from the initial equilibrium position z_0.

At the initial moment, the motion of an electron coincides with the direction of laser-pulse propagation; when the laser pulse outruns an electron, the electron moves in the reverse direction. As a result of the occurrence of the charge-separation field, plasma electrons perform a longitudinal oscillatory motion about the oscillation center z_0.

A laser pulse that propagates in the depth of plasma acts sequentially on every electron. In a physically analogous problem [14], numerical simulations have shown that a laser pulse that penetrates into and then propagates in a semi-bounded rarefied plasma virtually retains its initial shape, so that the temporal dynamics of the laser pulse in this case can be ignored. This corresponds to a quasi-static approximation, in which the wake-wave driver evolves over a time scale that is much greater than the scale of the plasma electron response. Therefore, at a constant group velocity V_{gr} of laser pulse propagation in homogeneous plasma, the pulse action on every next electron is fully analogous to that upon all of the preceding electrons. For this reason, all electrons that are located near the plasma boundary have similar motions upon interaction with the laser pulse. The difference between the trajectories of electrons that occur initially at a distance of Δz_0 from each other is only determined by the time delay $\Delta z_0/V_{gr}$ of the laser-pulse action.

The similarity of electron trajectories is retained as long as (i) the initial arrangement of electrons relative to each other is not modified and (ii) electrons that perform reverse motion upon interaction with a laser pulse do not travel outside of the ion background to the region of $z < 0$. Let us assume that the longitudinal size of a laser pulse is sufficiently small so that electrons leave the region of interaction with the pulse before crossing the vacuum–plasma boundary. Estimates show that at a relativistic oscillatory velocity of electrons in the longitudinal direction of rarefied plasma in which the laser-pulse group velocity is close to the velocity of light ($V_{gr} \approx c$); this assumption is readily satisfied provided that the laser pulse width does not exceed the amplitude of electron oscillations. The charge separation field E_z that acts on a given electron under the condition of a retained initial arrangement of electrons (i.e., that prior to the onset of laser-pulse action) depends on the shift of this electron relative to its initial position z_0. Unless the electron travels outside the region $z < 0$ of a constant ion background, the charge separation field E_z according to the Gauss theorem can be expressed as

$$E_z = 4\pi|e|n_0(z - z_0), \tag{3}$$

where n_0 is the plasma density and z is the current position of the electron on the z axis.

Thus, each electron is a relativistic oscillator that performs oscillatory motion relative to its oscillation center, which coincides with the initial position z_0 prior to the laser-pulse action. The equation of motion of this oscillator at the termination of the action of the laser pulse can be written as

$$\frac{dP}{dt} = -4\pi e^2 n_0(z - z_0) \tag{4}$$

and has the following integral that corresponds to the law of energy conservation:

$$\sqrt{m^2c^4 + c^2P^2} + 2\pi e^2 n_0(z - z_0)^2 = E_{os}, \tag{5}$$

where E_{os} is the oscillator energy.

Using relationship (5), it is possible to write the trajectory of any electron in the integral form for a time interval that begins at the moment of termination of the action of the laser pulse. This description will be valid as long as the given electron occurs within the ion background and the initial arrangement relative to neighboring electrons is retained. As will be shown below, violation of the order of electrons (or their mixing) can take place over a part of the trajectory where the electron (after reverse motion upon termination of the laser-pulse action) moves behind the pulse in the direction of its propagation. In this region, the trajectories of all electrons that did not cross the vacuum–plasma boundary are described by a general expression that takes the delay of the laser-pulse action on various electrons into account:

$$ct - ct_0 - \frac{cz_0}{V_{\mathrm{gr}}} = I(z, z_0), \qquad (6)$$

where

$$I(z, z_0) = \int_{z_0}^{z} \frac{dz}{\sqrt{1 - m^2c^4/[E_{\mathrm{os}} - 2\pi e^2 n_0 (z - z_0)^2]^2}},$$

$ct_0 = I(-A_m, A_m)$, and $A_m = \sqrt{(E_{\mathrm{os}} - mc^2)/(2\pi e^2 n_0)}$ is the amplitude of electron oscillations.

The integration constant in Eq. (6) that describes the set of trajectories is determined from the condition that an electron with an oscillation center that occurs initially at the origin would pass it at $t = 0$ with the velocity vector directed from the plasma to the vacuum.

As can be seen from the form of expression (6), the characteristics of the set of trajectories of plasma electrons that do not travel in the vacuum are determined prior to their mixing by the plasma density, the group velocity of the laser pulse, and the energy E_{os} of oscillators excited in plasma by the laser pulse. The same value of the plasma oscillator energy can be excited by laser pulses with various combinations of their maximum amplitude and duration. However, this issue is insignificant regarding the character of the trajectories of oscillating plasma electrons. Of the characteristics of laser pulses, only the group velocity V_{gr} explicitly enters into formula (6) and determines the phase shift between neighboring oscillators and the phase velocity $V_{\mathrm{ph}} = V_{\mathrm{gr}}$ of the plasma wave that lags behind the laser pulse that propagates in plasma.

3. THE MECHANISM OF THE SELF-INJECTION OF BACKGROUND PLASMA ELECTRONS INTO THE WAKE WAVE

A laser pulse that interacts with plasma on crossing its boundary and initiates the motion of background electrons plays essentially only the role of a factor that excites the system of plasma oscillators and determines their energy and phase delay relative to each other. All the subsequent physical phenomena that involve plasma electrons, including the generation of electron bunches, are determined entirely by the properties of these nonlinear relativistic oscillators.

As is known [13], one of these properties is that the mutual arrangement of oscillators can be violated during electron oscillations, which implies the intersection of their trajectories $Z = Z(z_0, t)$, i.e., the mixing of electrons. The mixing of electrons violates the regular structure of a plasma wave (wake field wave) excited by a laser pulse in plasma, which leads to breakage of this wave. All these processes are accompanied by the self-injection of electrons into the wake field of a laser pulse, which can trap these electrons and involve them in acceleration.

The condition of the mixing of electron trajectories, which can be written as $dZ/dz_0 = 0$ and occurs at some critical moment of time t_{cr}, determines the phase of oscillations at which the trajectories intersect. In the case of unbounded plasma, in which the motions of oscillators are not affected by boundaries, relationship (6) can be differentiated to show that the intersection of trajectories of electrons that oscillate with equal amplitudes determined by their energy and with the phase delay determined by the phase velocity V_{ph} of a plasma wave is only possible provided that the energy of oscillators excited by the laser pulse reaches a threshold value of $E_{\mathrm{os,th}} = mc^2\gamma_{\mathrm{ph}} = mc^2/\sqrt{1 - V_{\mathrm{ph}}^2/c^2}$.

The phase at which the trajectories intersect for the threshold oscillator energy $E_{\mathrm{os}} = E_{\mathrm{os,th}}$ corresponds to the moment of passage of an electron through its oscillation center, at which point the electron possesses the maximum velocity in the direction of laser-pulse propagation. This is consistent with the well-known fact [17] that a plasma wave in homogeneous plasma has a limiting value of the amplitude that occurs in the case when the maximum velocity of the electron component in the direction of wave propagation coincides with the phase velocity.

If the energy of the oscillators excited by the laser pulse exceeds the threshold value $E_{\mathrm{os,th}}$, the arrangement of oscillators is partly violated at some time and formula (3), which determines the restoring force $F_z = -|e|E_z$ driving electron toward the equilibrium position, is no longer valid. However, if only a small fraction of the electrons change their mutual arrangement, the influence of small violations in the order of electrons on their motion can be ignored and the electron trajectories can be considered unchanged and are described by formula (6). Then, the condition $dZ/dz_0 = 0$ for an electron with the oscillation center at z_0 determines its critical coordinate z_{cr} where the electron trajectory intersects with that of the neighboring electron:

$$z_{\rm cr} = z_0 - \sqrt{\frac{E_{\rm os} - \gamma_{\rm ph} mc^2}{2\pi e^2 n_0}}. \qquad (7)$$

According to integral relationship (5), it turns out that at the point of intersection of the trajectories, the energy of electron motion is $\sqrt{m^2c^4 + c^2P^2} = mc^2\gamma_{\rm ph}$; thus, the electron velocity is equal to the phase velocity of the wake wave.

The conditions under which formula (7) was obtained, which ignored the influence of the changes in the mutual arrangement of electrons on their motion, implicitly assumed that the mixing of electrons had a certain starting point. This allows the assumption that the effect of electron mixing on their motion is small for some period of time and can be ignored. This is impossible in unbounded plasma, where no any special region or point in space exists from which the process of electron mixing can start, while the presence of boundaries provides this possibility.

The vacuum–plasma boundary significantly influences the trajectories of electrons that cross it after interaction with the laser pulse. When the pulse approaches the plasma boundary and starts to penetrate into its volume, plasma electrons are initially pressed inward, while their mutual arrangement remains unchanged. The order of electrons is also retained when electrons perform the reverse motion after being overtaken by the laser pulse. During the reverse motion, some part of the electrons escape into the vacuum ($z < 0$), while still retaining their mutual order. However, their motion differs from that of electrons that remain within the ion background ($z > 0$) and is not described by the set of trajectories that correspond to formula (6). This is related to the fact that the charge-separation force in the region that contains no ions is not as large and escaped electrons perform a slower oscillatory motion at a lower frequency compared to that of electrons that do not cross the boundary. As a result, for $E_{\rm os} > E_{\rm os,th}$, the intersection of electron trajectories or their mixing begins with an electron that initially occurred at point $z_0 = A_m$ at a distance from the plasma boundary that is equal to the oscillation amplitude $A_m = \sqrt{(E_{\rm os} - mc^2)/(2\pi e^2 n_0)}$ of the oscillator excited by the laser pulse.

Figure 1 shows the results of numerical simulations of the interaction of plasma with the incident pulse of circularly polarized laser radiation with wavelength $\lambda_0 = 2\pi c/\omega_0 = 1$ μm and envelope type $a = a_0\cos^2[t/\tau]{\rm sign}(\pi\tau/2 - |t|)$, where $a_0 = |e|A_0/mc^2 = 4.95$ is the dimensionless vector potential amplitude and τ is the laser pulse duration that corresponds to the full width at the half maximum $\tau_{FWHM} = 1.143\tau = 12$ fs. The group velocity of laser pulse propagation in plasma corresponds to $\gamma_{\rm ph} = 1/\sqrt{1 - V_{\rm gr}^2/c^2} = 5$. The plasma density corresponds to $\omega_0/\omega_p = \gamma_{\rm ph} = 5$, where

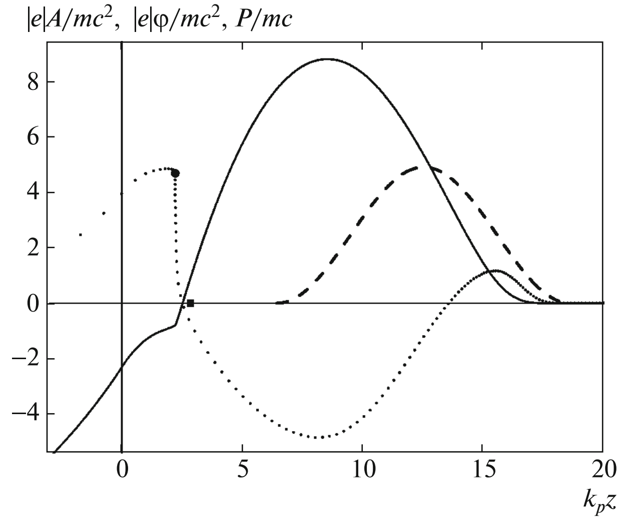

Fig. 1. The distribution of electrons (points) on the phase plane (z, P) at the onset of electron self-injection. The dashed and solid curves show the positions of the laser pulse and wake potential, respectively, on the z axis.

$\omega_p = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency, which yields $\tau = 3.96\omega_p^{-1}$. It should be noted that the adopted approximation of $\omega_0/\omega_p = \gamma_{\rm ph}$ does not take small nonlinear corrections to the group velocity of the laser pulse that generates the wake wave into account [18]; these corrections, related to the relativistic amplitude of a laser pulse, do not qualitatively change the physical picture of this phenomena.

A relativistic ($a_0 \gg 1$) laser pulse excites plasma oscillators up to the energy $E_{\rm os} = 5.04mc^2$, which is above the threshold value ($E_{\rm os,th} = 5mc^2$). Figure 1 shows the plots of the vector potential $|e|A/mc^2$ of the laser pulse (dashed curve) and the laser-induced wake potential $|e|\varphi/mc^2$ (solid curve) versus the dimensionless coordinate k_pz, where $k_p = \omega_p/c$. The points present the current values of coordinates and momenta of a selected family of plasma electrons, which were initially arranged in positions spaced at a step of $k_p\Delta z_0 = 0.15$ prior to the laser-pulse action.

Figure 1 illustrates the moment of time at which the initial arrangement of electrons is still retained, although it is close to the onset of the intersection of electron trajectories, which leads to breakage of the wake wave. The black circle represents an electron that is initially positioned at a distance equal to the oscillation amplitude A_m (shown by the black square) from the plasma boundary. This electron will be called the "leader." As can be seen from Fig. 1, electrons that occur initially on the right from the leader tend to approach it and form an accumulation point by the moment of the onset of the intersection of trajectories and the wake-wave breakage, which corresponds to an increasing density of the electron component. No accumulation point of comparable intensity is formed by electrons that escaped into the vacuum on the left from the electron leader. Therefore, the intersection of trajectories begins with the electron leader and develops predominantly for electrons that occur initially on the right from it. Numerical simulations showed that these electrons constituted the majority of electrons trapped by the wake wave and determined the characteristics of the injected bunch. In what follows, the main attention will be devoted to the motion of these electrons.

The results of modeling show that during motion after interaction with the laser pulse, electrons situated initially on the right of the leader enter one by one (strictly in the initial order) the process of trajectory intersection between neighboring electrons and are found on the left of the leader at the end of this process. As was indicated above, the energy of all these electrons at this time is close to $\gamma_{ph}mc^2$. Accordingly, their velocities are close to the phase velocity of the wake wave. The negative charge on the right of the leader decreases upon this change in the arrangement of electrons and hence, the force that acts on the leader from the wake field increases. As a result, the wake wave begins to accelerate this electron and its energy increases above $\gamma_{ph}mc^2$.

An analogous process develops for all of the other electrons after the onset of the intersection of their trajectories with those of the neighboring electrons; they follow the leader and their energies increase above $\gamma_{ph}mc^2$. Thus, the mixing of trajectories leads to the self-injection of electrons into the first period of the wake wave and their possible trapping by the wake field, since their velocities are close to the wake-wave velocity.

The length of the self-injected electron bunch turns out to be very small compared to the wake wavelength. This is due to the fact that the motion of an electron that initially occurs at point z_0 in front of the leader (having the initial coordinate $z_{0,\,\mathrm{ld}}$) proceeds with a delay that is proportional to the initial spacing $(z_0 - z_{0,\,\mathrm{ld}})/V_{ph}$. Therefore, the moment of time at which the trajectory of this electron intersects with the neighboring trajectories is also delayed by the same time relative to the moment of self-injection of the electron leader.

On the other hand, the velocities of the electrons at the instant of intersection of their trajectories are close to the phase velocity V_{ph} of the wake wave. With this velocity electrons are injected into the wake field of the laser pulse and move in this field during the entire process. According to formula (7), which determines the coordinates of the electron self-injection, each electron enters the process of mixing (self-injection) exactly at the moment when it is close to the electron leader and previously injected electrons in the wake wave.

On the whole, it appears as if an imaginary point (called the self-injection point) exists that moves behind the laser pulse at a velocity close to its group velocity, in which electrons that precede the leader enter the mixing process. Due to the specific features of the trajectories of previously trapped electrons and electrons that enter the process of mixing, spatial grouping of electrons occurs in the vicinity of the self-injection point, where a group of background plasma electrons that were initially situated on the right of the leader accumulates in a dense short bunch. All of the electrons in this bunch have a velocity that is close to the phase velocity of the wake wave and can be trapped by this wave.

It should be emphasized that this grouping of electrons does not occur because different forces act on various electrons in the bunch and collect them together. The grouping has a purely kinematic nature. On the contrary, the difference in the forces with which the electric field acts on various electrons in the trapped bunch leads to the termination of self-injection into the wake wave.

The length of the segment of background plasma electrons that are involved in the self-injection process and are eventually trapped by the wake wave can be estimated as follows. Formula (7) is exactly valid for the electron leader, from which the process of self-injection into the wake wave begins. Then, formula (3) can be used to determine the electric field at a point where the leader occurs at the onset of the self-injection of background plasma electrons into the wake wave:

$$E_z = -4\pi|e|n_0\sqrt{\frac{E_{os} - \gamma_{ph}mc^2}{2\pi e^2 n_0}}. \tag{8}$$

It follows from expression (8) that at the moment of self-injection of the electron leader, the integral charge on the right from the self-injection point is positive. As the laser pulse propagates and the self-injection point moves behind, the charge on the right decreases because an increasing number of electrons pass through the self-injection point and join the negatively charged electron bunch situated in front of this point. This accumulation process (i.e., self-injection of background electrons into the wake wave) ceases when the charge of the trapped bunch is such that the electric field that acts on the last trapped electron at the moment of its self-injection becomes zero. This condition yields the following formula for estimating the length of a plasma layer from which all electrons are self injected and trapped in the wake wave:

$$\Delta z_{tr} = \sqrt{\frac{E_{os} - \gamma_{ph}mc^2}{2\pi e^2 n_0}}. \tag{9}$$

In the case with a sharp plasma boundary, the layer of trapped electrons occurs under the surface of the plasma at a depth on the order of the oscillation amplitude of a laser-pulse-excited plasma oscillator, which depends on its energy. The thickness of the layer of background plasma electrons trapped in the wake wave is also determined by the energy of the plasma oscillator and the group velocity of the exciting laser pulse. It should also be noted that according to the adopted mechanism of self-injection of background plasma electrons into the wake wave, all electrons from this layer are trapped by the wake wave and involved in the acceleration process. This conclusion is confirmed by the results of numerical simulations.

Figure 2 presents the results of numerical simulations that show how the thickness of the layer of trapped electrons depends on the energy of plasma oscillators excited by the laser pulse. The circles and squares show the results of simulations for the threshold energies of electron self-injection and wake-wave breakage $E_{\text{os, th}} = 5mc^2$ and $7mc^2$, respectively. The solid curves present the results of analytic calculations by formula (9), which show good agreement with the data of numerical modeling.

A more detailed analysis of Fig. 2 shows that the accuracy of formula (9) somewhat decreases for a lower threshold energy of plasma oscillators. As will be shown below, this is related to the fact that the initial assumption, according to which the injected bunch consists entirely of background plasma electrons that were initially situated in front of the electron leader, is not quite correct. A small fraction of electrons from behind the leader are also involved in self-injection and this fraction increases with decreasing threshold energy $E_{\text{os, th}}$.

The data presented in Fig. 2 clearly demonstrate that the square-root dependence in formula (9) on the energy of oscillators excited by the laser pulse in a semi-bounded plasma leads to a rather fast growth in the thickness of the trapped electron layer, even for a relatively small excess of the oscillator energy over the threshold $E_{\text{os, th}}$ (i.e., to a sharp increase in the charge of the trapped bunch per unit cross-sectional area).

The dashed curves in Fig. 2 show the results of calculations using a formula proposed by the authors of this method for the injection of an electron bunch into the wake wave [16] for estimating the charge of a trapped bunch (as recalculated for the thickness of the corresponding layer of background plasma electrons).

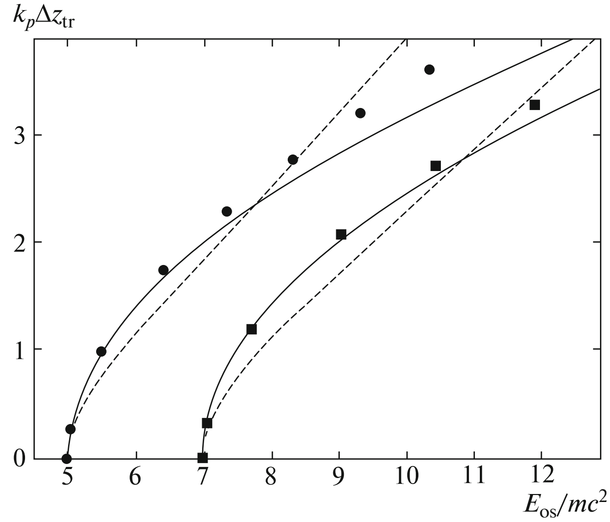

Fig. 2. The thickness of the layer of background plasma electrons vs. energy of plasma oscillators for threshold values $E_{\text{os, th}} = 5mc^2$ (circles) and 7 mc^2 (squares). The solid curves show the results of calculations by formula (9); the dashed curves are calculated by the formula from [16].

4. THE CHARACTERISTICS OF A BUNCH OF TRAPPED ELECTRONS

The length of a bunch generated by a laser pulse on crossing the plasma boundary and the energy spread of the electrons in the bunch are determined primarily by the fact that electrons are injected into the wake wave from various points of space, at various moments of time, and with different initial velocities. In addition, during self-injection and subsequent acceleration in the field of the wake wave, different electrons are accelerated by the different accelerating forces, which depend on the position of the electron in the bunch. For this reason, the characteristics of the bunch, in particular, its length and electron energy spread, are in the general case variable quantities that are dependent on the time of determination.

Figure 3 presents the results of numerical simulations of an electron bunch generated by a laser pulse with the parameters indicated above, which show the temporal variation of the length of the trapped electron bunch $L_{b,\text{ rms}} = 2\sigma_{L,\text{ rms}}$ (circles), the relative energy spread of the trapped electrons $\Delta E/\langle E\rangle = 2\sigma_{E,\text{ rms}}/\langle E\rangle$ (rhombs), and their average energy $\langle E\rangle$ (squares) during acceleration, where $\sigma_{L,\text{ rms}}$ and $\sigma_{E,\text{ rms}}$ are the mean-square characteristics of the distribution of the corresponding values in the bunch. As can be seen from these data, as the average electron energy in the bunch that is trapped and accelerated by the wake wave increases, the length of the bunch tends to a certain asymptotic value, while the relative electron energy spread at large distances varies rather slowly and can also be considered approximately constant.

The asymptotic value of the length of the bunch is attained quite rapidly because electrons that were injected into the wake wave field at the relativistic energy $E_{\text{inj}} \sim \gamma_{\text{ph}}mc^2$ and subsequently accelerated in the wake field to reach ultrarelativistic values that significantly exceed the initial energy ($E \gg \gamma_{\text{ph}}mc^2$). When the velocities of all of the electrons in the bunch become very close to the velocity of light, the mutual arrangement of electrons in the accelerated bunch

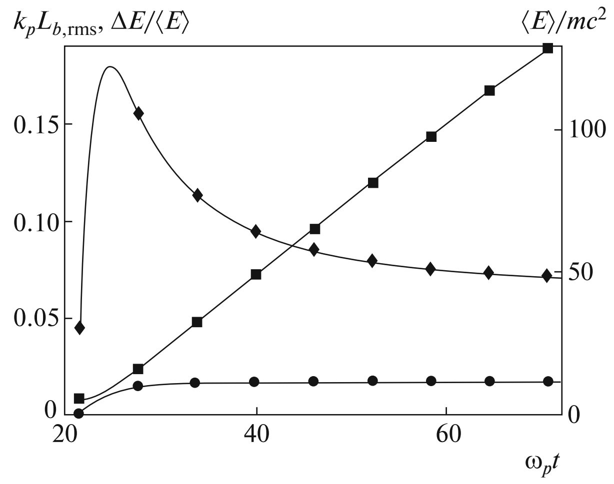

Fig. 3. The temporal variation of the length of the trapped electron bunch $L_{b,\ \mathrm{rms}}$ (circles), relative energy spread of trapped electrons $\Delta E/\langle E\rangle$ (rhombs), and their average energy $\langle E\rangle$ (squares) during acceleration of a bunch generated in plasma by a laser pulse with the parameters a_0 = 4.95, τ_{FWHM} = 12 fs, γ_{ph} = 5, and λ_0 = 1 µm.

ceases to change and the length of the bunch remains almost constant.

In order to find this asymptotic value of the length of the bunch, let us use the conventional approach [19–21], which allows the characteristics of accelerated electron bunches to be determined provided that electrons in the bunch have the integral of the equation of motion in the accelerating field. In this case, this integral in the frame of reference related to the wake wave for trapped electrons that initially occur in the front of the electron leader has the following form:

$$E' - |e|\varphi'(\xi) = E'_{\mathrm{inj}} - |e|\varphi'(\xi_{\mathrm{inj}}) - \gamma_{\mathrm{ph}}mc^2k_p(z_0 - z_{0,\mathrm{ld}})(\xi - \xi_{\mathrm{inj}}). \quad (10)$$

Here, $\varphi'(\xi)$ is the potential of a stationary wake wave that propagates in plasma at a constant phase velocity V_{ph} without changing shape; $\xi = k_p(z - V_{\mathrm{ph}}t)$ is the self-similar variable (wake-wave phase); $z_{0,\ \mathrm{ld}}$ and z_0 are the centers of oscillations of the electron leader and a particular electron trapped in the bunch, respectively, which is injected into the wake wave (during its breakage) with energy E'_{inj} and then accelerated to energy $E'(\xi)$; ξ_{inj} is the wake-wave phase in which the electron is self-injected, determined by its spatial position z_{inj} at time t_{inj} by the formula $\xi_{\mathrm{inj}} = k_p(z_{\mathrm{inj}} - V_{\mathrm{ph}}t_{\mathrm{inj}})$; the prime quantities here and below denote values in the frame of reference related to the wave.

The form of integral (10) is the ordinary energy conservation law [21] for an electron accelerated in the field of a stationary wake wave, where the additional term

$$\gamma_{\mathrm{ph}}mc^2k_p(z_0 - z_{0,\mathrm{ld}})(\xi - \xi_{\mathrm{inj}}),$$

corresponds to the repulsion of charges in the trapped bunch. This approach to allowance for the influence of the intrinsic charge on the acceleration of trapped electrons in the wake wave is valid as long as the correction term is small compared to the wake-wave field. This corresponds to the approximation in which the mixing of electrons weakly distorts the wake wave and, hence, can be acceptable with sufficient accuracy, at least for the head part of the trapped electron bunch.

In deriving integral (10), the fact was taken into account that trapped electrons that initially occurred in front of the electron leader changed their order to the opposite during the injection process. Numerical simulations showed that other electrons that were initially behind the electron leader were also trapped in the wake wave but in a much smaller quantity that constituted a small fraction of the total number of trapped electrons, provided that the energy of laser-pulse-excited oscillators was not much greater than the threshold oscillation energy ($E_{\mathrm{os}} - E_{\mathrm{os,\ th}} \le E_{\mathrm{os,\ th}}$). In qualitative analysis of the formation of a bunch of trapped electrons to obtain an estimate of the length of the bunch, this small fraction can be ignored.

Using the energy-conservation integral (10), it is possible to write the following general expression in quadratures for electron trajectories in a trapped bunch:

$$\gamma_{\mathrm{ph}}^{-2}ck_p(t - t_{\mathrm{inj}}) - \beta(\xi - \xi_{\mathrm{inj}}) = \int_{\xi_{\mathrm{inj}}}^{\xi} d\eta\left\{1 - \frac{m^2c^4}{[E'_{\mathrm{inj}} + |e|(\varphi'(\eta) - \varphi'(\xi_{\mathrm{inj}})) - \gamma_{\mathrm{ph}}mc^2k_p(z_0 - z_{0,\mathrm{ld}})(\eta - \xi_{\mathrm{inj}})]^2}\right\}^{-1/2}, \quad (11)$$

where $\beta = V_{\mathrm{ph}}/c$.

In the general case, determining the exact time t_{inj} of injection in the wake field of a laser pulse, the exact coordinate of injection z_{inj}, and the energy E_{inj} at this moment for each trapped electron is a difficult task, which requires allowance for the mutual influence of electrons and mixing of their trajectories during self-injection in the wake wave. However, at the initial stage of mixing of electron trajectories, where the influence of a change in the arrangement of electrons on their motion is insignificant, it can be assumed for electrons that initially occur in the front of the electron leader that the time of injection t_{inj} is determined by the distance from the electron oscillation center to that of the electron leader and, hence, is related to the moment of injection of the electron leader ($t_{inj,ld}$) as

$$t_{inj} = (z_0 - z_{0,ld})/V_{ph} + t_{inj,ld}. \qquad (12)$$

The energy of injected electrons and the injection phase ξ_{inj} are the same as those of the electron leader, so that $E_{inj} = \gamma_{ph}mc^2$. This assumption must be quite good for electrons that occur in the head of the trapped bunch, for which the process of self-injection begins upon breakage of the wake wave.

Now let us take the fact into account that the trajectories of trapped electrons (except for their small fraction that lag behind the accelerated bunch) remain close during the entire period of acceleration in the wake wave. This implies that expression (11) can be expanded at any time relative to the trajectory of the electron leader in terms of small perturbations $\delta\xi$ and δt_{inj} of the variables ξ and t_{inj} that describe the trajectories of other electrons in the bunch, with allowance for an additional parameter of the first order of smallness $\sim\delta z_0 = z_0 - z_{0,\,ld}$.

Taking the variation of relationship (11) with respect to $\delta\xi$, δt_{inj}, and δz_0, we eventually obtain as a first approximation that the deviation of an electron in a trapped bunch relative to the electron leader at an arbitrary moment of time is linearly related to its distance from the leader $\Delta z_0 = z_0 - z_{0,\,ld}$ in plasma prior to the laser-pulse action. The deviation can be approximately estimated as

$$\Delta z = z - z_{ld} \approx -\frac{\Delta z_0}{2\gamma_{ph}^2}(1 + I(\gamma_{ph})), \qquad (13)$$

where

$$I(\gamma_{ph}) = \gamma_{ph}^3 \int_0^{\Delta\xi_{acc}} \eta d\eta \left\{ \left[1 + \frac{|e|}{mc^2} \times\gamma_{ph}(\varphi(\xi_{inj,ld} + \eta) - \varphi(\xi_{inj,ld})) \right]^2 - 1 \right\}^{-3/2}, \qquad (14)$$

$\Delta\xi_{acc}$ is the wake-wave phase interval measured from the phase of the injection of the electron leader $\xi_{inj,\,ld}$, for which acceleration of the bunch of trapped electrons occurs.

Formula (13) clearly shows that electrons that initially occur in front of the electron leader prior to being trapped occur behind the leader after self-injection into the wake wave such that their order is retained but is opposite to the order in the plasma before the laser-pulse action. In addition, it follows from formula (13) that electrons in the trapped bunch are arranged more densely that in the initial plasma unperturbed by the laser pulse. Depending on γ_{ph}, the density of electrons in the trapped bunch can be several orders of magnitude higher than their initial density in unperturbed plasma.

In order to determine of the coefficient of proportionality in formula (13), it is necessary to calculate the value of the integral $I(\gamma_{ph})$. For this purpose, it is necessary to know how the wake potential $\varphi(\xi)$ depends on the wave phase. In the context of deriving formula (13), this integral is calculated over the trajectory of the electron leader, which is the first particle that was trapped and accelerated in the laser pulse wake field that was not disturbed by mixing with the trajectories of trapped electrons that occurred behind it. This wake field is described by an equation that coincides in form (at distances sufficiently large to neglect the direct influence of a laser field on the motion of plasma electrons) with an equation that describes a plasma wave as a mode of longitudinal plasma oscillations that propagate at constant velocity in unbounded plasma [17].

It is clear that for the conditions under consideration with the energy of electron oscillations $E_{os} > E_{os,\,th} = \gamma_{ph}mc^2$, this wave cannot exist in unbounded plasma because of the mixing of electron trajectories. Accordingly, the equation for a wake field behind the laser pulse in this case has no solution that has a physical meaning in the entire infinite space. However, a solution of this equation up to the point of wake-wave breakage exists and can be found. For numerical calculation of the wake potential, it is more convenient to use the following equation:

$$\frac{e^2}{2mc^2}\left(\frac{d\varphi}{d\xi}\right)^2 + V_{ph}P_{pl} + |e|\varphi = E_{os} - mc^2, \qquad (15)$$

which is the integral of the equation of plasma wave and corresponds to the law of energy conservation in the system of plasma with charged particles. Here, P_{pl} is the momentum of a background plasma electron in front of a trapped bunch, which can be expressed via the wake potential by the following formula [22]:

$$P_{pl}(\varphi) = \frac{mc}{1-\beta^2}\left\{\beta\left(1 + \frac{|e|\varphi}{mc^2}\right) - \sqrt{\left(1 + \frac{|e|\varphi}{mc^2}\right)^2 - 1 + \beta^2}\right\}. \qquad (16)$$

Equation (15) must be solved with the initial conditions determined for the phase at which the wake-wave breakage begins and self-injection of the electron leader occurs. At the moment of injection, the energy of the electron leader is exactly known to be $E_{inj,\,ld} = \gamma_{ph}mc^2$. Then, formula (16) yields the following initial condition for the integration of Eq. (15):

$$|e|\varphi(\xi_{inj,ld})/mc^2 = 1/\gamma_{ph} - 1. \qquad (17)$$

Once the function $\varphi(\xi)$ is numerically calculated, it is possible to determine the integral for the coefficient of proportionality in formula (13). It should be noted that the integrand function in $I(\gamma_{ph})$ quite rapidly decreases to zero with increasing upper integration limit $\Delta\xi_{acc}$. For this reason, the integral weakly depends on the upper limit for sufficiently large $\Delta\xi_{acc}$ values. This behavior physically corresponds to the

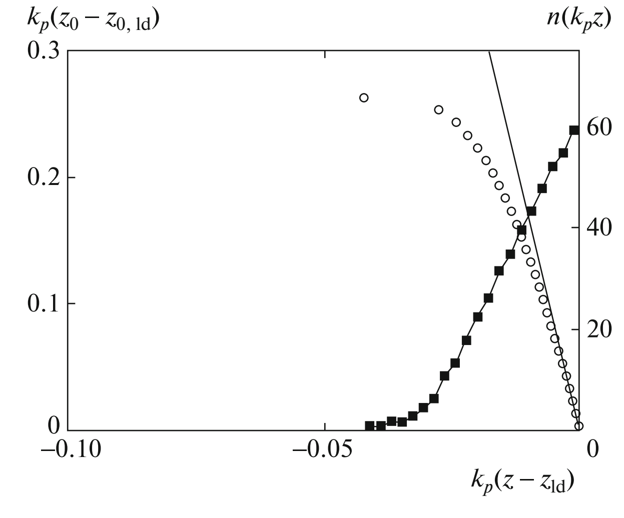

Fig. 4. The initial arrangement of self-injected electrons in plasma prior to their trapping by the wake field (left ordinate axis) depending on their position in the trapped bunch (the electron position is determined relative to that of the electron leader) for the bunch acceleration length corresponding to the average electron energy $\langle E\rangle/mc^2 \approx 200$. The circles show the results of numerical simulations; the solid line is calculated by formula (13); the squares show the distribution of the electron density (right ordinate axis) in the trapped bunch behind the electron leader. The laser-pulse parameters are the same as in Fig. 3.

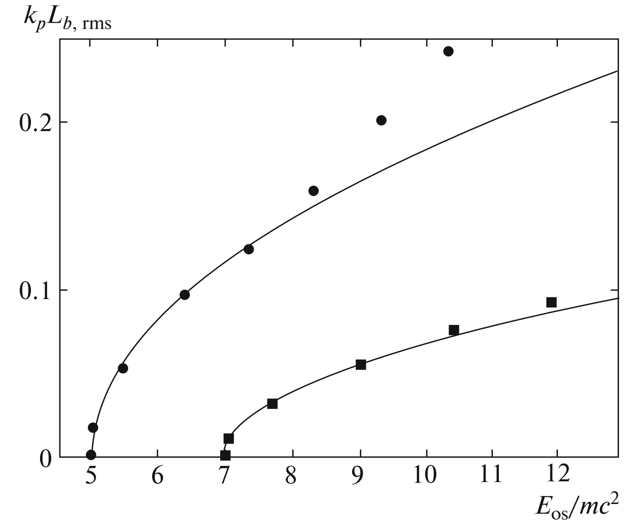

Fig. 5. The asymptotic length $L_{b,\,\mathrm{rms}}$ of an electron bunch vs. plasma oscillator energy for various values of the threshold energy (in the same notation as that in Fig. 2). The solid curves show the results of calculations by formula (18).

fact that the longitudinal size of a bunch ceases to depend on the electron acceleration length provided that electrons are sufficiently far from the phase of injection to the wake wave. The results of calculations show that in the range of γ_{ph} and E_{os} values that correspond to Fig. 2, the value of $I(\gamma_{\mathrm{ph}})$ in formula (13) does not exceed unity, while for $E_{\mathrm{os}}/mc^2 \gg \gamma_{\mathrm{ph}}$ this value is much smaller than unity and can be ignored.

Figure 4 shows the numerically simulated distribution of electrons in a bunch trapped and accelerated in the wake wave of a laser pulse with the parameters presented above for an acceleration length that corresponds to the average electron energy $\langle E\rangle/mc^2 \approx 200$. The circles show the arrangement (left ordinate axis) of trapped electrons (relative to the electron leader positioned at the origin) in the bunch for electrons that occur initially in plasma at a step of $k_p\Delta z_0 = 0.1$ in front of the electron leader. The solid line shows the same dependence calculated by formula (13) and plotted as $\Delta z_0(\Delta z)$.

As can be seen from Fig. 4, the results of simulations for electrons from the head of the trapped bunch satisfactorily agree with theoretical formula (13) and reveal the accumulation of a large fraction of trapped electrons near the electron leader. The electrons from the tail of the bunch lag significantly behind the electron leader, which is related to the fact that their self-injection into the wake wave is significantly influenced by the field of the charge of the previously trapped electrons. However, the fraction of electrons trapped in the tail of the bunch is relatively small, which is shown in Fig. 4 by the curve with squares, which corresponds to the electron-density distribution function $n(k_pz)$ normalized to unity.

The results of numerical simulations showed that a dense group of self-injected electrons near the electron leader determines the retention of a qualitative dependence of type (13) for the mean-square length $L_{b,\,\mathrm{rms}}$ of the bunch of trapped electrons in which the initial length of the bunch should be replaced by the corresponding thickness Δz_{tr} of their layer in plasma:

$$k_pL_{b,\mathrm{rms}} = \frac{\alpha k_p}{\gamma_{\mathrm{ph}}^2}\Delta z_{\mathrm{tr}} = \frac{\alpha}{\gamma_{\mathrm{ph}}^2}\sqrt{2\left(\frac{E_{\mathrm{os}}}{mc^2} - \gamma_{\mathrm{ph}}\right)}, \qquad (18)$$

where α is the fitting numerical coefficient.

The results of the simulations presented in Fig. 5 (in the same notation as that in Fig. 2) show the dependence of the mean-square length of the bunch $L_{b,\,\mathrm{rms}}$ on the energy of oscillators excited by the laser pulse for different values of the threshold energy $E_{\mathrm{os,\,th}}$. The statistical length of the bunch $L_{b,\,\mathrm{rms}} = 2\sigma_{L,\,\mathrm{rms}}$ was determined from the distribution function of all of the electrons in the trapped bunch, including those that occurred initially in the plasma behind the electron leader. The solid curves show the approximation of the data in the range of oscillator energies $E_{\mathrm{os}}/mc^2 - \gamma_{\mathrm{ph}} < 0.5\gamma_{\mathrm{ph}}$ by functions of type (18), where the fitting coefficient α in all cases turns out to be close to unity.

As can be seen from Fig. 5, the results of simulations for oscillator energies for $E_{\mathrm{os}}/mc^2 - \gamma_{\mathrm{ph}} \geq 0.5\gamma_{\mathrm{ph}}$

significantly deviate from dependence (18). This is related to the fact that under these conditions an increase in the oscillator energy E_{os} is accompanied by a gradual increase (up to approximately one-third) in the trapped bunch of a fraction of electrons that initially occurred behind the electron leader. The conditions of self-injection into the wake wave for these electrons differ from those for electrons that initially occurred in front of the electron leader, for which the effect of kinematic grouping described above is absent. As a result, the function of the electron density distribution in the trapped bunch becomes more smeared, provided that the number of electrons is large. Accordingly, the characteristic length of a trapped electron bunch also increases.

Nevertheless, the estimate of the length of the bunch by formula (18) remains quite satisfactory with respect to the order of magnitude. A comparison of Figs. 2 and 5 shows a large decrease in the length of the bunch as compared to the size of the plasma region occupied by electrons before trapping into the wake wave. This significantly influences the spread of electron energies in the accelerated bunch.

Estimation of the energy spread between electrons in the bunch can be obtained by variation of the integral of equation (10) of their motion in the wake field, by analogy with determining the trapped length of the bunch at the acceleration stage. As is known [6, 19–21, 23], the electron-energy spread in an accelerated bunch is determined by the initial spread that is present at the moment of injection and by factors that favor the accumulation of the energy spread in the course of acceleration. In the general case without the use of special methods [19–21, 23, 24] for the minimization of the energy spread that accumulated in the bunch during its acceleration the initial spread can be ignored in the case of a large length of acceleration. This is related to the fact that the electron-energy spread that accumulated over large distances is much greater than the initial spread that is present at the moment of injection.

The spread of electron energies in the accelerating bunch arises due to a difference of the wake field forces that act on electrons trapped in the head and tail of the bunch. This effect depends on the longitudinal size of the bunch [6, 21] and on its charge [23, 24]. In this case, the influence of the first factor is insignificant due to the small length of the bunch. The second factor is related to the repulsion of electrons in the bunch as determined by its charge. The repulsion is proportional to the charge and acts over the entire period of acceleration; thus it is proportional to the acceleration length. In the frame of reference related to the wake wave, the energy spread between the electron leader and last electron trapped in the bunch can be estimated by the formula

$$\frac{\Delta E'}{mc^2} = \gamma_{ph}(\xi - \xi_{inj,\,ld})k_p\Delta z_{tr},$$

where $\xi_{inj,\,ld}$ and ξ are the phase of the injection of the electron leader into the wake wave and the phase in which the acceleration of this electron ceases.

Thus, the energy spread of trapped electrons in an accelerating bunch continuously increases in magnitude. However, an increase in the acceleration length leads to growth of the average electron energy in the bunch. This value can be estimated by taking the fact into account that an electron bunch that is accelerated over a large length quite rapidly leaves the wake-wave phase region in which electrons are injected and shifts toward the region of phases for which the accelerating fields are close to the maximum values $E_{z,\,max}$, which can also be expressed via the plasma oscillator energy:

$$\frac{|eE_{z,max}|}{mc\omega_p} = \sqrt{2\left(\frac{E_{os}}{mc^2} - 1\right)}.$$

Taking the fact into account that under the condition that $E_{os}/mc^2 > E_{os,\,th}/mc^2 = \gamma_{ph} \gg 1$, the wake field is strongly nonlinear and the nonlinear plasma wavelength is much greater than the linear one, there is a large interval of wake-wave phases in which the accelerating field is close to its maximum (for estimation purposes, this value can be considered constant).

The increment of the average electron energy in an accelerated bunch can be estimated in its order of magnitude from the change in the energy of the electron leader. In the frame of reference related to the wake wave, the average energy of electrons over the phase interval of $\xi - \xi_{inj,\,ld}$ can be estimated as

$$\frac{E'}{mc^2} = \gamma_{ph}\sqrt{2\left(\frac{E_{os}}{mc^2} - 1\right)}(\xi - \xi_{inj,ld}).$$

From this result it follows that for sufficiently large acceleration lengths the average energy of the electrons in the bunch grows in proportion to the acceleration length in agreement with the results of numerical simulations presented in Fig. 3. On the other hand, as the acceleration length increases, the contribution of the initial (at the moment of injection) spread of the electron energy in the bunch decreases in comparison to the energy spread that accumulated in the course of acceleration, which also depends linearly on the acceleration length. As a result, the relative electron energy spread in the bunch gradually attains an approximately constant value (see Fig. 3). Under the condition that $E/mc^2 \gg \gamma_{ph}$, this provides the following estimate of the relative energy spread in the accelerating bunch:

$$\frac{\Delta E}{E} \approx \frac{\Delta E'}{E'} = \frac{k_p\Delta z_{tr}}{\sqrt{2(E_{os}/mc^2 - 1)}} = \sqrt{\frac{E_{os} - mc^2\gamma_{ph}}{E_{os} - mc^2}}. \quad (19)$$

The results of the simulations presented in Fig. 6 (in the same notation as that in Fig. 2) show the relative energy spread $\Delta E/\langle E\rangle$ in the electron bunch, where $\Delta E = 2\sigma_{E,\,rms}$ and $\sigma_{E,\,rms}$ is the mean-square deviation of the electron energy in the bunch. The

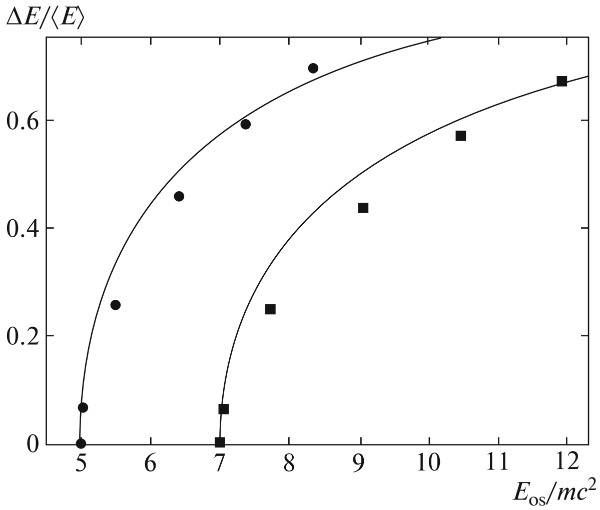

Fig. 6. The relative energy spread $\Delta E/\langle E\rangle$ in the electron bunch vs. plasma oscillator energy for various values of the threshold energy (in the same notation as that in Fig. 2). The solid curves show the results of calculations by formula (19).

solid curves show the energy spread calculated by formula (19). Note that this formula provides the correct vales of the energy spread in the bunch as long as the accelerating electrons are within the region of wake-wave phases in which the accelerating field is close to the maximum value $E_{z,\,\max}$. During further acceleration, the absolute spread increases at the same rate, while the average energy grows more slowly; as a result, the relative energy spread tends to increase.

All of the preceding formulas were obtained in the approximation of 1D motion of electrons trapped in the wake wave, i.e., the wake wave was assumed to be infinitely wide. However, the wake wave of a real laser pulse has a finite transverse size. Let us estimate the conditions under which the dynamics of motion of the majority of trapped electrons can be considered as approximately one dimensional and the transverse motion of electrons can be ignored. For this purpose, let us treat the transverse motion of an electron perpendicular to the z axis as a small nonrelativistic correction to its motion along the z axis.

First, it is necessary to compare the fields that act on the electron at the moment of its self-injection into the wake wave. For electrons from the head of the trapped bunch, the electric field that acts along the z axis can be estimated using formula (8) as

$$\frac{|eE_{z,\mathrm{inj}}|}{mc\omega_p} \approx \sqrt{2\left(\frac{E_{\mathrm{os}}}{mc^2} - \gamma_{\mathrm{ph}}\right)}.$$

The field that acts on the electron in the transverse direction depends on the characteristic transverse size w_0 of the wake wave, which can be estimated by the formula

$$\frac{|e|E_\perp}{mc\omega_p} \approx \frac{|e|\varphi(\xi)}{mc^2k_pw_0},$$

where ξ is the current position of an electron relative to the z axis.

At the moment of the self-injection of an electron, the wake potential can be estimated using formula (17) as $\varphi(\xi_{\mathrm{inj}}) \approx \varphi_{\min} = (1/\gamma_{\mathrm{ph}} - 1)mc^2/|e| < 0$, from which it follows that the electron is self-injected into a defocusing region of the wake-wave phase; thus, it will be repulsed away from the z axis. For the transverse motion of an electron to be weak, a necessary condition is

$$\sqrt{2\left(\frac{E_{\mathrm{os}}}{mc^2} - \gamma_{\mathrm{ph}}\right)} \gg \frac{|e\varphi_{\min}|}{mc^2k_pw_0}. \qquad (20)$$

During acceleration in the wake field, a trapped electron moves from the defocusing to the focusing phase region. It should be noted that the boundary between these regions for a nonlinear wake wave differs from the analogous boundary in the case of a linear wave because of a significant decrease in the size of this region. The above assumption that the boundary between the defocusing and focusing phase regions crosses the z axis in the same way as in the linear wake wave (i.e., at a point where the wake potential vanishes) significantly increases the influence of defocusing forces on the motion of electrons.

The above formulas for the length of the bunch (18) and electron energy spread (19) correspond to a stage of the process of electron bunch acceleration in the wake field of a laser pulse for which the energies of trapped electrons become ultrarelativistic ($E \gg E_{\mathrm{inj}} = \gamma_{\mathrm{ph}}mc^2$) and the velocities of electrons approach the velocity of light. The rate of energy gain by electrons trapped in the wake wave depends on the accelerating field, whose magnitude increases as the trapped electron phase deviates from the injection phase and reaches the maximum in the region where the wake potential $\varphi(\xi)$ crosses the zero level. For this reason, accelerating electrons mostly reach ultrarelativistic energies while moving near and after crossing the boundary of the focusing region of the wake wave.

For further estimates, let us consider the position of the boundary of the focusing region in the wake wave as an indicator for approximately (in its order of magnitude) determining the distance from the point of injection over which trapped electrons reach an ultrarelativistic energy. The force that drives an electron away from the z axis must be sufficiently weak such that the electron would not leave the wake field before falling into the focusing phase region. The equation for the motion of an electron in a transverse field can be written in the following form:

$$E(\xi)\frac{d^2r}{dt^2} \approx -|e|c^2\frac{\varphi(\xi)}{w_0}, \qquad (21)$$

where $E(\xi)$ and $\varphi(\xi)$ are the electron energy and wake potential, respectively, on the z axis, and ξ and r are the current coordinates. The values of $E(\xi)$ and $\varphi(\xi)$ vary when an electron moves along the z axis. The change of the wake potential up to the boundary of the focusing region amounts to $\Delta\varphi = |\varphi_{\min}| = (1/\gamma_{ph} - 1)mc^2/|e|$, the electron energy over this potential drop varies from $E_{inj} = \gamma_{ph}mc^2$ up to $E \approx 2\gamma_{ph}^2 mc^2$. In order to estimate the characteristic time $\Delta t_\perp$ for an electron to leave the wake field in the transverse direction, let us replace its energy in Eq. (21) by the average value $E(\xi) \approx \gamma_{ph}^2 mc^2$ on this interval and substitute $\varphi(\xi) = \varphi_{\min}/2$. Equation (21) then yields the following estimate:

$$\Delta t_\perp = \frac{2w_0\gamma_{ph}}{c\sqrt{|e\varphi_{\min}|/mc^2}} \approx \frac{2w_0\gamma_{ph}}{c}$$

provided that $\gamma_{ph} \gg 1$.

Let us now estimate the time that is necessary for an electron to reach the boundary of the focusing phase region while moving along the z axis for the electron leader. For this purpose, it is necessary first to estimate the phase distance $D\xi$ of the wake wave between the injection phase of the electron leader and the boundary of the focusing phase region. By solving Eq. (15), which determines the phase dependence of the wake potential to within the major terms, we obtain the following representation for $\varphi(\xi)$:

$$\frac{|e|\varphi(\xi)}{mc^2} \approx \frac{1}{\gamma_{ph}} - 1 + \sqrt{\frac{\sqrt{2}\gamma_{ph}^3}{\sqrt{E_{os}/mc^2 - \gamma_{ph}}}\frac{2}{3}(\xi - \xi_{inj,ld})^{3/2}}. \quad (22)$$

This formula leads to the following estimate of the order of magnitude for the interval of wake-wave phases in which the electron leader is accelerated up to ultrarelativistic energies:

$$\Delta\xi_{acc} \approx D\xi \approx \frac{(1-\gamma_{ph}^{-1})^{2/3}}{\gamma_{ph}}\sqrt[6]{2\left(\frac{E_{os}}{mc^2} - \gamma_{ph}\right)}.$$

Then, using an equation analogous to Eq. (11) but free of the term that describes the influence of the intrinsic charge of a bunch on the motion of electrons, the interval of time Δt_{acc} (in the laboratory frame of reference) that is necessary for an electron to pass over the phase difference $\Delta\xi_{acc}$ in the wake wave can be estimated as

$$\omega_p\Delta t_{acc} \approx \gamma_{ph}^2\left(\Delta\xi_{acc} + \frac{\sqrt{[1+\chi\Delta\xi_{acc}]^2 - 1}}{\chi}\right), \quad (23)$$

where $\chi = \gamma_{ph}|eE_{z,\,inj}|/mc\omega_p$. Under the condition of $\chi\Delta\xi_{acc} \gg 1$, Eq. (23) yields a simpler formula for estimating the time that is necessary for the electron leader to achieve ultrarelativistic energy:

$$\omega_p\Delta t_{acc} \approx 2\gamma_{ph}(1-\gamma_{ph}^{-1})^{2/3}\sqrt[6]{2\left(\frac{E_{os}}{mc^2} - \gamma_{ph}\right)}.$$

In order that the electron does not leave the wake field in the defocusing region during this period of time, it is necessary that $\Delta t_{acc} \ll \Delta t_\perp$. Eventually, with a retained condition of $\chi\Delta\xi_{acc} \gg 1$, we arrive at the following inequality:

$$\frac{(k_p w_0)^3}{(1-\gamma_{ph}^{-1})^{7/2}} \gg \sqrt{2\left(\frac{E_{os}}{mc^2} - \gamma_{ph}\right)}. \quad (24)$$

For a sufficiently large characteristic width w_0 of the wake field, the relationships (20) and (24) can be valid with a large margin. In this case, forces that act on an electron in the transverse direction relative to the z axis produce a weak action and the electron motion has a nearly 1D character during acceleration to ultrarelativistic energies. Subsequently, the 1D character is favored by the "relativistic" mass gain of an electron accelerated in the focusing phase region of the wake wave. Estimates (20) and (24) were verified by numerical simulations of the acceleration of test electrons.

For justified use of the 1D geometry in investigations of the mechanism of the self-injection of an electron into the wake field generated by a non-1D laser pulse with an envelope of the type

$$a = a_0\exp\left(-\frac{r^2}{\sigma^2}\right)\cos^2\left[\frac{t}{\tau}\right]\mathrm{sgn}\left(\frac{\pi\tau}{2} - |t|\right),$$

it is also necessary to determine the conditions under which the motion of background plasma electrons can also be considered as nearly one dimensional. As is known [25, 26], the transverse size of a laser pulse determines the character of the dynamics of the motion of background plasma electrons in the so-called "bubble" regime of laser pulse propagation in plasma to a considerable degree. In this case, the ponderomotive force of the laser field drives plasma electrons in the transverse direction and leads to the formation of a cavern with a characteristic size w_0 as estimated from the condition of equilibrium between the ponderomotive force and Lorentz force with which the cavern acts on the electron [26]. This characteristic size is estimated as $k_pw_0 \sim \sqrt{a_0}$ and is dependent on the laser-pulse amplitude. Due to the correspondence between the cavern size w_0 and laser pulse width σ, $k_pw_0 \sim k_p\sigma$, the laser pulse amplitude and width in a stable bubble regime must also be mutually consistent: $k_p\sigma \sim \sqrt{a_0}$. Numerical simulations [27] have refined this relationship as $k_p\sigma = 2\sqrt{a_0}$.

The regime of laser pulse–plasma interaction considered in this work, which is accompanied by the self-injection of background plasma electrons into a wake wave, assumes that slow (not high-frequency) transverse motion of background plasma electrons during this interaction can be ignored in comparison to their longitudinal motion. This implies that in order to exclude the bubble regime it is necessary that the laser pulse width at a given pulse amplitude a_0 obey the condition $k_p\sigma \gg 2\sqrt{a_0}$. In this case, it can be expected that the 2D motion of background plasma electrons would be close to one dimensional.

This estimate was checked and successfully confirmed by the authors of the proposed method for electron bunch injection into a wake wave in [16], where 2D numerical simulations were performed for plasma that interacts with a linearly polarized laser pulse. The laser pulse had an envelope $a = a_0\exp(-r^2/\sigma^2)\cos^2[t/\tau]\mathrm{sign}(\pi\tau/2 - |t|)$ and duration $\tau_{FWHM} = 12.13$ fs at $\gamma_{\mathrm{ph}} = 5$–7, $\lambda_0 = 1$ μm, and characteristic transverse size $\sigma = 20\lambda_0$. Under these conditions, a laser pulse of the indicated width meets a limitation on the amplitude, $a_0 \ll 4 \times 10^3/\gamma_{\mathrm{ph}}^2$, which has been demonstrated [16] to provide good coincidence of the energy characteristics of electron bunches generated in 1D and 2D geometries by this laser pulse interacting with semi-bounded plasma. It was pointed out that although the motion of background plasma electrons under these conditions is effectively one-dimensional (in the sense that the electron distance r from z axis during the interaction with laser pulse remains unchanged), the character of electron motion at different radial distances from the axis varies because the exciting laser-pulse amplitude depends on r.

The charge of an electron bunch trapped in a wake wave also depends on the width σ of the laser pulse. Based on the results of 1D calculations, the charge on an electron bunch generated in plasma by a laser pulse that has an amplitude distributed in the radial direction as $a = a_0\exp(-r^2/\sigma^2)$ with characteristic transverse size σ can be estimated using the following formula:

$$\begin{aligned} Q_{\mathrm{tr}} &\approx -|e|n_0\sigma^2\Delta z_{\mathrm{tr}}\ln\frac{a_0}{a_{\mathrm{th}}} \\ &= -|e|k_p^{-1}n_0\sigma^2\sqrt{2\left(\frac{E_{\mathrm{os}}(a_0)}{mc^2} - \gamma_{\mathrm{ph}}\right)}\ln\frac{a_0}{a_{\mathrm{th}}}, \end{aligned} \tag{25}$$

where $a_{0,\mathrm{th}}$ is the threshold amplitude of a laser pulse that corresponds to the threshold plasma oscillator energy $E_{\mathrm{os,th}}$. Calculations show that for a circularly polarized laser-radiation pulse with $\lambda_0 = 1$ μm and $\tau_{FWHM} = 12$ fs at $\gamma_{\mathrm{ph}} = 5$, the threshold amplitude value is approximately $a_{0,\mathrm{th}} \approx 4.92$. Then, according to estimation using formula (25), a laser pulse with envelope amplitude $a_0 = 5.3$ (corresponding to $E_{\mathrm{os}}/mc^2 \approx 5.48$) and characteristic transverse size $\sigma = 20\lambda_0$ (i.e., a power of $P \approx 0.48$ PW) generates an electron bunch with a charge of $Q_{\mathrm{tr}} \approx 167$ pC. In asymptotics, the electron bunch is characterized by a short duration of approximately 140 as and a relative electron energy spread of approximately 25%.

According to formulas (19) and (20), a laser pulse of smaller amplitude would generate electron bunches with a smaller spread of relative electron energies, but their charge will be lower (see also [28]). Therefore, to obtain bunches with a greater charge, this method for generating short electron bunches is of interest primarily as an injector to the next stage of a laser-plasma accelerator, in which the accelerating fields can significantly exceed the wake-field strength in the injector that generates the primary bunch. In this case, despite a large relative energy spread of electrons at the moment of bunch injection, the smallness of its length as compared to the wake wavelength and a large wake field strength in the accelerating stage of the laser-plasma accelerator make it possible to provide monoenergetic acceleration of electric bunches with a charge close to 1 nC up to energies on the level of several gigaelectronvolts.

5. CONCLUSIONS

The results of the investigation of the process of the generation of a short electron bunch by a laser pulse of relativistic intensity that passes through a sharp boundary of plasma revealed some principal features of the physical mechanism that underlies this process.

It has been demonstrated that the process of the self-injection of an electron into the wake wave is fully determined by the characteristics of the plasma oscillators that are excited by the laser pulse that passes through a semi-bounded plasma. A necessary condition for the self-injection of electrons in the first period of the wake field is that the energy of plasma oscillators should exceed the gamma-factor of the wake wave of the laser pulse.

It has been established that the process of electron self-injection into the wake wave starts with an electron that initially occurred in the depth of the plasma at a distance from its boundary equal to the amplitude of its subsequent oscillations caused by the interaction with the laser pulse. In what follows, this electron becomes a leader, that is, the first particle in the head part of the bunch trapped in the wake wave. The major fraction of electrons trapped in the wake wave are electrons that initially (in the plasma that is not disturbed by the laser pulse) occurred in front of the electron leader; their order in the trapped bunch is opposite to that in the undisturbed plasma.

The length of the trapped electron bunch is determined by the effect of kinematic grouping, which consists in the fact that electron self-injection into the wake wave occurs at the point of space and the moment of time when the electron-leader along with the previously trapped electrons is close to this point.

Subsequently, during acceleration of trapped electrons in the wake wave, the length of the bunch increases as a result of the initial spread in the conditions of electron injection and the mutual repulsion of electrons in the bunch, but tends asymptotically to a certain limit. The asymptotic value of the bunch length is determined by the characteristics of the oscillator excited by a laser pulse in a plasma.

The spread of energy between electrons in the bunch due to their repulsion increases monotonically during their acceleration in the wake wave in proportion to the length of acceleration.

However, the relative electron energy spread in the bunch over large acceleration lengths can be minimized due to analogous growth in the average energy of electrons in the bunch.

Estimations show that a laser pulse of relativistic intensity that interacts with semi-bounded plasma according to the mechanism that was considered in this work can generate electron bunches with a duration below 1 fs and a charge of several hundred picoCoulombs.

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