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PHYSICAL PROPERTIES OF CRYSTALS

Manifestation of Optical Activity at Oblique Incidence of Light in Crystals of Classes $\bar{4}2m$ and $\bar{4}$

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Abstract—The polarization azimuths and ellipticities of the reflected and transmitted light have been calculated as functions of the angle of incidence for transparent crystals of classes $\bar{4}2m$ and $\bar{4}$. Analytical expressions for these parameters are obtained. It is shown that in the general case the polarization azimuths and ellipticities for a plate cut parallel to the optical axis differ for positive and negative angles of incidence of light.

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INTRODUCTION

Although the optical activity in uniaxial crystals has been well studied, there are still some open questions. Investigations have generally been performed at normal incidence of light onto a crystal. The influence of the optical activity at oblique incidence of light has been considered much more rarely. This is especially holds true for crystals of classes $\bar{4}2m$ and $\bar{4}$, in which the optical activity manifests itself in a peculiar way [1]: the rotation of the plane of polarization in the direction of the optical axis is absent. The optical activity in these classes was found for the first time for $AgGaS_2$ (class $\bar{4}2m$) and $CdGa_2S_4$ (class $\bar{4}$) crystals, which have an isotropic point [2, 3]. In this case, the refractive indices of the ordinary and extraordinary waves coincide at a certain wavelength, and one can observe rotation of the plane of polarization in the directions different from the optical axis. A new method for determining the components of the gyration tensor was later proposed, which can be applied for crystals without an isotropic point [4, 5]. Manifestation of the optical activity in crystals of classes $\bar{4}2m$ and $\bar{4}$ under at incidence of light was theoretically described in detail in [6–8].

Polarization of transmitted light at oblique incidence for $\bar{4}2m$ crystals was considered in [9]; however, no analytical expressions were given therein. In this paper, we report the results of a more thorough investigation of polarizations of the reflected and transmitted light for such crystals at oblique incidence of light.

POLARIZATION OF REFLECTED LIGHT IN $\bar{4}2m$ AND $\bar{4}$ CRYSTALS

Gyration Tensor in $\bar{4}2m$ and $\bar{4}$ Crystals

For crystals of classes $\bar{4}2m$ and $\bar{4}$, the gyration tensor has the form [1]

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{12} & -\alpha_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

For class $\bar{4}2m$, we have $\alpha_{12} = 0$ if the X and Y coordinate axes are directed along the second-order symmetry axes oriented parallel to the [100] and [010] crystallographic directions. If X and Y are chosen as perpendiculars to the symmetry planes, we obtain $\alpha_{11} = 0$ and $\alpha_{12} \neq 0$. The symmetry elements for this crystal and cut of the gyration surface by a plane oriented perpendicular to the $\bar{4}$ axis are shown in Fig. 1a [10, 11].

A crystal of class $\bar{4}$ has only the $\bar{4}$ symmetry axis. The gyration surface has the same form as for crystal of class $\bar{4}2m$; however, the symmetry axes of the gyration surface for class $\bar{4}$ do not coincide with the [100] and [010] crystallographic directions (Fig. 1b). Gyration tensor (1) takes a diagonal form ($\alpha_{12} = 0$) after rotation of the coordinate system around the Z axis by an angle determined by the condition $\tan 2\psi_{max} = -\alpha_{12}/\alpha_{11}$; here, diagonal elements of (1) are equal to $\pm\sqrt{\alpha_{11}^2 + \alpha_{12}^2}$. It is obvious that, after the rotation, the X and Y axes will coincide with the symmetry axes of the gyration surface. Thus, the directions corresponding to the maxima of the gyration-surface cross section

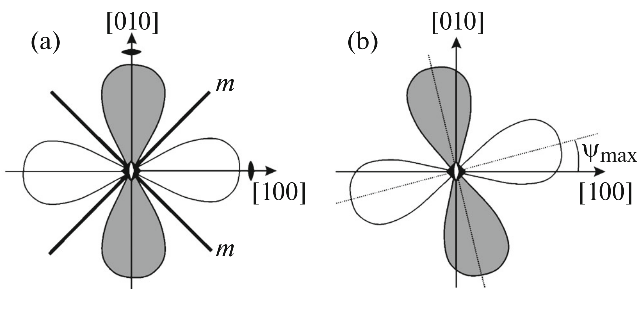

Fig. 1. Cut of the gyration surface and symmetry elements of the crystals of classes (a) $\bar{4}2m$ and (b) $\bar{4}$. The $\bar{4}$ axis (optical axis) is oriented perpendicular to the drawing plane. For the crystal of class $\bar{4}$, only the $\bar{4}$ axis is retained, and the [100] and [010] directions are shifted with respect to the maxima of the gyration surface cut by angle ψ_{max}.

deviate from the [100] and [010] directions by angle ψ_{max}.

Polarization of Reflected Light

Let us calculate the polarization azimuths χ_r and ellipticities K_r of reflected light for *p*- and *s*-polarized incident light. To this end, one should know components E_r of the reflected-wave electric field.

To find the E_r values, we should solve the boundary problem of light reflection and transmission through a crystalline plate. The Maxwell equations, coupling equations, and boundary conditions are required. Exact solutions to the boundary problem were obtained by the Berreman method [12, 13] using the Wolfram Mathematica 7.0 program package. However, the exact solutions are too complex and cannot be written explicitly. To obtain simpler dependences, we performed the solution within the approximation of light reflection from a semi-infinite medium with only first-order values with respect to α_{ij} retained (because α_{ij} is a small value).

The polarization azimuths χ_r and ellipticities K_r of reflected light can be written as [1]

$$(K_r)_{p,s} = \tan(\gamma_{rp,rs}), \quad \sin 2\gamma_{rp,rs} = \frac{2\,\mathrm{Im}\,\kappa_{rp,rs}}{1+|\kappa_{rp,rs}|^2},$$
$$\tan 2(\chi_r)_{p,s} = \frac{2\,\mathrm{Re}\,\kappa_{rp,rs}}{1-|\kappa_{rp,rs}|^2}, \tag{2}$$
$$\kappa_{rp} = \frac{E_{r(ps)}}{E_{r(pp)}}, \quad \kappa_{rs} = \frac{E_{r(sp)}}{E_{r(ss)}}.$$

Here, $E_{r(pp)}$, $E_{r(ps)}$, $E_{r(sp)}$, and $E_{r(ss)}$ are the components of the reflected-wave electric field (the first subscripts *p* and *s* indicate polarization of the incident wave, while the second subscripts indicate polarization of the reflected wave).

Optical Axis is Oriented Perpendicular to the Plate Surface

Using the solution to the boundary problem of light reflection from a semi-infinite medium, we obtained the following expressions for the components of the reflected-wave electric field for *p*- and *s*-polarized incident light:

$$E_{r(pp)} = E_{ip}\frac{\varepsilon_e\left(\varepsilon_o\eta_i - n_i^2\eta_2\right) - i\alpha_{12}\left(\varepsilon_e\eta_i\eta_2 + n_i^2\xi^2\right)}{\varepsilon_e\left(\varepsilon_o\eta_i + n_i^2\eta_2\right) - i\alpha_{12}\left(\varepsilon_e\eta_i\eta_2 - n_i^2\xi^2\right)},$$
$$E_{r(ps)} = E_{ip}\frac{2in_i\eta_i\alpha_{11}\varepsilon_e(\varepsilon_e\eta_2 - \varepsilon_o\eta_1)}{(\varepsilon_e-\varepsilon_o)(\eta_1+\eta_i+i\alpha_{12})\left(\varepsilon_e\left(\varepsilon_o\eta_i + n_i^2\eta_2\right) - i\alpha_{12}\left(\varepsilon_e\eta_i\eta_2 - n_i^2\xi^2\right)\right)},$$
$$E_{r(ss)} = -E_{is}\frac{\eta_1 - \eta_i + i\alpha_{12}}{\eta_1 + \eta_i + i\alpha_{12}}, \tag{3}$$
$$E_{r(sp)} = -E_{is}\frac{2in_i\eta_i\alpha_{11}\varepsilon_e(\varepsilon_e\eta_2 - \varepsilon_o\eta_1)}{(\varepsilon_e-\varepsilon_o)(\eta_1+\eta_i+i\alpha_{12})\left(\varepsilon_e\left(\varepsilon_o\eta_i + n_i^2\eta_2\right) - i\alpha_{12}\left(\varepsilon_e\eta_i\eta_2 - n_i^2\xi^2\right)\right)},$$
$$\xi = n_i\sin\varphi, \quad \eta_i = n_i\cos\varphi, \quad \eta_1 = \sqrt{\varepsilon_o - \xi^2}, \quad \eta_2 = \sqrt{\varepsilon_o - \xi^2\varepsilon_o/\varepsilon_e}, \quad \varepsilon_o = n_o^2, \quad \varepsilon_e = n_e^2.$$

Here, n_o and n_e are the principal refractive indices of the crystal, n_i is the refractive index of the environment, φ is the angle of incidence of light, and E_{ip} and E_{is} are the components of the incident-wave electric field for *p* and *s* polarizations. It can be seen that there are not only a reflected wave with the same polarization as for the incident one ($E_{r(pp)}$ and $E_{r(ss)}$) but also a reflected wave with a different polarization ($E_{r(ps)}$ and $E_{r(sp)}$); however, the amplitude of the latter is small (proportional to α_{11}).

Using the calculated $E_{r(pp)}$, $E_{r(ps)}$, $E_{r(sp)}$, and $E_{r(ss)}$ values, we can write κ_{rp} and κ_{rs} in the form

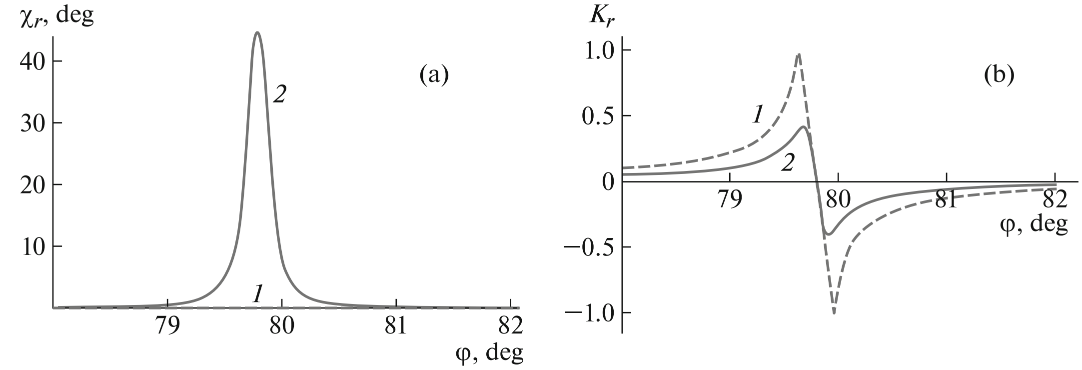

Fig. 2. Dependences of the (a) azimuth χ_r and (b) ellipticity K_r of reflected light for the p polarization of incident light on the angle of incidence φ in the vicinity of the Brewster angle for the KH_2PO_4 crystal (class $\bar{4}2m$): (*1*) $\psi = 0°$, $\alpha'_{11} = 1.48 \times 10^{-4}$, and $\alpha'_{12} = 0$ [4] and (*2*) $\psi = 30°$, $\alpha'_{11} = 0.74 \times 10^{-4}$, and $\alpha'_{12} = 1.28 \times 10^{-4}$. Refractive indices of the crystal are $n_o = 1.5095$ and $n_e = 1.4684$ [14], light is incident from a medium with the refractive index $n_i = 1.467$, and wavelength is $\lambda = 0.589$ μm.

$$\varkappa_{rp} = \frac{2in_i\eta_i\alpha_{11}\varepsilon_e(\varepsilon_e\eta_2 - \varepsilon_o\eta_1)}{(\varepsilon_e - \varepsilon_o)\left[\varepsilon_e(\eta_1 + \eta_i)\left(\varepsilon_o\eta_i - n_i^2\eta_2\right) + i\alpha_{12}\left((\eta_1 + \eta_i)\left(\varepsilon_e\eta_i\eta_2 + n_i^2\xi^2\right) - \varepsilon_e\left(\varepsilon_o\eta_i - n_i^2\eta_2\right)\right)\right]},$$
$$\varkappa_{rs} = -\frac{2in_i\eta_i\alpha_{11}\varepsilon_e(\varepsilon_e\eta_2 - \varepsilon_o\eta_1)}{(\varepsilon_e - \varepsilon_o)\left[\varepsilon_e(\eta_1 - \eta_i)\left(\varepsilon_o\eta_i + n_i^2\eta_2\right) - i\alpha_{12}\left((\eta_1 - \eta_i)\left(\varepsilon_e\eta_i\eta_2 - n_i^2\xi^2\right) - \varepsilon_e\left(\varepsilon_o\eta_i + n_i^2\eta_2\right)\right)\right]}. \quad (4)$$

The χ_r and K_r values are determined by formula (2). It follows from (2) and (4) that $\chi_r = 0$ and $K_r = 0$ at $\alpha_{11} = 0$ and $\chi_r = 0$ and $K_r \neq 0$ at $\alpha_{12} = 0$. With this orientation, the relations $\chi_r(-\varphi) = \chi_r(\varphi)$ and $K_r(-\varphi) = K_r(\varphi)$ are always valid.

Let us consider a crystal of class $\bar{4}2m$. We assume that light is incident in the *XOZ* plane. If the *X* axis is oriented parallel to the [100] axis and the *Y* axis is oriented parallel to the [010] axis (Fig. 1a), tensor α has a diagonal form ($\alpha_{12} = 0$) according to (1). If the *X* and *Y* axes deviate from the [100] and [010] directions by angle ψ, we find for tensor α in the *XYZ* coordinate system that

$$\alpha'_{11} = \alpha_{11}\cos 2\psi, \quad \alpha'_{12} = \alpha_{11}\sin 2\psi. \quad (5)$$

Thus, formula (4) for a crystal of class $\bar{4}2m$ can be obtained by replacing (in correspondence with (5)) α_{11} and α_{12} with $\alpha_{11}\cos2\psi$ and $\alpha_{11}\sin2\psi$, respectively.

For a crystal of class $\bar{4}$, we have the following expressions for the components of tensor α in the *XYZ* coordinate system if the *X* and *Y* axes make angle ψ with the [100] and [010] directions:

$$\alpha'_{11} = \alpha_{11}\cos 2\psi - \alpha_{12}\sin 2\psi,$$
$$\alpha'_{12} = \alpha_{11}\sin 2\psi + \alpha_{12}\cos 2\psi. \quad (6)$$

We find from (2) and (4) that $K_r \sim \alpha_{11}$ and $\chi_r \sim \alpha_{11}\alpha_{12}$; therefore, the χ_r and K_r values are very small in most cases. The χ_r and K_r values would be rather large if one considers incidence of light from a medium with a refractive index close to the principal refractive index of the crystal rather than from air. Large χ_r and K_r values occur only for *p*-polarized incident light and angles of incidence close to the Brewster angle, which is equal in this case to

$$\tan^2\varphi_B = \frac{n_e^2(n_o^2 - n_i^2)}{n_i^2(n_e^2 - n_i^2)}.$$

An example of the dependences $\chi_r(\varphi)$ and $K_r(\varphi)$ for KH_2PO_4 crystal is shown in Fig. 2. The refractive indices for KH_2PO_4 and the α_{11} value were taken from [14] and [4], respectively. The Brewster angle is $\varphi_B = 79.8°$. At $\alpha'_{12} = 0$ ($\psi = 0°$), we have $\chi_r(\varphi) = 0$ and $K_r(\varphi)$ reaches ±1 in the vicinity of φ_B ($K_r(\varphi_B) = 0$) (Figs. 2a, 2b, curves *1*). At $\varphi = \varphi_B$ and $\psi \neq 0$, the χ_r value is maximum (Fig. 2a, curve *2*). At $\psi \neq 0$, the maxima of $K_r(\varphi)$ are not equal to ±1 anymore (Fig. 2b, curve *2*). In the vicinity of angle $\varphi = -\varphi_B$, the dependences $\chi_r(\varphi)$ and $K_r(\varphi)$ have the same form. For *s*-polarized incident light, the values to be found are small at any angles of incidence.

For a crystal of class $\bar{4}$ at $\psi = \psi_{max} = \arctan(-\alpha_{12}/\alpha_{11})/2$, we obtain $\alpha'_{12} = 0$, $\chi_r(\varphi) = 0$, and $K_r(\varphi) \neq 0$; the dependence $K_r(\varphi)$ near the Brewster angle has a form similar to that presented in Fig. 2b (curve *1*). At $\psi = \psi_0 = \arctan(\alpha_{11}/\alpha_{12})/2$, we have $\alpha'_{11} = 0$ and, accordingly, $\chi_r(\varphi) = 0$ and $K_r(\varphi) = 0$. In the general case of position

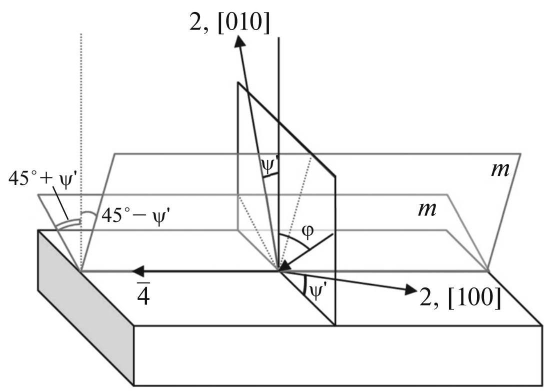

Fig. 3. Orientation of the symmetry elements for the plate cut from the crystal of class $\bar{4}2m$ parallel to the optical axis.

of the plane of incidence of light (at $\psi \neq \psi_{max} \pm \pi l/2$ and $\psi \neq \psi_0 \pm \pi l/2$, where l is integer), we obtain the dependences $\chi_r(\varphi)$ and $K_r(\varphi)$ that are similar to those presented in Fig. 2 (curves *2*).

Optical Axis of the Crystal Is Oriented Parallel to the Surface and Perpendicular to the Plane of Incidence of Light

The plate can be cut differently with respect to the [100] and [010] crystallographic directions to be oriented parallel to the optical axis. Let ψ' be the angle between the [100] direction and the crystal surface. Figure 3 shows the position of the symmetry elements of $\bar{4}2m$ crystal with the orientation under consideration. The [100] and [010] directions coincide with the second-order axes for crystals of class $\bar{4}2m$ (Fig. 3). For a crystal of class $\bar{4}$, only the $\bar{4}$ symmetry axis remains.

The components of the reflected-wave electric field for *p*- and *s*-polarized incident light waves were calculated by solving the boundary problem of light reflection:

$$E_{r(pp)} = E_{ip}\frac{\varepsilon_o\,\eta_i - n_i^2\eta_1}{\varepsilon_o\eta_i + n_i^2\eta_1},\quad E_{r(ps)} = E_{ip}\frac{2in_i\eta_i\left[\alpha_{12}(\eta_1-\eta_2)n_i\sin\varphi + \alpha_{11}\left(\eta_1\eta_2 + n_i^2\sin^2\varphi\right)\right]}{(\eta_1+\eta_2)(\eta_2+\eta_i)\left(\varepsilon_o\,\eta_i + n_i^2\eta_1\right)},$$
$$E_{r(ss)} = E_{is}\frac{\eta_2-\eta_i}{\eta_2+\eta_i},\quad E_{r(sp)} = E_{is}\frac{2in_i\eta_i\left[\alpha_{12}(\eta_1-\eta_2)n_i\sin\varphi - \alpha_{11}\left(\eta_1\eta_2 + n_i^2\sin^2\varphi\right)\right]}{(\eta_1+\eta_2)(\eta_2+\eta_i)\left(\varepsilon_o\,\eta_i + n_i^2\eta_1\right)},\quad \eta_{1,2} = \sqrt{\varepsilon_{o,e} - \xi^2}. \tag{7}$$

The polarization azimuth χ_r and ellipticity K_r of the reflected light are calculated from formula (2). Using the calculated electric-field components, we have the following expressions for the transparent crystal:

$$\varkappa_{rp} = \frac{2in_i\eta_i\left[\alpha_{12}(\eta_1-\eta_2)n_i\sin\varphi + \alpha_{11}\left(\eta_1\eta_2 + n_i^2\sin^2\varphi\right)\right]}{(\eta_1+\eta_2)(\eta_2+\eta_i)\left(\varepsilon_o\,\eta_i - n_i^2\eta_1\right)},$$
$$\varkappa_{rs} = \frac{2in_i\eta_i\left[\alpha_{12}(\eta_1-\eta_2)n_i\sin\varphi - \alpha_{11}\left(\eta_1\eta_2 + n_i^2\sin^2\varphi\right)\right]}{(\eta_1+\eta_2)(\eta_2-\eta_i)\left(\varepsilon_o\eta_i + n_i^2\eta_1\right)},\quad (\chi_r)_{p,s} = 0, \tag{8}$$

$(K_r)_{p,s} = -i\kappa_{rp,rs}$ if $|\kappa_{rp,rs}| \le 1$ or $(K_r)_{p,s} = i/\kappa_{rp,rs}$ if $|\kappa_{rp,rs}| > 1$.

At $\psi' \neq 0°$, the gyration-tensor components used in (8) are calculated from formula (5) replacing ψ with ψ' (for a crystal of class $\bar{4}2m$) and from formula (6) (for the crystal class $\bar{4}$).

It can be seen that the contribution from parameter α_{12} is opposite for positive and negative angles of incidence; therefore, we have $|K_r(-\varphi)| \neq |K_r(\varphi)|$ at $\alpha_{12} \neq 0$. Component α_{12} affects the result to a less extent in comparison with α_{11}, because it is multiplied by $\eta_1 - \eta_2$ (which is proportional to the birefringence). It follows from (8) that α_{12} does not affect the $K_r(\varphi)$ value if the refractive indices are equal (isotropic point). Figures 4a and 4b show the dependences $K_r(\varphi)$ for KH_2PO_4 crystal (class $\bar{4}2m$) in the case of *p*-polarized incident light and angles φ close to the Brewster angle ($\tan^2\varphi_B = \varepsilon_o/n_i^2$). In this case, the dependences $K_r(\varphi)$ have two narrow peaks in the vicinity of the Brewster angle with maximum values equal to ±1. At $\alpha'_{11} = 0$ ($\psi' = 45°$), we obtain $K_r(-\varphi) = -K_r(\varphi)$; the peaks are very narrow in this case (Figs. 4a, 4b, curves *1*). At $\psi' = 30°$, we have $|K_r(-\varphi)| \neq |K_r(\varphi)|$; however, the difference between the $K_r(-\varphi)$ and $K_r(\varphi)$ values is very small and imperceptible in the figure (Figs. 4a, 4b, curves *2*). The corresponding dependences for *s* polarization are shown in Fig. 4c. As for *p* polarization, the dependences $K_r(\varphi)$ are antisymmetric at $\alpha'_{11} = 0$ ($\psi' = 45°$) ($K_r(-\varphi)) = -K_r(\varphi)$, curve *1* in Fig. 4c), whereas at $\psi' = 30°$ we have $|K_r(-\varphi)| \neq |K_r(\varphi)|$ (Fig. 4c, curve *2*).

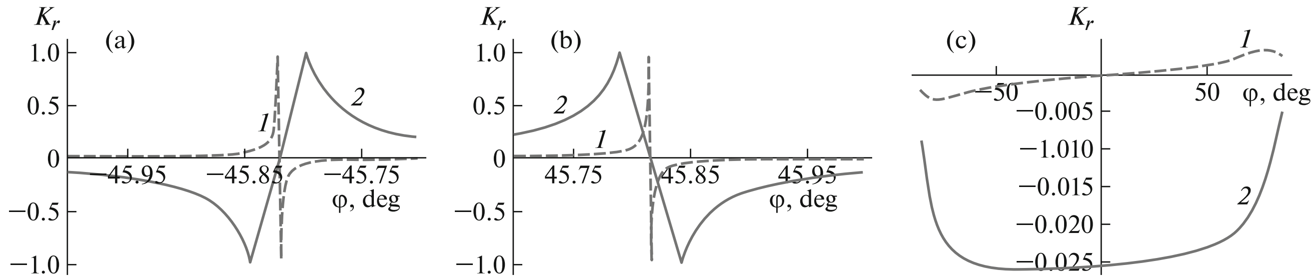

Fig. 4. Dependences $K_r(\varphi)$ for the KH_2PO_4 crystal (class $\bar{4}2m$) at (a, b) p-polarized incident light and angles of incidence φ close to the Brewster angle φ_B and (c) s-polarized incident light: (*1*) $\psi' = 45°$, $\alpha'_{11} = 0$, and $\alpha'_{12} = 1.48 \times 10^{-4}$ and (*2*) $\psi' = 30°$, $\alpha'_{11} = 0.74 \times 10^{-4}$, and $\alpha'_{12} = 1.28 \times 10^{-4}$; $n_o = 1.5095$, $n_e = 1.4684$ [14], $n_i = 1.467$, and $\lambda = 0.589$ μm.

The values obtained at $\alpha'_{11} = 0$ are much smaller than those in the presence of both gyration-tensor components.

For the crystal of class $\bar{4}$, we obtain antisymmetric dependences $K_r(\varphi)$ (Fig. 4, curves *1*) at $\alpha'_{11} = 0$ and $\psi' = \psi_0 = \arctan(\alpha_{11}/\alpha_{12})/2$. At $\psi' = \psi_{max} = \arctan(-\alpha_{12}/\alpha_{11})/2$, we have $\alpha'_{12} = 0$ and $K_r(-\varphi) = K_r(\varphi)$. In the general case, the $K_r(-\varphi)$ and $K_r(\varphi)$ values differ in magnitude; however, this difference is small (Fig. 4, curves *2*).

POLARIZATION OF TRANSMITTED LIGHT IN $\bar{4}2m$ AND $\bar{4}$ CRYSTALS

Expressions for the polarization azimuths χ_t and ellipticities K_t of the transmitted light can be obtained from formulas (2) by replacing subscripts r with t and components of the reflected-wave electric field with the corresponding components for the transmitted wave. To calculate components E_t of the transmitted-wave electric field, we first solved the problem of light reflection and transmission for the isotropic medium–semi-infinite crystal interface. The obtained refracted-wave amplitudes, multiplied by the corresponding phase factors, were considered as amplitudes of the waves incident on the second interface and used for determining E_t. Multiple light reflections in the plate were disregarded, and the calculation was performed in the first order with respect to the α_{ij} values.

Optical Axis Is Oriented Perpendicular to the Plate Surface

Let us consider the polarization azimuths χ_t and ellipticities K_t of transmitted light at oblique incidence. To find χ_t and K_t, we calculated the transmitted-wave electric field components $E_{t(pp)}$, $E_{t(ps)}$, $E_{t(sp)}$, and $E_{t(ss)}$ (the first subscripts p and s indicate the polarization of the incident wave, while the second subscripts indicate the polarization of the transmitted wave) in the following form:

$$
\begin{aligned}
E_{t(pp)} &= E_{ip}\frac{4\varepsilon_o\varepsilon_e n_i n_t \eta_2\eta_i e^{-2i\pi d\eta_2/\lambda}}{\varepsilon_e\left(n_i^2\eta_2+\varepsilon_o\eta_i\right)\left(n_t^2\eta_2+\varepsilon_o\eta_t\right)-i\alpha_{12}\varepsilon_o\varepsilon_e\left(n_t^2\eta_i-n_i^2\eta_t\right)},\\
E_{t(ps)} &= E_{ip}\frac{4i n_i\eta_i\alpha_{11}e^{-2i\pi d\eta_2/\lambda}\left[\eta_1\left(\varepsilon_e\eta_2+\varepsilon_o\eta_i\right)\left(n_t^2\eta_2+\varepsilon_o\eta_t\right)e^{i\Delta}-\varepsilon_o\eta_2\left(\eta_1+\eta_i\right)\left(n_t^2\eta_1+\varepsilon_e\eta_t\right)\right]}{\left(\varepsilon_e-\varepsilon_o\right)\left(\eta_1+\eta_i\right)\left(\eta_1+\eta_t\right)\left(n_i^2\eta_2+\varepsilon_o\eta_i\right)\left(n_t^2\eta_2+\varepsilon_o\eta_t\right)},\\
E_{t(ss)} &= E_{is}\frac{4\eta_1\eta_i e^{-2i\pi d\eta_1/\lambda}}{\left(\eta_1+\eta_i+i\alpha_{12}\right)\left(\eta_1+\eta_t-i\alpha_{12}\right)},\\
E_{t(sp)} &= E_{is}\frac{-4i\alpha_{11}\eta_i n_t e^{-2i\pi d\eta_1/\lambda}\left[\varepsilon_o\eta_2\left(n_i^2\eta_1+\varepsilon_e\eta_i\right)\left(\eta_1+\eta_t\right)e^{-i\Delta}-\eta_1\left(n_i^2\eta_2+\varepsilon_o\eta_i\right)\left(\varepsilon_e\eta_2+\varepsilon_o\eta_t\right)\right]}{\left(\varepsilon_e-\varepsilon_o\right)\left(\eta_1+\eta_i\right)\left(\eta_1+\eta_t\right)\left(n_i^2\eta_2+\varepsilon_o\eta_i\right)\left(n_t^2\eta_2+\varepsilon_o\eta_t\right)},
\end{aligned}
\quad (9)
$$

where $\eta_t = \sqrt{n_t^2 - \xi^2}$, $\Delta = \frac{2\pi d}{\lambda}(\eta_2 - \eta_1)$, and n_t is the refractive index of the lower medium; the other parameters are the same as in formula (4).

The χ_t and K_t values can be calculated from formulas (2). Having retained only α_{ij} in the first power, we obtain the following expressions for transparent crystals of classes $\bar{4}$ and $\bar{4}2m$ at p and s polarizations:

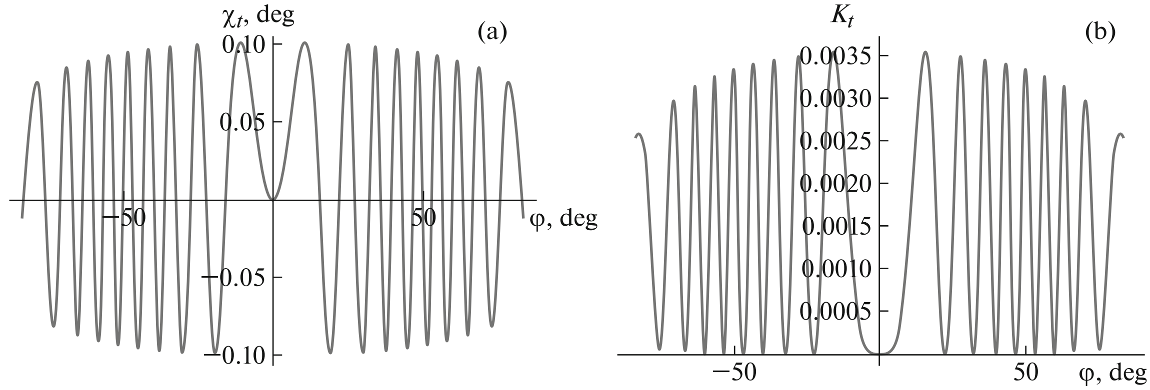

Fig. 5. Dependences of the (a) azimuth χ_t and (b) ellipticity K_t of the transmitted light on the angle of incidence φ for *p*-polarized incident light and the KH_2PO_4 crystal (class $\bar{4}2m$). The optical axis is oriented perpendicular to the plate plane, $\psi = 0°$, $n_i = 1$, $n_t = 1$, $d = 200$ μm, and $\lambda = 0.589$ μm.

$$\begin{gathered}
\chi_{tp} = \frac{\alpha_{11}\eta_1(\varepsilon_e\eta_2 + \varepsilon_o\eta_i)\left(n_t^2\eta_2 + \varepsilon_o\eta_t\right)\sin\Delta}{\varepsilon_o\eta_2 n_t(\varepsilon_e - \varepsilon_o)(\eta_1 + \eta_i)(\eta_1 + \eta_t)}, \\
K_{tp} = \frac{\alpha_{11}\left[\eta_1(\varepsilon_e\eta_2 + \varepsilon_o\eta_i)\left(n_t^2\eta_2 + \varepsilon_o\eta_t\right)\cos\Delta - \varepsilon_o\eta_2(\eta_1 + \eta_i)\left(n_t^2\eta_1 + \varepsilon_e\eta_t\right)\right]}{\varepsilon_o\eta_2 n_t(\varepsilon_e - \varepsilon_o)(\eta_1 + \eta_i)(\eta_1 + \eta_t)}, \\
\chi_{ts} = -\frac{\alpha_{11} n_t\varepsilon_o\eta_2\left(n_i^2\eta_1 + \varepsilon_e\eta_i\right)(\eta_1 + \eta_t)\sin\Delta}{\eta_1(\varepsilon_e - \varepsilon_o)\left(n_i^2\eta_2 + \varepsilon_o\eta_i\right)\left(n_t^2\eta_2 + \varepsilon_o\eta_t\right)}, \\
K_{ts} = -\frac{\alpha_{11} n_t\left[\varepsilon_o\eta_2\left(n_i^2\eta_1 + \varepsilon_e\eta_i\right)(\eta_1 + \eta_t)\cos\Delta - \eta_1\left(n_i^2\eta_2 + \varepsilon_o\eta_i\right)(\varepsilon_e\eta_2 + \varepsilon_o\eta_t)\right]}{\eta_1(\varepsilon_e - \varepsilon_o)\left(n_i^2\eta_2 + \varepsilon_o\eta_i\right)\left(n_t^2\eta_2 + \varepsilon_o\eta_t\right)}.
\end{gathered} \tag{10}$$

It can be seen that parameter α_{11} enters expression (10), whereas component α_{12} does not. The χ_t and K_t values have different signs for the *p* and *s* polarizations. With this orientation, χ_t and K_t are always identical at positive and negative angles of incidence: $\chi_t(-\varphi) = \chi_t(\varphi)$ and $K_t(-\varphi) = K_t(\varphi)$.

For a crystal of class $\bar{4}2m$, we replace α_{11} with $\alpha_{11}\cos 2\psi$ according to (5) (ψ is the angle between the [100] direction and the *X* axis). At $\psi = 0°$, the plane of incidence of light is oriented parallel to one of the second-order symmetry axes and the [100] direction. In this case, as follows from expressions (5) and (10), the $\chi_t(\varphi)$ and $K_t(\varphi)$ values (Fig. 5) vary in the widest range. At normal incidence of light, $\chi_t = 0$ and $K_t = 0$. At $\psi = 90°$, the plane of incidence of light is oriented parallel to the other second-order axis and the [010] direction. In this case, $\chi_t(\varphi)$ and $K_t(\varphi)$ also reach maximum values but with opposite (in comparison with $\psi = 0°$) signs. This fact follows from formulas (5) and (10) and from the cut of the gyration surface in Fig. 1a. If $\psi = 45°$, the plane of incidence of light is oriented parallel to one of the symmetry planes; in this case, $\chi_t(\varphi) = 0$ and $K_t(\varphi) = 0$.

For a transparent crystal of class $\bar{4}$, the dependences $\chi_t(\varphi)$ and $K_t(\varphi)$ do not radically change. At incidence of light in the planes oriented parallel to the [100] and [010] directions, the $\chi_t(\varphi)$ and $K_t(\varphi)$ values are not maximum anymore (Fig. 1b). It follows from the gyration-surface form that one can choose the plane of incidence of light in which the optical activity does not manifest itself and $\chi_t(\varphi) = K_t(\varphi) = 0$. These planes of incidence make angles $\psi_{01} = \arctan(\alpha_{11}/\alpha_{12})/2$ and $\psi_{02} = \arctan(\alpha_{11}/\alpha_{12})/2 + 90°$ with the [100] direction. At incidence of light in the planes rotated by 45° with respect to the planes in which $\chi_t(\varphi) = K_t(\varphi) = 0$, the $\chi_t(\varphi)$ and $K_t(\varphi)$ values will be maximum (at an identical Δ value). These planes make angles $\psi_{\max 1} = \arctan(-\alpha_{12}/\alpha_{11})/2$ and $\psi_{\max 2} = \arctan(-\alpha_{12}/\alpha_{11})/2 + 90°$ with the [100] direction.

Optical Axis of the Crystal Is Oriented Parallel to the Surface and Perpendicular to the Plane of Incidence of Light Refractive Indices

Let us calculate the refractive indices of the waves propagating in crystals of classes $\bar{4}$ and $\bar{4}2m$ with the

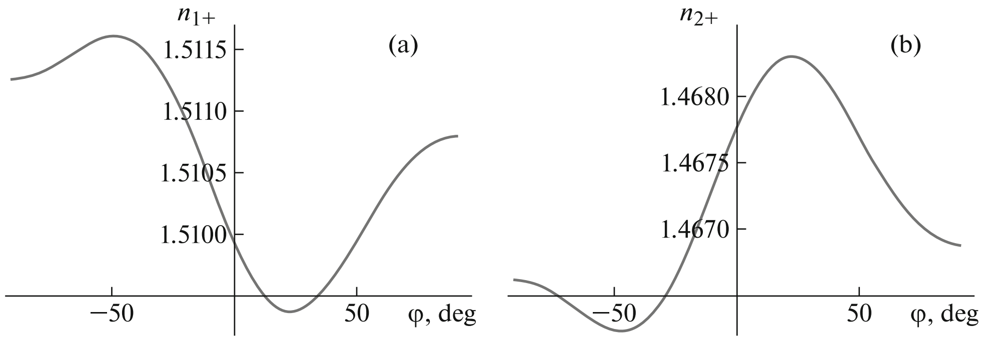

Fig. 6. Dependences of the refractive indices (a) n_{1+} and (b) n_{2+} of the refracted waves on the angle of incidence φ; $n_o = 1.5095$, $n_e = 1.4684$, $\alpha_{11} = 0.95 \times 10^{-2}$, and $\alpha_{12} = 1.65 \times 10^{-2}$.

orientation shown in Fig. 3. Using the equation of normals for optically active crystals [1], at an arbitrary angle of incidence of light, we obtain a quartic equation with respect to parameter $x = \eta = \sqrt{n^2 - \sin^2\varphi}$ (n is the refractive index), which has the following form at $n_i = n_t = 1$:

$$\begin{gathered} Ax^4 + Bx^3 + Cx^2 + Dx + E = 0, \\ A = n_o^2 - \alpha_{11}^2, \quad B = 4\alpha_{11}\alpha_{12}\sin\varphi, \\ C = -\left(n_e^2 + n_o^2\right)\left(n_o^2 - \alpha_{11}^2 - \alpha_{12}^2\right) \\ + 2\left(n_o^2 + \alpha_{11}^2 - 2\alpha_{12}^2\right)\sin^2\varphi, \\ D = -4\alpha_{11}\alpha_{12}\sin^3\varphi, \\ E = n_e^2\left(n_o^2 - \alpha_{11}^2 - \alpha_{12}^2\right)^2 - \left(n_e^2 + n_o^2\right) \\ \times\left(n_o^2 - \alpha_{11}^2 - \alpha_{12}^2\right)\sin^2\varphi + \left(n_o^2 - \alpha_{11}^2\right)\sin^4\varphi. \end{gathered} \tag{11}$$

The found equation has four roots. Under normal incidence of light (φ = 0°), the equation of normals is biquadratic [1]. At φ ≠ 0°, Eq. (11) is not biquadratic, because coefficients B and D at x and x^3 are nonzero. In addition, B and D are small values because they are proportional to the product $\alpha_{11}\alpha_{12}$. Positive and negative solutions to Eq. (11) correspond, respectively, to the refracted waves and to the waves reflected from the second face of the plate back into the crystal.

For optically inactive crystals ($\alpha_{11} = \alpha_{12} = 0$), the equation of normals has the roots

$$x_{01\pm} = \eta_{1\pm} = \pm\sqrt{n_o^2 - \sin^2\varphi},$$
$$x_{02\pm} = \eta_{2\pm} = \pm\sqrt{n_e^2 - \sin^2\varphi}.$$

For optically active crystals, we obtain $x_{1,2} = x_{01,02} + \delta x_{1,2}$, $\delta x_{1,2} \sim \alpha_{ij}^2$, and the refractive indices have the form $n_{1,2}^2 = (x_{01,02})^2 + 2x_{01,02}\delta x_{1,2} + \sin^2\varphi$.

The expressions for the refractive indices can be written as

$$\begin{gathered} n_{1\pm}^2 = n_o^2 - \frac{1}{n_o^2\left(n_e^2 - n_o^2\right)}\Big\{\alpha_{11}^2\left(n_e^2 n_o^2 - 4n_o^2\sin^2\varphi\right. \\ \left. + 4\sin^4\varphi\right) + \alpha_{12}^2\left[\left(n_o^2 - 2\sin^2\varphi\right)^2 - n_o^2 n_e^2\right] \\ \pm 4\alpha_{11}\alpha_{12}\left(n_o^2 - 2\sin^2\varphi\right)\sqrt{n_o^2 - \sin^2\varphi}\,\sin\varphi\Big\}, \\ n_{2\pm}^2 = n_e^2 + \frac{1}{n_o^2\left(n_e^2 - n_o^2\right)} \\ \times\Big\{\alpha_{11}^2\left(n_e^2 n_o^2 - 4n_e^2\sin^2\varphi + 4\sin^4\varphi\right) \\ - \alpha_{12}^2\left[\left(n_e^2 - 2\sin^2\varphi\right)^2 - n_o^2 n_e^2\right] \\ \mp 4\alpha_{11}\alpha_{12}\left(n_e^2 - 2\sin^2\varphi\right)\sqrt{n_e^2 - \sin^2\varphi}\,\sin\varphi\Big\}. \end{gathered} \tag{12}$$

The plus and minus signs correspond, respectively, to the refracted waves and to the waves reflected from the second face. In follows from the obtained expressions that the refractive-index component containing $\alpha_{11}\alpha_{12}$ is opposite for the refracted waves and the waves reflected from the second face. It can also be seen that the refractive indices $n_{1,2+}$ and $n_{1,2-}$ swap when replacing φ with −φ.

For the crystal of class $\bar{4}2m$, one can replace α_{11} with $\alpha_{11}\cos2\psi'$ and α_{12} with $\alpha_{11}\sin2\psi'$ in formula (12), where ψ' is the angle of deviation of the second-order axis oriented parallel to the [100] direction from the surface (Fig. 3); ψ' and positive values of angle φ are counted in the same quarter.

Figure 6 shows the refractive indices for the refracted waves in the crystal, depending on the angle of incidence. Since the difference of the indices from n_o and n_e, being proportional to α_{ij}^2, is very small, it is of interest only at large α_{11} and α_{12} values.

Polarization Azimuths and Ellipticities of the Transmitted Light

Transmitted-wave electric field components for p- and s-polarized incident waves were obtained from the solution to the boundary problem with multiple reflections neglected:

$$E_{t(pp)} = E_{ip}\frac{4n_i n_t \varepsilon_o \eta_1 \eta_i e^{-2i\pi d\eta_1/\lambda}}{\left(n_i^2\eta_1 + \varepsilon_o\eta_i\right)\left(n_t^2\eta_1 + \varepsilon_o\eta_t\right)},$$

$$E_{t(ps)} = E_{ip}e^{-2i\pi d\eta_1/\lambda}$$

$$\times\frac{4in_i\eta_i\left[\alpha_{11}\left(p_1e^{-i\Delta} + p_3\right) + \alpha_{12}\left(p_2e^{-i\Delta} + p_4\right)\sin\varphi\right]}{(\varepsilon_e-\varepsilon_o)(\eta_2+\eta_i)(\eta_2+\eta_t)\left(n_i^2\eta_1+\varepsilon_o\eta_i\right)\left(n_t^2\eta_1+\varepsilon_o\eta_t\right)},$$

$$p_1 = \eta_2\left(n_t^2\eta_1 + \varepsilon_o\eta_t\right)$$

$$\times\left[\eta_1(\varepsilon_e + \eta_1\eta_i) - (2\eta_1 + \eta_i)n_i^2\sin^2\varphi\right], \quad (13)$$

$$p_2 = -n_i\eta_2\left(n_t^2\eta_1 + \varepsilon_o\eta_t\right)\left(\eta_1^2 + \eta_2^2 + 2\eta_1\eta_i\right),$$

$$p_3 = -\eta_1(\eta_2 + \eta_i)$$

$$\times\left[\eta_2\left(n_t^2\eta_1^2 + \varepsilon_o\eta_2\eta_t\right) - \left(n_t^2\eta_2 + \varepsilon_o\eta_t\right)n_i^2\sin^2\varphi\right],$$

$$p_4 = n_i\eta_1(\eta_2 + \eta_i)\left[n_t^2\left(\eta_1^2 + \eta_2^2\right) + 2\varepsilon_o\eta_2\eta_t\right],$$

$$E_{t(ss)} = E_{is}\frac{4\eta_2\eta_i e^{-2i\pi d\eta_2/\lambda}}{(\eta_2 + \eta_i)(\eta_2 + \eta_t)},$$

$$E_{t(sp)} = E_{is}e^{-2i\pi d\eta_2/\lambda}$$

$$\times\frac{4i\eta_i[\alpha_{11}\left(q_1e^{i\Delta} + q_3\right) + \alpha_{12}\left(q_2e^{i\Delta} + q_4\right)\sin\varphi]}{(\varepsilon_e-\varepsilon_o)(\eta_2+\eta_i)(\eta_2+\eta_t)\left(n_i^2\eta_1+\varepsilon_o\eta_i\right)\left(n_t^2\eta_1+\varepsilon_o\eta_t\right)},$$

$$q_1 = -n_t\eta_1(\eta_2 + \eta_t)$$

$$\times\left[\eta_2\left(n_i^2\eta_1^2 + \varepsilon_o\eta_2\eta_i\right) - \left(n_i^2\eta_2 + \varepsilon_o\eta_i\right)n_i^2\sin^2\varphi\right],$$

$$q_2 = n_i n_t\eta_1(\eta_2 + \eta_t)\left[n_i^2\left(\eta_1^2 + \eta_2^2\right) + 2\varepsilon_o\eta_2\eta_i\right], \quad (14)$$

$$q_3 = n_t\eta_2\left(n_i^2\eta_1 + \varepsilon_o\eta_i\right)$$

$$\times\left[\eta_1(\varepsilon_e + \eta_1\eta_t) - (2\eta_1 + \eta_t)n_i^2\sin^2\varphi\right],$$

$$q_4 = -n_i n_t\eta_2\left(n_i^2\eta_1 + \varepsilon_o\eta_i\right)\left[\eta_1^2 + \eta_2^2 + 2\eta_1\eta_t\right],$$

$$\eta_{1,2} = \sqrt{\varepsilon_{o,e} - \xi^2}.$$

The polarization azimuths χ_t and ellipticities K_t of the transmitted light for the crystals of classes $\bar{4}2m$ and $\bar{4}$ are calculated from formulas (2) and can be written (in the first order with respect to α_{ij}) as

$$\chi_{tp} = -\frac{(\alpha_{11}p_1 + \alpha_{12}p_2\sin\varphi)\sin\Delta}{(\varepsilon_e - \varepsilon_o)n_t\varepsilon_o\eta_1(\eta_2 + \eta_i)(\eta_2 + \eta_t)},$$

$$K_{tp} = \frac{\alpha_{11}(p_1\cos\Delta + p_3) + \alpha_{12}(p_2\cos\Delta + p_4)\sin\varphi}{(\varepsilon_e - \varepsilon_o)n_t\varepsilon_o\eta_1(\eta_2 + \eta_i)(\eta_2 + \eta_t)}, \quad (15)$$

$$\chi_{ts} = -\frac{(\alpha_{11}q_1 + \alpha_{12}q_2\sin\varphi)\sin\Delta}{(\varepsilon_e - \varepsilon_o)\eta_2\left(n_i^2\eta_1 + \varepsilon_o\eta_i\right)\left(n_t^2\eta_1 + \varepsilon_o\eta_t\right)},$$

$$K_{ts} = \frac{\alpha_{11}(q_1\cos\Delta + q_3) + \alpha_{12}(q_2\cos\Delta + q_4)\sin\varphi}{(\varepsilon_e - \varepsilon_o)\eta_2\left(n_i^2\eta_1 + \varepsilon_o\eta_i\right)\left(n_t^2\eta_1 + \varepsilon_o\eta_t\right)}. \quad (16)$$

At normal incidence of light, the χ_t and K_t values have different signs for p- and s-polarized incident light (at $\varphi = 0°$, we have $p_1, p_4 > 0$, $p_2, p_3 < 0$, $q_1, q_4 < 0$, $q_2, q_3 > 0$). If $n_i = n_t = 1$, then $p_1 = q_3$, $p_2 = q_4$, $p_3 = q_1$, $p_4 = q_2$.

To determine the α_{11} value in the crystal of class $\bar{4}2m$ at normal incidence of light, one should use a plate cut parallel to the [010] direction ($\psi' = 90°$); here, the light is incident in the [100] direction. Then, one can find α_{11} from the χ_t and k_t values at $\varphi = 0°$ using the following approximate formulas for χ_t and k_t [15]:

$$\tan 2\chi_t = -2k\sin\Delta,$$

$$\sin 2\gamma = -2k(1 - \cos\Delta), \quad K_t = \tan\gamma, \quad (17)$$

$$\Delta = 2\pi d(n_e - n_o)/\lambda, \quad k = \frac{\alpha_{11}}{2(n_e - n_o)},$$

where k is the ellipticity of eigenwaves. A plate cut parallel to the [100] direction ($\psi' = 0°$) can also be used; in this case, the $-\alpha_{11}$ value is obtained.

Two plates are required to determine the gyration-tensor components in the crystal of class $\bar{4}$. Using the plate cut parallel to the [010] direction, we find α_{11} from formula (17); to determine α_{12}, we should take the plate cut at an angle of 45° with the [100] direction ($\psi' = 45°$).

Figure 7 shows the dependences of χ_t and K_t on angles φ and ψ' for a KH_2PO_4 crystal of class $\bar{4}2m$. It can be seen that the surfaces obtained are not symmetric with respect to the $\varphi = 0$ plane. If the plate is cut parallel to one of the second-order axes ($\psi' = 0°$ or 90°), the χ_t and K_t values are identical for positive and negative angles of incidence: $\chi_t(-\varphi) = \chi_t(\varphi)$ and $K_t(-\varphi) = K_t(\varphi)$. At $\psi' = 90°$, the χ_t and K_t values have equal magnitudes and opposite signs, as compared with the values calculated at $\psi' = 0°$. If the plate is cut parallel to one of the symmetry planes ($\psi' = \pm 45°$), the χ_t and K_t values at positive and negative angles of incidence are opposite: $\chi_t(-\varphi) = -\chi_t(\varphi)$ and $K_t(-\varphi) = -K_t(\varphi)$. In the case where the plate is cut parallel to none of the second-order axes and none of the symmetry planes, the χ_t and K_t values change in magnitude with a change in the sign of the angle of incidence: $|\chi_t(-\varphi)| \neq |\chi_t(\varphi)|$, $|K_t(-\varphi)| \neq |K_t(\varphi)|$.

Note that the investigations in [4, 5] were performed only at normal incidence of light ($\varphi = 0°$).

In contrast to the crystal of class $\bar{4}2m$, for which the χ_t and K_t values at positive and negative angles of incidence are identical at $\psi' = 0°$ and opposite at $\psi' = 45°$,

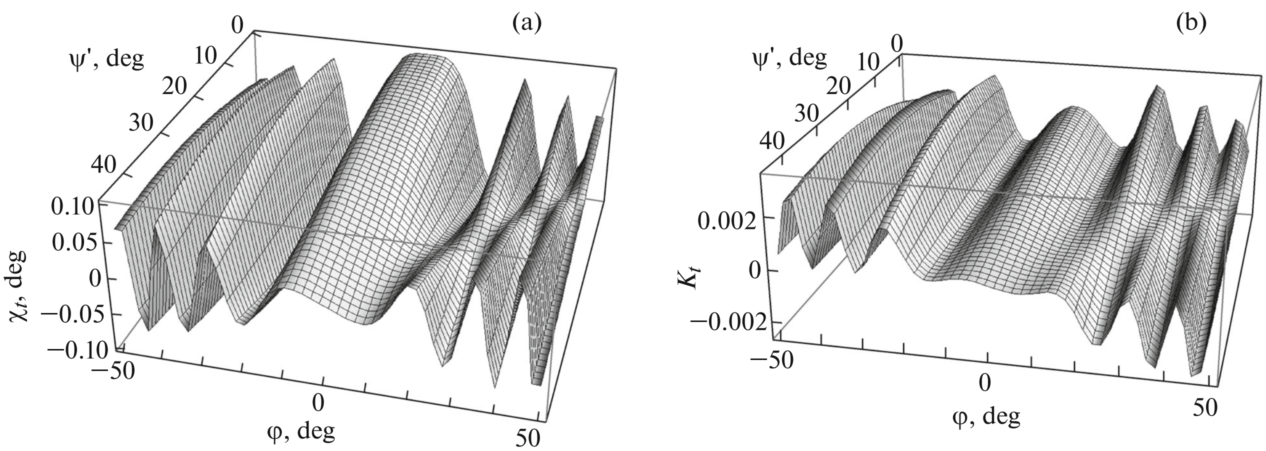

Fig. 7. Dependences of the (a) polarization azimuth χ_t and (b) ellipticity K_t of the transmitted light for the KH_2PO_4 crystal (class $\bar{4}2m$) and *p*-polarized incident light on the angle of incidence φ and angle ψ' between the [100] direction and the plate surface; $d = 268.7$ μm, $\lambda = 0.589$ μm, $n_i = 1$, and $n_t = 1$.

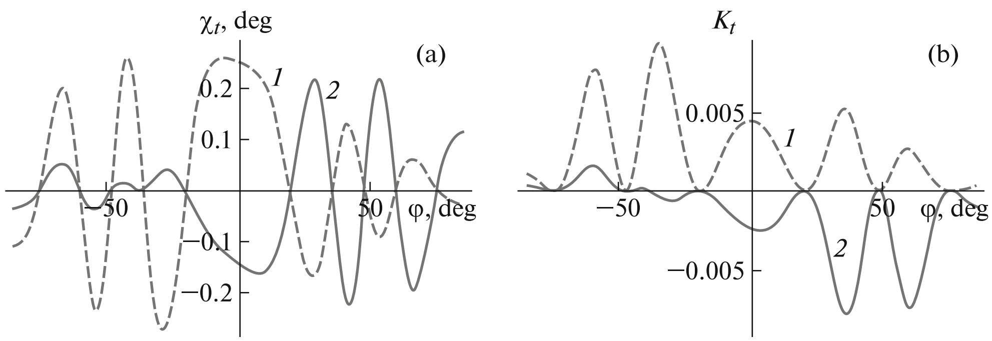

Fig. 8. Dependences (a) $\chi_t(\varphi)$ and (b) $K_t(\varphi)$ for the $CdGa_2S_4$ crystal (class $\bar{4}$) at $\lambda = 0.589$ μm: (*1*) $\psi' = 0°$ and (*2*) $\psi' = 45°$. The plate thickness is $d = 5617.3$ μm, $n_o = 2.4643$, $n_e = 2.4616$, $\alpha_{11} = 2.36 \times 10^{-5}$, $\alpha_{12} = 1.33 \times 10^{-5}$ [16], $n_i = 1$, and $n_t = 1$.

for class $\bar{4}$ we have the χ_t and K_t values at φ and −φ that are different in magnitude for both ψ' = 0° and 45°, as shown in Fig. 8 for a $CdGa_2S_4$ crystal (the refractive indices and α_{11} and α_{12} values are taken from [16]). For the crystal of class $\bar{4}$, we obtain symmetric dependences $\chi_t(\varphi)$ and $K_t(\varphi)$, i.e., $\chi_t(-\varphi) = \chi_t(\varphi)$ and $K_t(-\varphi) = K_t(\varphi)$, at $\psi' = \psi_{max} = \arctan(-\alpha_{12}/\alpha_{11})/2$ and antisymmetric dependences $\chi_t(-\varphi) = -\chi_t(\varphi)$ and $K_t(-\varphi) = -K_t(\varphi)$ at $\psi' = \psi_0 = \arctan(\alpha_{11}/\alpha_{12})/2$. If the plate is cut parallel to the [100] direction ($\psi' = 0°$), the α_{11} and α_{12} values can be calculated from the formulas (for *p* polarization)

$$\begin{aligned} &\chi_{tp}(-\varphi) - \chi_{tp}(\varphi) \\ &= \frac{2\alpha_{12} p_2 \sin\varphi \sin\Delta}{(\varepsilon_e - \varepsilon_o) n_t \varepsilon_o \eta_1 (\eta_2 + \eta_i)(\eta_2 + \eta_t)}, \\ &\chi_{tp}(-\varphi) + \chi_{tp}(\varphi) \\ &= -\frac{2\alpha_{11} p_1 \sin\Delta}{(\varepsilon_e - \varepsilon_o) n_t \varepsilon_o \eta_1 (\eta_2 + \eta_i)(\eta_2 + \eta_t)}. \end{aligned} \quad (18)$$

Similar expressions can be written for the *s* polarization using (16).

Thus, in the crystals of classes $\bar{4}2m$ (Fig. 7) and $\bar{4}$ (Fig. 8), the polarization azimuths and ellipticities of the transmitted light generally differ at positive and negative angles of incidence.

Manifestation of Optical Activity Near the Isotropic Point

Some crystals may have an isotropic point: the refractive indices of the ordinary and extraordinary waves coincide at a certain wavelength. Hobden found the optical activity for classes $\bar{4}2m$ ($AgGaS_2$, [2]) and $\bar{4}$ ($CdGa_2S_4$, [3]) in specifically these crystals. Crystals with an isotropic point were theoretically considered in detail in [6–8].

Let us consider the polarization azimuths χ_t of the transmitted light in the crystals of classes $\bar{4}2m$ and $\bar{4}$ in the presence of isotropic point. Figure 9a shows the

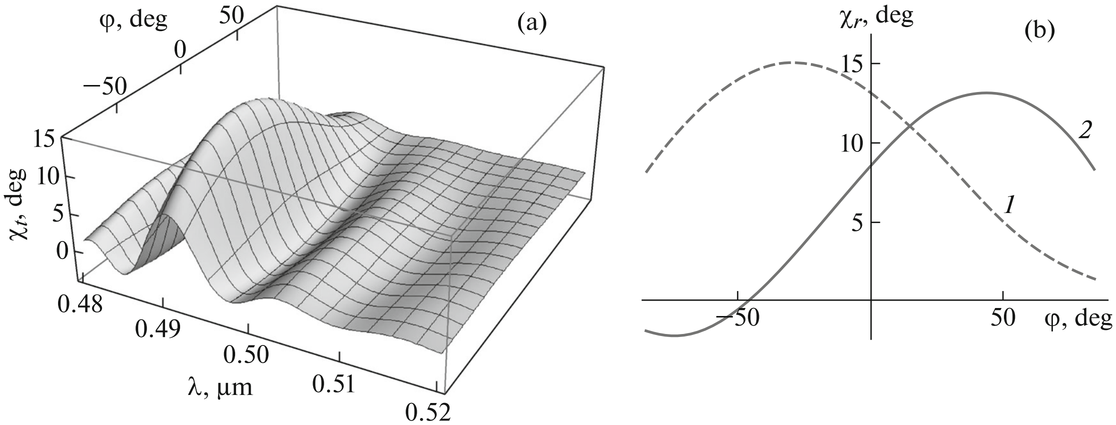

Fig. 9. (a) Dependence $\chi_t(\varphi, \lambda)$ for the $CdGa_2S_4$ crystal of class $\bar{4}$ having the isotropic point for the plate cut parallel to the [010] direction ($\psi' = 90°$) and (b) dependence $\chi_t(\varphi)$ at the isotropic point $\lambda = 0.4907$ μm: (*1*) $\psi' = 90°$ and (*2*) $\psi' = 45°$; $d = 1000$ μm, $n_i = 1$, and $n_t = 1$.

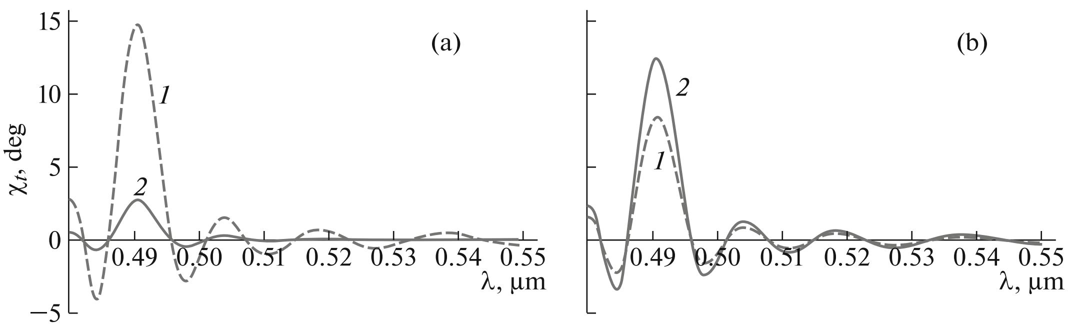

Fig. 10. Dependences $\chi_t(\varphi)$ in the presence of the isotropic point for the $CdGa_2S_4$ crystal (class $\bar{4}$) at the angles of incidence of light φ = (a) −30° and (b) 30°: (*1*) $\psi' = 90°$ and (*2*) $\psi' = 45°$; $d = 1000$ μm, $n_i = 1$, and $n_t = 1$.

dependence $\chi_t(\varphi, \lambda)$ for the $CdGa_2S_4$ crystal of class $\bar{4}$ (the plate is cut parallel to the [010] direction ($\psi' = 90°$),and Fig. 9b shows the dependences $\chi_t(\varphi)$ at the isotropic point ($\lambda = 0.4907$ μm) at $\psi' = 90°$ and $\psi' = 45°$. The dispersions of the refractive indices and the α_{11} and α_{12} values of the $CdGa_2S_4$ crystal, used in the calculations, are taken from [16]. The $\chi_t(\varphi)$ value at the isotropic point becomes much larger than that in the presence of birefringence, and $K_t(\varphi)$ tends to zero. The dependences $\chi_t(\lambda)$ (Fig. 10) have a peak in the vicinity of the isotropic point; however, the height of this peak differs for positive and negative angles of incidence of light (Figs. 10a, 10b).

CONCLUSIONS

Analytical expressions for the polarization azimuths $\chi_{r,t}$ and ellipticities $K_{r,t}$ of reflected (*r*) and transmitted (*t*) light waves as functions of the angle of incidence were obtained for crystals of classes $\bar{4}2m$ and $\bar{4}$. The influence of the diagonal (α_{11}) and off-diagonal (α_{12}) components of the gyration tensor on the polarization of the reflected and transmitted lights was considered. For the plate cut perpendicular to the optical axis the α_{12} value does not affect the polarization azimuths χ_t and ellipticities K_t of the transmitted light but influences the corresponding values χ_r and K_r for the reflected light: we obtained $\chi_r \neq 0$ only at $\alpha_{12} \neq 0$. If the plate is cut parallel to the optical axis, the $\chi_{r,t}$ and $K_{r,t}$ values differ at positive and negative angles of incidence of light for a nonzero α_{12} value. This is especially pronounced for the transmitted light because α_{11} and α_{12} make a comparable effect on the result in this case. For the reflected light, the influence of α_{12} is much lower and proportional to the product of the α_{12} value and the birefringence.

For the crystals of class $\bar{4}2m$, the dependences of χ_t and K_t on the angle of incidence for a plate cut parallel to the optical axis have radically different forms at different orientations of the second-order axes and the symmetry planes with respect to the plate surface. If a plate is cut parallel to one of the second-order symmetry axes, the χ_t and K_t values do not change with a change in the sign of the angle of incidence. If the plate is cut parallel to one of the symmetry planes, the χ_t and K_t values are opposite at positive and negative angles of incidence. In the other cases, χ_t and K_t differ in magnitude with a change in the sign of the angle of incidence. For the crystal of class $\overline{4}$, all these situations may occur at different orientations of the plate with respect to the [100] and [010] crystallographic directions. At the isotropic point, values of rotation of the plane of polarization are different at positive and negative angles of incidence.

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