ISSN 1063-7737, Astronomy Letters, 2018, Vol. 44, No. 3, pp. 202–211.

Two Populations of Sunspots: Differential Rotation

Yu. A. Nagovitsyn[1,2*], A. A. Pevtsov[3], and A. A. Osipova[1,2]

[1] *Pulkovo Astronomical Observatory, Russian Academy of Sciences, Pulkovskoe sh. 65, St. Petersburg, 196140 Russia*

[2] *St. Petersburg State University of Aerospace Instrumentation, Bol'shaya Morskaya ul. 67, St. Petersburg, 190000 Russia*

[3] *National Solar Observatory, Sunspot, NM 88349, USA*

Received July 27, 2017

Abstract—To investigate the differential rotation of sunspot groups using the Greenwich data, we propose an approach based on a statistical analysis of the histograms of particular longitudinal velocities in different latitude intervals. The general statistical velocity distributions for all such intervals are shown to be described by two rather than one normal distribution, so that two fundamental rotation modes exist simultaneously: fast and slow. The differentiality of rotation for the modes is the same: the coefficient at $\sin^2$ in Faye's law is 2.87–2.88 deg/day, while the equatorial rotation rates differ significantly, 0.27 deg/day. On the other hand, an analysis of the longitudinal velocities for the previously revealed two differing populations of sunspot groups has shown that small short-lived groups (SSGs) are associated with the fast rotation mode, while large long-lived groups (LLGs) are associated with both fast and slow modes. The results obtained not only suggest a real physical difference between the two populations of sunspots but also give new empirical data for the development of a dynamo theory, in particular, for the theory of a spatially distributed dynamo.

DOI: 10.1134/S1063773718020056

Keywords: *Sun, solar activity, sunspots, solar rotation.*

1. INTRODUCTION

One of the main effects that play a role in the solar dynamo mechanism is differential rotation: mechanical shearing plasma motions lead to magnetic field variations. The dependence of the solar rotation rate on the distance from the center has been obtained through helioseismological studies, which show the steepest gradient at the base of the convection zone and in the subphotospheric layers. Two populations of sunspots may be associated with these two toroidal magnetic field accumulation regions (Nagovitsyn et al. 2012, 2016, 2017; Nagovitsyn and Pevtsov 2016).

A difference in rotation rates as a function of group age (Newton and Nunn 1951), magnetic structure (Balthasar et al. 1986), and size (Ward 1966) was pointed out even in the classic studies of solar surface differential rotation, where sunspot groups were used as tracers. Howard et al. (1984) considered the rotation of sunspot groups as a function of their area and found that large groups >15 millionths of a solar hemisphere (m.s.h.) rotate slower than small ones <5 m.s.h. by 2%. The phenomenological model of the solar activity cycle by Gokhale (1979) associates the activity with two clusters of magnetic flux tubes, where small sunspot groups are generated by the decay of deeply anchored flux tubes and their motion to the surface. A more recent paper (Javaraiah and Gokhale 1997) supports this interpretation, finding that the initial depth of origin of sunspot groups goes down into the Sun by 21 000 km per day, while the anchoring depth rises at the same rate since their emergence on the surface. The theory of Gokhale (1979) was supported by the study of Gokhale and Sivaraman (1981), where for eight solar cycles the distribution of sunspot groups with respect to their maximum areas was shown to have two components behaving differently in the solar cycle. Based on the boundary area of 100 m.s.h. obtained by Gokhale and Sivaraman (1981), Hiremath (2002) showed that for both components the change in differential rotation rate agrees with the change in solar angular velocity as a function of radius. The rotation rate is believed to decrease as the sunspot groups age (Gokhale and Hiremath 1984; Balthasar et al. 1982, 1986; Tuominen and Virtanen 1987; Zappala and Zuccarello 1991; Zuccarello 1993), tending to the rotation rate of the

*E-mail: nag@gaoran.ru

surrounding photospheric gas. Zappala and Zuccarello (1991) argue that the differences in sunspot group rotation rates are evolutionary, with the key lifetime parameter. However, after the separation of sunspot groups in lifetime, Sivaraman et al. (2003), on the contrary, find acceleration of the rotation with time.

The two populations of sunspot groups to be discussed in this paper are also observed from the bimodality of the distribution of areas with an average boundary of ∼40 m.s.h., while Nagovitsyn and Pevtsov (2016) proposed to use the lifetime as a separation parameter. Short- and long-lived groups can be associated with the subsurface layers and the base of the convection zone, respectively (Hiremath 2002; Sivaraman et al. 2003). The differential rotation of the two populations of sunspots can give important information about the depths of origin of the magnetic flux tubes responsible for the appearance of sunspots (Gilman and Foukal 1979; Balthasar et al. 1986; Hiremath 2002).

2. OBSERVATIONAL DATA AND METHODS OF ANALYSIS

In this paper we used the Greenwich data on the daily areas and coordinates of sunspot groups from 1874 to 1976.

The velocities in both heliographic coordinates were calculated from the coordinates of groups with a lifetime of more than two days by the least-squares method. For groups with a lifetime of two days the velocities were determined as the difference in positions on these days divided by the time interval between their recordings. One-day groups were excluded from consideration.

The regression method, where the curve for the argument $\sin\varphi \equiv \mathrm{SF}$ was drawn through a cloud of points, was used in the overwhelming majority of studies of differential rotation (and meridional motions). In this case, naturally, the constructed curve was assumed to be unique.

We took a different path or, more specifically, analyzed the statistical distributions. The occurrence histogram of velocities was calculated for each latitude interval with a step $\Delta\mathrm{SF} = 0.05$. It was then fitted and the maximum of the fitting distribution (with an estimate of the confidence interval) was taken as the values for a given latitude interval. Thereafter, we considered the global distribution in SF. In our paper we used the approximation of Faye's law $a + b\sin^2\varphi$ for this purpose.

3. IS THE DIFFERENTIAL ROTATION OF SUNSPOT GROUPS UNIMODAL?

First consider all sunspot groups of the sample (more than 20 000 values). Let us construct the occurrence histograms of rotation rate residuals relative to the Carrington one $\omega - \omega_C$ for various SF intervals. Our initial hypothesis is that the residuals (including those due to the random measurement errors) must represent a random process similar to a Gaussian one. Therefore, we fit the derived distributions for each SF interval by one Gaussian (Fig. 1). The law of sidereal differential rotation corresponding to Fig. 1 is

$$\omega = (14.4508 \pm 0.0080) - (2.717 \pm 0.076)\sin^2\varphi. \quad (1)$$

Here and below, the global law of angular velocity is determined from the values of the mean for the fitting distribution (Gaussian) by taking into account its error for each SF.

It can be seen from Fig. 1 that for almost all latitudes the observed statistical distribution has an excess with respect to the fitting Gaussian. Let us make a test for the asymmetry and excess of all Gaussians in Fig. 1.

The asymmetry and excess are calculated as follows (Gmurman 1979):

$$A = \frac{1}{ns^3}\sum_{i=1}^{n} n_i(x_i - \overline{x})^3, \quad (2)$$

$$E = \frac{1}{ns^4}\sum_{i=1}^{n} n_i(x_i - \overline{x})^4 - 3.$$

Their dispersions are

$$D(A) = \frac{6(n-1)}{(n+1)(n+3)}, \quad (3)$$

$$D(E) = \frac{24(n-2)(n-3)n}{(n-1)^2(n+3)(n+5)}.$$

It is assumed that if

$$\mathrm{dev}A \equiv |A| - 3\sqrt{D(A)} \le 0, \quad (4)$$

$$\mathrm{dev}E \equiv |E| - 5\sqrt{D(E)} \le 0,$$

then the data are consistent with the hypothesis that the distribution is normal.

In fact, according to our data, (4) appear as presented in Fig. 2. It can be seen from this figure that for all statistically significant histograms ($N \gtrsim 500$), either dev$A$ or dev$E$ and, more often, both these quantities are greater than zero, suggesting a deviation from the normal distribution.

In general, the distribution of rotation rates in Fig. 1 can be represented as an asymmetric but unimodal (single-component) one. However, it is not

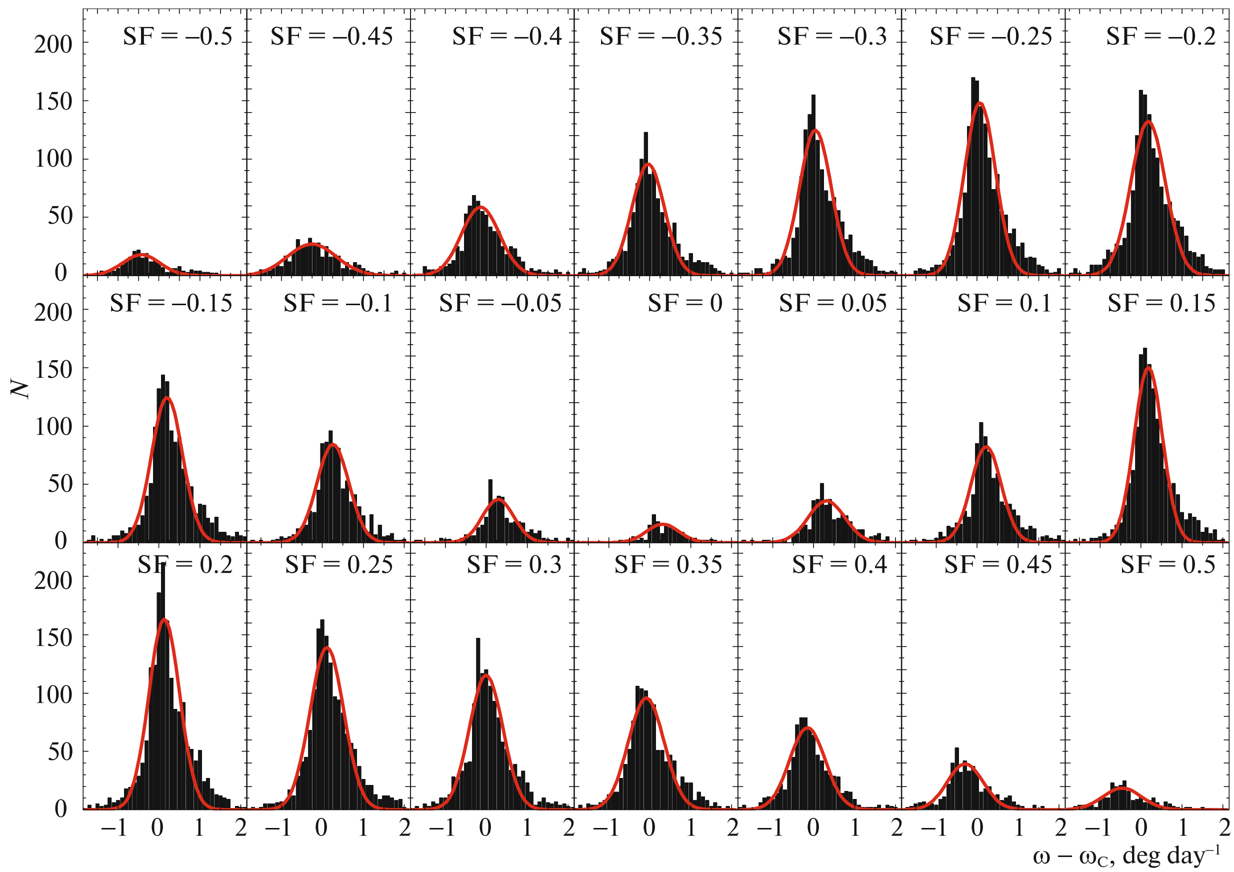

Fig. 1. (Color online) Occurrence histograms of rotation rates for all sunspot groups relative to the Carrington grid for various sines of the heliographic latitude SF and their unimodal Gaussian fit.

very clear precisely what fitting function we must use. Therefore, we will take a different path. We will adopt the hypothesis that the above fact is explained by two distributions of rotation rates (asymmetry) different in half-width (excess). Let us perform a two-Gaussian analysis (Fig. 3).

Obviously, for all latitudes there are two typical Gaussian distributions of rotation rates that we wall call T1 (wide, faster rotation) and T2 (narrow, slower rotation).

Let us test the hypothesis that the distribution of rotation rates is bimodal using the chi-square test (Agekyan 1974). We calculate

$$U = \sum_{i=1,k} \frac{(m_i - np_i)^2}{np_i}, \qquad (5)$$

where m_i are the frequencies in the histograms and np_i are the "theoretical" frequencies in the fitting distributions. To eliminate the influence of the error at a small number of values in a histogram column, we consider only those values for which $np_i > 5$.

Substituting the experimental U and the number of independent parameters $\xi = k - l - 1$ ($\xi = 3$ and 6 for one and two Gaussians, respectively) into the right-hand side,

$$P(u, \xi) = \frac{1}{2^{\xi/2}\Gamma(\frac{\xi}{2})} \int\limits_u^\infty t^{\xi/2-1} e^{-\frac{1}{2}t} dt, \qquad (6)$$

we obtain P. According to Agekyan, the hypothesis is rejected if $P < 0.05$ and the data are consistent with the hypothesis if $P > 0.05$ (respectively, at a 5% significance level).

The values of P obtained are illustrated by Fig. 4. The result is: the hypothesis that the normal distribution of rotation rates is unimodal is rejected for all SF, except for -0.5 and -0.45; on the whole, however, it is still rejected. At the same time, the data are consistent with the hypothesis about two existing normal distributions (except for SF $= 0.3$) at a 5% significance level.

In addition to the classical criteria, there are more modern ones that allow us to choose the most probable one from two descriptions, unimodal and bimodal.

For example, the Akaike information criterion (*AIC*), a criterion used to choose from several statistical models, is well known (Grasa 1989). Its value

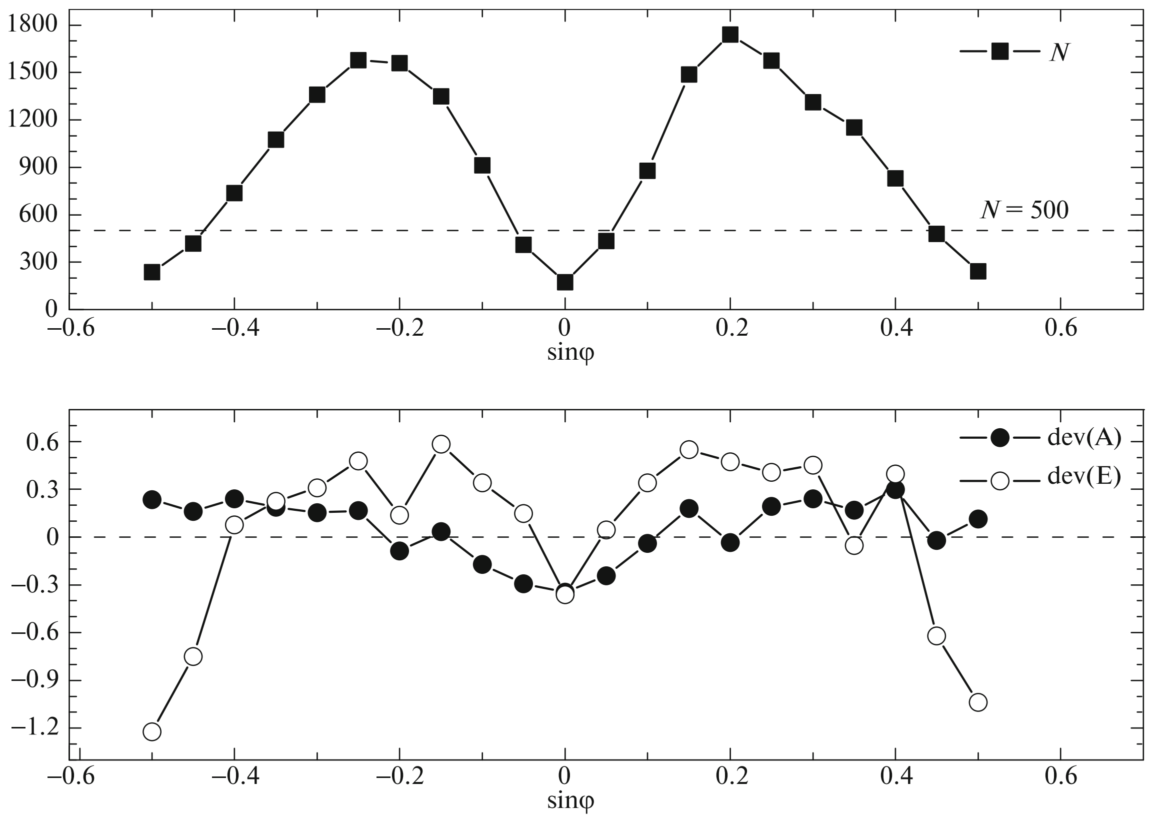

Fig. 2. The number of sunspot groups (top) and the variations of parameters (4) (bottom) for various sines of the heliographic latitude.

is defined as

$$AIC = 2k - 2\ln L, \tag{7}$$

where k is the number of model parameters and L is the maximized likelihood function of the model. The likelihood function for a model with several parameters for a discrete random variable is defined as follows:

$$L(x_1, x_2, \ldots, x_n; \Theta_1, \ldots, \Theta_k) \tag{8}$$
$$= p(x_1; \Theta_1, \ldots, \Theta_k)$$
$$\times\, p(x_2; \Theta_1, \ldots, \Theta_k) \ldots p(x_n; \Theta_1, \ldots, \Theta_k),$$

where p is the probability that the quantity will take x_i as a result of the trial. The functions L and $\ln L$ reach a maximum for the same set of parameters $\{\Theta_i\}_{i=1}^{k}$.

There is an experimental sample $y_i =$ count $(\omega \in [\omega_i, \omega_{i+1}])$, where we have a graph of the frequencies of occurrence of the differential sunspot group rotation rates for 21 latitude intervals.

Denote $x_i = (\omega_i + \omega_{i+1})/2$ (the middle of the interval). There are two models from which it is proposed to choose the one that describes better the experimental data. They can be described by one Gaussian:

$$f_1(x) = \frac{A \exp\left[-\frac{4\ln 2(x-x_c)^2}{w^2}\right]}{w\sqrt{\frac{\pi}{4\ln 2}}}, \quad k = 3, \tag{9}$$

or two Gaussians:

$$f_2(x) = \frac{A_1 \exp\left[-\frac{4\ln 2(x-x_{c,1})^2}{w_1^2}\right]}{w_1\sqrt{\frac{\pi}{4\ln 2}}} \tag{10}$$
$$+ \frac{A_2 \exp\left[-\frac{4\ln 2(x-x_{c,2})^2}{w_2^2}\right]}{w_2\sqrt{\frac{\pi}{4\ln 2}}}, \quad k = 6.$$

The probability $p(x_i; A, x_c, w) = f_1(x_i; A, x_c, w)/n$; similarly for $f_2(x_i; A_1, x_{c,1}, w_1, A_2, x_{c,2}, w_2)$.

We obtain $\ln L = \sum_{i=1}^{n} \ln p(x_i)$.

We know the model parameters and we can calculate the likelihood function for both models for each of the 21 samples; then we calculate the AIC. Plus we will take into account the fact that for small-size samples, where $n/k < 40$ (and this is precisely our case), the corrected AIC is commonly assumed to be more sensitive to an increase in the number of model parameters:

$$AIC_c = AIC + \frac{2k(k+1)}{n-k-1}, \tag{11}$$

where n is the sample size.

In addition, the Bayes (Schwarz) information criterion (BIC) proposed as a modification of the AIC, more sensitive to the number of model parameters, is

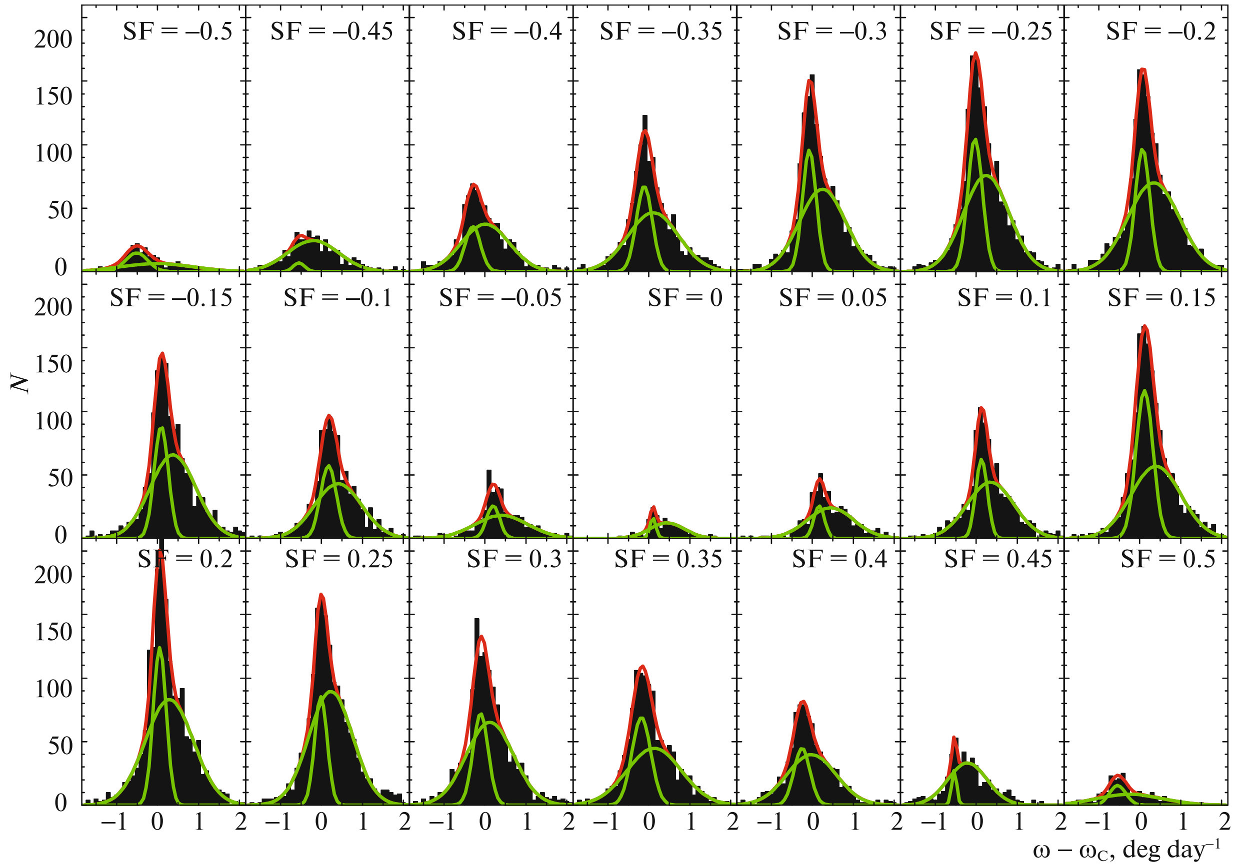

Fig. 3. (Color online) Occurrence histograms of rotation rates for all sunspot groups relative to the Carrington grid for various sines of the heliographic latitude SF and their bimodal Gaussian fit.

well known (Grasa 1989). (It is preferable to use the BIC for samples with $n \gg k$):

$$BIC = k \ln n - 2 \ln L. \quad (12)$$

AIC and BIC tend to give preference to models with a larger and smaller number of parameters, respectively.

The results obtained are illustrated by Table 1. The first column in Table 1 shows the middle of the interval of sines of the heliolatitude under consideration $\Delta \sin\varphi = 0.05$ in width, the second column shows the difference of the AIC values for the fits of the sample by one and two Gaussians $\Delta AIC = AIC_1 - AIC_2$, the third column shows the analogous difference of the corrected AIC values $\Delta AIC_c = AIC_{1,c} - AIC_{2,c}$, and the fourth column shows the analogous difference of the BIC values $\Delta BIC = BIC_1 - BIC_2$. Since the information criteria are smaller for the model that describes better the sample, a positive value in the table suggests that the fit by two Gaussians describes better the sample in the latitude interval under consideration. Negative values can be explained by the small sample size; the corresponding latitudes are italicized in Table 1. The observational data are better fitted by the sum of two normal distributions in the overwhelming majority of cases (>80%).

Thus, on the whole, the distribution of solar angular velocities is described by a bimodal rather than unimodal normal distribution for all latitudes.

4. TWO POPULATIONS OF SUNSPOT GROUPS AND THEIR ROTATION

As we have already noted, the sunspot groups were shown (Nagovitsyn et al. 2012, 2016, 2017; Nagovitsyn and Pevtsov 2016) to be reliably separated by their physical properties (areas, magnetic fields, lifetimes) into two populations: small short-lived (SSG) and large long-lived (LLG) ones.

Let us divide our data set into these populations and construct the occurrence histograms of angular velocities for them separately (Fig. 5). We see that the SSG and LLG sunspots show unimodal and bimodal distributions, respectively.

Let us check this conclusion using the chi-square test at a significance level $P = 0.05$, as before (Fig. 6). Apart from one latitude interval for LLGs and two for SSGs, the data are consistent with the

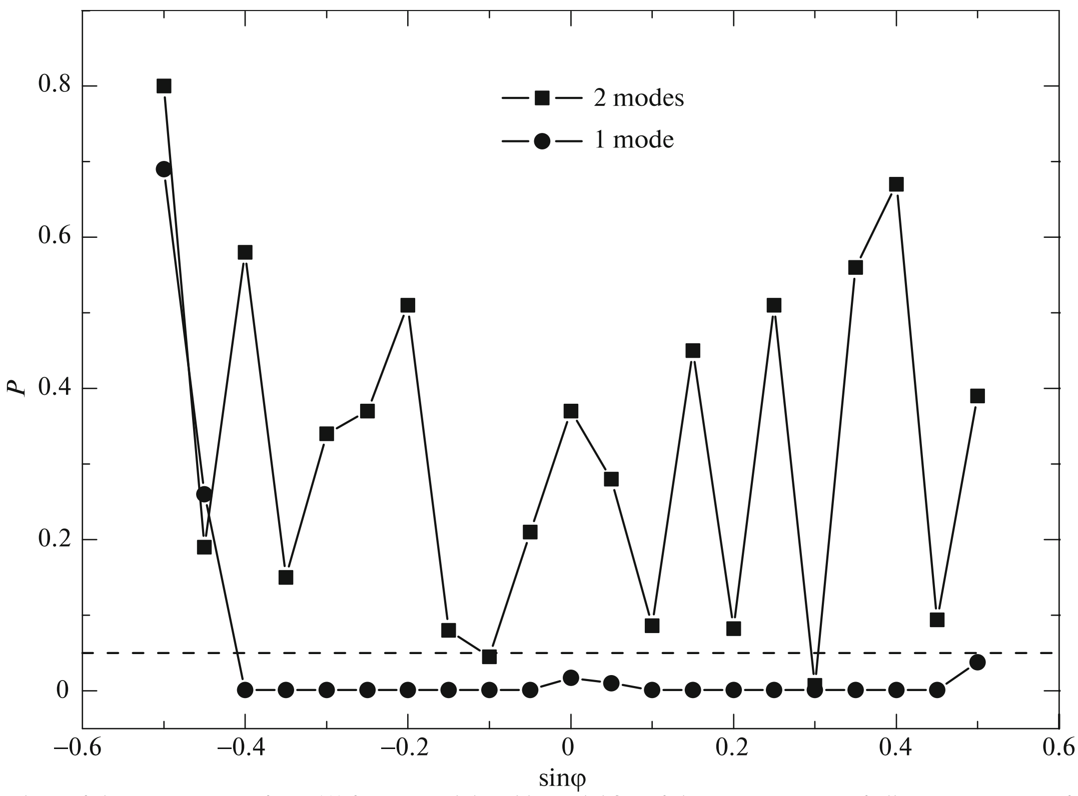

Fig. 4. Values of the parameter P from (6) for unimodal and bimodal fits of the rotation rates of all sunspot groups for various sines of the heliographic latitude.

Table 1. Results of our calculations

$\sin\varphi$	ΔAIC	ΔAIC_c	ΔBIC
−0.5	183	181	178
−0.45	26	24	21
−0.4	73	71	68
−0.35	15	13	10
−0.3	139	138	134
−0.25	151	149	145
−0.2	115	113	110
−0.15	145	143	140
−0.1	134	132	129
−0.05	183	181	177
0	*−6*	*−8*	*−11*
0.05	62	60	57
0.1	173	171	168
0.15	225	223	220
0.2	170	168	165
0.25	*−107*	*−109*	*−112*
0.3	*−19*	*−21*	*−24*
0.35	119	117	114
0.4	*0*	*−1*	*−5*
0.45	114	112	109
0.5	164	162	158

$\sin\varphi$ is the middle of the latitude interval under consideration ($\Delta\sin\varphi = 0.05$), ΔAIC is the difference of the AIC values for the fits of the sample by one and two Gaussians, ΔAIC_c is the analogous difference of the corrected AIC values, and ΔBIC is the analogous difference of the BIC values. The latitude intervals for which one or more criteria are less than zero and the negative values of the criteria themselves are italicized.

hypotheses about two-Gaussian and one-Gaussian distributions for LLGs and SSGs, respectively, at the selected significance level. We will designate the SSG rotation mode as S1, the fast LLG rotation mode as L1, and the slow one as L2.

5. TWO SYSTEMS OF DIFFERENTIAL ROTATION OF SUNSPOT GROUPS

Let us plot the data on the rotation of the S1, L1, and L2 modes on the graphs together with the T1 and T2 modes from Section 3. S1 and L1 coincide with T1, L2 coincides with T2 (Fig. 7). Thus, two sunspot group rotation modes exist on the Sun: "fast" T1 and "slow" T2:

$$\omega = (14.616 \pm 0.013) - (2.88 \pm 0.13)\sin^2\varphi, \quad (13)$$
$$\omega = (14.3499 \pm 0.0039) - (2.869 \pm 0.043)\sin^2\varphi.$$

For the modes (13) the rotation differentiality coefficients coincide with a high accuracy, the equatorial rotation rates differ by tens of sigma.

6. RESULTS AND CONCLUSIONS

In contrast to the previous studies, the result obtained here is based on an analysis of the statistical distributions of angular velocities for various latitude intervals. It is interpreted as the existence of two

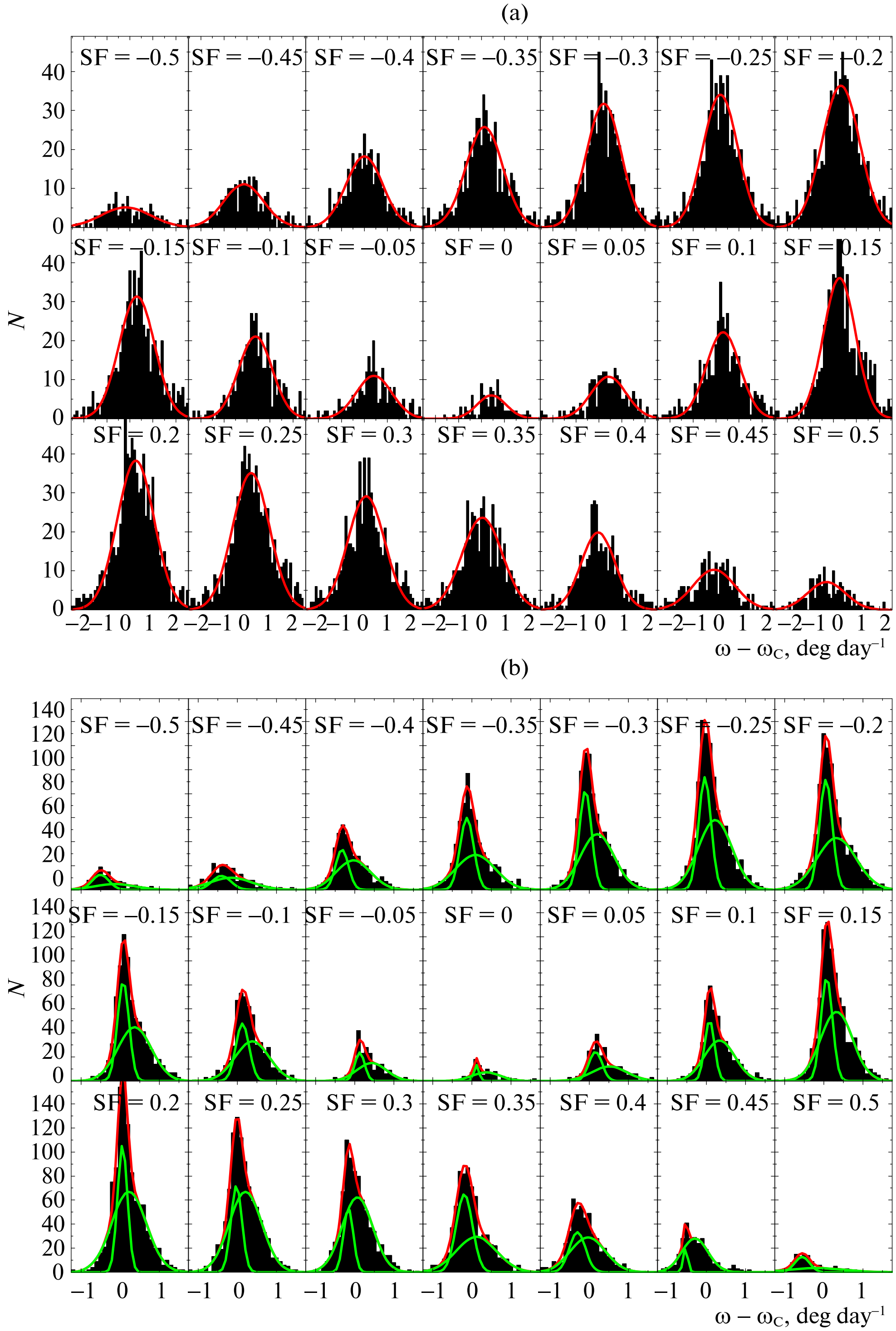

Fig. 5. (Color online) (a) Occurrence histograms of SSG rotation rates for various sines of the heliographic latitude SF and their unimodal Gaussian fit; (b) the same for the LLG population and their bimodal fit by two Gaussians.

fundamental sunspot group rotation modes (components) on the Sun with the same differentiality but different equatorial rotation rates. Note in parentheses that the bimodality of rotation for higher layers of the solar atmosphere, the solar corona, was detected by Badalyan and Sýkora (2005, 2006) and Badalyan (2009). However, the pictures of rotation differ significantly in the papers of these authors and in our paper (the difference of the mode periods is several days in the paper by Badalyan and Sýkora and less than one day in our paper).

The conclusion that follows from Fig. 7 is that

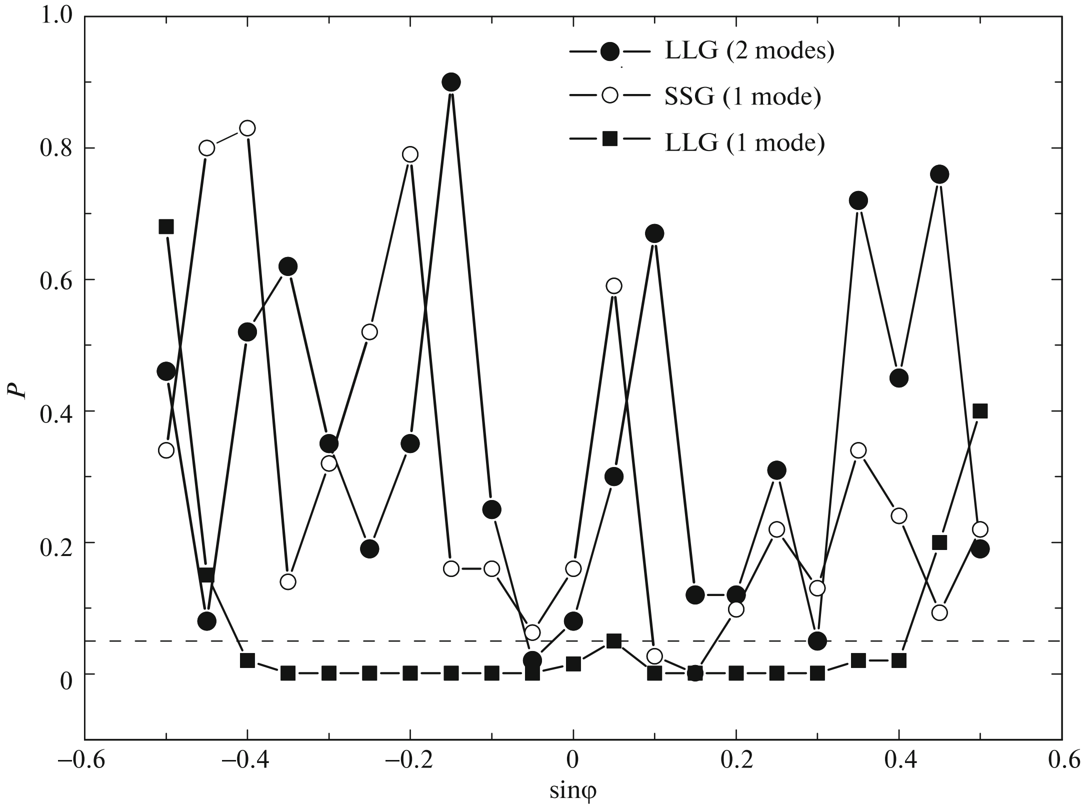

Fig. 6. Values of the parameter P from (6) for the rotation rates of the SSG population in a unimodal fit and the LLG population in a bimodal one for various sines of the heliographic latitude. Values of P in a unimodal fit for LLGs are given for comparison.

the rotation of SSGs is determined by the fast T1 mode, while that of LLGs is determined by both T1 and T2 (slow) modes. Indeed, if we consider the mean differences in the rotation of the corresponding populations with the fundamental modes, then we will obtain

$$\overline{S1-T1} = -0.031 \pm 0.075, \quad (14)$$
$$\overline{L1-T1} = -0.020 \pm 0.082,$$
$$\overline{L2-T2} = -0.010 \pm 0.024,$$

i.e., the corresponding modes do coincide. Recall that we identified the T1 and T2 rotation modes based on the entire data set, without any separation into the populations, and S1, L1, and L2 after the separation.

The fast and slow modes (13) differ significantly. Let us ask the question: Can the existence of two (fast and slow) synodic differential rotation modes of sunspot groups (that underlies our observations and calculations of the rotation rates) be due to the slightly different velocities of the Earth around the Sun at aphelion (summer season) and perihelion (winter season)? Let us estimate this residual.

We will denote the yearly mean parameters and those at perihelion and aphelion by the indices 0, 1, and 2, respectively. According to https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html, the orbital velocity of the Earth is $V_0 = 29.78$ km s^{-1}, $V_1 = 30.29$ km s^{-1}, and $V_2 = 29.29$ km s^{-1}.

Due to the conservation of angular momentum MVr, the distances from the Earth to the Sun will be $r_0 = 1.496 \times 10^8$ km, $r_1 = r_0V_0/V_1 = 1.471 \times 10^8$ km, and $r_2 = r_0V_0/V_2 = 1.521 \times 10^8$ km.

In one day the Earth will traverse the following distances in its orbit: $l_0 = 2.573 \times 10^6$ km, $l_1 = 2.617 \times 10^6$ km, and $l_2 = 2.531 \times 10^6$ km.

These will correspond to the angular velocity residuals (sidereal minus synodic)

$$\Delta\omega_0 = 2\arctan(l_0/(2r_0)) \quad (15)$$
$$= 0.9854 \text{ deg/day},$$
$$\Delta\omega_1 = 1.019 \text{ deg/day},$$
$$\Delta\omega_2 = 0.9534 \text{ deg/day}.$$

The scatter of residuals at perihelion and aphelion relative to the mean is 0.03 deg/day. Recall that we constructed the histograms of rotation rates with a step in rotation rate of 0.1 deg/day. So, the effects related to the difference of the Earth's velocities at aphelion and perihelion are smaller than the accuracy of constructing our histograms. On the other hand,

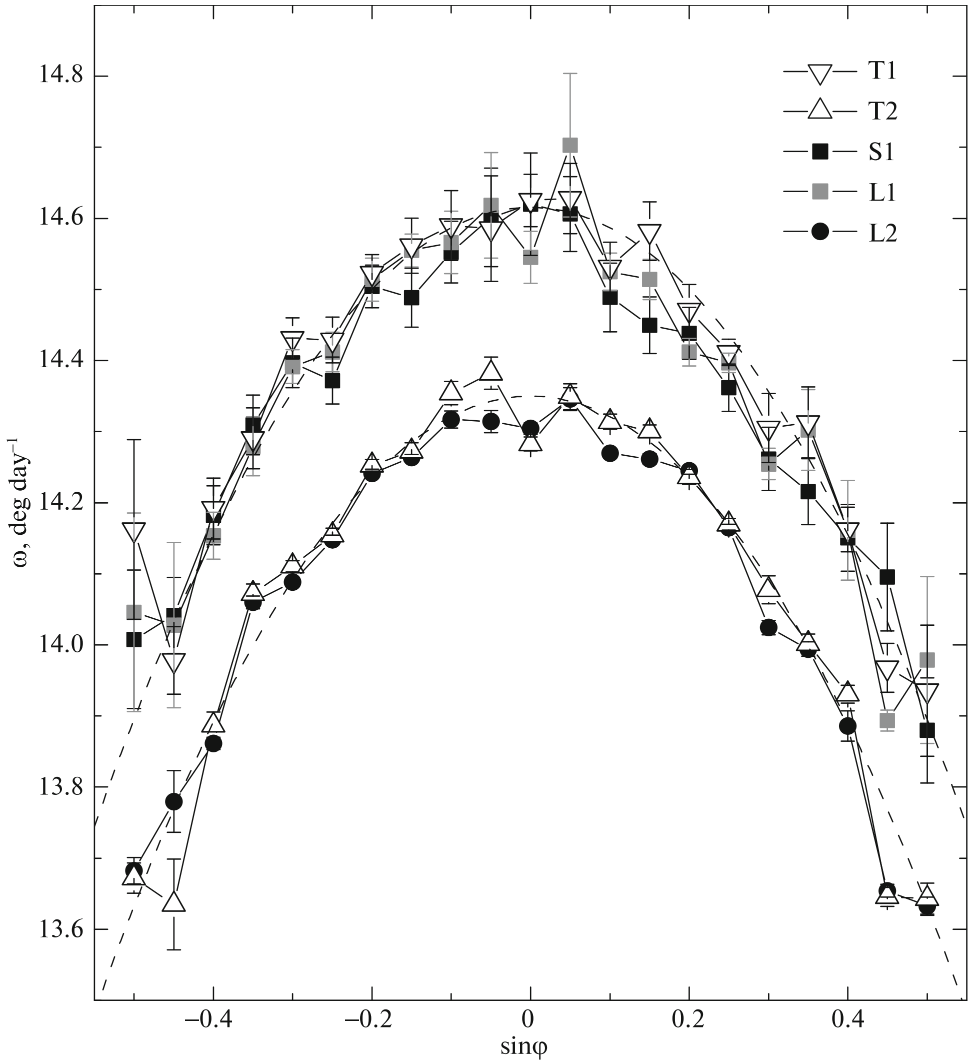

Fig. 7. Differential rotation of sunspot groups (for a description of the legend, see the text).

they cannot be responsible for the two different distributions of rotation rates per se: their difference is 0.06 deg/day, while we obtained 0.27 deg/day.

The question also arises as to how to compare our results with those of other authors, which are close to but not always coincide with our results. Most of the authors applied the regression methods insensitive to the distribution of individual rotation rates when constructing the rotation curves, while our statistical method allows one to diagnose the distribution being studied and to obtain the necessary parameter estimates from it.

To summarize, the revealed fundamental solar rotation modes are closely related to the existence of two populations of sunspot groups (14), while the populations themselves differ not only in areas, magnetic field, and lifetime (Nagovitsyn et al. 2012, 2016, 2017; Nagovitsyn and Pevtsov 2016), but also in differential rotation rate. All of this not only suggests a real physical difference between the two revealed populations of sunspot groups but also gives new empirical data for the development of a dynamo theory, in particular, for the theory of a spatially distributed dynamo.

ACKNOWLEDGMENTS

We are grateful to the referees for their valuable remarks. This work was supported in part by the Russian Foundation for Basic Research (project no. 16-02-00090), the Presidium of the Russian Academy of Sciences, and the Program for Support of Leading Scientific Schools (NSh-7241.2016.2).

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Translated by V. Astakhov