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Relationship between the North–South Asymmetry of Sunspot Formation and the Amplitude of 11-Year Solar Activity Cycles

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Abstract—A relationship between the north–south asymmetry of sunspot formation and the amplitude of 11-year cycles has been established from the RGO/USAF/NOAA data on sunspots. It is shown that the higher the solar cycle amplitude, the smaller the absolute value of the north–south asymmetry. The revealed pattern has been investigated in a numerical dynamo model with irregular variations of the alpha-effect.

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INTRODUCTION

The solar activity manifests itself equally in the northern and southern solar hemispheres only in a rough approximation. A detailed study of various sunspot formation indices shows that there exists quite a significant north–south asymmetry, i.e., "asynchrony" in the operation of the northern and southern solar hemispheres is observed. The north–south asymmetry is usually determined from the formula (Vitinskii 1973)

$$A = \frac{N - S}{N + S}, \quad (1)$$

where N and S are the activity indices for the northern and southern solar hemispheres.

On the scales of the 11-year cycle, this asymmetry is reduced primarily to an excess of the total area and the number of sunspot groups in one of the hemispheres as well as asynchrony in the latitudinal distribution of sunspot formation centers (Badalyan 2009, 2011; Nagovitsyn et al. 2010) and a difference in the epochs of extrema and the shape of the 11-year cycle curves in different hemispheres (Vitinskii 1973). The north–south asymmetry also manifests itself as a longitudinal inhomogeneity of sunspots. The active longitudes of the northern and southern hemispheres are shifted relative to one another (Vitinskii 1966; Marish 1971). The active longitudes of one and the other hemispheres dominate at the rise and decline phases, respectively (Plyusnina 2004, 2010). The north–south asymmetry is observed in the magnetic field polarity reversal at the north and south solar poles. A significant north–south asymmetry of sunspot activity is responsible for the asynchrony of the polar field reversal (Kitchatinov and Khlystova 2014; Mordvinov and Yazev 2014; Mordvinov et al. 2015).

In addition to the classical asymmetry index, the absolute asymmetry is also occasionally used:

$$|A| = \frac{|N - S|}{N + S}. \quad (2)$$

It characterizes the degree of "imbalance" between the hemispheres without specifying precisely which hemisphere dominates and shows how strongly the configuration of the toroidal magnetic field generated in the Sun's subphotospheric layers deviates from the dominant mode (a dipole or a quadrupole is generated when $A \to 0$, there is a mixed type of symmetry when $A \to 1$, and the sunspot activity is observed in one of the hemispheres at $A = 1$); on the Sun, as observations show, the dipolar type of symmetry dominates (below, by the asymmetry we mean precisely its absolute value). This quantity has a distinct 11-year variation and reaches its maximum values near the minima of activity cycles (Badalyan 2009). However, since the minima are characterized by a small number of events, the statistical significance of the asymmetry indices, which reaches the largest values at this time, is doubtful. The asymmetry of solar magnetic activity at the cycle maximum is of greatest value for the dynamo theory. These values of the asymmetry are easy to estimate from the available observations of the areas of sunspot groups or from the Wolf numbers.

The north–south asymmetry is also a characteristic of the grand minima of solar activity. At the epoch of the Maunder minimum, the north–south

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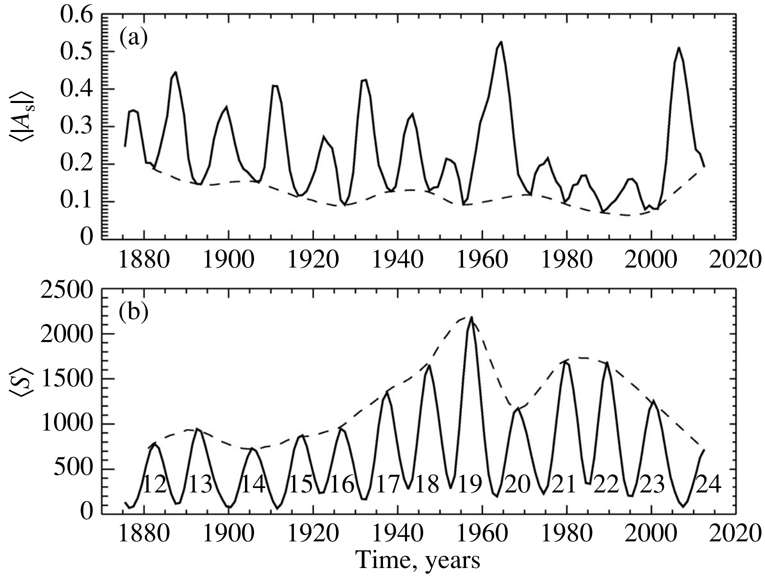

Fig. 1. (a) Absolute north–south asymmetry of yearly mean total sunspot areas. (b) The total sunspot area in m.s.h. The dashed lines envelope the local minima of the absolute asymmetry $\langle|A_S|\rangle$ and the 11-year cycle amplitudes $\langle S\rangle$. The correlation coefficient between the asymmetry minima and cycle amplitudes is −0.64. The 11-year cycle numbers are specified.

asymmetry of sunspot activity reached relatively large values during several solar cycles; sunspots were observed only in the southern hemisphere (see, e.g., Nesme-Ribes et al. 1994). Studies of the asymmetry based on the reconstructed series of solar activity over 2000 years (Nagovitsyn and Kuleshova 2015) have also shown that the most significant asymmetry extrema are observed in the periods of grand minima. Simulations (Olemskoy and Kitchatinov 2013) reveal a tendency for the equatorial symmetry (or parity) index P to increase when passing to grand minima:

$$P = \frac{E^{\mathrm{q}} - E^{\mathrm{d}}}{E}, \quad (3)$$

where E is the total magnetic energy represented as a sum of the energies of the dipolar and quadrupolar magnetic field components: $E = E^{\mathrm{q}} + E^{\mathrm{d}}$.

Simulations show that the smaller the cycle amplitude, the greater the deviation from the dipolar field configuration. The correlation coefficient between the cycle amplitude and the parity index (3) is −0.3. In contrast to the asymmetry (2), the parity index (3) shows precisely which mode dominates ($P = 1$ and -1 correspond to the quadrupolar and dipolar field configurations, respectively). In this paper, based on the observations of sunspots at the present epoch (activity cycles 12–24), we have revealed a similar pattern in the behavior of "high" activity cycles: the smaller the cycle amplitude, the greater the north–south asymmetry of sunspot formation, i.e., the greater the deviation from the dipolar type of symmetry.

RESULTS AND DISCUSSIONS

We calculated the correlation between the amplitudes of 11-year cycles expressed by the total sunspot area and the local minima of the absolute asymmetry in sunspot area. For the calculations, we used the Greenwich-USAF/NOAA (http://solarscience.msfc.nasa.gov/greenwch.shtml) observations of sunspot groups over 13 solar activity cycles. The data processing technique consisted in calculating the yearly mean total sunspot areas separately for the northern and southern solar hemispheres, whereupon the north–south asymmetry $|A_S|$ of the total sunspot area was calculated from Eq. (2). From the times series of S and $|A_S|$ we then calculated the moving averages of these quantities with a 5-year window. To identify the 11-year maxima of S and minima of $|A_S|$, we applied a filter with weights 1-2-2-1 (Vitinskii 1973). Figure 1 shows the smoothed absolute values of the asymmetry $\langle|A_S|\rangle$ and the total sunspot area $\langle S\rangle$ as a function of time.

For the clarity of the effect, the dashed curves indicate the envelope of the 11-year cycle maxima (Fig. 1b) and the envelope of the absolute asymmetry minima (Fig. 1a). We see that the curves change with time in antiphase; the correlation coefficient between the local minima of the asymmetry and the amplitudes of 11-year solar activity cycles expressed by the total sunspot area is −0.64. It should be noted that a similar result was also obtained when using the Wolf numbers; the correlation coefficient is −0.62.

A local 11-year minimum of the absolute asymmetry (Fig. 1a) corresponds to each 11-year cycle

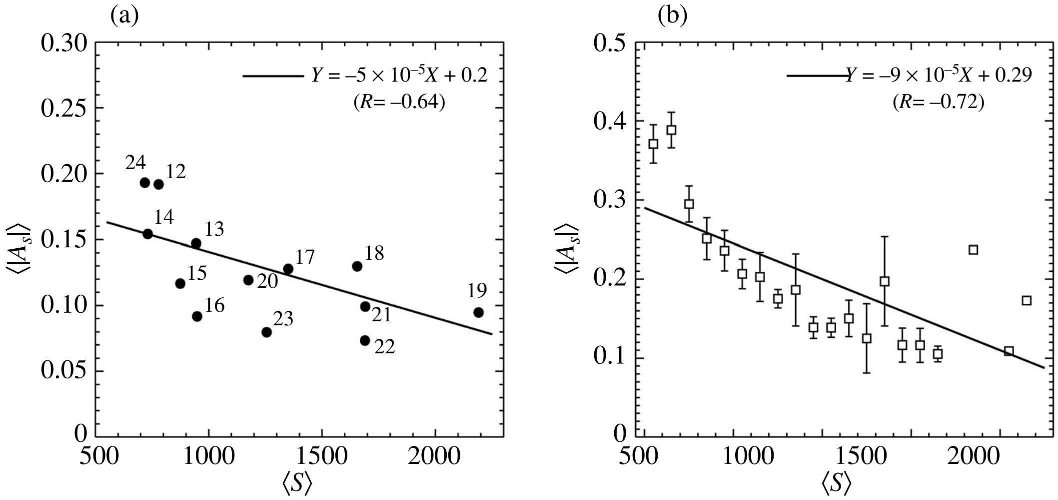

Fig. 2. (a) Local minima of the north–south asymmetry $\langle|A_S|\rangle$ versus amplitudes of 11-year solar activity cycles expressed by $\langle S\rangle$. The 11-year cycle numbers corresponding to Fig. 1 are specified. (b) Yearly mean $\langle|A_S|\rangle$ versus smoothed yearly mean total sunspot area $\langle S\rangle$. The vertical bars indicate the confidence intervals. The linear regression equations and correlation coefficients are presented in the figures.

maximum (Fig. 1b). The 11-year minima of the absolute asymmetry are of greatest interest for the dynamo theory, because the values of $\langle|A_S|\rangle$ at the minima characterize the solar cycle at its maximum development and show how much the toroidal field configuration deviates from the dipolar symmetry.

Figure 2a shows an anticorrelation of the local minima of $\langle|A_S|\rangle$ with the amplitudes of 11-year solar activity cycles expressed by the total sunspot area $\langle S\rangle$. Figure 2b presents a plot of the smoothed yearly mean $\langle|A_S|\rangle$ against $\langle S\rangle$. The range of $\langle S\rangle$ was divided into equal bins with a step of 100 millionths of a solar hemisphere (m.s.h.). The arithmetic mean $\langle|A_S|\rangle$ was found for each bin of sunspot areas. To estimate the confidence intervals of the means, we calculated their standard errors. The standard error of the mean was not calculated for the last three points in Fig. 2b, because there is one event for each of these bins of sunspot areas, which is clearly shown by the histogram in Fig. 3a.

As can be seen from Fig. 2, the asymmetry index, on average, decreases with increasing activity. The negative correlation (−0.64 for the local extrema and −0.72 for the yearly means) can be interpreted as follows. As observations show, the dipolar mode is predominantly generated on the Sun. Random fluctuations of the alpha-effect, irregular in time and space, can transfer part of the magnetic energy to the quadrupolar modes. The quadrupolar modes are a subcritical regime for the excitation of such magnetic configurations, and these modes rapidly decay. The results from Galitskii et al. (2005) confirm that the dipolar magnetic configuration grows more rapidly than the quadrupolar one, but, at the same time, the complex growth rates of the dipolar and quadrupolar configurations are close. This points to the possibility of long-term existence of a nondipolar magnetic field configuration on the Sun. Deviations from the dipole reduce the magnetic energy. Large deviations can promote the transitions to grand minima. Thus, deviations of the large-scale solar magnetic field from the equatorially antisymmetric mode, i.e., large values of the asymmetry at the maxima of 11-year cycles, can be an indicator of a decline in magnetic activity and evidence for the transition to a grand minimum.

If not the absolute asymmetry but the asymmetry calculated from the standard formula (1) is used in the calculations, then no correlation of the asymmetry with the solar activity level is found (the correlation coefficient between $\langle A_S\rangle$ and $\langle B\rangle$ is −0.14). Studies of the north–south asymmetry based on the reconstructed series of solar activity over 2000 years (Nagovitsyn and Kuleshova 2015) did not show any correlation of the asymmetry calculated from the classical formula (1) with the solar activity level either.

Figure 3a presents the histogram of the number of cases of yearly mean $\langle S\rangle$. As follows from the figure, the statistics of events is very inhomogeneous. The total number of events is 135, which roughly corresponds to the number of years of observations, 50% of the values of $\langle S\rangle$ lie within the range of areas 0–600 m.s.h., the total sunspot area reaches 1500–2200 m.s.h. in 13% of the cases. Nevertheless, we will show below that inhomogeneous statistics does not affect the relationship of the asymmetry to the power of 11-year cycles and that this relationship is a nonrandom process. The distribution of

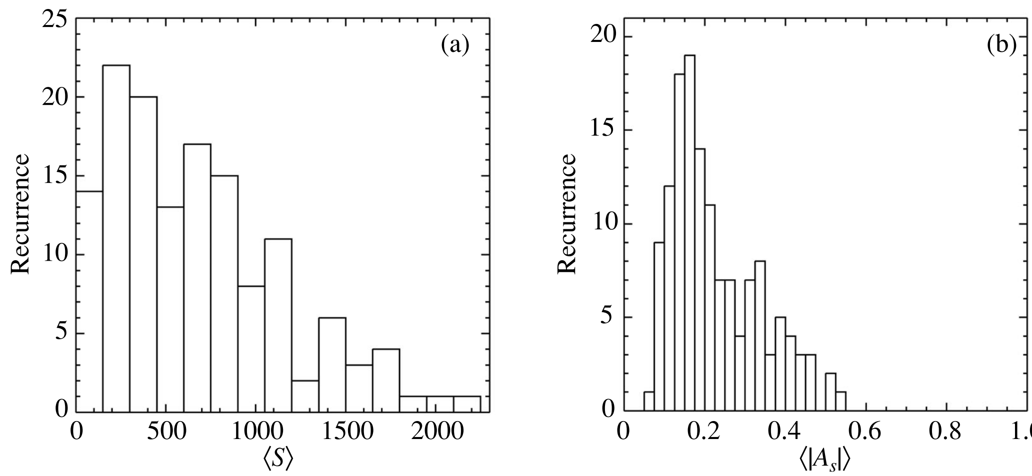

Fig. 3. (a) Recurrence histogram of yearly means of the total sunspot area $\langle S\rangle$ (with a bin width of 150 m.s.h.). (b) Recurrence histogram of yearly means of the north–south asymmetry in the total sunspot area $\langle |A_S|\rangle$ (with a bin width of 0.025).

smoothed yearly means of the north–south asymmetry $\langle |A_S|\rangle$ is shown in Fig. 3b. For the present epoch of "high" activity cycles, the hemispheric asymmetry of magnetic activity predominantly oscillates within the range 0.1–0.3 and rarely exceeds 0.5 (three cases in 135 years); larger values of the asymmetry probably occur in the years of 11-year cycle minima.

North–South Asymmetry in a Dynamo Model with Irregular Variations of the Alpha-Effect

The negative correlation of the north–south asymmetry with the magnetic activity level revealed by the observational data was investigated in a dynamo model with a nonlocal alpha-effect varying irregularly in time and space and diamagnetic pumping of the field (Kitchatinov and Olemskoy 2011a, 2012). There are also other dynamo models with fluctuations in governing parameters that reproduce the well-known empirical relations between various activity cycle parameters (see, e.g., Pipin et al. 2012). The model with a fluctuating alpha-effect used in this paper reproduces consistently a number of observed global characteristics for the 11-year cycle and the variations of solar activity on long time scales. The alpha-effect fluctuation parameters were determined from the observations of sunspots (Kitchatinov and Olemskoy 2011b; Olemskoy et al. 2013). The dominant mode in the model is a dipole. The magnetic flux B is calculated in the model as an activity index separately for the northern, $B_{\rm N}$, and southern, $B_{\rm S}$, solar hemispheres, which is close in physical meaning to the total sunspot area:

$$B = B_{\rm N} + B_{\rm S}, \qquad (4)$$

$$B_{\rm N} = \int\limits_{x_0}^{1}\int\limits_{0}^{\pi/2} \sin\theta\, x\, \phi_{\rm b}(x)\, |\beta(x,\theta)|\, {\rm d}x\, {\rm d}\theta,$$

$$B_{\rm S} = \int\limits_{x_0}^{1}\int\limits_{\pi/2}^{\pi} \sin\theta\, x\, \phi_{\rm b}(x)\, |\beta(x,\theta)|\, {\rm d}x\, {\rm d}\theta.$$

Here, x and θ are the relative radius and the latitude, respectively; x_0 is the base of the convection zone; $\beta(x,\theta)$ is the toroidal field at the corresponding point of the convection zone; the function $\phi_{\rm b}(x)$ specifies the layer near the bottom of the convection zone whose toroidal fields form active regions on the surface as magnetic loops rise to the surface and contribute to the alpha-effect (Kitchatinov and Olemskoy 2011a, 2012); the coefficient $\sin\theta$ allows for the latitude dependence of the length of flux tubes for the toroidal field (the probability of the rise of magnetic loops to the surface with the formation of sunspots is assumed to be proportional to the length of the magnetic flux tube). The absolute value of $\beta(x,\theta)$ enters into the integrand of (4), because the sunspot polarity is disregarded in the determination of the total area.

To compare the simulations with the observational data, we converted the dimensionless magnetic flux to m.s.h., the units of total sunspot area; the conversion coefficient is 9.2×10^5. The simulation time is $\sim$10 000 years. To process the simulated data series, we applied a technique similar to that used to process the observations: we found the yearly means of B, $B_{\rm N}$, and $B_{\rm S}$ and calculated the north–south asymmetry of the magnetic flux $|A_B|$ from Eq. (2). The values of B and $|A_B|$ were smoothed and filtered. Figure 4

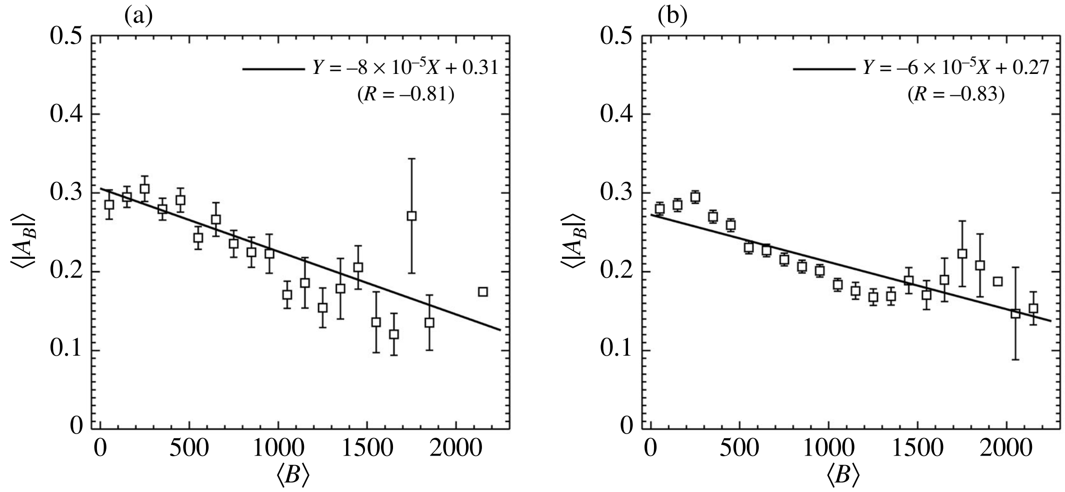

Fig. 4. Simulations: (a) local minima of the north–south asymmetry $\langle|A_B|\rangle$ versus amplitudes of 11-year solar activity cycles expressed by $\langle B\rangle$; (b) yearly mean $\langle|A_B|\rangle$ versus smoothed yearly mean magnetic flux $\langle B\rangle$. The vertical bars indicate the confidence intervals. The linear regression equations and correlation coefficients are presented in the figures.

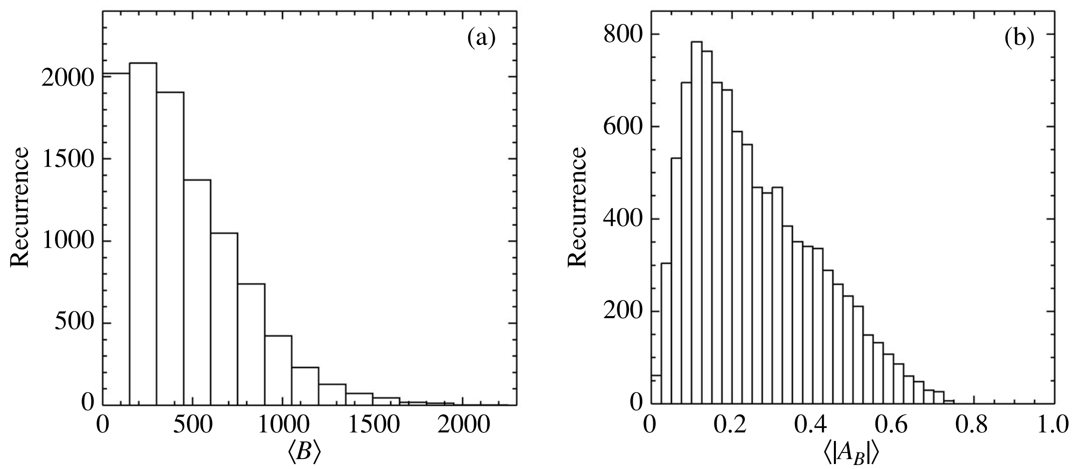

Fig. 5. Simulations: (a) recurrence histogram of yearly means of the magnetic flux $\langle B\rangle$ (with a bin width of 150 m.s.h.); (b) recurrence histogram of yearly means of the north–south asymmetry in the magnetic flux $\langle|A_B|\rangle$ (with a bin width of 0.025).

presents the results of our regression analysis of the smoothed $\langle B\rangle$ and $\langle|A_B|\rangle$ and their extrema.

Figure 4a shows an anticorrelation of the north–south asymmetry $\langle|A_B|\rangle$ with the amplitudes of 11-year solar activity cycles expressed by $\langle B\rangle$. In Fig. 4b, the smoothed yearly mean $\langle|A_B|\rangle$ are plotted against $\langle B\rangle$. Similarly to Fig. 2b, the range of $\langle B\rangle$ was divided into equal bins with a step of 100 m.s.h. We found the arithmetic mean $\langle|A_B|\rangle$ in each bin and calculated their standard errors; the confidence intervals are indicated by the vertical bars. In our simulations, the statistics of events exceeds the statistics of sunspot observations by two orders of magnitude; therefore, the statistical significance of the results obtained is higher. Our simulations (Fig. 4a) reproduce the linear negative correlation of the north–south asymmetry with the 11-year cycle amplitudes revealed by the observational data (Fig. 2a). The yearly means of the asymmetry $\langle|A_B|\rangle$ (Fig. 4b) also show a similar correlation with the solar activity level; the correlation coefficient is $R = -0.83$. The pair regression coefficients in our simulations (Fig. 4) and from the observational data (Fig. 2), have the same order of magnitude, 10^{-5}.

Figure 5 shows the recurrence histograms of yearly mean $\langle B\rangle$ and $\langle|A_B|\rangle$. The histograms qualitatively correspond to those in Fig. 3 constructed from the observational data, 50% of the values of $\langle B\rangle$ recur in the range 0–450 m.s.h. (Fig. 5a), the range of $\langle|A_B|\rangle$ (Fig. 5b) is slightly wider than the range

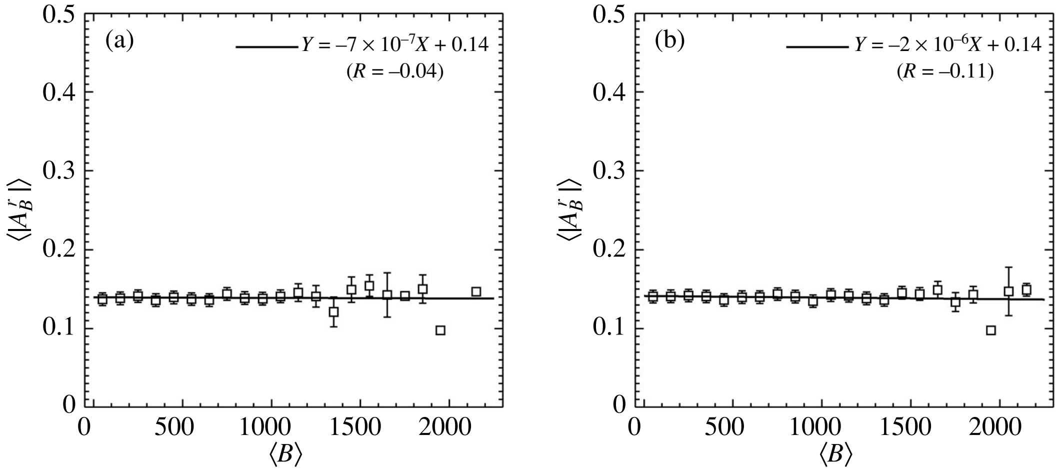

Fig. 6. Simulations, the north–south asymmetry $\langle|A_B^r|\rangle$ was calculated as a random process: (a) local minima of the north–south asymmetry $\langle|A_B^r|\rangle$ versus amplitudes of 11-year solar activity cycles expressed by $\langle B\rangle$; (b) yearly mean $\langle|A_B^r|\rangle$ versus smoothed yearly mean magnetic flux $\langle B\rangle$. The vertical bars indicate the confidence intervals. The linear regression equations and correlation coefficients are presented in the figures.

of $\langle|A_S|\rangle$, because the time series of the asymmetry in our simulations exceeds considerably the period of observations.

As follows from Fig. 5a, the statistics of events generated in our simulations is as inhomogeneous as that in the observations. This series of yearly mean magnetic fluxes B computed in the dynamo model over a period of ∼10 000 years was used for our calculation and statistical estimation of the random asymmetry, when the ratio of the magnetic fluxes B between the northern and southern hemispheres was determined by a random function r.

North–South Asymmetry due to Statistical Noise

Figure 6 presents the dependence of the north–south asymmetry on $\langle|B|\rangle$ calculated as a random process:

$$B_{\rm N}^r = Br, \quad B_{\rm S}^r = B - B_{\rm N}^r, \qquad (5)$$
$$B = B_{\rm N}^r + B_{\rm S}^r = B_{\rm N} + B_{\rm S},$$

where r is a random variable with a uniform distribution of values in the interval $0 \le r \ge 1$, and B is the magnetic flux (4) randomly distributed between the northern, $B_{\rm N}^r$, and southern, $B_{\rm S}^r$, solar hemispheres. The technique described above was used to smooth and filter the computational data. The north–south asymmetry $\langle|A_B^r|\rangle$ of the magnetic flux was calculated from Eq. (2).

As can be seen from Fig. 6, for the random asymmetry there is no correlation with the solar activity level, $\langle|A_B^r|\rangle$ does not correlate with the cycle amplitudes and the yearly mean $\langle B\rangle$, the correlation coefficients are -0.04 and -0.11, respectively. The regression line runs parallel to the OX axis, implying that Y does not change as X changes, i.e., $\langle|A_B^r|\rangle$ does not depend on $\langle B\rangle$. The confidence interval for all points of both figures does not exceed ± 0.03, while for those intervals in which the statistics of events is more than 100 cases, the error oscillates within ± 0.008, suggesting a high statistical significance of the results obtained.

Thus, the anticorrelation of the north–south asymmetry with the solar activity level revealed by the observational data (Fig. 2) is more deterministic in character and to a lesser degree is a stochastic process. The solar dynamo model completely reproduces the negative correlation of the north–south asymmetry with the power of 11-year cycles revealed by the observations (Fig. 4). As has already been noted above, the growth of the north–south asymmetry as the solar activity level decreases is explained by the fact that quadrupolar modes can be excited due to the random fluctuations of the alpha-effect; the generation of such modes for the Sun is a supercritical regime, i.e., it is harder to excite such modes for the Sun and more magnetic energy is required, which leads to a reduction in the overall magnetic energy level of the solar activity. A gradual decline in solar activity level and a transition to the regime of a grand minimum similar to the Maunder minimum can be the result of such mode beating.

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