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# Spectroscopic Studies of Three Cepheids with High Positive Pulsation Period Increments: SZ Cas, BY Cas, and RU Sct

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**Abstract**—Three high-resolution spectra have been taken at different times with the 6-m SAO RAS telescope (LYNX and PFES spectrographs) for three Cepheids exhibiting high positive period increments: the small-amplitude (DCEPS) SZ Cas and BY Cas and the classical (DCEP) RU Sct. SZ Cas and RU Sct are members of the Galactic open clusters $\chi$ and h Per and Trump 35, respectively. Analysis of the spectra has shown that the interstellar Na I D1 and D2 lines in all objects are considerably stronger than the atmospheric ones and are redshifted in SZ Cas and BY Cas and blushifted in RU Sct. The core of the H$\alpha$ absorption line in BY Cas has an asymmetric knifelike shape, while RU Sct exhibits an intense emission in the blue wing of this line. Such phenomena are observed in long-period Cepheids and bright hypergiants with an extended envelope. In this case, the strong Mg Ib 5183.62 Å and Ba II 5853.67, 6141.713, and 6496.90 Å lines with low $\chi_{\rm low}$ in SZ Cas and RU Sct also show characteristic knifelike profiles with an asymmetry in the red region, while the Ba II 4934.095 Å line shows similar profiles in the blue one. The absorption lines of neutral atoms and singly ionized metals with different lower-level excitation potentials exhibit different degrees of asymmetry: from a pronounced one with secondary components in BY Cas (similar to those in the small-amplitude Cepheid BG Cru pulsating in the *first* overtone and having an envelope) to its insignificance or virtual absence in SZ Cas and RU Sct. Analysis of the secular changes in mean $T_{\rm eff}$ determined from photometric color indices and spectra over the last 55 years for these stars has revealed periodic fluctuations of 200 K for SZ Cas and BY Cas and 500 K for RU Sct. For SZ Cas and RU Sct, $T_{\rm eff}$ determined in some years from some color indices show much lower values, which together with the temperature fluctuations can be associated with mass loss and dust formation. Based on these facts, we hypothesize the existence of circumstellar envelopes around all three Cepheids. We have determined the atmospheric parameters and chemical composition of the program Cepheids. An appreciable carbon underabundance, a nitrogen overabundance (the result was obtained only for BY Cas), a nearly solar oxygen abundance, a sodium overabundance, and solar magnesium and aluminum abundances have been revealed in all stars, suggesting that these yellow supergiants has already passed the first dredge-up. The abundances of the Fe-peak elements, $\alpha$-elements, and r- and s-process elements are nearly solar. [Fe/H] = −0.05 dex for SZ Cas and [Fe/H] = +0.05 dex for RU Sct can be used to estimate the metallicities of the open clusters $\chi$ and h Per and Trump 35, respectively.

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## INTRODUCTION

In 2001, we (Usenko et al. 2001) began a program of spectroscopic studies for classical Cepheids and main-sequence (MS) stars belonging to Galactic open clusters. Since Cepheids in clusters are mostly calibration objects for the period–luminosity–color relation, one of our tasks was to determine the metallicity of these objects and to take into account its influence on this relation. It was important to establish the evolutionary status of these objects using the abundances of the so-called key elements of the evolution of yellow supergiants: carbon, nitrogen, oxygen, sodium, magnesium, and aluminum. All three objects (SZ Cas, BY Cas, and RU Sct) were initially assumed to be members of Galactic open clusters according to the General Catalogue of Variable Stars (GCVS) (Kholopov et al. 1986). Subsequently, it emerged that one of the objects, BY Cas, is not a

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cluster member. Nonetheless, all these three objects share one common feature: they have high *positive* secular pulsation period increments.

**SZ Cas**, according to the GCVS (Kholopov et al. 1986), is a small-amplitude Cepheid (DCEPS) with a relatively long ($P = 13^{d}.6$) pulsation period of spectral type F6 I–G4 Ib that is a member of the corona of the open cluster $\chi$ and h Per. It has always been used as a calibration object for the period–luminosity–color relation for classical Cepheids of the Galaxy. Iben and Tuggle (1972) also used SZ Cas as one of the calibration objects among 13 Cepheids in Galactic open clusters to compute pulsation models for Cepheids with various masses and chemical compositions: with $T_{\text{eff}} = 5943$ K and a mass of 4.8 $M_{\odot}$. According to Fricke et al. (1972), the mean $T_{\text{eff}} = 5960$ K, $M_{\text{ev}} = 8.2\ M_{\odot}$, $M_{\text{pul}} = 5.5\ M_{\odot}$, and $R_{\text{pul}} = 79.8\ R_{\odot}$ was taken for their theoretical model. Cox (1979) calculated the theoretical, evolutionary, and pulsational masses of Cepheids and adopted the mean $T_{\text{eff}} = 5919$ K, $M_{\text{ev}} = 8.77\ M_{\odot}$, $M_{\text{th}} = 9.04\ M_{\odot}$, and $M_{\text{pul}} = 7.89\ M_{\odot}$ for SZ Cas. If SZ Cas is a *first*-overtone pulsator, then $M_{\text{ev}} = 8.77\ M_{\odot}$ and $M_{\text{th}} = 10.89\ M_{\odot}$, respectively.

Wielen (1973) estimated the age of the Cepheid to be $\approx 2.6 \times 10^{7}$ yr, while the mean age of the cluster $\chi$ and h Per was estimated by him to be $(0.8–1.6) \times 10^{7}$ yr. According to the photometric studies by Schmidt (1976), the mass of the Cepheid lies within the range (4.4–5.1) $M_{\odot}$ at the color excess $E_{B-V} = 0^{m}.88$. Cogan (1978) estimated the radius of SZ Cas to be (65–79.8) $R_{\odot}$. Having applied the Baade–Wesselink method based on ten spectrograms and using the red $V - R$ color indices, Coker et al. (1989) obtained its unexpectedly low mean $T_{\text{eff}} = 5351$ K and radius $R = 38 \pm 6\ R_{\odot}$.

Madore and Fernie (1980) suspected the existence of a hot companion to the Cepheid based on the phase shift of the minima in the $V$ and $U - B$ curves. Using a color–color $(U - B)$–$(B - V)$ diagram, Usenko (1990) determined the spectral type of the companion, B3–B4 V. The existence of a companion was confirmed by Szabados (1991) based on the change of the $O - C$ curve. Observations in the range (2.3–12.6)$\mu$ revealed no significant infrared excess in the Cepheid (Gehrz and Hackwell 1974). Polarimetric observations of SZ Cas in 1983–1984 showed its polarization parameters on the $p_x p_y$ diagram to form a characteristic, in shape, rosette with a rather significant area, $\Delta = 0.61 \pm 0.105\%$ (Polyakova 1987). Similar rosettes with such a significant area were also observed in some long-period Cepheids as well as in W Vir and RR Lyr, which may suggest the existence of an extended circumstellar envelope. According to Erleksova and Irkaev (1982), SZ Cas exhibits a rather high pulsation period increment, $\Delta P/P = +1.691$ s yr$^{-1}$.

Three spectrogram for SZ Cas were taken by Luck and Lambert (2011) using the Hobby–Eberly Telescope (HET) (spectral range 4400–7850 Å, resolution $R = 30\,000$) in 2009–2010 at various pulsation phases: $0^{P}.447$, $0^{P}.020$, and $0^{P}.508$. $T_{\text{eff}}$ were 6173, 5710, and 6222 K; $\log g$ were 2.08, 1.77, and 2.22; $V_t$ were 4.74, 4.68, and 4.89 km s$^{-1}$, respectively. The abundances of 21 chemical elements were estimated.

**BY Cas** is a small-amplitude short-period Cepheid (DCEPS) of spectral type F5 I–F7 I (Kholopov et al. 1986). Malik (1965) was the first to note the possible association of BY Cas with the open cluster NGC 663. After performing careful $UBV$ photometry for the Cepheid itself and some stars, cluster members, he pointed out that the Cepheid is behind a thick absorbing cloud and could be close to the cluster. He found the color excess to be $E_{B-V} = 0^{m}.8 \pm 0^{m}.33$ and estimated the distance to be from 1.25 to 1.61 kpc. Phelps and Janes (1994) determined the distance to NGC 663 of 2.8 kpc and its age of $2 \times 10^{7}$ yr. Based on infrared photometry, Majaess et al. (2009) determined $E_{B-V} = 0^{m}.6$ for BY Cas, its distance of 1.7 kpc, and its age of $10^{9}$ yr. Based on the radial velocities for the cluster $V_r$, from $-27.3$ to $-39.6$ km s$^{-1}$, measured from stars of spectral types B6 Ia–B9 V (Liu et al. 1991) and the mean radial velocity of the Cepheid, $-58$ km s$^{-1}$ (Gorynya et al. 1994), the authors pointed out BY Cas is most likely *not* a member of NGC 663. Nevertheless, Turner et al. (2013) supposes that BY Cas is still a cluster member. Based on infrared photometry from Fouqué et al. (2007), Luck and Lambert (2011) determined the color excess for the Cepheid, $E_{B-V} = 0^{m}.76$, and its distance of 1.55 kpc.

Analyzing the behavior of the $O - C$ curve for the Cepheid, Szabados (1991) and, subsequently, Berdnikov and Pastukhova (1994) pointed out cyclic variations of the pulsation period that, according to Usenko (1990), could be associated with the existence of a close companion of spectral type B5 V. Based on their radial-velocity measurements for the Cepheid, Gorynya et al. (1994, 1995) revealed such a companion with an orbital period from 553 to 563 days.

Based on a Fourier analysis of the photometric data for the Cepheid, Welch et al. (1995) found its amplitude ratio $R_{21}$ to correspond to a *first*-overtone pulsation, while Turner et al. (2010) supposes that BY Cas crosses the Cepheid instability strip for the *first* time. Erleksova and Irkaev (1982) pointed out a sharp increase in the pulsation period increment for the Cepheid, $\Delta P/P = +0.774$ s yr$^{-1}$, with the period having decreased before 1940 and, subsequently, having begun to increase sharply. Neilson et al. (2012) noted that the estimate of $\Delta P/P$ for BY Cas is comparable to that for Polaris ($\alpha$ UMi), 4.3–4.5 s $yr^{-1}$, in which the mass loss rate is known to be $\Delta M \approx 10^{-6}\ M_{\odot}\ yr^{-1}$. They concluded that BY Cas might also have *enhanced mass loss*.

The s10301 spectrum taken in 1995 at phase $0^{P}.977$ was investigated previously (Kovtyukh et al. 1996), but such important atmospheric parameters as the effective temperature and microturbulent velocity were determined by different methods. The first was determined by combining the estimates obtained from the $(B-V)_0 - T_{\rm eff}$ relation and the absence of any relation between the derived neutral-iron abundances and their excitation potential. The second was determined from the absence of any relation between the neutral-iron abundances and their equivalent widths. The following atmospheric parameters were obtained: $T_{\rm eff} = 6400 \pm 100$ K, $\log g = 2.50$, and $V_t = 3.50$ km $s^{-1}$. The abundances were estimated for ten elements from a small number of lines. The authors also pointed out the presence of a very strong interstellar sodium doublet component ($\lambda$ 5890–5896 Å) redshifted relative to the stellar components by 0.9 Å. It would be quite natural to redetermine the atmospheric parameters by a more perfect method and to recalculate the abundances of elements using an expanded list of their lines.

One spectrum at phase $0^{P}.664$ was taken by Luck and Lambert (2011) using the HET with atmospheric parameters $T_{\rm eff} = 6110 \pm 72$ K, $\log g = 2.27$, and $V_t = 3.42$ km $s^{-1}$. The abundances were estimated for 21 elements.

**RU Sct** was identified as a member of the open cluster Trumpler 35 by Tsarevskii et al. (1966). The cluster membership was subsequently confirmed by Turner (1980): $M_V = -5^m.19$, $E_{B-V} = (0^m.95 - 1^m.03) \pm 0^m.2$, and an age of $(9 \pm 3) \times 10^6$ yr. Pel (1978) noted a negative slope of all color loops on the color–color diagrams for this Cepheid based on $VBULUW$ photometry, while Sollazzo et al. (1981) determined the mean radius of the Cepheid, $R = 108.2 \pm 23.1\ R_{\odot}$, based on the same photometry. Using the Baade–Wesselink method, Laney and Stobie (1995) determined the radius of the Cepheid, 120.5 $R_{\odot}$, while Sachkov et al. (1998) found it to be 105 $R_{\odot}$. The surface brightness method gave an estimate of 120.5 $R_{\odot}$ (Arellano Ferro and Rosenzweig 2000). Eggen (1985) determined $M_V = -5^m.01$ using Strömgren photometry, while Gieren and Fouque (1993) found $-5^m.26$ based on cluster OB stars.

Erleksova and Irkaev (1982) also pointed out a very high pulsation period increment, $\Delta P/P = +1.004$ s $yr^{-1}$, while Butler et al. (1987) noted a considerable infrared excess based on IRAS observations ($1^m.12$) and significant mass loss, $1.12 \times 10^{-8}\ M_{\odot}\ yr^{-1}$, which may suggest the existence of an envelope around the Cepheid. Having analyzed their radial velocity measurements, Gorynya et al. (1996) and, subsequently, Szabados (1996) concluded that RU Sct is a spectroscopic binary with a $\gamma$-velocity amplitude of more than 10 km $s^{-1}$. According to Turner (1995), the progenitor of RU Sct was a main-sequence star of spectral type B3 V with a mass $\approx$10 $M_{\odot}$. Sachkov (2002) supposes that the Cepheid pulsates in the *fundamental* mode. Eggen (1985) estimated the metallicity of RU Sct, [Fe/H] = +0.18 dex, based on Strömgren photometry.

One spectrum at phase $0^{P}.407$ was taken by Luck and Lambert (2011) using the HET with atmospheric parameters $T_{\rm eff} = 5441 \pm 72$ K, $\log g = 1.32$, and $V_t = 4.53$ km $s^{-1}$. As in the previous cases, the abundances of 21 elements were estimated.

In recent years, the search for gaseous envelopes around Cepheids and the investigation of their structure have been among the priority directions in Cepheid research. An unusual behavior of the H$\alpha$ absorption line in Cepheids with pulsation periods longer than $10^{d}$ was detected (Wallerstain and Jacobsen 1981; Gillet 1993): an almost stable radial velocity of the line core at each pulsation phase (changes by no more than a few km $s^{-1}$), while the absorption lines of metals change their radial velocities with phase. In addition, secondary variable absorption and emission features were observed in the H$\alpha$ wings. To understand the cause of this strange behavior, Nardetto et al. (2008) analyzed their high-resolution spectra for Cepheids in a wide range of pulsation periods. They established that the peculiarities in the H$\alpha$ line (the secondary variable absorption and emission features in its wings) are actually typical of Cepheids with periods longer than 10 days, while no such peculiarities are observed for stars with shorter periods. It was hypothesized that such an effect arose for two reasons: either due to peculiar dynamics of the upper atmospheric layers or due to the presence of a *circumstellar envelope*, because the H$\alpha$ line is formed in the atmosphere fairly high. The second reason seems most plausible, because the observations by Kerevella et al. (2009) for two Cepheids, l Car и RS Pup, confirmed the presence of such envelopes around them. The problem is that such an envelope must be not only optically thick in the H$\alpha$ core but also "transparent" for most of the metal absorption lines. Hence an important assumption is made: *in this case, the core of the H$\alpha$ absorption line represents the radial velocity of this envelope, while the other, weaker* *lines show the Doppler velocities of the pulsating atmospheric layers* (Fokin et al. 2009). If this assumption is valid, then the observed emission in the wings forming deeper than the absorption core can be produced by the passage of shock waves through the upper atmospheric layers. In addition, enhanced mass loss, from $10^{-10}$ to $10^{-6}$ $M_{\odot}$ yr$^{-1}$, was detected in long-period Cepheids, with its rate increasing with pulsation period (Wilson et al. 1988; Kerevella et al. 2009). This is a confirmation the fact that *continuous mass loss* is responsible for the existence of dense envelopes around long-period Cepheids.

The paper by Gallenne et al. (2012), which is devoted to the results of infrared measurements for Cepheids with the VISIR thermal photometer, showed that not only long-period Cepheids but also objects with pulsation periods shorter than $10^{d}$ exhibit an infrared excess at wavelengths longer than 10 $\mu$m. The authors associated this fact with the envelope emission of dust around the supergiants produced by mass loss. Mathias et al. (2006) drew attention to the Cepheid X Sgr ($P = 7^{d}$), whose spectrum exhibits a splitting of metal absorption lines initially into two components, redshifted and blueshifted, with weaker additional components in the same red and blue regions appearing over the pulsation period. This phenomenon was extended only partially to strong lines with low excitation potentials (the authors pointed out the Na D doublet and Ba II 4934.095 Å and the absence of such a phenomenon in the H$\alpha$ line). The absence of a phase shift between the weak lines of metals forming deep in the stellar atmosphere and the H$\alpha$ line forming much higher was pointed out. As an example, the authors showed the Fe I 6055.99 Å absorption line profile to change with pulsation phase. Usenko et al. (2013a) used the same line as an example in their work, while pointing out a slight change in the radial velocities of the H$\alpha$ and H$\beta$ cores. Mathias et al. (2006) showed that such peculiarities in the absorption lines of metals could be produced by two consecutive shock waves: the first is driven by the $\kappa$-mechanism, while an *envelope or extended atmosphere is needed* for the generation of the second one (the cases of shock generation due to nonradial pulsations and the existence of a close companion were also considered, but, according to the authors, they are less plausible). Investigating southern-hemisphere Cepheids, Usenko et al. (2014a, 2014b) detected similar peculiarities of metal absorption lines in the short-period small-amplitude Cepheids BG Cru AV Cir, BP Cir, and LR TrA, not only in strong absorption lines of neutral atoms with low excitation potentials (as the Fe I 6055.99 Å line mentioned above) but also in ions (the Fe I 6369.46 Å and Si II 6371.355 Å lines were given as an example). A pulsation model was computed for BG Cru, based on which the authors concluded that this Cepheid pulsates in an *overtone*, while the peculiar behavior of the metal lines is attributable to the existence of a circumstellar envelope. The behavior of the H$\alpha$ and H$\beta$ absorption lines with pulsation phase is analogous to the case with X Sgr mentioned above.

Analyzing a dynamical model atmosphere for the yellow hypergiant $\rho$ Cas, Klochkova et al. (2014) showed the blue components of split metal lines with low excitation potentials (as, for example, the above-mentioned Na D doublet lines, Ba II 4554.036, 4934.095, 6141.713 Å and others) to be characteristic indicators of a gaseous envelope. No such splitting is observed in Cepheids, but asymmetries in the profiles of such lines arise in objects with long pulsation periods.

Thus, the following can serve as the main spectroscopic indicators of circumstellar envelopes around Cepheids:

(1) A slight change in the radial velocity of the H$\alpha$ core with pulsation phase compared to that determined from the metal lines (for Cepheids of all periods).

(2) The appearance of secondary variable absorption (the so-called knifelike profiles) and emission components in the red and blue H$\alpha$ wings at various phases of the pulsation period (for Cepheids with pulsation periods longer than $10^{d}$).

(3) The appearance of red and blue components at various pulsation phases in strong lines of metals (neutral atoms and ions) with low excitation potentials, which impart a noticeable asymmetric shape to the lines (for Cepheids with periods shorter than $10^{d}$).

(4) Asymmetries of strong metal absorption lines with low excitation potentials (for Cepheids with pulsation periods longer than $10^{d}$).

Hence it follows that the above facts about the possible enhanced mass loss by these Cepheids and, as a consequence, the existence of gaseous envelopes around them can be checked by means of modern spectroscopy.

## OBSERVATIONS AND PRIMARY REDUCTION

The spectra of all three Cepheids were taken with the 6-m BTA telescope at SAO RAS using the following echelle spectrographs: LYNX (Panchuk et al. 1993) for BY Cas and PFES (Panchuk et al. 1997) for SZ Cas and RU Sct, respectively. The spectrum for BY Cas was taken with a resolution $R \approx 24\,000$ and a signal-to-noise ratio $\approx$100 for

**Table 1.** Observations, light-curve elements, and radial-velocity estimates for the program Cepheids

| Cepheid | Spectrum number | HJD 2440000+ | $V$, mag | $P$, days | Init. time of max. 2430000+ | Phase | Exposure time, min | $V_r$, km s$^{-1}$ | Number of lines |
|---|---|---|---|---|---|---|---|---|---|
| SZ Cas | s11507 | 9888.4410 | 9.853 | 13.63775 | 1756.750[1] | 0.266 | 108 | $-50.40 \pm 1.84$ | 312 |
| BY Cas | s10301 | 9709.1667 | 10.366 | 3.22330 | 1933.033[1] | 0.977 | 100 | $-61.18 \pm 2.86$ | 112 |
| RU Sct | s11505 | 9888.3681 | 9.466 | 19.70062 | 3117.486[2] | 0.689 | 108 | $+9.13 \pm 1.60$ | 390 |

1—Data from Berdnikov and Pastukhova (1994).
2—Data from Berdnikov (1986).

29 spectral orders in the spectral range 5500–7850 Å, each with a length of ≈50 Å. For SZ Cas and RU Sct, the resolution was $R \approx 15\,000$ for 28 orders with a length of about 110 Å in the wavelength range 4900–6970 Å at the same signal-to-noise ratio. A thorium–argon lamp was used for the wavelength calibration. The extraction of one-dimensional spectra and all of the necessary procedures (cosmic-ray hit removal, linearization, slice image summation) were performed using a modified (Yushkin and Klochkova 2005) ECHELLE context of the MIDAS package. The subsequent work with the spectra, including the continuum level normalization, the measurements of equivalent widths with $W_\lambda \leq 160$ mÅ and radial velocities, was performed using the DECH20 software package (Galazutdinov 1992). A log of observations is given in Table 1. The times of maxima for the light curves of the program Cepheids and their radial-velocity estimates with the number of spectral lines used are also given there.

## MODEL PARAMETERS

To calculate the chemical composition of a star, it is first necessary to determine its effective temperature $T_{\rm eff}$, surface gravity $\log g$, and microturbulent velocity $V_t$.

**Table 2.** Atmospheric parameters for the program Cepheids

| Object | Spectrum number | $T_{\rm eff}$, K | $\log g$ | $V_t$, km s$^{-1}$ | Phase |
|---|---|---|---|---|---|
| SZ Cas | s11507 | $6211 \pm 17$ | 1.60 | 5.20 | 0.266 |
| BY Cas | s10301 | $6287 \pm 30$ | 2.10 | 4.00 | 0.977 |
| RU Sct | s11505 | $5509 \pm 26$ | 1.30 | 8.50 | 0.689 |

The effective temperatures $T_{\rm eff}$ were determined by a method based on the depth ratios of selected pairs of spectral lines most sensitive to the temperature. Several spectroscopic criteria (Kovtyukh 2007) were applied in this case. This method provides an internal accuracy of ∼10–30 K for $T_{\rm eff}$ (the error of the mean).

The microturbulent velocity $V_t$ was determined from the condition for the Fe II abundance derived from a set of lines being independent of their equivalent widths (Kovtyukh and Andrievsky 1999).

The surface gravity $\log g$ was determined from the ionization equilibrium condition for Fe I and Fe II atoms.

The atmospheric parameters that we determined from the spectra of the program Cepheids are listed in Table 2. When estimating the atmospheric parameters and chemical composition, we used the solar oscillator strengths (Kovtyukh and Andrievsky 1999) and model atmospheres from Kurucz (1992).

## CHEMICAL COMPOSITION

We determined the chemical composition of the program stars in the LTE approximation using the WIDTH9 code and a grid of models from Kurucz (1992). Tables 3–5 present the derived elemental abundances relative to the Sun [El/H] with their errors $\sigma$ and give the number of lines NL used for each element. Complete information about the influence of uncertainties in determining the atmospheric parameters on the elemental abundance estimates is described in Berdnikov et al. (2010).

## SECULAR CHANGES IN THE MEAN EFFECTIVE TEMPERATURES OF THE PROGRAM CEPHEIDS

The assumption made by Neilson et al. (2012) about the possible enhanced mass loss in BY Cas prompted us to check the presence of this phenomenon in all three Cepheids based on the secular changes of their mean effective temperatures.

**Table 3.** Elemental abundances for SZ Cas

| Species | Our paper | | | LL |
|---|---|---|---|---|
| | [El/H] | $\sigma$ | NL | [El/H] |
| C I | −0.40 | 0.08 | 6 | −0.28 |
| O I | +0.05 | 0.38 | 3 | +0.08 |
| Na I | +0.20 | 0.08 | 2 | +0.34 |
| Mg I | +0.12 | 0.32 | 3 | +0.20 |
| Al I | +0.06 | 0.26 | 2 | +0.08 |
| Si I | +0.06 | 0.16 | 14 | +0.14 |
| S I | −0.10 | 0.17 | 4 | – |
| Ca I | −0.01 | 0.09 | 9 | +0.07 |
| Sc II | −0.17 | 0.30 | 2 | – |
| Ti I | +0.14 | 0.21 | 19 | +0.22 |
| Ti II | −0.12 | 0.09 | 3 | – |
| V I | +0.21 | 0.12 | 11 | – |
| V II | −0.22 | 0.23 | 4 | – |
| Cr I | −0.00 | 0.24 | 15 | +0.14 |
| Cr II | −0.05 | 0.15 | 9 | – |
| Mn I | −0.29 | 0.15 | 8 | −0.06 |
| Fe I | −0.05 | 0.10 | 60 | +0.07 |
| Fe II | −0.05 | 0.12 | 16 | +0.07 |
| Co I | +0.10 | 0.19 | 8 | +0.04 |
| Ni I | −0.01 | 0.20 | 35 | −0.03 |
| Cu I | +0.17 | 0.51 | 4 | – |
| Y II | +0.14 | 0.33 | 7 | +0.15 |
| Zr II | +0.10 | 0.28 | 5 | – |
| La II | +0.12 | 0.36 | 3 | +0.46 |
| Ce II | −0.02 | 0.24 | 8 | +0.30 |
| Pr II | −0.23 | 0.20 | 3 | −0.14 |
| Nd II | +0.08 | 0.23 | 6 | +0.31 |
| Eu II | +0.13 | 0.05 | 2 | +0.28 |
| Gd II | +0.33 | – | 1 | – |

LL—Data from Luck and Lambert (2011).

**Table 4.** Elemental abundances for BY Cas

| Species | Our paper | | | KAUK | LL |
|---|---|---|---|---|---|
| | [El/H] | $\sigma$ | NL | [El/H] | [El/H] |
| C I | −0.58 | 0.16 | 7 | −0.20 | −0.19 |
| N I | +0.45 | 0.02 | 2 | +0.33 | +0.39 |
| O I | +0.05 | – | 1 | – | +0.31 |
| Na I | +0.06 | 0.12 | 2 | +0.08 | +0.26 |
| Al I | +0.08 | 0.15 | 4 | +0.59 | +0.11 |
| Si I | −0.03 | 0.22 | 13 | +0.05 | +0.21 |
| Si II | −0.24 | 0.07 | 2 | −0.14 | – |
| S I | −0.35 | 0.10 | 6 | – | – |
| K I | +0.40 | – | 1 | – | – |
| Ca I | −0.17 | 0.22 | 6 | −0.12 | +0.09 |
| Sc II | −0.19 | – | 1 | +0.03 | – |
| Ti I | +0.24 | 0.10 | 8 | – | +0.24 |
| Ti II | −0.22 | 0.04 | 2 | −0.19 | – |
| V I | +0.25 | 0.23 | 3 | – | – |
| V II | −0.22 | – | 1 | – | – |
| Cr I | +0.41 | 0.07 | 2 | – | +0.13 |
| Cr II | +0.01 | – | 1 | – | – |
| Mn I | +0.08 | 0.20 | 4 | – | −0.08 |
| Fe I | +0.05 | 0.14 | 72 | −0.02 | +0.12 |
| Fe II | +0.05 | 0.12 | 13 | −0.04 | +0.12 |
| Co I | +0.08 | 0.20 | 4 | – | +0.05 |
| Ni I | −0.05 | 0.20 | 27 | +0.07 | +0.03 |
| Cu I | −0.05 | – | 1 | – | – |
| Zn I | +0.24 | – | 1 | – | – |
| Y II | +0.27 | 0.05 | 2 | – | +0.25 |
| Zr II | −0.14 | – | 1 | – | – |
| Ce II | +0.16 | −3 | 1 | – | +0.29 |
| Nd II | −0.18 | – | 1 | – | +0.23 |
| Eu II | +0.38 | 0.23 | 2 | – | +0.21 |

KAUK—Data from Kovtyukh et al. (1996).
LL—Data from Luck and Lambert (2011).

**Table 5.** Elemental abundances for RU Sct

| Species | Our paper | | | LL |
|---|---|---|---|---|
| | [El/H] | $\sigma$ | NL | [El/H] |
| C I | −0.25 | 0.18 | 7 | −0.08 |
| O I | −0.03 | 0.12 | 3 | +0.31 |
| Na I | +0.41 | 0.20 | 3 | +0.33 |
| Mg I | +0.25 | 0.04 | 2 | +0.07 |
| Al I | +0.29 | 0.32 | 2 | +0.17 |
| Si I | +0.05 | 0.13 | 22 | +0.15 |
| Si II | −0.14 | – | 1 | – |
| S I | −0.01 | 0.06 | 4 | – |
| K I | +0.20 | – | 1 | – |
| Ca I | −0.04 | 0.23 | 6 | +0.00 |
| Sc I | +0.14 | 0.20 | 5 | – |
| Sc II | −0.09 | 0.15 | 5 | – |
| Ti I | +0.05 | 0.21 | 38 | +0.07 |
| Ti II | −0.13 | 0.06 | 2 | – |
| V I | +0.10 | 0.07 | 17 | – |
| V II | −0.00 | 0.26 | 4 | – |
| Cr I | +0.04 | 0.19 | 23 | +0.08 |
| Cr II | −0.01 | 0.12 | 5 | – |
| Mn I | −0.13 | 0.13 | 10 | −0.14 |
| Fe I | +0.05 | 0.15 | 152 | +0.11 |
| Fe II | +0.05 | 0.10 | 17 | +0.11 |
| Co I | +0.05 | 0.18 | 22 | −0.04 |
| Ni I | −0.05 | 0.12 | 56 | −0.05 |
| Cu I | −0.08 | 0.00 | 2 | – |
| Y I | +0.19 | 0.30 | 2 | – |
| Y II | +0.07 | 0.23 | 5 | +0.14 |
| Zr II | +0.05 | 0.10 | 4 | – |
| Ru I | +0.10 | – | 1 | – |
| La II | +0.04 | 0.17 | 2 | +0.17 |
| Ce II | −0.05 | 0.25 | 8 | +0.08 |
| Pr II | +0.04 | 0.37 | 2 | −0.16 |
| Nd II | −0.03 | 0.23 | 14 | +0.09 |
| Eu II | −0.05 | 0.16 | 2 | +0.10 |
| Gd II | −0.09 | – | 1 | – |

LL—Data from Luck and Lambert (2011).

Previously, we used the method of investigating the secular changes in the mean effective temperatures of classical Cepheids based on their estimates obtained from both photometry and spectroscopy for such well-known Cepheids as $\alpha$ UMi (Usenko et al. 2005) and SU Cas (Usenko et al. 2013b). We used the estimates obtained from both spectroscopy and photometry in various photometric systems—the works of various authors over the last 55 years (see Tables 6–8). To determine the effective temperature $T_{\rm eff}$ from $(B-V)$, we used the calibration relation for yellow supergiants from Gray (1992):

$$\log T_{\rm eff} = 3.988 - 0.881(B-V) + 2.142(B-V)^2,$$
$$-3.614(B-V)^3 + 3.2637(B-V)^4,$$
$$-1.4727(B-V)^5 + 0.2600(B-V)^6.$$

We took the color excess $E_{B-V} = 0\overset{m}{.}949$ for SZ Cas from Kovtyukh et al. (2008), $E_{B-V} = 0\overset{m}{.}76$ for BY Cas from Malik (1965), and $E_{B-V} = 0\overset{m}{.}998$ for RU Sct from Bersier (2002). Since some of the series of observations were obtained using the Walraven $WULBV$, Strömgren $uvby\beta$, Geneva $UB_1BB_2V_1VG$, and Washington $CMT_1T_2V$ photometric systems, we converted them to the Johnson $UBV$ system in accordance with the transformations from Moro and Munari (2000) (see Tables 6–8). Usually, we try to integrate all observations in series with a duration of several months. In some cases, these time intervals occupy a period up to a year because of the small number of estimates spanning the entire Cepheid pulsation period.

All of these data for each Cepheid are shown in Figs. 1–3. Some variations of the mean $T_{\rm eff}$ for each object are noticeable at a glance. For example, there are differences in the estimates by different authors whose observations were performed in 1959. They are particularly pronounced for the Cepheids SZ Cas and BY Cas, probably because different photometric standards were used. Subsequent observations show significant fluctuations of the mean effective temperature in the 1960s–1990s. The results obtained by Harris (1980) in 1977–1979 for SZ Cas and RU Sct in the Washington system are particularly interesting. The mean $T_{\rm eff}$ derived from the red $T_1 - T_2$ color indices turned out to be considerably lower than those derived from Johnson photometry over the same period. A very close estimate of $T_{\rm eff}$ was obtained for SZ Cas from spectrophotometry by Coker et al. (1989). The same low estimates, but for RU Sct, were obtained by Arellano Ferro et al. (1998) and Bersier (2002) using Strömgren and Geneva photometry, respectively. On the whole, the averaged effective temperatures for all three objects are: 6084 ± 39 K for SZ Cas, 6154 ± 55 K for BY Cas, and 5600 ± 35 K for RU Sct.

**Table 6.** Mean color indices and effective temperatures of SZ Cas over 55 years

| Year | HJD 2400000+ | $(B-V)_0$ | Number of estimates | $T_{\rm eff}$, K | Method | Reference |
|---|---|---|---|---|---|---|
| 1959 | $36827.952 \pm 21.96$ | 0.542 | 16 | $6077 \pm 40$ | 1 | Oosterhoff (1960) |
| 1959 | $36833.832 \pm 23.95$ | 0.468 | 19 | $6319 \pm 35$ | 1 | Weaver et al. (1960) |
| 1959 | $36853.292 \pm 27.54$ | 0.569 | 16 | $5991 \pm 45$ | 1 | Bahner et al. (1962) |
| 1959 | $36894.310 \pm 88.34$ | 0.594 | 56 | $5979 \pm 90$ | 1 | Mitchell et al. (1964) |
| 1959 | $36962.636 \pm 19.99$ | 0.637 | 10 | $5782 \pm 45$ | 1 | Bahner et al. (1962) |
| 1977 | $43454.976 \pm 34.42$ | 0.477 | 5 | $6289 \pm 80$ | 1 | Szabados (1981) |
| 1977 | $43411.860 \pm 52.93$ | 0.834 | 10 | $5235 \pm 50$ | 1a | Harris (1980) |
| 1978 | $43839.412 \pm 95.84$ | 0.509 | 6 | $6183 \pm 80$ | 1 | Szabados (1981) |
| 1978 | $43762.395 \pm 44.46$ | 0.849 | 5 | $5289 \pm 50$ | 1a | Harris (1980) |
| 1979 | $44140.951 \pm 74.42$ | 0.528 | 7 | $6122 \pm 80$ | 1 | Szabados (1981) |
| 1979 | $44182.853 \pm 39.87$ | 0.469 | 4 | $6315 \pm 25$ | 1 | Moffett and Barnes (1984) |
| 1980 | $44559.215 \pm 55.90$ | 0.518 | 4 | $6154 \pm 25$ | 1 | Moffett and Barnes (1984) |
| 1981 | $44962.733 \pm 75.86$ | 0.522 | 19 | $6141 \pm 25$ | 1 | Moffett and Barnes (1984) |
| 1983 | $45675.781 \pm 31.36$ | 0.581 | 11 | $5954 \pm 15$ | 1 | Berdnikov (2008) |
| 1986 | $46731.303 \pm 108.33$ | – | 9 | $5368 \pm 47$ | 2, 3 | Coker et al. (1989) |
| 1988 | $47417.981 \pm 16.53$ | 0.536 | 34 | $6096 \pm 15$ | 1 | Berdnikov (2008) |
| 1991 | $48513.384 \pm 9.95$ | 0.544 | 19 | $6070 \pm 15$ | 1 | Berdnikov (1992) |
| 1992 | $48872.924 \pm 20.45$ | 0.567 | 26 | $5998 \pm 15$ | 1 | Berdnikov (1993) |
| 1994 | $49626.400 \pm 9.06$ | 0.589 | 20 | $5929 \pm 15$ | 1 | Berdnikov and Vozyakova (1995) |
| 1995 | 49888.441 | – | 1 | $6211 \pm 17$ | 4 | This paper |
| 1997 | $50327.286 \pm 31.94$ | 0.554 | 31 | $6039 \pm 15$ | 1 | Berdnikov (2008) |
| 2009 | $55156.722 \pm 20.85$ | – | 3 | $6101 \pm 72$ | 4 | Luck and Lambert (2011) |

1—$T_{\rm eff}$ from the Johnson $(B-V)$ color index.
1a—$(B-V)$ converted from the Washington $(T_1-T_2)$ color index.
2—$T_{\rm eff}$ from spectrophotometry.
3—From the grid of models by Gustafsson et al. (1975).
4—From the grid of models by Kurucz (1992).

**Table 7.** Mean color indices and effective temperatures of BY Cas over 55 years

| Year | HJD 2400000+ | $(B-V)_0$ | Number of estimates | $T_{\rm eff}$, K | Method | Reference |
|---|---|---|---|---|---|---|
| 1959 | 36792.964 ± 28.44 | 0.500 | 18 | 6213 ± 40 | 1 | Oosterhoff (1960) |
| 1959 | 36826.368 ± 32.42 | 0.521 | 19 | 6144 ± 35 | 1 | Weaver et al. (1960) |
| 1959 | 36859.276 ± 21.50 | 0.596 | 12 | 5907 ± 45 | 1 | Bahner et al. (1962) |
| 1959 | 36887.284 ± 94.32 | 0.532 | 62 | 6109 ± 90 | 1 | Mitchell et al. (1964) |
| 1959 | 36961.159 ± 20.44 | 0.553 | 15 | 6042 ± 45 | 1 | Bahner et al. (1962) |
| 1964 | 38408.676 ± 19.96 | 0.513 | 45 | 6170 ± 45 | 1 | Malik (1965) |
| 1967 | 39786.911 ± 23.39 | 0.514 | 11 | 6167 ± 80 | 1 | Szabados (1977) |
| 1972 | 41634.399 ± 47.87 | 0.460 | 14 | 6346 ± 80 | 1 | Szabados (1977) |
| 1973 | 41947.478 ± 14.03 | 0.460 | 5 | 6346 ± 80 | 1 | Szabados (1977) |
| 1976 | 43092.910 ± 47.35 | 0.470 | 5 | 6312 ± 80 | 1 | Szabados (1977) |
| 1992 | 48874.405 ± 19.94 | 0.543 | 26 | 6074 ± 15 | 1 | Berdnikov (1993) |
| 1994 | 49709.167 | – | 1 | 6287 ± 30 | 2 | This paper |
| 1996 | 50328.324 ± 20.95 | 0.560 | 29 | 6020 ± 15 | 1 | Berdnikov (2008) |
| 2008 | 54786.740 | – | 1 | 6110 ± 44 | 2 | Luck and Lambert (2011) |

1—$T_{\rm eff}$ from the Johnson $(B-V)$ color index.
2—From the grid of models by Kurucz (1992).

## SPECTRAL PECULIARITIES OF THE PROGRAM CEPHEIDS

As has been mentioned in the Introduction, extended envelopes around Cepheids and probable overtone and nonradial pulsations can be detected spectroscopically by changes in hydrogen and metal absorption line profiles. Figures 4–6 show portions of the spectrum containing the Na I D1 and D2 (5889.95 and 5895.92 Å, respectively) doublet absorption lines. It can be seen from Figs. 4 and 5 that the interstellar sodium doublet lines in SZ Cas and BY Cas are much stronger than the stellar ones and are redshifted, on average, by −36.43 and −41.74 km s$^{-1}$, respectively. The reverse is true for RU Sct: the same stronger interstellar lines are blueshifted, while the difference with the stellar components is +24.64 km s$^{-1}$. It can be seen from Table 1 that the mean radial velocity estimated from the metal absorption lines for SZ Cas is −50.40 km s$^{-1}$. The same estimate but obtained from the H$\alpha$ line differs insignificantly: −51.17 km s$^{-1}$. The same insignificant difference in estimates is also observed for BY Cas: −61.18 and −59.01 km s$^{-1}$, respectively. However, this difference is enormous for RU Sct: +9.13 km s$^{-1}$ from the metal lines and +31.59 km s$^{-1}$ from the H$\alpha$ line. The H$\alpha$ core is redshifted due to the very intense emission in the blue line wing (see Fig. 7), while the difference between the emission and absorption line cores is 83.64 km s$^{-1}$. Such differences are observed, for example, in the bright hypergiant $\rho$ Cas surrounded by an extended hydrogen envelope (Klochkova et al. 2012). The

**Table 8.** Mean color indices and effective temperatures of RU Sct over 55 years

| Year | HJD 2400000+ | $(B-V)_0$ | Number of estimates | $T_{\rm eff}$, K | Method | Reference |
|---|---|---|---|---|---|---|
| 1956 | $35472.630 \pm 163.23$ | 0.623 | 10 | $5825 \pm 90$ | 1 | Mitchell et al. (1964) |
| 1959 | $36791.251 \pm 55.39$ | 0.764 | 21 | $5418 \pm 35$ | 1 | Weaver et al. (1960) |
| 1959 | $36791.744 \pm 54.90$ | 0.764 | 21 | $5389 \pm 90$ | 1 | Mitchell et al. (1964) |
| 1970 | $40804.434 \pm 57.82$ | 0.626 | 33 | $5816 \pm 35$ | 1a | Pel (1976) |
| 1971 | $41131.967 \pm 31.43$ | 0.687 | 13 | $5634 \pm 35$ | 1a | Pel (1976) |
| 1979 | $43932.150 \pm 237.12$ | 0.904 | 9 | $5065 \pm 50$ | 1d | Harris (1980) |
| 1979 | $44076.374 \pm 30.41$ | 0.771 | 7 | $5400 \pm 25$ | 1 | Moffett and Barnes (1984) |
| 1980 | $44439.302 \pm 55.37$ | 0.686 | 18 | $5638 \pm 25$ | 1 | Moffett and Barnes (1984) |
| 1982 | $45192.795 \pm 284.20$ | 0.574 | 6 | $5976 \pm 25$ | 1 | Moffett and Barnes (1984) |
| 1982 | $44836.912 \pm 33.41$ | 0.633 | 10 | $5795 \pm 15$ | 1 | Berdnikov (2008) |
| 1983 | $45501.846 \pm 13.50$ | 0.801 | 13 | $5320 \pm 15$ | 1 | Berdnikov (1986) |
| 1984 | $45879.326 \pm 7.99$ | 0.742 | 15 | $5479 \pm 15$ | 1 | Berdnikov (1986) |
| 1989 | $47755.726 \pm 20.44$ | 0.783 | 41 | $5367 \pm 15$ | 1 | Berdnikov (2008) |
| 1992 | $48949.410 \pm 149.55$ | 1.012 | 20 | $4828 \pm 25$ | 1c | Arellano Ferro et al. (1998) |
| 1994 | $49511.413 \pm 3.51$ | 1.142 | 7 | $4573 \pm 25$ | 1c | Arellano Ferro et al. (1998) |
| 1994 | $49571.671 \pm 42.90$ | 0.970 | 16 | $4917 \pm 60$ | 1b | Bersier (2002) |
| 1995 | 49888.368 | – | 1 | $5509 \pm 26$ | 2 | This paper |
| 1996 | $50148.112 \pm 181.47$ | 1.045 | 7 | $4761 \pm 60$ | 1b | Bersier (2002) |
| 2008 | 55318.967 | – | 1 | $5441 \pm 72$ | 2 | Luck and Lambert (2011) |

1—$T_{\rm eff}$ from the Johnson $(B-V)$ color index.
1a—$(B-V)$ converted from the Walraven $(V-B)$ color index.
1b—$(B-V)$ converted from the Geneva $(B_2-V_1)$ color index.
1c—$(B-V)$ converted from the Strömgren $(b-y)$ color index.
1d—$(B-V)$ converted from the Washington $(T_1-T_2)$ color index.
2—From the grid of models by Kurucz (1992).

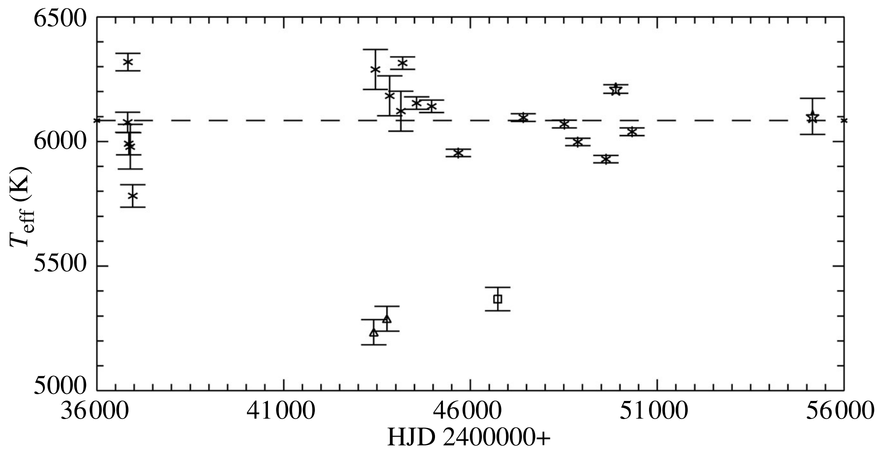


**Fig. 1.** Secular changes in the mean effective temperature of the Cepheid SZ Cas over the last 55 years: the data from Johnson photoelectric photometry (six-point stars); the data from Washington photometry (empty triangle); the data from spectrophotometry and model atmospheres (empty square); the data from spectroscopy using model atmospheres (empty five-point stars). The dashed line indicates the mean $T_{\text{eff}} = 6084$ K.

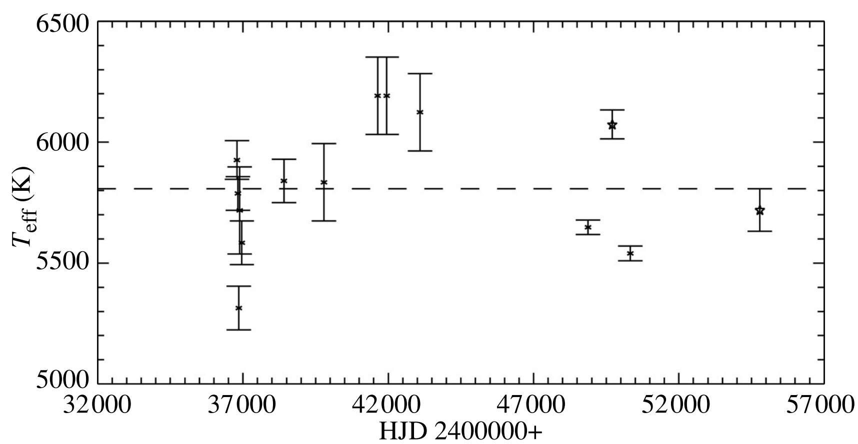


**Fig. 2.** Secular changes in the mean effective temperature of the Cepheid BY Cas over the last 55 years: the data from Johnson photoelectric photometry (six-point stars); the data from spectroscopy using model atmospheres (empty five-point stars). The dashed line indicates the mean $T_{\text{eff}} = 6154$ K.

H$\alpha$ profile for SZ Cas is symmetric, without any emission. For BY Cas only the blue half of the profile fell into the spectral order, but a characteristic knifelike shape of the core is noticeable (Fig. 8).

The behavior of absorption lines of neutral atoms and ions in the spectra of the program Cepheids is very interesting. Figures 9 and 10 show the profiles of absorption lines of neutral iron atoms, 6055.990 Å ($\chi_{\text{low}} = 4.7$ eV), and a pair of lines of singly ionized iron, 6369.460 Å ($\chi_{\text{low}} = 2.9$ eV), and silicon, 6371.355 Å ($\chi_{\text{low}} = 8.1$ eV), atoms for all three program Cepheids. The same absorption lines from the spectrum of the Cepheid BG Cru pulsating in the *first* overtone and probably having a gaseous envelope (Usenko et al. 2014a) are presented in the upper part of the figures for comparison. Just as in BG Cru, there is a significant asymmetry of the absorption lines and the secondary components are most pronounced in the spectrum of BY Cas. Such an asymmetry is noticeable only for the Si II 6371.355 Å line in SZ Cas and is virtually absent for RU Sct. The main reason may probably have to do with more than a factor of 1.5 lower resolution of the spectrograph.

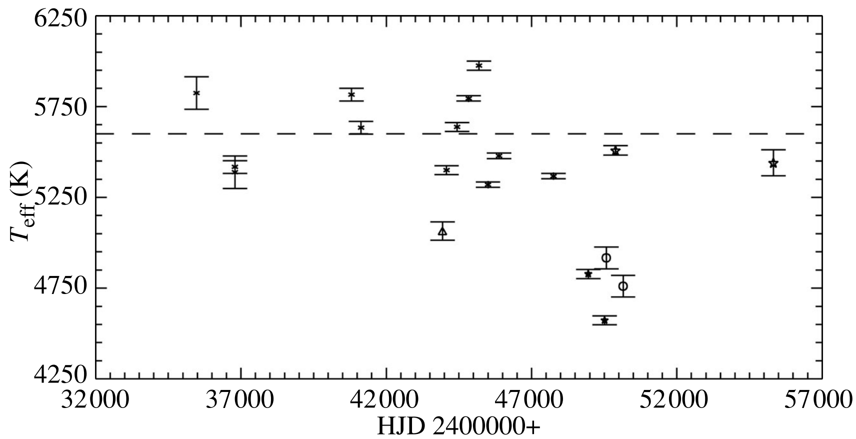


**Fig. 3.** Secular changes in the mean effective temperature of the Cepheid RU Sct over the last 55 years: the data from Johnson photoelectric photometry (six-point stars); the data from Strömgren photoelectric photometry (filled five-point stars); the data from Geneva photometry (empty circles); the data from spectroscopy using model atmospheres (empty five-point stars). The dashed line indicates the mean $T_{eff} = 5600$ K.

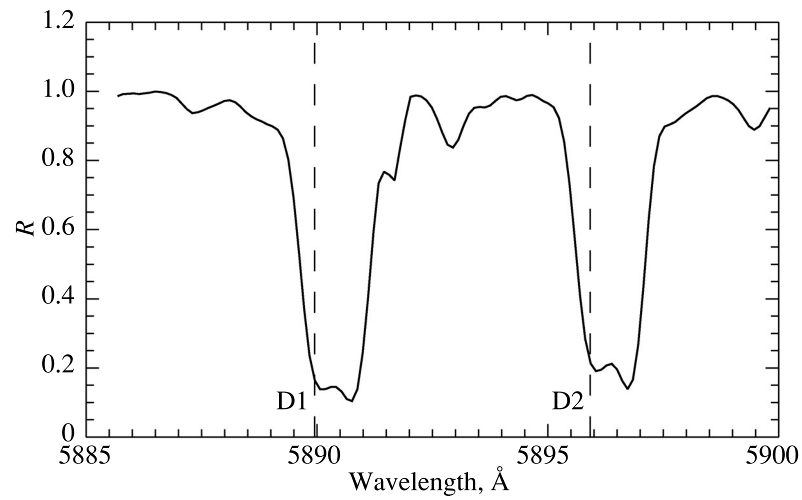


**Fig. 4.** Na I D1 and D2 5889.95 and 5895.92 Å doublet absorption lines in SZ Cas.

Figures 11–15 show the Mg Ib 5183.618 Å ($\chi_{low} = 2.7$ eV) and Ba II 4934.095 Å ($\chi_{low} = 0.0$ eV), 5853.67 ($\chi_{low} = 0.6$ eV), 6141.713 Å ($\chi_{low} = 0.7$ eV), and 6496.90 ($\chi_{low} = 0.6$ eV) line profiles for SZ Cas and RU Sct (in the case of BY Cas, these lines did not fall into the spectral order). The strong lines of neutral magnesium and singly ionized barium with low lower-level excitation potentials can form components in the gaseous envelope and can serve as indicators of its presence around the Cepheid. For example, the split Ba II line components in the spectra of $\rho$ Cas reflected the presence of such an envelope (Klochkova et al. 2014). It can be seen from Fig. 11 that both Mg Ib line profiles have a distinct knifelike shape with a clear presence of the secondary component on the red side. The Ba II 5853.67, 6141.713, and 6496.90 Å lines have a similar shape, with this being most pronounced in the last two lines as the stronger ones (see Figs. 13–15). However, an asymmetry in the strongest Ba II 4934.095 Å line (Fig. 12) is observed on the blue side. Unfortunately, based on one spectrum for each cepheid, we cannot assert that additional red and blue absorption components may be present in these lines—several higher-resolution spectra taken over all pulsation periods are needed for these purposes.

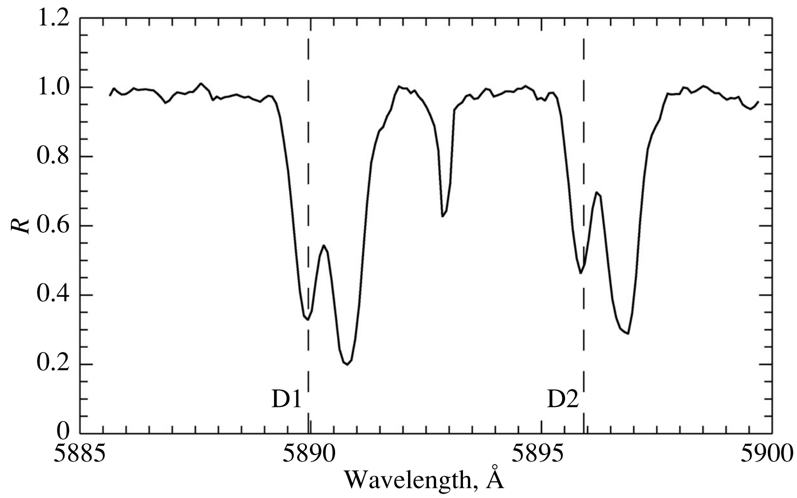


**Fig. 5.** Na I D1 and D2 5889.95 and 5895.92 Å doublet absorption lines in BY Cas.

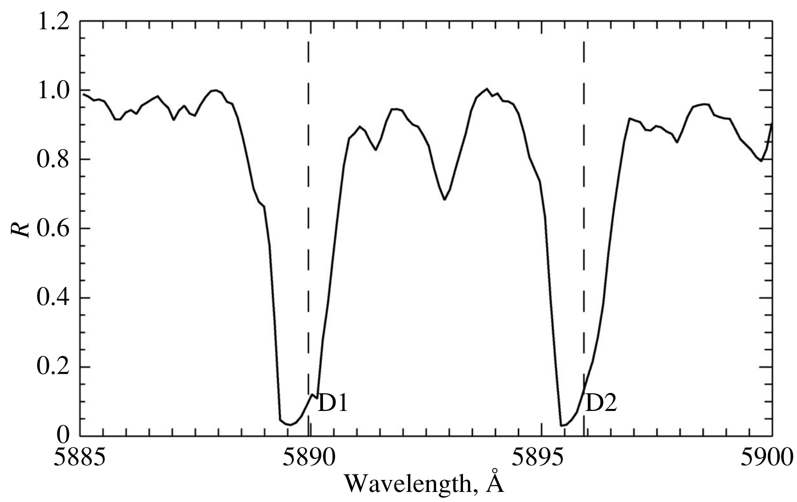


**Fig. 6.** Na I D1 and D2 5889.95 and 5895.92 Å doublet absorption lines in RU Sct.

## DISCUSSION

**SZ Cas**. For this Cepheid, there are a maximum number of spectra (four) from which its atmospheric parameters and chemical composition were determined. The problem is that $T_{\rm eff}$, $\log g$, and $V_t$ from Luck and Lambert (2011) do not correspond to the pulsation phases presented in their paper at all: the minimum values of these parameters occur at the maximum of the light curve and vice versa. For some reason, the authors did not provide the ephemeris of the Cepheid's light curve in their paper. If the ephemeris from Table 1 is used, then approximately the same result is obtained: $0^{\rm P}_{.}543$, $0^{\rm P}_{.}116$, and $0^{\rm P}_{.}604$, respectively. If we take into account our atmospheric parameters at phase $0^{\rm P}_{.}266$, then the mean effective temperature determined from all four spectroscopic observations will be $6129 \pm 58$ K, and it is also close to $T_{\rm eff} = 6084 \pm 39$ K averaged over all parameters (see Fig. 1). On the whole, however, the fluctuations in the effective temperature for most periods of Cepheid observations do not exceed 200 K, except for the periods 1977–1978 and 1986, when its sharp reductions were observed. Since the red color indices of the Washington system and $V - R$ were used in these cases to determine $T_{\rm eff}$, it may be assumed that these reductions can be associated with mass loss and dust formation in the Cepheid's gaseous envelope.

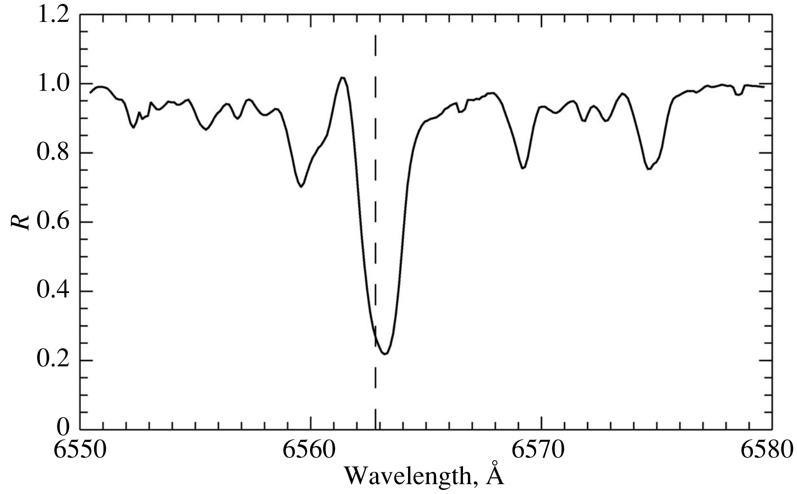


**Fig. 7.** H$\alpha$ absorption line profile for RU Sct.

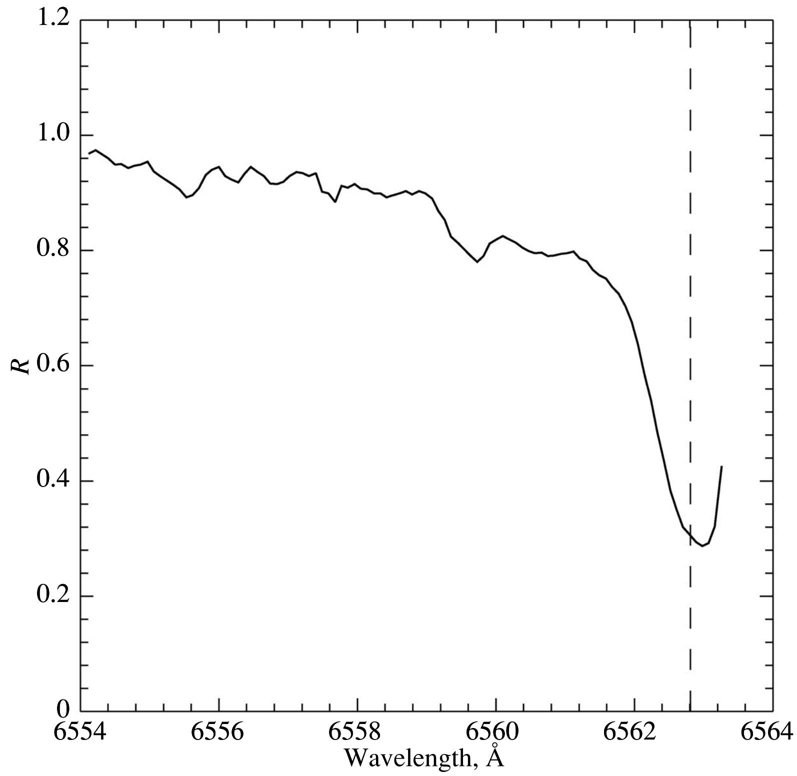


**Fig. 8.** Fragment of the H$\alpha$ absorption line profile for BY Cas.

The H$\alpha$ absorption line profile shows neither emission in the line wings nor noticeable asymmetry. As has been mentioned above, judging by Figs. 9 and 10, only a slight asymmetry in the absorption lines of singly ionized metals (see the Si II 6371.355 Å line) is visible, which is probably related to an insufficiently high resolution of the spectrograph. For comparison, analogous absorption line profiles were added in Figs. 9 and 10 for BG Cru, a *first-overtone* pulsator with an envelope (Usenko et al. 2014a). Since they

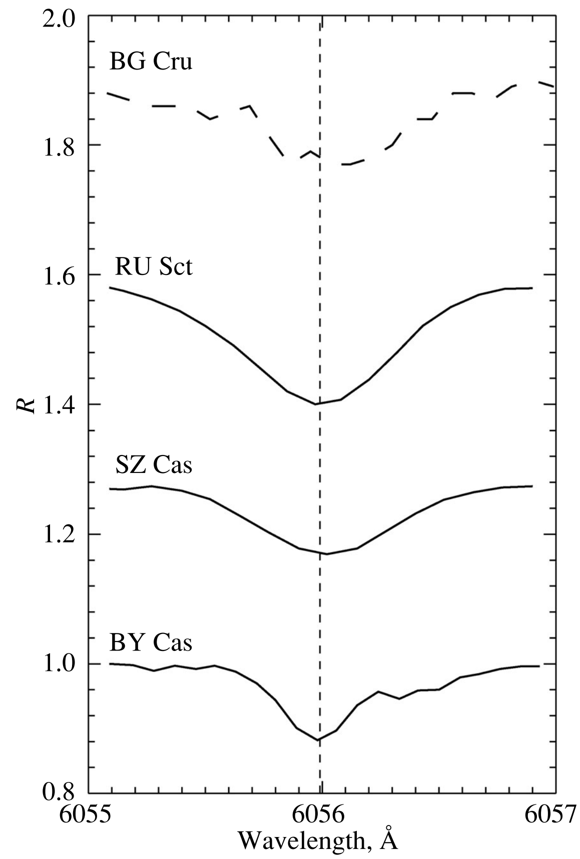


**Fig. 9.** Fe I 6055.99 Å absorption line profiles for the program Cepheids. An analogous line profile for BG Cru at phase $0^{\rm p}.264$ is given in the upper part for comparison (Usenko et al. 2014a).

differ sharply in shape, it is rather difficult to judge the presence of *overtone* pulsations in the Cepheid. The very strong redshifted interstellar sodium D1 and D2 absorption lines (Fig. 4) may belong to the envelope around the star. The same can also be said about the secondary red component in the Mg I 5183.62 Å (Fig. 11) and Ba II 5853.67, 6141.713, and 6496.90 Å (Figs. 13–15) lines. Nevertheless, this secondary component is blue in the Ba II 4934.095 Å line (Fig. 12). Another indirect confirmation of the existence of an envelope around SZ Cas is the presence of a polarimetric "rosette," which has already been mentioned in the Introduction. If no significant infrared excess was recorded in the Cepheid before 1974, then it can be suggested that the gaseous envelope formed in the period 1977–1986.

Judging by the chemical composition of the Cepheid, there is approximate agreement between our data and those from Luck and Lambert (2011) on a nearly solar metallicity of SZ Cas. Since the Cepheid belongs to the open cluster $\chi$ and h Per, this estimate can be compared with those for other Cepheids in this cluster: [Fe/H] = +0.04 dex for UY Per, [Fe/H] = +0.02 dex for VX Per, and [Fe/H] = +0.04 dex for VY Per (Luck and Lambert 2011). The agreement is the same, within the error limits.

The abundances of the key elements of the evolution of yellow supergiants (an appreciable carbon underabundance, a nearly solar oxygen abundance, a sodium and magnesium overabundance, and a solar aluminum abundance) suggest that the Cepheid has already passed the first dredge-up. At the same time, our C abundance estimate shows a slightly larger underabundance. Interestingly, the C abundance estimate for the Cepheid VX Per close in pulsation period ($P = 10.^{\rm d}89$), according to Luck and Lambert (2011), is the same, but a difference for nitrogen is noticeable, while the agreement for oxygen is excellent, including that with our estimate. The same agreement is also observed for the sodium, magnesium, and aluminum abundances (VX Per: [Na/H] = +0.27 dex; [Mg/H] = +0.06 dex; [Al/H] = +0.09 dex).

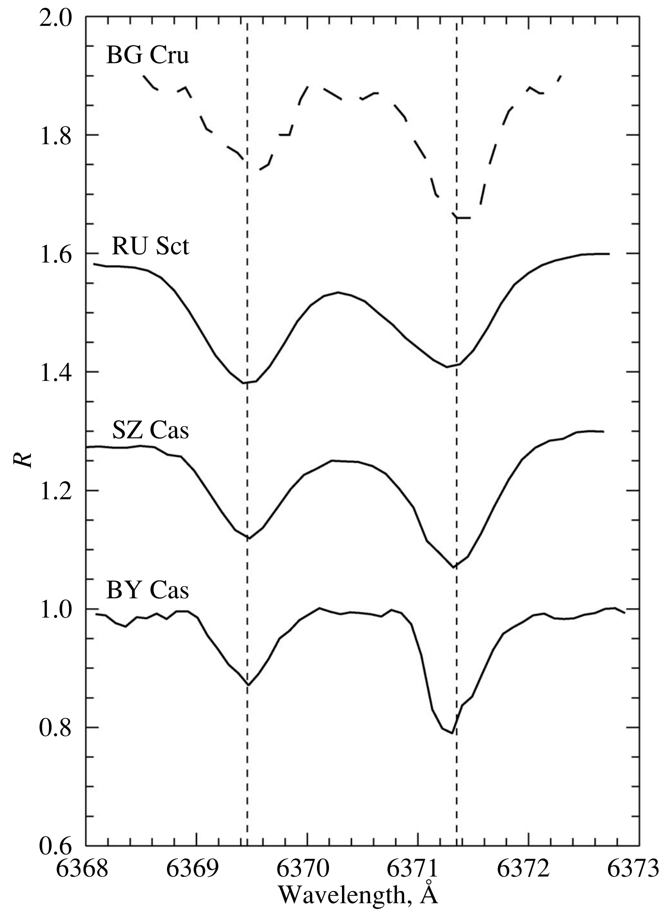


**Fig. 10.** Fe II 6369.46 Å and Si II 6371.355 Å absorption line profiles for the program Cepheids. An analogous line profile for BG Cru at phase $0^{P}.264$ is given in the upper part for comparison (Usenko et al. 2014a).

The abundances of the $\alpha$-elements are approximately identical; the same can also be said about the iron-peak elements (except for a larger manganese underabundance in our paper). For the r-process, heavy and light s-process elements, a significant difference is observed only in the lanthanum, cerium, and neodymium abundances. On the whole, the abundances of all these elements that we determined are nearly solar.

**BY Cas**. There are two spectra for this Cepheid: our spectrum taken near maximum light and the spectrum from Luck and Lambert (2011) taken approximately in the middle of the light curve (if the phase is calculated using the ephemeris from Table 1). Their $T_{\rm eff} = 6110 \pm 72$ K from the latter paper (the spectrum was taken approximately in the middle of the ascending branch of the light curve) is close to the value of $6154 \pm 72$ K averaged over all parameters over 55 years. It can be seen from Fig. 2 that the effective temperature fluctuations in BY Cas over this period are, as in SZ Cas, $\sim$200 K.

The H$\alpha$ core has an asymmetric knifelike shape (Fig. 8). The absorption lines of metals in the spectrum of BY Cas are highly unusual and also have strong asymmetric profiles with the possible presence of additional components, such as, for example, in BG Cru (Usenko et al. 2014a). Just as for SZ Cas, Figs. 9 and 10 present the Fe I 6055.99 Å, Fe II 6369.46 Å, and Si II 6371.355 Å absorption line profiles for BY Cas. Their similarity to those for BG Cru is obvious, which is particularly clearly seen in Fig. 8 for the neutral iron line. Since BG Cru, as has been said above, pulsates in the *first* overtone and has an extended gaseous envelope, it is highly likely that BY Cas is also a *first-overtone* pulsator and such an envelope can also be present around it, as a confirmation of the assumption made by Neilson et al. (2012) about enhanced mass loss by the Cepheid. The intense Na I D1 and D2 lines should also be remembered; they may also correspond to the envelope (Fig. 5).

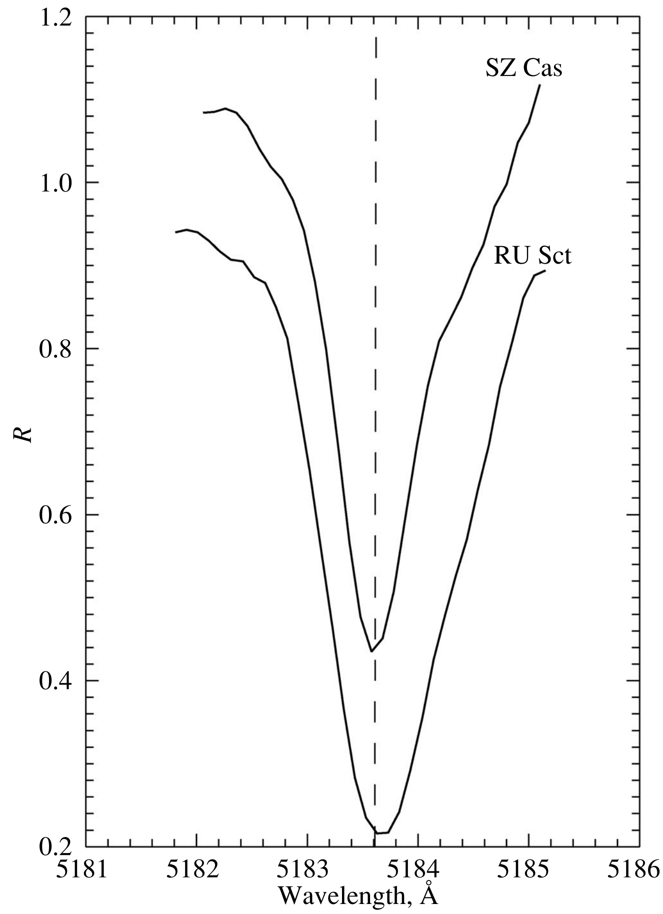


**Fig. 11.** Mg Ib 5183.62 Å absorption line profiles for the Cepheids SZ Cas and RU Sct.

A significant difference in the carbon abundances in our paper and Kovtyukh et al. (1996), −0.58 dex and −0.20 dex, can be noticed when comparing the abundance estimates for BY Cas, although the latter is close to that from Luck and Lambert (2011) (see Table 4). The reason for such a difference lies not only in the fact that Kovtyukh et al. (1996) used four carbon lines ($\lambda$ 7111.480, 7113.180, 7132.110, and 7837.110 Å) for their analysis, while we used three more lines ($\lambda$ 7115.190, 7116.990, 7119.670, and 7837.110 Å), although the abundance difference from the common four lines is −0.35 dex. The main reason is a difference in atmospheric parameters (our estimates of $T_{\rm eff}$ and $\log g$ are lower), and a much larger carbon underabundance is obtained when computing the model with such parameters. Nitrogen shows a larger overabundance and is close to the estimate from Luck and Lambert (2011), while our oxygen abundance estimate is nearly solar, in contrast to the large overabundance obtained by these authors. The sodium and aluminum abundances determined from two and four lines, respectively, are nearly solar, in contrast to the slight overabundance in Luck and Lambert (2011). Judging by the CNO abundance estimates for BY Cas, it can be concluded that this small-amplitude Cepheid has also passed the first dredge-up.

According to our data, the metallicity of BY Cas is nearly solar. The abundances of the $\alpha$-elements are either nearly solar or exhibit a moderate underabundance (sulfur) or an overabundance (potassium). Our estimates for the Fe-peak elements are approximately identical to those from Luck and Lambert (2011), except for chromium. The very significant abundance differences for neutral atoms and ions for titanium, vanadium, and chromium from our spectrum are related to the small number of lines used in our analysis because of the short spectral orders. On the whole, the iron-peak elements show nearly solar abundances, except for titanium. As regards the r-process elements and heavy and light s-process elements, here there is good agreement with the data from Luck and Lambert (2011): either a slight over-

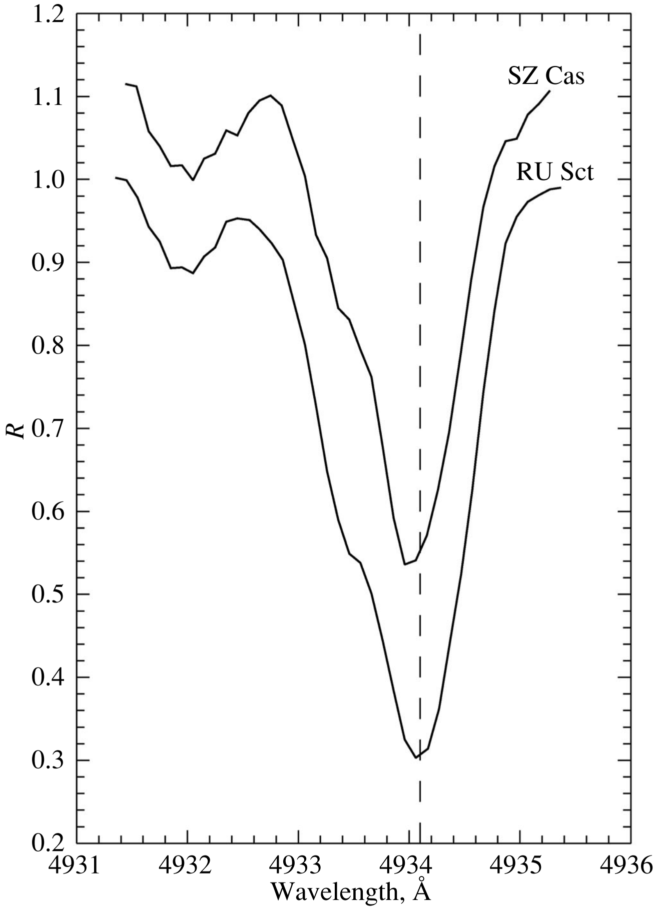


**Fig. 12.** Ba II 4934.0957 Å absorption line profiles for the Cepheids SZ Cas and RU Sct.

abundance or a nearly solar abundance is observed for most of them.

**RU Sct**. Just as for the preceding Cepheid, there are also two spectra: our spectrum in the middle of the ascending branch of the light curve and that from Luck and Lambert (2011) near minimum light ($0^{\rm P}.407$, according to the authors). Taking into account the value of $T_{\rm eff} = 5600 \pm 35$ K averaged over all parameters (Fig. 3), it can be seen that our estimate of the effective temperature from the spectrum is close to it. As with the preceding Cepheids, fluctuations in averaged $T_{\rm eff}$, but within $\sim$500 K, are observed. However, as with SZ Cas, sharp reductions in the mean effective temperature are noticeable in 1979, 1992–1994, and 1996. Similarly, the red color indices were used to determine $T_{\rm eff}$: the assumption that these values can be associated with mass loss and dust formation in the Cepheid's gaseous envelope follows from this. The existence of such an envelope is confirmed by the presence of an intense emission in the blue H$\alpha$ wing (Fig. 7) and significant mass loss estimated by Butler et al. (1987) from the infrared excess. The blueshifted interstellar sodium D1 and D2 lines can presumably also belong to the envelope (Fig. 6). A secondary red component, the same as that in SZ Cas, is observed in the Mg I 5183.62 Å (Fig. 11) and Ba II 5853.67, 6141.713, and 6496.90 Å lines (Figs. 13–15), while a blue component is observed in the Ba II 4934.095 Å line (Fig. 12), which may also be indicative of its existence. As regards the absorption lines of neutral atoms and ions, they show no noticeable asymmetry (only the Si II 6371.355 Å line in Fig. 10 constitutes an exception). As has been mentioned above, this can be due to an insufficiently high resolution of the spectrograph.

If our abundance data (Table 5) are compared with those from Luck and Lambert (2011), then it should be noted that a nearly solar metallicity is observed in the Cepheid (although the [Fe/H] estimate from the above paper is slightly larger). However, our carbon abundance estimate shows a noticeable underabundance, while Luck and Lambert (2011) give a nearly solar carbon abundance. The discrepancy

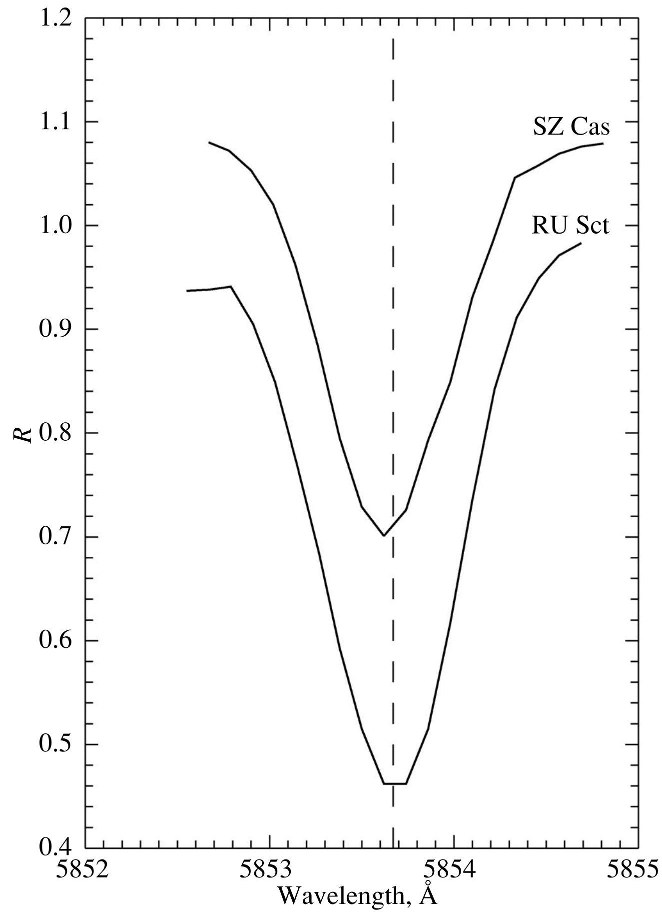


**Fig. 13.** Ba II 5853.67 Å absorption line profiles for the Cepheids SZ Cas and RU Sct.

in data on oxygen is the same: [O/H] = −0.03 dex and +0.31 dex. There is good agreement on the sodium abundance, an overabundance. The same overabundance (it is appreciably larger in our paper) is observed for magnesium and aluminum. If our abundances of the key elements of the evolution of yellow supergiants (by adding the estimate of [N/H] = +0.35 dex from Luck and Lambert (2011)) are taken as a basis, then it can be concluded that RU Sct has also passed the first dredge-up.

The abundances of the $\alpha$-elements and Fe-peak elements are approximately identical and nearly solar, except for titanium and manganese in Luck and Lambert (2011). Our estimates for the r-process elements and heavy and light s-process elements are nearly solar. The differences with the data from Luck and Lambert (2011) are insignificant.

## CONCLUSIONS

Based on our reduction of the spectra for three Cepheids with the largest *positive* secular period increments, SZ Cas, BY Cas, and RU Sct, taken with the 6-m SAO RAS telescope, we obtained the following results:

(1) The atmospheric parameters and chemical composition of the program Cepheids were determined (for SZ Cas and RU Sct) and redetermined (for BY Cas). These estimates have been obtained for the first time for some of the elements.

(2) Our abundance analysis for these objects revealed a carbon underabundance (especially in BY Cas), a nearly solar oxygen abundance, a sodium overabundance, and nearly solar (or slightly enhanced) magnesium and aluminum abundances in all of them. All of this suggests that these Cepheids have already passed the first dredge-up.

(3) The values of [Fe/H] for all of the program Cepheids are nearly solar. Given the membership of SZ Cas and RU Sct in the open clusters $\chi$ and h Per and Trumpler 35, respectively, these metallicity estimates can also be applied to the open clusters themselves.

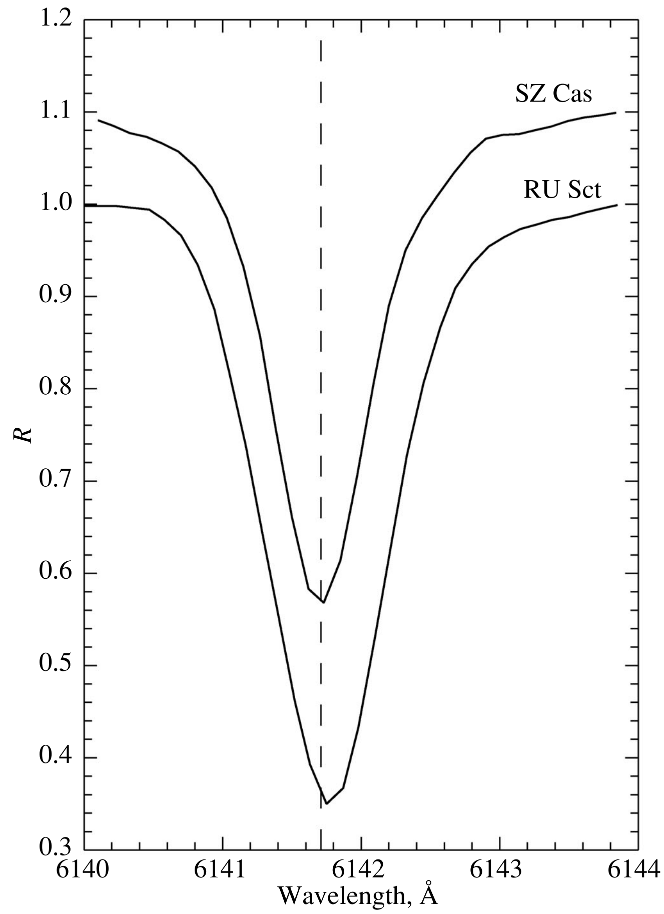


**Fig. 14.** Ba II 6141.713 Å absorption line profiles for the Cepheids SZ Cas and RU Sct.

(4) The abundances of the $\alpha$-elements, iron-peak elements, and r- and s-process elements for all Cepheids turned out to be nearly solar.

(5) Our elemental abundance estimates mostly agree with the data obtained by Luck and Lambert (2011) for the $\alpha$-elements, Fe-peak elements, r-process elements, and heavy and light s-process elements. For the key elements of the evolution of yellow supergiants, there is a significance difference in C and O abundances in BY Cas and RU Sct.

(6) The H$\alpha$ line profile for the small-amplitude Cepheid SZ Cas is symmetric, BY Cas exhibits a knifelike asymmetry in the core, while RU Sct exhibits a strong emission in the blue line wing. As a rule, short- and long-period Cepheids (for example, X Sgr and l Car) and bright yellow hypergiants ($\rho$ Cas) with extended envelopes exhibit such peculiarities.

(7) In SZ Cas and RU Sct, the strong magnesium and barium lines with low lower-level excitation potentials exhibit a significant asymmetry of the absorption lines of neutral atoms and ions: the presence of secondary red and blue components is noticeable in them. This asymmetry is barely noticeable or absent in weak lines. However, this asymmetry is significant in BY Cas. It is quite likely that there exist *first*-overtone nonradial pulsations in BY Cas, just as in the small-amplitude Cepheid BG Cru. According to one hypothesis, the presence of additional components in the absorption lines of metals can also be explained by the existence of gaseous envelopes around the Cepheids.

(8) In all three Cepheids, the interstellar sodium D1 and D2 lines are much stronger than the atmospheric ones, with them being redshifted in SZ Cas and BY Cas and blueshifted in RU Sct. It is quite likely that these lines can also correspond to the envelopes of the program Cepheids.

(9) Studies of the secular changes in the mean effective temperatures of the program Cepheids over 55 years have shown that they all exhibit temperature fluctuations from 200 to 500 K. For SZ Cas and RU Sct, the effective temperatures determined from the red photometric color indices of the Washington

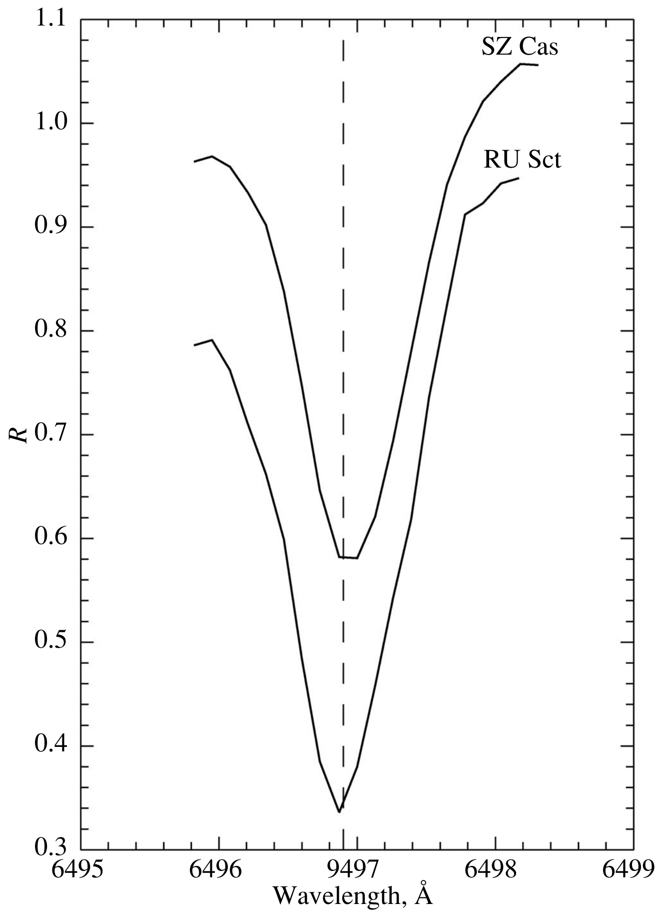


**Fig. 15.** Ba II 6496.90 Å absorption line profiles for the Cepheids SZ Cas and RU Sct.

system, $V - R$, and the Geneva and Strömgren systems showed much lower values in some years. It was hypothesized that these underestimated values of $T_{\text{eff}}$ and the fluctuations in the mean effective temperatures could be associated with mass loss and dust formation in the Cepheid envelopes.

(10) The existence of envelopes around the program Cepheids can be directly related to their evolutionary status, the observed significant increase in the positive pulsation period increment; therefore, careful multiphase spectroscopic observations with a fairly high resolution are urgently needed for them in future.

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## REFERENCES

1. E. Antonello, E. Poretti, and L. Reduzzi, Astron. Astrophys. **236**, 138 (1990).
2. A. Arellano Ferro, Mon. Not. R. Astron. Soc. **209**, 481 (1984).
3. A. Arellano Ferro and P. Rosenzweig, Mon. Not. R. Astron. Soc. **315**, 296 (2000).
4. A. Arellano Ferro, E. Rojo Arellano, S. Gonzalez-Bedolla, and P. Rosenzweig, Astrophys. J. Suppl. Ser. **117**, 167 (1998).
5. K. Bahner, W. A. Hiltner, and R. P. Kraft, Astrophys. J. Suppl. Ser. **6**, 319 (1962).
6. L. N. Berdnikov, Perem. Zvezdy **22**, 369 (1986).
7. L. N. Berdnikov, Astron. Lett. **18**, 42 (1992).
8. L. N. Berdnikov, Astron. Lett. **19**, 84 (1993).
9. L. N. Berdnikov, *Photoelectric Observations of Cepheids in UBV(RI)c,* on-line data catalog (2008).
10. L. N. Berdnikov, V. V. Ignatova, E. N. Pastukhova, and D. J. Turner, Astron. Lett. **23**, 177 (1997).
11. L. N. Berdnikov, A. Yu. Knyazev, I. A. Usenko, V. V. Kovtyukh, and V. V. Kravtsov, Astron. Lett. **36**, 490 (2010).
12. L. N. Berdnikov and E. N. Pastukhova, Astron. Lett. **20**, 479 (1994).

13. L. N. Berdnikov and O. V. Vozyakova, Astron. Lett. **21**, 308 (1995).
14. D. Bersier, Astrophys. J. Suppl. Ser. **140**, 465 (2002).
15. C. J. Butler, H. P. Deasy, and P. A. Waymann, IAU Symp. **122**, 227 (1987).
16. B. C. Cogan, Astrophys. J. **221**, 635 (1978).
17. R. Coker, B. Madore, J. Mould, J. B. Oke, A. Picard, J. Huchra, and D. Latham, Publ. Astron. Soc. Pacif. **101**, 5947 (1989).
18. A. N. Cox, Astrophys. J. **229**, 212 (1979).
19. O. J. Eggen, Astron. J. **90**, 1278 (1985).
20. G. E. Erleksova and B. N. Irkaev, Perem. Zvezdy **21**, 715 (1982).
21. A. B. Fokin, N. Nardetto, and P. Mathias, EAS Publ. Ser. **38**, 151 (2009).
22. P. Fouqué, P. Arriagada, J. Storm, T. G. Barnes, N. Nardetto, A. Mérand, P. Kervella, W. Gieren, D. Bersier, G. F. Benedict, and B. E. McArthur, Astron. Astrophys. **476**, 73 (2007).
23. K. Fricke, R. S. Stobie, and P. A. Strittmatter, Astrophys. J. **171**, 593 (1972).
24. G. A. Galazutdinov, Publ. SAO RAN **92**, 1 (1992).
25. A. Gallenne, P. Kervella, and A. Merand, Astron. Astrophys. **538**, 24 (2012).
26. R. D. Gehrz and J. A. Hackwell, Astrophys. J. **193**, 385 (1974).
27. W. P. Gieren and P. Fouque, Astron. J. **106**, 734 (1993).
28. D. Gillet, Private commun. (1993).
29. N. A. Gorynya, N. N. Samus, and A. S. Rastorguev, Inform. Bull. Var. Stars **4130**, 1 (1994).
30. N. A. Gorynya, N. N. Samus, L. N. Berdnikov, and A. S. Rastorguev, Inform. Bull. Var. Stars **4199**, 1 (1995).
31. N. A. Gorynya, N. N. Samus, A. S. Rastorguev, and M. E. Sachkov, Astron. Lett. **22**, 175 (1996).
32. D. Gray, *Observation and Analysis of Stellar Atmospheres,* 2nd ed. (Cambrige Univ. Press, 1992).
33. H. C. Harris, Ph.D. Thesis (Univ. of Washington, 1980).
34. J. Iben, Jr. and R. S. Tuggle, Astrophys. J. **173**, 135 (1972).
35. P. Kerevella, A. Merand, and A. Gallenne, Astron. Astrophys. **498**, 425 (2009).
36. P. N. Kholopov, N. N. Samus, M. S. Frolov, et al., *General Catalogue of Variable Stars* (Nauka, Moscow, 1986), vol. 3 [in Russian].
37. V. G. Klochkova, V. E. Panchuk, N. S. Tavolzhanskaya, and I. A. Usenko, Astron. Rep. **58**, 101 (2014).
38. V. V. Kovtyukh, Mon. Not. R. Astron. Soc. **378**, 617 (2007).
39. V. V. Kovtyukh and S. M. Andrievsky, Astron. Astrophys. **351**, 597 (1999).
40. V. V. Kovtyukh, S. M. Andrievsky, I. A. Usenko, and V. G. Klochkova, Astron. Astrophys. **316**, 155 (1996).
41. V. V. Kovtyukh, C. Soubiran, R. E. Luck, D. G. Turner, S. I. Belik, S. M. Andrievsky, and F. A. Chekhonadskikh, Mon. Not. R. Astron. Soc. **389**, 1336 (2008).
42. R. L. Kurucz, in *Model Atmospheres for Populational Synthesis, The Stellar Populations of Galaxies, Proceedings of the IAU Symposium No. 149,* Ed. by B. Barbuy and A. Renzini (Kluwer Acad. Publ., Dordrecht, 1992), p. 225.
43. C. D. Laney and R. S. Stobie, Astrophys. J. **274**, 337 (1995).
44. T. Liu, K. A. Janes, and T. M. Bania, Astron. J. **102**, 1103 (1991).
45. R. E. Luck and D. L. Lambert, Astron. J. **142**, 136 (2011).
46. R. E. Luck, S. M. Andrievsky, V. V. Kovtyukh, W. Gieren, and D. Graczyk, Astron. J. **142**, 51 (2011).
47. B. F. Madore and J. D. Fernie, Publ. Astron. Soc. Pacif. **92**, 315 (1980).
48. D. J. Majaess, D. G. Turner, and D. J. Lane, J. Am. Assoc. Var. Star. Observ. **37**, 179 (2009).
49. G. M. Malik, Astron. J. **70**, 94 (1965).
50. P. Mathias, D. Gillet, A. B. Fokin, N. Nardetto, P. Kervella, and D. Mourard, Astron. Astrophys. **457**, 575 (2006).
51. R. I. Mitchell, B. Iriarte, D. Steinmetz, and H. L. Johnson, Bol. Obs. Tonantzintla Tacubaya **3**, 153 (1964).
52. T. J. Moffett and T. G. Barnes III, Astrophys. J. Suppl. Ser. **55**, 389 (1984).
53. D. Moro and U. Munari, Astron. Astrophys. **147**, 361 (2000).
54. N. Nardetto, J. H. Groth, S. Kraus, F. Millour, and D. Gillet, Astron. Astrophys. **489**, 1263 (2008).
55. H. R. Neilson, S. G. Engle, E. Guinan, N. Lancer, R. P. Wasatonic, and D. B. Williams, Astrophys. J. **745**, 32 (2012).
56. R. T. Oosterhoff, Bull. Astr. Neth. **15**, 199 (1960).
57. V. E. Panchuk, V. G. Klochkova, G. A. Galazutdinov, V. P. Ryadchenko, and E. L. Chentsov, Astron. Lett. **19**, 431 (1993).
58. V. E. Panchuk, I. D. Najdenov, V. G. Klochkova, A. B. Ivanchik, S. V. Yermakov, and V. A. Murzin, Bull. Spec. Astrophys. Observ. **44**, 127 (1997).
59. J. W. Pel, Astron. Astrophys. **24**, 413 (1976).
60. J. W. Pel, Astron. Astrophys. **62**, 75 (1978).
61. J. H. Peña, A. Arellano Ferro, R. Peña-Miller, A. Alvarez, Y. Rosas, H. Garsia, G. Muños, B. Vargas, J. P. Sareyan, C. A. Guerrero, and A. Renteria, Rev. Mex. Astron. Astrof. **46**, 291 (2010).
62. R. L. Phelps and K. A. Janes, Astrophys. J. Suppl. Ser. **90**, 31 (1994).
63. T. A. Polyakova, Astrofizika **26**, 469 (1987).
64. M. E. Sachkov, Astron. Lett. **28**, 589 (2002).
65. M. E. Sachkov, A. S. Rastorguev, N. N. Samus, and N. A. Gorynya, Astron. Lett. **24**, 337 (1998).
66. E. G. Schmidt, Astrophys. J. **203**, 466 (1976).
67. C. Sollazzo, G. Russo, A. Onnembro, and B. Caccin, Astron. Astrophys. **99**, 66 (1981).
68. L. Szabados, Budapest. Mitt. **70**, 1 (1977).
69. L. Szabados, Budapest. Mitt. **94**, 1 (1989).
70. L. Szabados, Budapest. Mitt. **96**, 125 (1991).
71. L. Szabados, Astron. Astrophys. **311**, 189 (1996).
72. G. S. Tsarevskii, V. Urekhe, and Yu. N. Efremov, Astron. Tsirk. **367**, 1 (1966).

73. D. G. Turner, Astrophys. J. **240**, 137 (1980).
74. D. G. Turner, J. R. Astron. Soc. Canada **90**, 82 (1995).
75. D. G. Turner, J. Am. Assoc. Var. Star. Observ. **26**, 101 (1998).
76. D. G. Turner, D. J. Majaess, J. R. Percy, D. A. English, and R. Huziak, Odessa Astr. Publ. **23**, 125 (2010).
77. D. G. Turner, V. V. Kovtyukh, R. E. Luck, and L. N. Berdnikov, Astrophys. J. **772**, 10 (2013).
78. I. A. Usenko, Kinem. Fiz. Nebesn. Tel **6**, 91 (1990).
79. I. A. Usenko, A. Yu. Knyazev, L. N. Berdnikov, V. V. Kravtsov, and A. B. Fokin, Astron. Lett. **39**, 432 (2013a).
80. I. A. Usenko, V. G. Klochkova, and N. S. Tavolzhanskaya, Astron. Lett. **39**, 634 (2013b).
81. I. A. Usenko, A. Yu. Knyazev, L. N. Berdnikov, A. B. Fokin, and V. V. Kravtsov, Astron. Lett. **40**, 435 (2014a).
82. I. A. Usenko, A. Yu. Knyazev, L. N. Berdnikov, and V. V. Kravtsov, Astron. Lett. **40**, 800 (2014b).
83. I. A. Usenko, V. V. Kovtyukh, V. G. Klochkova, V. E. Panchuk, and S. V. Yermakov, Astron. Astrophys. **367**, 831 (2001).
84. I. A. Usenko, A. S. Miroshnichenko, V. G. Klochkova, and M. V. Yushkin, Mon. Not. R. Astron. Soc. **362**, 1219 (2005).
85. G. Wallerstain and T. Jacobsen, Bull. Am. Astron. Soc. **13**, 804 (1981).
86. H. Weaver, D. Steinmetz, and R. Mitchell, Lowell Obs. Bull. **5**, 30 (1960).
87. D. L. Welch, C. Alcock, D. P. Bennett, K. H. Cook, H. A. Allsman, T. S. Axelrod, K. C. Freeman, B. A. Peterson, et al., ASP Conf. Ser. **83**, 232 (1995).
88. R. Wielen, Astron. Astrophys. **25**, 285 (1973).
89. L. A. Wilson, J. H. Bowen, and C. Struk-Marcell, Comm. Astrophys. **12**, 17 (1988).
90. M. V. Yushkin and V. G. Klochkova, Preprint SAO RAN No. 206 (Spets. Astrofiz. Observatoriya, Nizh. Arkhyz, 2005).

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