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New Characteristics of the Solar Cycle and Dynamo Theory

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Abstract—Based on an analysis of the observational data for solar cycles 12–23 (Royal Greenwich Observatory—USAF/NOAA Sunspot Data), we have studied various parameters of the "Maunder butterflies." Based on the observational data for cycles 16–23, we have found that BT/L and S depend linearly on each other, where B is the mean magnetic field of the cycle, T is the cycle duration, S is the cycle strength, and L is the mean sunspot latitude in the cycle (the arithmetic mean of the absolute values of the mean latitudes in the north and south). The connection of the observed quantities with the α–ω-dynamo theory is discussed.

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INTRODUCTION

At present, there exist a number of regularities connecting various solar activity indices. These primarily include the Gnevyshev–Ohl rule (GOR) (Gnevyshev and Ohl 1948), according to which the area under the curve of Wolf numbers for an even cycle SR_{2N} correlates with that for the succeeding odd one SR_{2N+1}, while SR_{2N-1} does not correlate with SR_{2N}, with the GOR in such a formulation being valid for all cycles without exception (Nagovitsyn et al. 2009). Its more popular but less accurate formulation states that each odd cycle is larger in amplitude than the preceding even one. Another well-known regularity is the Waldmeier rule (effect) (WR) that exists in two formulations: (a) the amplitude of the solar cycle correlates negatively with the duration of the rise phase: a stronger cycle corresponds to a shorter rise phase (according to Karak and Choudhuri (2011), the "first Waldmeier effect"); (b) the cycle amplitude correlates positively with its rise rate: a stronger cycle corresponds to a higher rise rate (the "second effect" or, according to Komitov et al. (2010), the Waldmeier "rule"). Note that the correlation coefficient in the second case is much larger. The GOR and WR allow one to successfully predict the activity of future cycles and to reconstruct the activity of past ones. Cameron and Schüssler (2007), Pipin and Kosovichev (2011), and Pipin et al. (2012) showed these rules to be reproduced within the framework of a solar dynamo model.

The magnetic field of a sunspot is responsible for its emergence and is one of its main characteristics (neither the GOR nor the WR use directly this parameter). Tlatov (2013) and Tlatova et al. (2013) digitized the magnetic fields of sunspots based on observations at the Mount Wilson Observatory (MWO) and revealed temporal regularities in the magnetic fields of sunspots with various sizes for cycles 16–23. Based on these data, here we derive a linear relationship between four quantities that are among the most important parameters characterizing the solar cycle: the cycle strength S, the mean sunspot magnetic field strength B, the cycle duration T, and the mean sunspot latitude L. Although the activity of the current cycle cannot be predicted from its initial phase based on the derived relationship (because all these quantities can be calculated only after the cycle), the correlation found can contribute to our understanding of the solar dynamo theory and the existence of an 11-year solar activity cycle. In addition, the magnetic activity in those years when the magnetic fields of sunspots had not yet been measured directly can be reconstructed using the correlation found.

As is well known, sunspots appear at the beginning of the cycle in the region of mid-latitudes and gradually approach the equator during approximately eleven years. Such a latitude–time distribution of sunspots is called a butterfly diagram or

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"Maunder butterflies." According to the dynamo theory, the appearance of the butterflies is associated with the motion of the toroidal magnetic field component (dynamo wave) from high latitudes toward the equator in each hemisphere. The latitude–time diagram of toroidal magnetic field isolines obtained using even the simplest dynamo models qualitatively reproduces the observational butterfly diagrams. The characteristics of the theoretical butterfly diagrams (the butterfly shape, the amplitude and period of the magnetic field oscillations) for the Sun and other stars depend significantly on the control parameters in dynamo models (Brandenburg and Subramanian 2005). In various works on the dynamo theory (Brandenburg and Subramanian 2005; Dikpati and Gilman 2001; Dikpati et al. 2006; Choudhuri et al. 1995; Popova et al. 2008; Popova 2009; Muñoz-Jaramillo et al. 2011), the influence of various model parameters on the evolution of the magnetic field was analyzed. For example, it was shown numerically (Choudhuri et al. 1995; Dikpati et al. 2006; Dikpati and Gilman 2001) and analytically (Popova et al. 2008; Popova and Sokoloff 2008; Popova 2009) that the meridional circulation directed oppositely to the propagation of the dynamo wave could decelerate significantly its motion. In addition, intense meridional circulation "blow away" the dynamo wave toward the poles. Muñoz-Jaramillo et al. (2011) showed the turbulent diffusivity to be also capable of affecting the duration of the solar activity cycle. In other works (see, e.g., Brandenburg and Subramanian 2005), the latitude profile of the α-effect was shown to also affect the shape of the Maunder butterfly.

An attempt to reproduce the observational latitude–time distributions for the magnetic fields of stars by choosing appropriate values of the control parameters and their dependence on coordinates and time can give information about the physics of the process being studied. In this case, however, it should be remembered that the physics of the process will be restricted by the chosen model. On the other hand, not all of the observed quantities enter into the dynamo model. One can try to correlate some of the solar cycle parameters with model quantities only indirectly, and this correlation can be be rather controversial in some cases. Therefore, an analysis of various characteristics of the observational butterfly diagrams may turn out to be useful for such a correlation and for elucidating the physics of magnetic field generation in celestial bodies.

The goal of this paper is to analyze the evolution of various parameters of the butterfly diagrams obtained from the observational data for solar cycles 12–23 (Royal Greenwich Observatory—USAF/NOAA Sunspot Data). We investigate the possible relationships of these parameters to one another and compare the observational results with the dynamo theory within the framework of such an analysis.

CHARACTERISTICS OF THE OBSERVATIONAL BUTTERFLY DIAGRAMS

In this paper, we used the RGO–NASA/Marshall data on the monthly mean and daily sunspot activity indices and sunspot coordinates over the period 1878–2008, which completely spans solar activity cycles 12–23, as well as for the cores over the period 1878–1976 (cycles 12–20, respectively). Based on these data, we constructed the butterfly diagrams for the northern and southern solar hemispheres for each cycle. For each butterfly "wing," both in the north and in the south, we plotted the dependence of the latitude on the monthly mean sunspot area and made a linear fit.

There is a problem here: "sunspots from the preceding cycle," i.e., those with low latitudes, continue to go for some time at the beginning of the current cycle. Similarly, "sunspots from the succeeding cycle," i.e., those with high latitudes, begin to appear at the end of the current cycle. Although the relative number of groups with such sunspots (with respect to the total number of groups in the cycle) is small, their presence would introduce a distortion into the linear fit. Therefore, the sunspots belonging to the "adjacent" cycles, i.e., the sunspots with low latitudes at the beginning of the cycle and the sunspots with high latitudes at the end of the cycle, were removed in each wing.

Table 1 presents the cycle durations. The first column gives the cycle numbers, the second column gives the total cycle durations (according to the NGDC/NOAA data), the third column gives the cycle durations for the northern hemisphere after the removal of the groups belonging to the "adjacent" cycles as well as the number of groups removed at the beginning of the cycle (the first number in parentheses), the end of the cycle (the second number), and the total number in the cycle (the third number), the fourth columns gives the same data as those in the third column but for the southern hemisphere.

As can be seen from Table 1, the number of removed groups is very small in comparison with the total number of groups in the cycle (of the order of twenty thousand), so that their removal barely changed the mean latitude of the cycle for each hemisphere. It is also interesting to note that the northern and southern wings of the butterflies "pass" to the

Table 1. Cycle durations

	Total	Northern wings	Southern wings
12	12.1878–02.1890	06.1879–01.1890 $(4+1=5)$	01.1879–01.1890 $(2+2=4)$
13	03.1890–01.1902	03.1890–08.1901 $(0+2=2)$	03.1890–01.1902 $(0+0=0)$
14	02.1902–07.1913	02.1902–06.1913 $(0+6=6)$	02.1902–07.1913 $(0+0=0)$
15	08.1913–07.1923	10.1913–07.1923 $(3+0=3)$	08.1913–06.1923 $(0+2=2)$
16	08.1923–08.1933	08.1923–08.1933 $(0+0=0)$	08.1923–08.1933 $(0+0=0)$
17	09.1933–01.1944	09.1933–01.1944 $(3+0=3)$	10.1933–12.1943 $(7+17=24)$
18	02.1944–03.1954	07.1944–03.1954 $(19+0=19)$	02.1944–03.1954 $(1+0=1)$
19	04.1954–09.1964	05.1954–09.1964 $(1+0=1)$	06.1954–09.1964 $(6+0=6)$
20	10.1964–05.1976	10.1964–05.1976 $(0+0=0)$	04.1965–05.1976 $(37+0=37)$
21	06.1976–08.1986	06.1976–08.1986 $(13+0=13)$	06.1976–08.1986 $(0+0=0)$
22	09.1986–04.1996	09.1986–04.1996 $(5+0=5)$	10.1986–04.1996 $(12+0=12)$
23	05.1996–12.2008	05.1996–09.2008 $(0+24=24)$	05.1996–12.2008 $(17+0=17)$

next cycle not simultaneously but independently. Using these data, we constructed the butterfly tilt angle for each hemisphere in each cycle.

Based on the Origin software package, we calculated the mean latitude L for each wing as the sum of pairwise products of the latitude by the area divided by the sum of all sunspot areas. Then, we calculated the strength of each cycle S as a mean of the monthly mean sunspot (core) areas.

Analysis of the relationships between these quantities showed the arithmetic mean of the tangents of the butterfly wing tilt angles in the north and south to be proportional to the cycle strength (the correlation coefficient is $R = 0.66$) (Fig. 1a). For the cores in cycles 12–20, such a relationship holds with the correlation coefficient $R = 0.69$ (Fig. 1b). The mean latitude of the wing in the southern hemisphere virtually coincides with its mean latitude in the northern hemisphere for each cycle ($R = 0.89$) (Fig. 2). Note that this latitude changes from cycle to cycle, while no clear periodicity has been detected in its change. On the whole, cycles with a shorter duration have a larger tilt angle. The ratio of the core area to the sunspot area is directly proportional to the cycle strength; for the groups in cycles 12–17, the increase in the ratio of the areas with increasing strength is greater than that for the groups in cycles 18–20.

Tlatov (2012) calculated the mean magnetic fields of cycles based on the MWO data for sunspots with an area of more than 100 msh (millionths of a solar hemisphere) as an arithmetic mean of the sum of the field strengths for such sunspots. Since the bulk of the magnetic flux is contained in such sunspots, neglecting small sunspots does not introduce a significant error. Using these data for cycles 16–23, we found BT/L and S to depend linearly on each other, where B is the mean magnetic field of the cycle, T is the cycle duration, S is the cycle strength, and L is the mean sunspot latitude in the cycle (the arithmetic mean of the absolute values of the mean latitudes in the north and south). This dependence can be described by the following equation: $S = 2539 - 0.89BT/L$, $R = 0.87$, the rems deviation is $\sigma = 132$ (Fig. 3a). The data are summarized in Table 2.

If the mean area of not the sunspots but the cores is used as the cycle strength S, then the dependence is even closer to a linear one. In this case, however, we have only five cycles (16–20) for which both the core area and the magnetic field were calculated (Fig. 3b). In the case of cores, the dependence takes the form $S = 500 - 0.20BT/L$, $R = 0.95$, $\sigma = 16$.

Let us analyze the result obtained. Other things being equal, a larger cycle strength S corresponds either to a smaller magnetic field B, or to a shorter

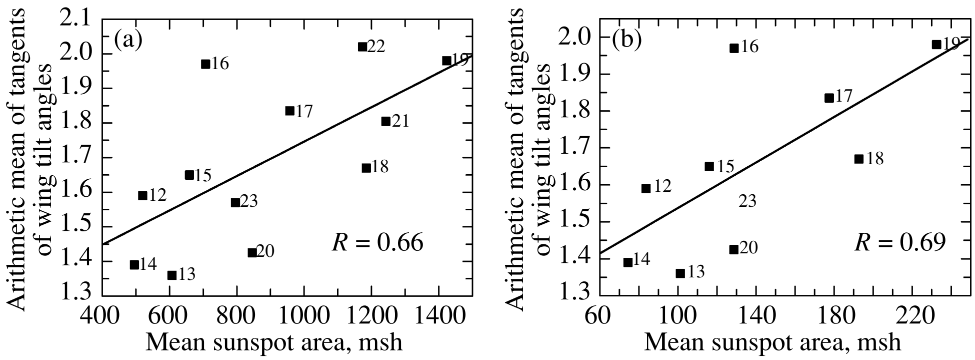

Fig. 1. Tilt angle versus mean sunspot (a) and core (b) area.

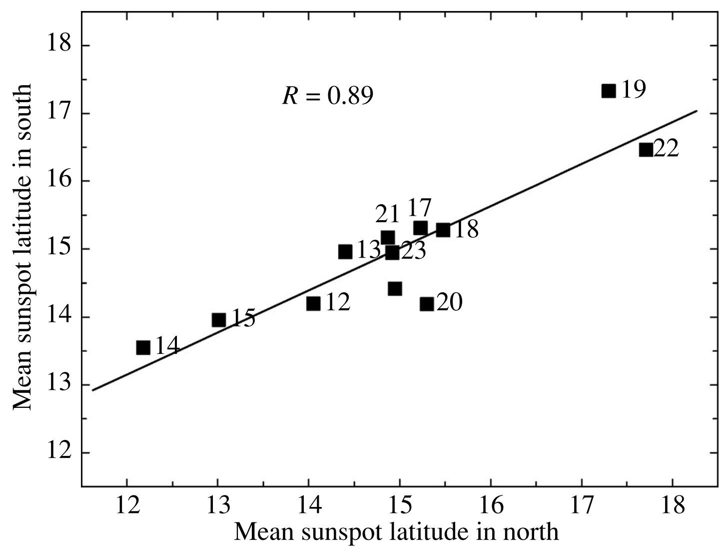

Fig. 2. Relationship between the mean sunspot latitudes in the north and south.

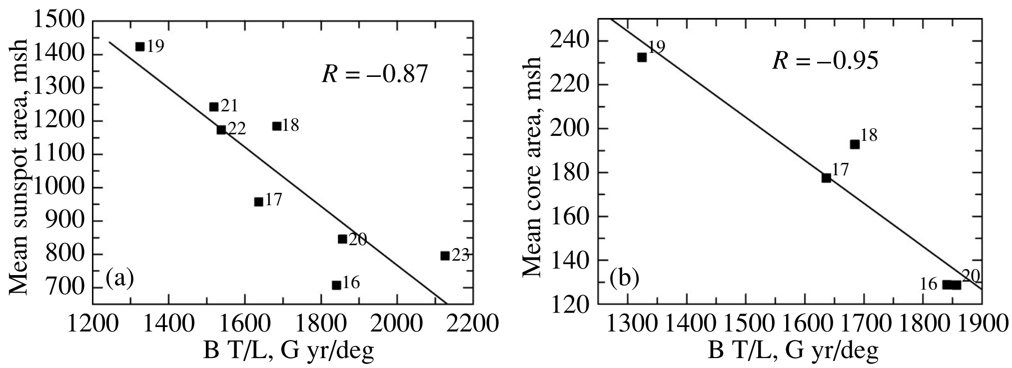

Fig. 3. Relationship between the magnetic field, cycle duration, mean sunspot (a) and core (b) areas, and their mean latitudes.

cycle duration T, or to a higher latitude L. This suggests that approximately the same magnetic energy is spent on each cycle, but the ways of magnetic energy realization in each cycle are different. For example, an enhanced sunspot activity generally leads to the fact that sunspots will be generated at higher latitudes, but the field strength in sunspots will be small, while the duration of the cycle itself will be short.

In fact, all four quantities change in combination, because there is no relationship whatsoever between any pair of individual quantities. The only exception is the pair of quantities (S and L) for which a weak dependence with the correlation coefficient $R = 0.74$

Table 2. Basic cycle parameters

Cycle number	S, msh (sunspots)	S, msh (cores)	B, G	T, yr	L, deg
16	707	129	2695	10.03	14.7
17	957	178	2400	10.41	15.3
18	1185	193	2555	10.14	15.4
19	1424	233	2190	10.47	17.3
20	846	129	2350	11.65	14.7
21	1242	–	2230	10.23	15.0
22	1174	–	2720	9.66	17.1
23	796	–	2520	12.60	14.9

is observed (Fig. 4a). However, this dependence becomes much stronger if not the mean sunspot area in the cycle but the relative fraction of groups containing very large sunspots is taken as the parameter S (Otkidychev 2014; Otkidychev and Skorbezh 2014). Figure 4b shows the relationship between the mean sunspot latitude and the relative fraction of groups (in comparison with the total number of groups in the cycle) containing maximal sunspots with an area of more than 800 msh. This suggests that the meridional circulation responsible for the shift of the butterfly wings in latitude is associated with the total magnetic energy.

The solid and dashed lines in Fig. 5 indicate, respectively, the time dependence of the ratio of the yearly mean core areas to the yearly mean sunspot areas and the time dependence of the yearly mean sunspot areas. It can be seen that the ratio of the core areas to the sunspot areas in years tend to increase at minima and to decrease at maxima. Thus, starting from some value, the sunspots “grow” more rapidly than do the cores in these sunspots, but no regularity is observed. Starting from cycle 17, there is a clear tendency for the ratio of the core areas to the sunspot areas to decrease.

On the whole, the cycle strength is directly proportional to the tangent of the butterfly tilt angle, although no strict dependence is observed. No dependence of the magnetic field amplitude on the cycle duration, mean latitude, tangent of the tilt angle, and cycle strength was found.

COMPARISON WITH THE DYNAMO THEORY

We will compare the observational results obtained with the simplest α–ω-dynamo model with meridional circulation.

According to the dynamo theory, the 11-year solar activity cycle is connected with the magnetic dynamo action whose mechanism is based on the joint operation of the α-effect and differential rotation (Parker 1955). The solar magnetic field is assumed to have two components: poloidal and toroidal. The toroidal magnetic field is obtained from the poloidal one under the action of differential rotation inside the solar convective zone. The reverse process of transformation of the toroidal magnetic field to the poloidal one results from convection mirror symmetry breaking in a rotating body. The action of the Coriolis force on rising and expanding (sinking and contracting) vortices leads to a predominance of right-handed vortices in the northern hemisphere (left-handed ones in the southern hemisphere). The electromotive force resulting from the action of Faraday electromagnetic induction after its averaging over the velocity pulsations acquires a component parallel to the mean magnetic field. It closes the self-excitation circuit in the Parker dynamo.

The Parker dynamo equations are derived from a complete system of equations of mean-field electrodynamics (Krause and Rädler 1980) under the assumption that the dynamo wave propagates in a thin spherical shell. When these equations are derived, the magnetic field is averaged along the radius within

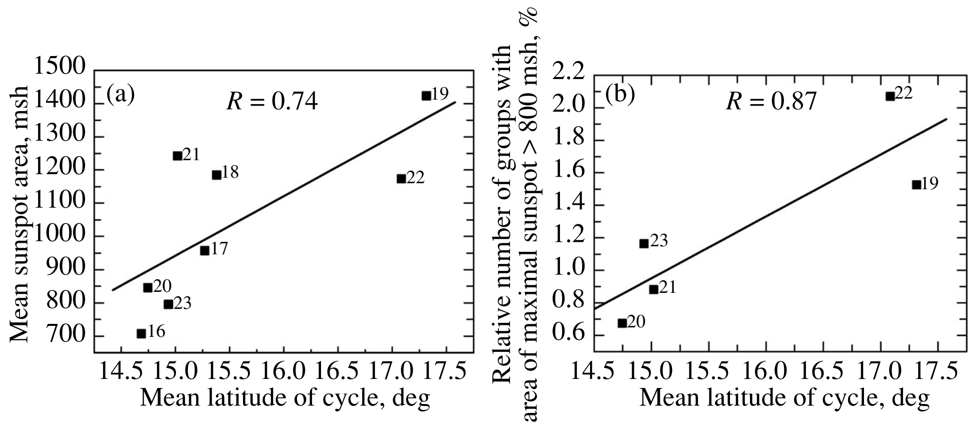

Fig. 4. Mean sunspot area (a) and relative number of sunspot groups with a size of the maximal sunspot more than 800 msh (b) versus mean sunspot latitude.

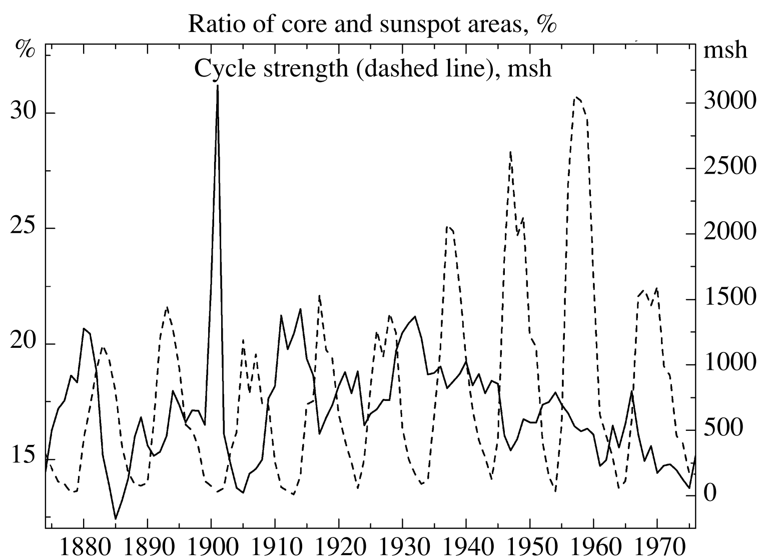

Fig. 5. Ratio of the core and sunspot areas in years. The solid and dashed lines indicate, respectively, the time dependence of the ratio of the yearly mean core areas to the yearly mean sunspot areas and the time dependence of the yearly mean sunspot areas.

some spherical shell where the dynamo action takes place, and the terms describing the curvature effects near the pole are discarded. A formal procedure for deriving the dynamo equations was described by Sokoloff et al. (1995). Meridional flows can also be taken into account in dynamo models. Popova et al. (2008), Popova and Sokoloff (2008), and Popova (2009) derived and investigated the dynamo equations with meridional circulation:

$$\frac{\partial A}{\partial t} = \alpha B + \beta \frac{\partial^2 A}{\partial \theta^2} - V \frac{\partial A}{\partial \theta}, \qquad (1)$$

$$\frac{\partial B}{\partial t} = -D \cos\theta \frac{\partial A}{\partial \theta} + \beta \frac{\partial^2 B}{\partial \theta^2} - \frac{\partial (VB)}{\partial \theta}. \qquad (2)$$

Here, B is the toroidal magnetic field (measured in units of the equipartition field), A is proportional to the toroidal component of the vector potential that defines the poloidal magnetic field, θ is the latitude measured from the equator, t is the time, $V(-\theta) = -V(\theta)$ is the meridional circulation, and β is the turbulent diffusivity. The distances are measured in units of the solar radius R, and the time is in units of the diffusion time R^2/β_0, where β_0 is the diffusivity to which the normalization occurs. The factor $\cos\theta$

corresponds to a decrease in the length of the parallel near the pole (Kuzanyan and Sokoloff 1995). The term αB describes the contribution of the α-effect. The amplitudes of the angular velocity gradient R_ω and the α-effect R_α enter into the dynamo number D as follows: $D = R_\omega R_\alpha$. The small contribution of the α-effect is discarded in the second equation, i.e., the so-called $\alpha\omega$-approximation is used. The curvature effects are discarded in the diffusion terms. The radial angular velocity gradient is assumed to be constant as θ changes.

In this model, it is assumed that $\alpha = \alpha_0(\theta)(1 + \xi^2 B^2)^{-1}$, where α_0 is the helicity in an unmagnetized medium, and $B_0 = \xi^{-1}$ is the magnetic field at which the α-effect is suppressed significantly.

From symmetry considerations ($\alpha(-\theta) = -\alpha(\theta)$), Eqs. (1) and (2) may be considered only for one (northern) hemisphere with antisymmetry (dipole symmetry) or symmetry (quadrupole symmetry) conditions on the equator. In this paper, we restrict our analysis to the dipole symmetry with the simplest kinematic helicity in an unmagnetized medium, $\alpha_0 = \sin\theta$. We use the conditions $A(0) = B(0) = A(\pi) = B(\pi) = 0$ as the boundary conditions at the poles, because here we are interested in the solutions with the dipole symmetry.

Following Popova (2009), we consider the latitude profile of meridional circulation $V(\theta) = v \sin 2\theta$. Since the latitude in the model is measured from the equator, a value with the positive sign corresponds to meridional circulation directed oppositely to the dynamo-wave propagation.

Since the parameters of the Maunder butterflies change from cycle to cycle, it seems quite likely that the parameters responsible for their shape depend on time. No periodic dependence of their change with time was found.

According to this simplest dynamo theory, an increase in the amplitude of meridional circulation directed oppositely to the dynamo-wave propagation vector leads to an increase in the cycle duration and a shift of the Maunder butterflies to higher latitudes. If the shifts of the butterflies on the solar surface are assumed to be associated with the action of meridional circulation, then, according to the data obtained, their synchronous motion in latitude can be associated with the fact that the meridional circulation is the same in magnitude in both hemispheres.

As the turbulent diffusivity decreases, the cycle duration grows, the field amplitude increases, and the butterfly is not shifted in latitude.

The butterfly tilt angle is affected by both meridional circulation and turbulent diffusivity. Thus, the fact that cycles with a shorter duration have a larger tilt angle can be reproduced in the model through an increase in meridional circulation or a decrease in turbulent diffusivity.

According to the data obtained, there is no clear dependence of the mean butterfly latitude on the cycle duration. According to the dynamo theory, if only the amplitude of meridional circulation changed from cycle to cycle, then the cycle duration would be directly proportional to the mean latitude. It is possible that, apart from the change in the amplitude of meridional circulation, the turbulent diffusivity can also change with time. Thus, the absence of any dependence of the mean butterfly latitude on the cycle duration stems from the fact that these parameters change in combination on the Sun.

The mean latitude at which the butterfly wing is located is about 15°. The maximum deviation from the mean latitude in the observed cycles is 18%. The maximum deviation from the 11-year cycle duration is about 11%; the deviation from the mean field amplitude of 2455 G is 13%.

We considered typical parameters for the Sun: $D \approx -10\,000$, an amplitude of meridional circulation $\approx$0.5 model units ($\approx$2 m s^{-1}), and $\beta \approx 1$. At such parameters, the 11-year cycle is reproduced in model (1)–(2). We solved Eqs. (1)–(2) numerically using the Mathcad 11 package. Our numerical analysis showed that the observed deviation from the mean latitude could be reached when the amplitude of meridional circulation changed by $\Delta v \approx 0.1$ model units (the deviation from the mean is 20%) or $\Delta\beta \approx 0.25$ (the deviation from the mean is 25%). The observed deviation from the mean cycle duration can be reached when the amplitude of meridional circulation changes by $\Delta v \approx 1.5$ model units (the deviation from the mean is 300%) or $\Delta\beta \approx 0.05$ (the deviation from the mean is 5%). To obtain the observed deviation from the mean amplitude of the toroidal magnetic field, the dynamo number D ($\Delta D \approx 2000$ or 20%), or the meridional circulation v ($\Delta v \approx 2.5$ or 500%), or the turbulent diffusivity β ($\Delta\beta \approx 0.01$ or 1%) must change.

CONCLUSIONS

An increase in meridional circulation always leads to a shift of the butterflies to higher latitudes and an increase in cycle duration. However, the field amplitude initially decreases and then increases with a further increase in meridional circulation. As the turbulent diffusivity decreases, the cycle duration increases, the field amplitude grows, and the butterfly is not shifted in latitude. It follows from the dynamo theory that these parameters may change in combination in the observed cycles. Therefore, for example, there is no clear dependence of the mean butterfly latitude on the cycle duration, because, according to the dynamo theory, if only the meridional circulation changed, then the cycle duration would be directly proportional to the mean latitude.

The linear dependence $f((BT/L), S)$ from cycle to cycle may be realized, because the same amount of energy is spent on each cycle, but it is redistributed differently in each cycle; therefore, a change in other cycle characteristics is observed. The dynamo number is responsible for the intensity of the generation of magnetic fields in dynamo models. It is then quite likely that it does not change from cycle to cycle.

The changes in cycle characteristics inferred from observations are not that large, and, therefore, it seems highly likely that the changes in control parameters with time are small and random in nature.

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