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Trajectories of Bodies in Triple Systems with Moderate Lifetimes

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Abstract—The general three-body problem with equal masses and zero initial velocities is considered. Sets of initial conditions for which the triple systems decay over comparatively short times, $10T_{\rm cr} < T < 20T_{\rm cr}$, are distinguished ($T_{\rm cr}$ is the mean crossing time for a component of the system). These sets form distinct families of structures in the domain of the initial conditions. The properties of trajectories of the bodies within some of these structures are described. It is shown that the set of families is no more than countable. A new classification for families of trajectories in decaying triple systems is proposed. Some problems in classifying trajectories in the three-body problem that must be addressed in the future are formulated.

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1. INTRODUCTION

Interest has been expressed recently in mathematics and celestial mechanics in the three-body problem based on Newtonian gravitational interactions. Both analytical and numerical-simulation methods aimed at studying the dynamics of triple systems have been considered (see, e.g., the books by Marchal [1], Valtonen and Karttunen [2], and Martynova et al. [3]). Researchers have considered both special cases (such as the rectilinear three-body problem, when the bodies always move along a stationary straight line) and more general cases (in particular, motion of the bodies in a plane or in three-dimensional space).

One interesting case is the three-body problem with equal masses and zero initial velocities (the so-called "free-fall three-body problem"). The first numerical simulations of this problem carried out by Agekyan and Anosova [4] in 1967 showed that the evolution of such triple systems ends in decay—one of the components leaves the final binary formed by the two remaining bodies along a hyperbolic orbit.

Agekyan and Anosova [4] present examples of trajectories of the bodies in triple systems from the initial time to their decay [4, Fig. 2]. Binary and triple encounters, component ejections, and complex interations can occur in a triple system. The evolution of a triple system ends in decay following a close triple encounter of the three bodies. A subsequent detailed analysis of the motions of the bodies has revealed a number of interesting regularities in the orbital evolution and in the triple encounters leading to the decays of the triple systems (see, e.g., [5, 6]). For a more rigorous analysis of the trajectories in triple systems, Szebehely [7] and Agekyan and Martynova [8] proposed a classification of states in the general three-body problem, and derived criteria describing a triple encounter, a simple interaction, and an ejection.

In our earlier work [9], we investigated the trajectories of bodies in the free-fall three-body problem for the case of rapid decay of the triple systems. Some properties of the dynamical evolution of these systems were identified, and several families of trajectories distinguished. Our current study continues the analysis of [9]. Here, we consider triple systems with moderate decay times and aim to distinguish the main properties of the orbits in such systems.

2. FORMULATION OF THE PROBLEM

As in [9], we consider here triple systems with components with equal masses and zero initial velocities. At the initial time, the first body A is located at the point with coordinates $(-0.5, 0)$, the second body B at the coordinates $(+0.5, 0)$, and the third body C at the coordinates (ξ, η) inside a domain D bounded by the coordinate axes and the arc of a circle of unit radius with its center at the point $(-0.5, 0)$. Since the angular momentum of the triple system is zero, the motion will proceed in the plane of the initial triangular configuration.

As in [9], we used the following system of dynamical units:

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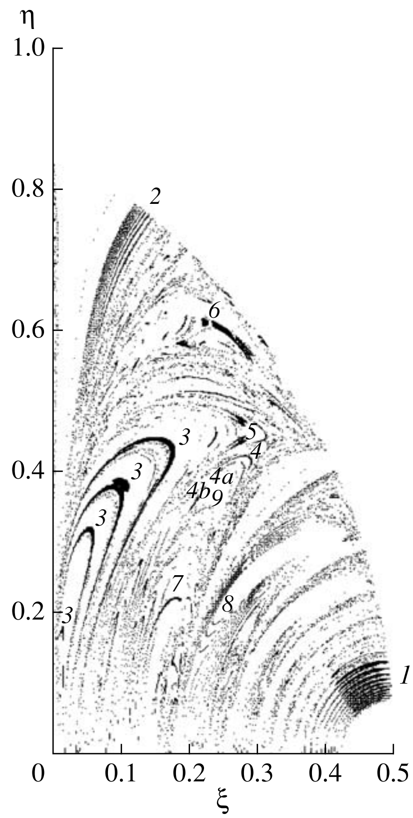

Fig. 1. Initial coordinates (ξ, η) for triple systems with lifetimes $10T_{\rm cr} < T < 20T_{\rm cr}$.

1) the unit of distance was taken to be the mean size of the triple system,

$$d = \frac{G \sum_{i<j} m_i m_j}{|E|},$$

where G is the gravitational constant, $m_i (i = 1, 2, 3)$ are the masses of the bodies, and E is the total energy of the triple system;

2) the unit of time was taken to be the mean crossing time for a component of the triple system,

$$T_{\rm cr} = \frac{\sqrt{G}(\sum_{i=1}^{3} m_i)^{5/2}}{(2|E|)^{3/2}}.$$

We adopted units such that $G = 1$, and the masses of the bodies were also taken to be unity.

In [9], we considered triple systems with decay times $T < 10T_{\rm cr}$ (we used the criteria of Standish [10] to identify the decay time). Here, we consider triple systems with somewhat longer lifetimes: $10T_{\rm cr} < T < 20T_{\rm cr}$. To carry out a detailed study of the motions of bodies in such systems, we numerically integrated the equations of motion for the general three-body problem in barycentric coordinates (the coordinate origin was located at the center of mass of the triple system). We applied the method of Bulirsch and Stoer [11] to numerically solve the system of differential equations. We reduced the uncertainties during close binary encounters using the regularization technique of Aarseth and Zare [12]. All the computations were carried out using the TRIPLE software compiled by Aarseth [13]. When constructing the trajectories, we specified the accuracy parameter $\varepsilon = 1.6 \times 10^{-16}$. For the three last trajectories (see Figs. 16–18 below), we used $\varepsilon = 2.0 \times 10^{-16}$.

3. RESULTS

Figure 1 shows the initial positions (ξ, η) for the triple systems with $10T_{\rm cr} < T < 20T_{\rm cr}$. Zones of rapid decay were distinguished in the domain D of initial conditions (ξ, η) in [9]. These often correspond to triple systems that decay after the first close triple encounter of the three components, preceded by n binary encounters of components B and C, with $n = 0, 1, 2, \ldots$. These zones form a system of concentric arcs concentrated toward the lower-right corner of the domain D. Several such arcs can be seen in Fig. 1, which presents the points (ξ, η) corresponding to triple systems with decay times $10T_{\rm cr} < T < 20T_{\rm cr}$. As was shown in [5] and confirmed in [9], the zones of departure from a given family are dynamically equivalent, and differ only in the number of binary encounters of components B and C before the close triple encounter that leads to the departure of one of the three components. The number n of binary encounters is one smaller than the number assigned to the zone. Figure 1 presents zones of a given family (denoted in the figure by the number *1*), beginning from the ninth zone ($n = 8$). In the previous zones with $n = 0, 1, 2, \ldots 7$, the decay times are $T < 10T_{\rm cr}$, and these zones were considered in [9].

In the upper part of any zone of this family, in the vicinity of the circle $(\xi + 0.5)^2 + \eta^2 = 1$ bounding the domain D on the right, body A departs along a nearly linear orbit during the decay (an example is shown in Fig. 2). A detailed analysis of the properties of trajectories from this family and the character of the internal structure of each of the arc-like regions is given in [9]. Here, we will not consider this family further, and instead turn to our analysis of other families that are clearly visible in Fig. 1.

One such family (labeled *2* in Fig. 1) is associated with the boundary of the domain D, the circle $(\xi + 0.5)^2 + \eta^2 = 1$. This family was already discussed in [9] for short decay times, $T < 10T_{\rm cr}$. It is characterized by several passages of component A between bodies B and C along nearly linear orbits. This problem is close to the isosceles case (the isosceles case is precisely realized on the circle bounding the domain D). One example of such motions is shown in Fig. 3.

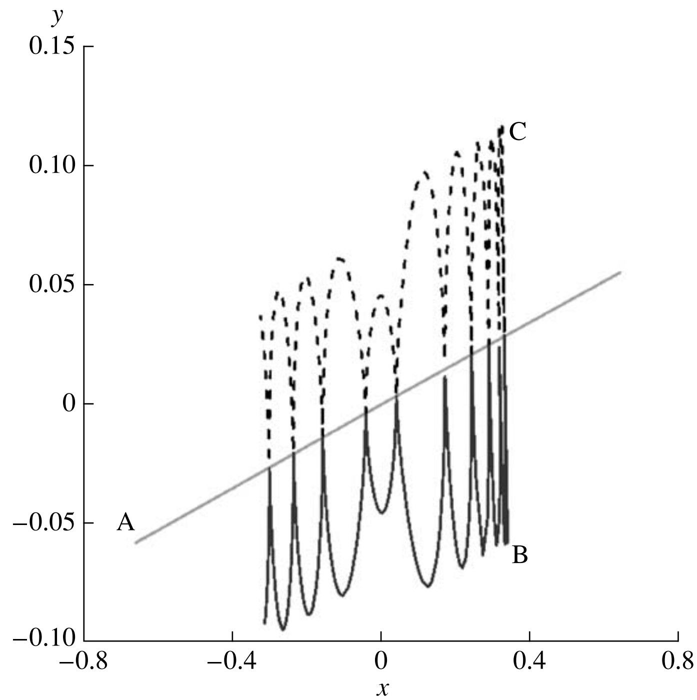

Fig. 2. Trajectories of bodies in a triple system from Family *1* with initial coordinates $(\xi, \eta) = (0.485, 0.175)$ over a time $t = 3.5T_{\text{cr}}$.

In this case, after a close binary encounter between components B and C, component A passes near the center of masses of the triple system between bodies B and C (the first wide "flyby" triple encounter according to the classification of [5]). As a result, component A is flung outward a considerable distance to the right and upward, while bodies B and C are sent in the opposite direction. Nine close binary encounters of bodies B and C occur during this process. After this there is a second flyby close triple encounter, after which body A leaves the triple system. In all, two flyby triple encounters took place before the decay, with the second encounter leading to the decay of the triple system.

Further consider Family *3*. Figure 4 presents an example of trajectories from the upper part of the outer zone of this family. In this case, the first wide triple encounter is an "exchange" type (according to the classification of [5]). The triple encounter consists of two binary encounters: first, a wide binary encounter of components A and C occurs (as a result, the trajectory of A has a cusp, conserving the direction of its convexity), after which there is a close binary encounter of components B and C. There is a successive transfer of kinetic energy from A to C, and then from C to B, in the process of these two binary encounters. As a result, component B is flung outward, and the temporary binary AC forms. Three binary encounters between A and C occur during this outward ejection of component B. After the return of B, there is a close binary encounter between B and A, accompanied by a flyby of C between bodies A and B, with its subsequent departure. The decay begins after the second close triple encounter of the components. While the first exchange triple encounter leads only to one of the bodies being flung outward, the second, closer flyby triple encounter leads to the departure of body C. Note that the trajectory of component C describes an angular path just before the flyby. The trajectory remains smooth—both the radius vector of C and its time derivative remain continuous. Such angular paths are rather typical, and often precede a flyby triple encounter. Note also that angular paths are often observed in the figures of trajectories together with (although at different times than) cusps in the trajectory of another body. The angles formed by the trajectory of bodies A and C lie on opposite sides of the axis of symmetry of the ejection of the third body B.

The departure of component C always occurs in the upper part of the outer zone of Family *3*. Moving within this zone downward along the right or left branch, we reach the point of a triple collision [6]. The structure of the branches becomes layered in the vicinity of the triple-collision point, and zones from which component B departs appear (Fig. 5). Thus, the triple-collision point represents a sort of division between the departure zones for the different bodies

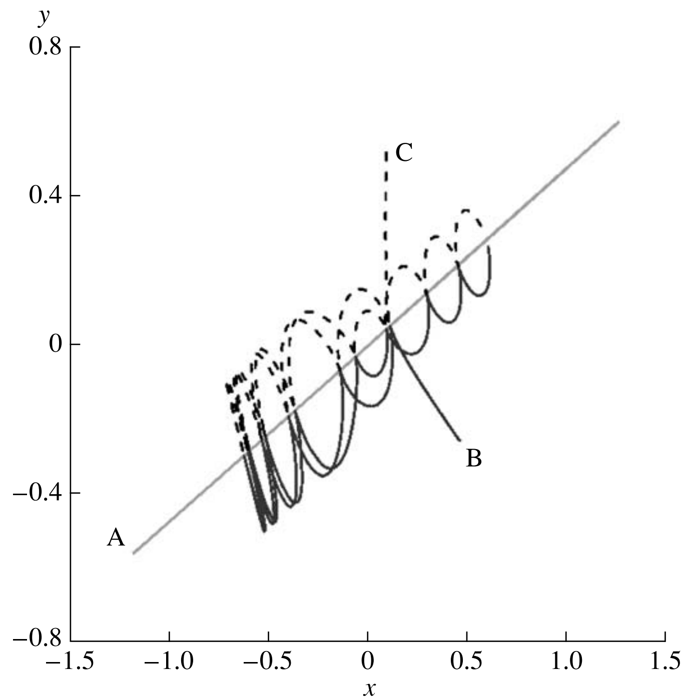

Fig. 3. Trajectories of bodies in a triple system from Family *2* with the initial coordinates $(\xi, \eta) = (0.130, 0.775)$ over a time $t = 5.0T_{\rm cr}$.

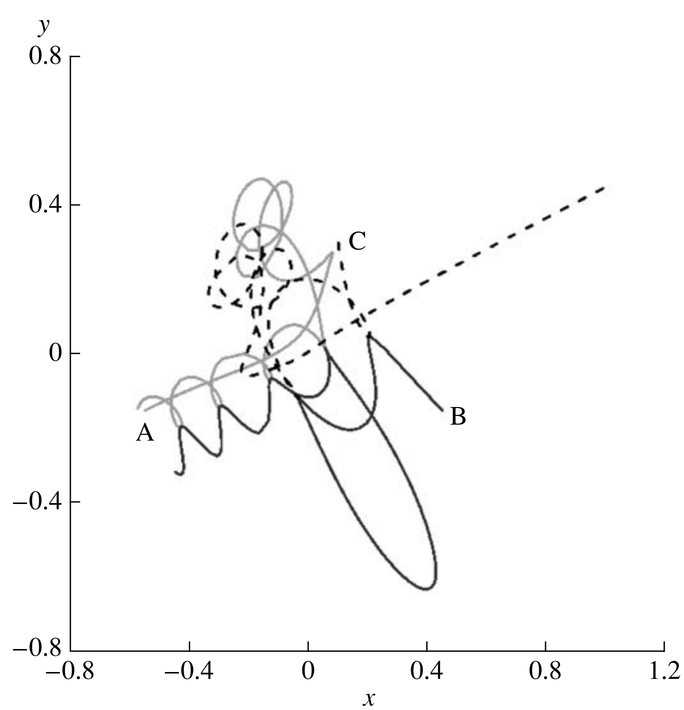

Fig. 4. Trajectories of bodies in a triple system from Family *3* with the initial coordinates $(\xi, \eta) = (0.150, 0.450)$ over a time $t = 4.0T_{\rm cr}$. Departure of component C.

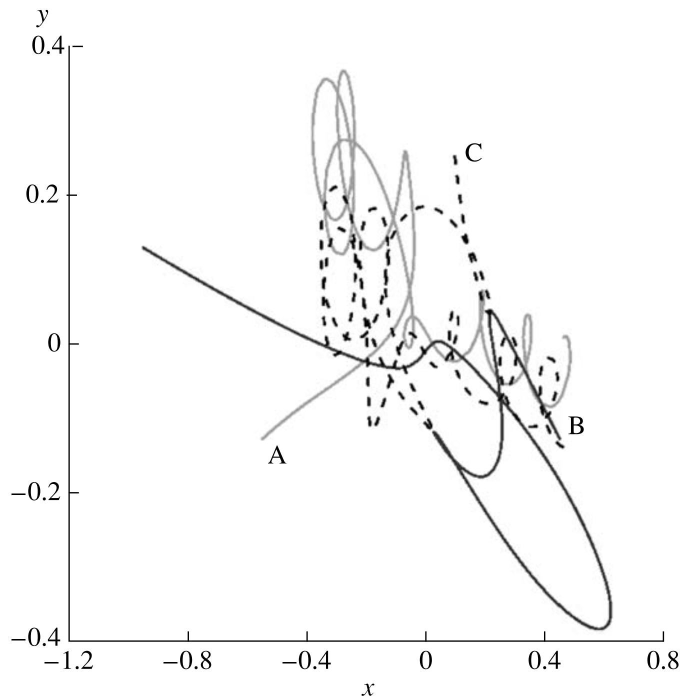

Fig. 5. Trajectories of bodies in a triple system from Family *3* with the initial coordinates $(\xi, \eta) = (0.147, 0.380)$ over a time $t = 3.7T_{\rm cr}$. Departure of component B.

within the domains of a single type of evolution of the triple systems.

The character of the evolution of the triple systems in the other zones of Family *3* is mainly similar to that described above (Figs. 6–8). A comparison of Figs. 6–8 shows that the characters of the motions in the upper parts of these zones are the same, and differ only in the number of binary encounters of components A and C preceding the close triple encounter that leads to the decay of the triple system. In all three cases, component A departs as a result of the triple encounter.

Family *4* consists of several sparsely populated zones arranged along single-peaked curves. An example of trajectories of bodies with initial conditions near the maximum of the outermost zone is presented in Fig. 9. In this case, there are three triple encounters. The first (exchange) and second (flyby) triple encounters lead to components B and C being flung outward, respectively. The third close triple encounter, which is also a flyby encounter, leads to the departure of component C. The trajectory of the departing component displays appreciable curvature. Note that symmetrical similar structures formed by windings of the trajectories at various times are observed in this figure. For example, the trajectory of component A during its approach toward the binary BC at the beginning of the evolution in Fig. 9 forms a loop, which is symmetrical to the loop of this same body at the end of the evolution, before the close triple encounter that leads to the departure of component C. This loop makes a transition to a winding of the trajectory of component A in the final binary AB. Another example is the loops of the trajectories of A and B during the outward excursion of component C. The trajectory of A first forms a narrow loop before the interaction with component B in the binary, while component B at this time moves through a wide loop in the same direction, moving through a narrow loop in the opposite direction after the interaction in the binary, symmetrical to the loop undergone by component A. Note that component C departs in a direction approximately orthogonal to the direction of motion of A at the beginning of the evolution of the triple system.

In the next structure, Family *4* (Fig. 10), the character of the motions at the beginning of the evolution is similar to that in the previous case (Fig. 9). In contrast to the previous case, the trajectory of component B forms a downward loop after its return from being flung outward, after which this component undergoes a binary encounter with component A and the trajectory forms an upward loop parallel to the downward loop. Further, body B undergoes another binary encounter with component A, traces out another small loop to the left, and then forms the final binary with loops to the right. In this case, the third close triple encounter, which is a flyby, already leads

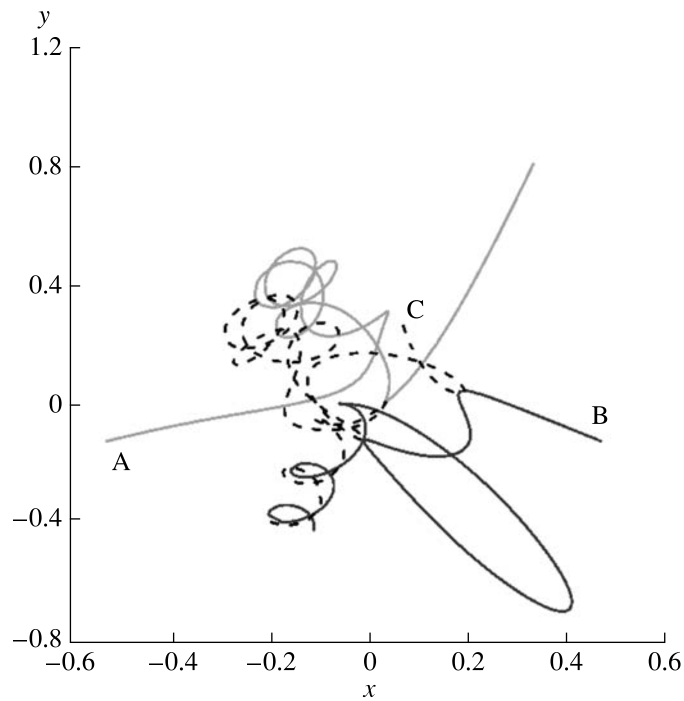

Fig. 6. Trajectories of bodies in a triple system from Family *3* with initial coordinates $(\xi, \eta) = (0.100, 0.391)$ over a time $t = 4.0T_{\rm cr}$.

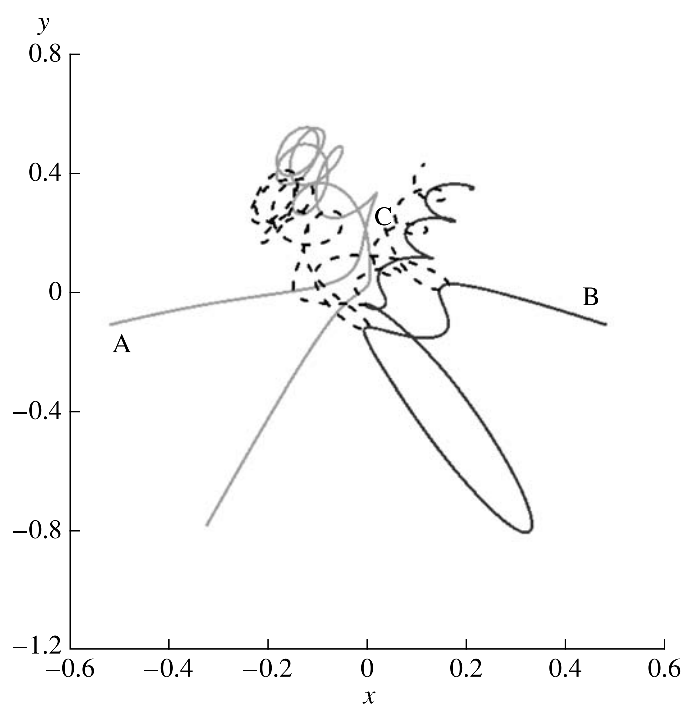

Fig. 7. Trajectories of bodies in a triple system from Family *3* with initial coordinates $(\xi, \eta) = (0.055, 0.323)$ over a time $t = 4.5T_{\rm cr}$.

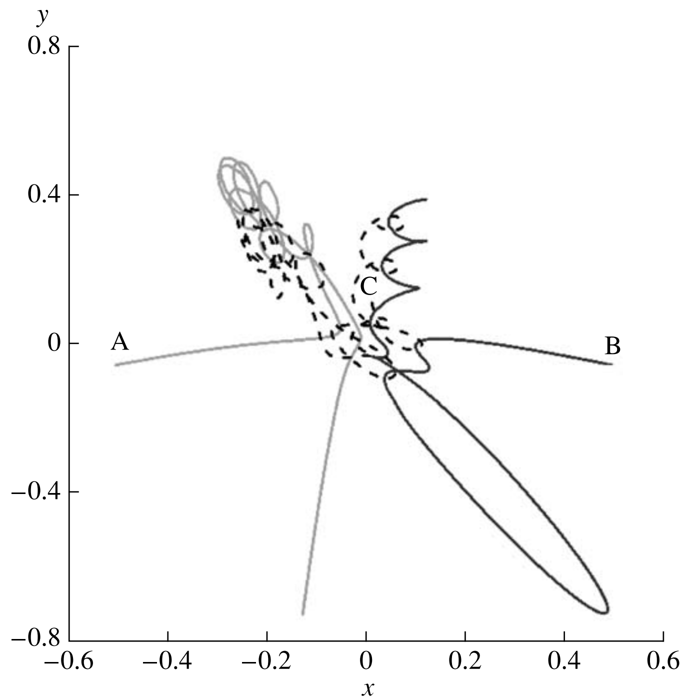

Fig. 8. Trajectories of bodies in a triple system from Family *3* with initial coordinates $(\xi, \eta) = (0.016, 0.169)$ over a time $t = 4.5T_{\rm cr}$.

to the departure of component A after its "angular" reversal. The departure is parallel to the motion after the reversal. The trajectory of A has an inflection point at the center of mass of the triple system. Note that, as in the previous case, the departure of component A is roughly orthogonal to the initial motion of this component. Such approximately "orthogonal" departures are observed rather often in the triple systems considered. This gives the impression that a triple system "remembers" the initial direction of acceleration of the components.

Two subfamilies branch off from Family *4*, denoted in Fig. 1 as *4a* and *4b* (Figs. 11 and 12). The initial evolution in triple systems from Family *4a* resembles the evolution of systems from Family *4* described above: there are two wide exchange and flyby triple encounters that lead successively to outward ejections of components B and C. However, in contrast to the main Family *4*, the third triple encounter does not lead to the decay of the triple system, and instead leads to component A being flung roughly vertically upward. Only the next close triple encounter, which is a flyby encounter with an inflection point at the center of mass, leads to the departure of component A approximately vertically downward, roughly orthogonal to the initial motion of this component.

The situation is somewhat different in Family *4b*. In contrast to Family *4a*, the second triple encounter here leads to component C being flung out, rather than component A. The third triple encounter sends component C approximately vertically downward, and the fourth (flyby) triple encounter leads to the departure of this body vertically upward, again approximately orthogonal to the motion of component A at the beginning of the triple's evolution. In both Families *4a* and *4b*, the decay of the triple system is preceded by four triple encounters. The first three wide triple encounters fling out various bodies, while the fourth close triple (flyby) encounter leads to the departure of one of the components.

Family *5* consists of two paired elongated structures. A typical example of the evolution of a triple system from this family is shown in Fig. 13. Three triple encounters occur during the evolution of the system. The first wide (exchange) triple encounter causes component B to be flung outward, leading to the formation of the temporary binary AC. The second triple encounter is a flyby encounter with component A flung outward and then reversing its motion by 180°. The narrow, needle-like ejection of this body is approximately vertical. The temporary binary BC that is formed has a strongly elongated orbit; components B and C undergo two close binary encounters during body A's outward excursion. After the return of component A, there is a close triple encounter with a flyby of component B along a strongly curved trajectory, with an inflection point at the center of mass of the triple system. The general character of the evolution

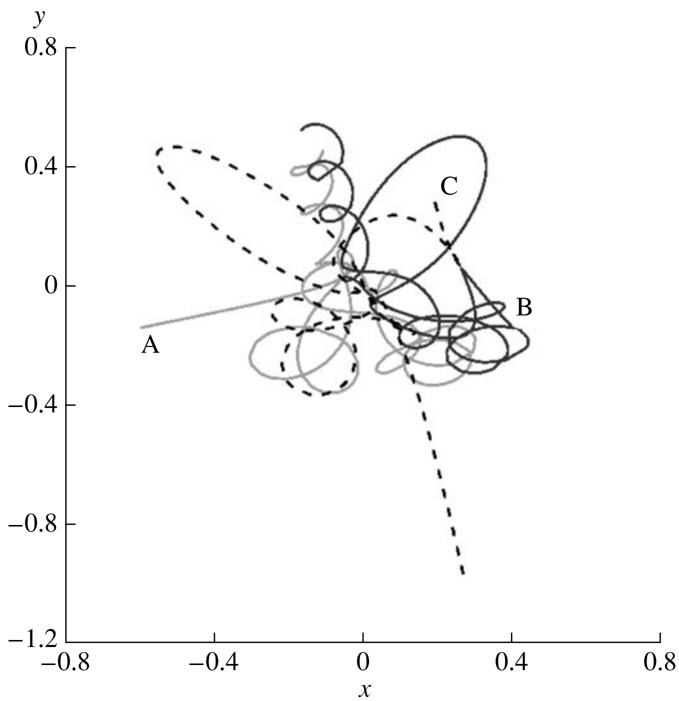

Fig. 9. Trajectories of bodies in a triple system from Family *4* with initial coordinates $(\xi, \eta) = (0.292, 0.418)$ over a time $t = 5.5T_{\rm cr}$.

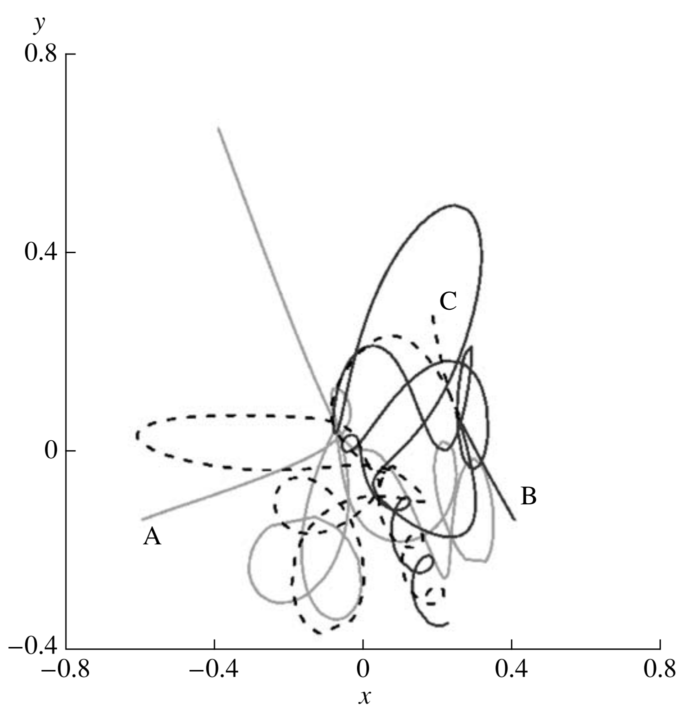

Fig. 10. Trajectories of bodies in a triple system from Family *4* with initial coordinates $(\xi, \eta) = (0.280, 0.410)$ over a time $t = 7.5T_{\rm cr}$.

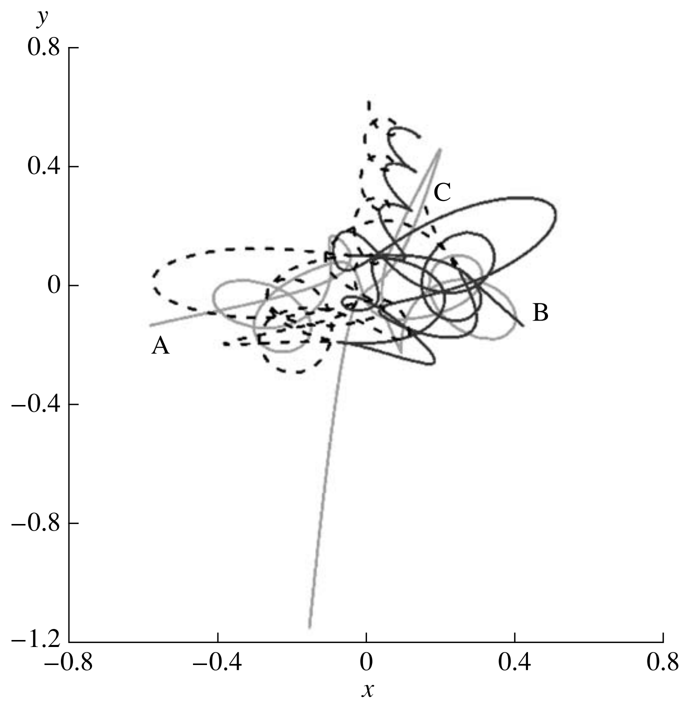

Fig. 11. Trajectories of bodies in a triple system from Family *4a* with initial coordinates $(\xi, \eta) = (0.238, 0.400)$ over a time $t = 6.5T_{\rm cr}$.

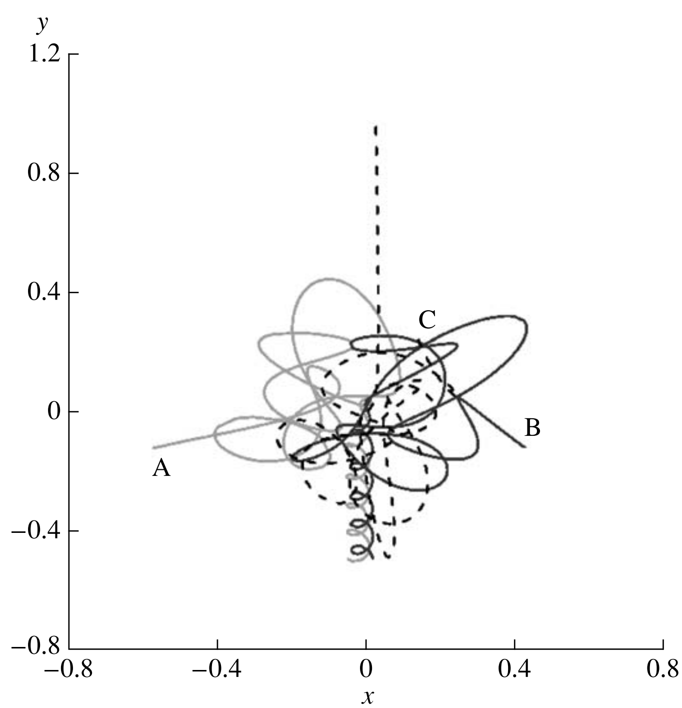

Fig. 12. Trajectories of bodies in a triple system from Family *4b* with initial coordinates $(\xi, \eta) = (0.212, 0.363)$ over a time $t = 5.2T_{\rm cr}$.

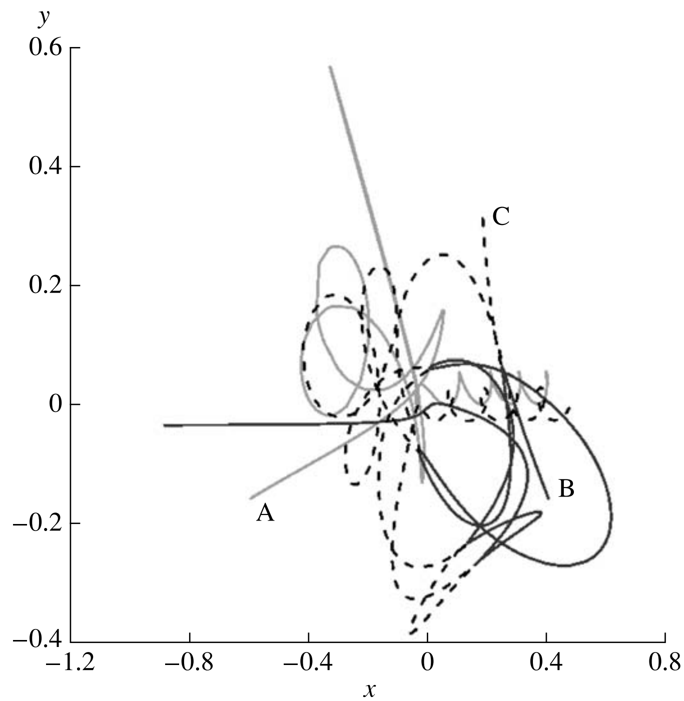

Fig. 13. Trajectories of bodies in a triple system from Family *5* with initial coordinates $(\xi, \eta) = (0.279, 0.470)$ over a time $t = 7.0T_{\text{cr}}$.

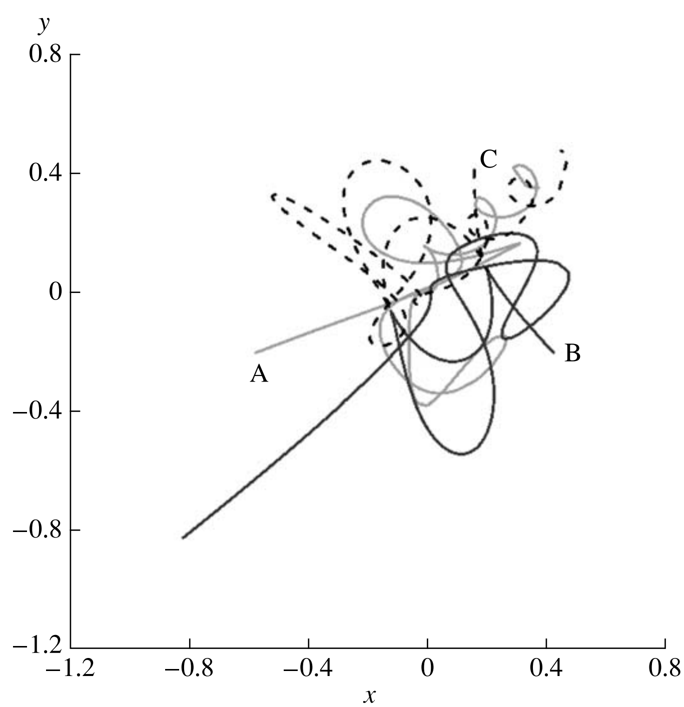

Fig. 14. Trajectories of bodies in a triple system from Family *6* with initial coordinates $(\xi, \eta) = (0.233, 0.615)$ over a time $t = 4.5T_{\text{cr}}$.

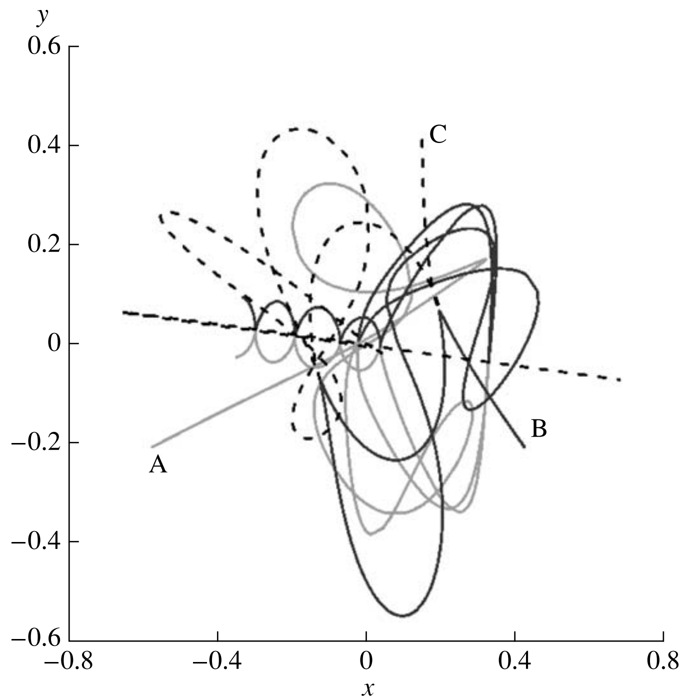

Fig. 15. Trajectories of bodies in a triple system from Family *6* with initial coordinates $(\xi, \eta) = (0.225, 0.620)$ over a time $t = 9.0T_{\rm cr}$.

for other initial conditions from Family *5* is similar, although the final triple encounter can lead to the departure of a different component.

Family *6* is an elongated formation consisting of several fragments. Figure 14 shows a characteristic example of the evolution of a triple system with initial conditions from this family. The initial stage of the evolution resembles the motion in the isosceles problem (Family *2*; Fig. 3); component A passes near the center of mass of the triple system, with a small shift toward component C. Due to this shift, the expected flyby triple encounter becomes a wide exchange triple encounter, with component B flung outward and the formation of the wide pair AC. The second triple encounter is an exchange encounter, in which component C is flung outward and the pair AB is formed. Each of these triple encounters consists of two binary encounters whose joint action leads to one of the bodies being flung outward. The third encounter includes a flyby along a curved trajectory with an inflection point near the center of mass of the triple system, which leads to the departure of body B.

The motion in the left fragment of Family *6* also displays a similar character (Fig. 15). However, in this case, the final close flyby triple encounter already leads to the departure of component C. Note that this flyby occurs after a very narrow, needle-like ejection of this component: during its departure, body C moves along a nearly linear orbit without an inflection point near the center of mass of the triple system. Note also that, compared with Fig. 14, another close triple encounter has appeared in Fig. 15, which led to the needle-like excursion referred to above. Thus, in this case, the decay of the triple system occurred after the fourth triple encounter. Here, as in Family *4*, we observe a certain "breaking up" of Family *5* as a result of the appearance of another triple encounter. This behavior of regions with different numbers of triple encounters preceding the decay of the triple system was qualitatively predicted in a brief note by Agekyan and Anosova [14]. This phenomenon was also observed for some other families indicated in Fig. 1.

Since the initial conditions for the triple systems from Family *7* are located near the Oξ axis, various properties of motions in the rectilinear three-body problem are manifest here: the central body C undergoes oscillatory motions in which it approaches first one, then the other outer body (Fig. 16). It is interesting that these oscillations occur nearly in vertically, while, in the purely rectilinear problem (with all three bodies on the Oξ axis at the initial time), the oscillations should occur along the horizontal axis. In the final evolution, there is a close flyby triple encounter leading to the departure of the central body C. Before the flyby, component C undergoes a short needle-like ejection approximately opposite to the direction of its departure during the decay. A number of short, nearly vertical structures that are related to the rectilinear

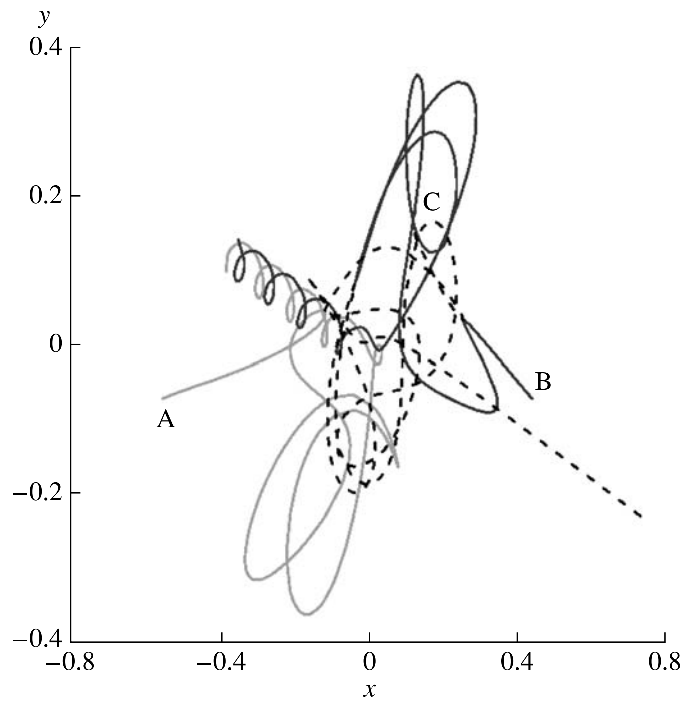

Fig. 16. Trajectories of bodies in a triple system from Family *7* with initial coordinates $(\xi, \eta) = (0.182, 0.223)$ over a time $t = 4.7T_{\text{cr}}$.

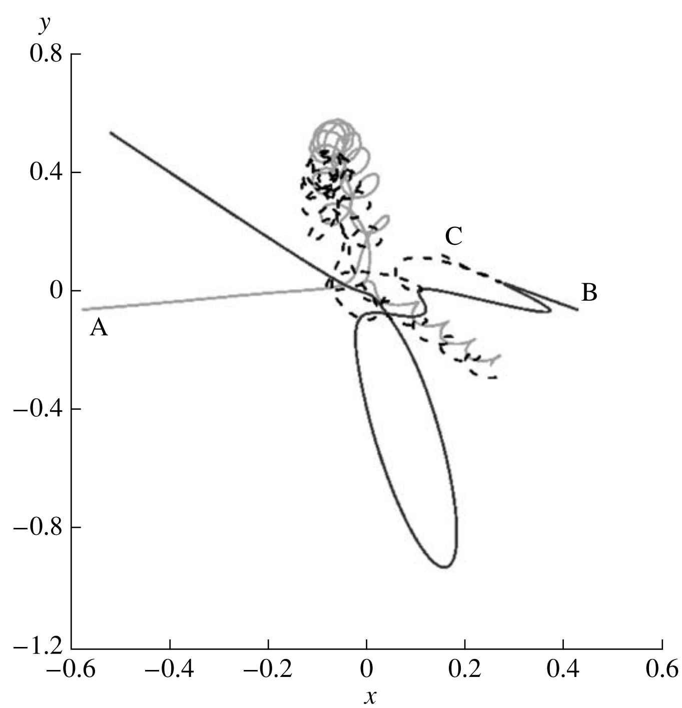

Fig. 17. Trajectories of bodies in a triple system from Family *8* with initial coordinates $(\xi, \eta) = (0.228, 0.185)$ over a time $t = 6.4T_{\text{cr}}$.

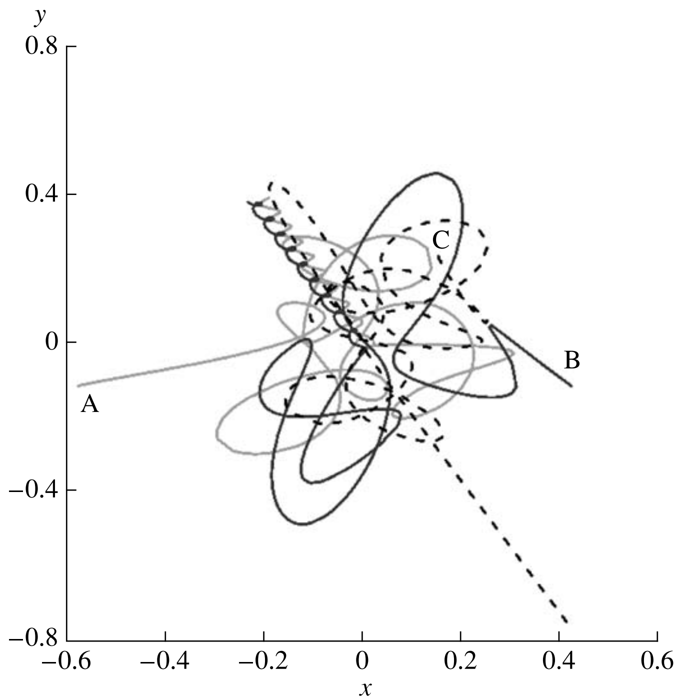

Fig. 18. Trajectories of bodies in a triple system from Family *9* with initial coordinates $(\xi, \eta) = (0.230, 0.351)$ over a time $t = 5.67T_{\rm cr}$.

three-body problem are observed near the horizontal axis. A detailed analysis of the motions of these structures will be conducted in a separate study.

The individual elongated zones of Family *8* are oriented with their apices downward. A typical example of trajectories from this family is shown in Fig. 17. At the beginning of the evolution, after the first wide exchange triple encounter, component B is flung outward to a substantial distance from the center of mass of the triple system. During this excursion, a temporary binary composed of components A and C forms. After the return of B, there is a close flyby triple encounter leading to the departure of this body from the triple system. Individual zones from Family *8* differ in the number of binary approaches between components A and C in the temporary binary. The trajectories from Family *8* are qualitatively similar to those from Family *3* (Fig. 4). They differ only in the number of close approaches of bodies B and C preceding the first triple encounter. One binary approach of these bodies occurs in Family *3*, while two are observed in Family *8*. In both cases, the first triple encounter leads to the ejection of component B to a substantial distance and the formation of the temporary binary AC. The result of the close triple encounter could be different: either component B (Fig. 17) or component C (Fig. 4) could depart. The possible departure of component A can also not be excluded.

An interesting class of motions is exhibited by Family *9* (Fig. 18). The triple system undergoes four triple encounters before its decay. The first encounter is an exchange encounter that results in component B being flung approximately vertically upward and to the right; the second encounter is a flyby encounter leading to an excursion of this same component in the opposite direction; the third is a flyby encounter that flings component C vertically upward and to the left; and finally the fourth is a flyby encounter that leads to the departure of this body in the opposite direction (downward and to the right). Note that the ejections of the bodies and the departure of component C form a symmetrical, cross-like structure. Note also that the ejections were so short that the bodies forming the temporary binary left behind were able to undergo only one binary approach between triple encounters leading to the ejections. This suggests that the entire evolution of the triple system occurred within the sphere of simple interactions [8]. A detailed analysis of the roles of the spheres of triple encounters, simple interactions, and outward ejections in the evolution of triple systems will be presented in a separate study.

4. CONCLUSION

We have considered triple systems with initial stationary bodies of equal mass, and studied properties of the evolution of triple systems with decay times

$10T_{\rm cr} < T < 20T_{\rm cr}$. Nine families of trajectories have been distinguished in the domain of the initial conditions, (ξ, η), within which the orbits share characteristic properties (Fig. 1). A family can be described by certain important characteristics: the total number of triple encounters of the bodies preceding the decay of the system, and the sequence of excursions of the bodies leading up to the last triple encounter that leads to the decay of the system. Some families consist of several zones, which differ in a single parameter, such as the number of binary encounters undergone by a particular pair of bodies while the third body is undergoing an ejection. Subfamilies were distinguished in some families (such as Family *4*), in which the number of triple encounters preceding the decay of the triple system is one greater than for the corresponding main family.

Some zones of some families (such as the zones of Families *1*, *2*, and *3*) contain the triple-collision points found in [6]. The presence of these points leads to a layering of the corresponding zones into regions of departure of various bodies after the same number of triple encounters. This raises the question of whether all the zones form such layers. This question requires additional studies using both numerical simulations and analytical (in particular, geometrical) constructions.

Another important question is the total number of different families. Note that Family *1* consists of a countable number of zones, each corresponding to a certain number of binary encounters of components B and C precedng the triple encounter that leads to the decay of the triple system. Other families with different total numbers of triple encounters of the bodies and specific sequences of excursions of specific bodies preceding the last triple encounter that leads to decay of the system are located between the first zone of this family, which is elongated along the $O\eta$ axis, and the second zone in [9, Fig. 1]. This is suggestive of a self-similar structure: analogous families are located between the second and third zones of Family *1*, the third and fourth zones, etc. This hypothesis requires additional verification using specially designed numerical simulations.

If we suppose that each family corresponds to a specific number of triple encounters separated by specific sequences of outward excursions of specific bodies, and that all the sequences actually occur in reality (i.e., there are no forbidden sequences), the number of families should be no greater than countable. Of course, another question then arises: whether it is possible to represent any trajectory as a sequence of triple encounters and excursions. If, following Tanikawa and Umehara [6], we consider a triple encounter as a time when the moment of inertia of the triple system is minimum and assume that there exists a finite minimum time interval Δt separating two successive triple encounters, the total number of triple encounters, and consequently the total number of excursions, during the evolution of a triple system will be no more than countable. Thus, the number of all possible families of trajectories will likewise be no greater than countable.

Based on the above, we can propose a new classification for the families of trajectories displayed by decaying triple systems. We present this classification as a symbolic sequence, using four symbols: "0" represents a triple encounter (e.g., a time when the moment of inertia of the triple system is minimum), and "A", "B", and "C" correspond to outward excursions or departures of the corresponding components from the triple system. Let the beginning of the evolution correspond to the first triple encounter (0) and the end of the evolution to the departure of component A, B, or C (denoted by the corresponding symbol). Then, all the trajectories in Family *1* can be described by the sequences 0A, 0B, or 0C—the decay of the system occurs after the first triple encounter. The trajectories for Family *2* correspond to the sequences 0A0A, 0A0B, or 0A0C, and so forth. It is of interest to distinguish the families derived in this way in the domain D.

A number of questions arise when this type of scheme is used. Can any trajectory be described by a symbolic sequence, and will that correspondence be unique? Does a triple encounter always have a single minimum moment of inertia, and what should be done if there are several such minima? Is a situation possible in which there is an exchange of distant compoments during an ejection (i.e., one body overtakes the other)? Binary encounters of the bodies are not taken into account in our classification—could this limit the value of this classification scheme? We will consider these and other related questions in future studies.

The situation could change if we consider only triple encounters within a sphere of a specified radius, for example, in the sphere of triple encounters introduced by Agekyan and Martynova [8] (or the ejection sphere, also introduced in [8]). Then, in principle, situations in which the entire evolution of the triple system proceeds within this sphere—i.e., in which there are no triple encounters—are possible. Such triple systems do not fit into our classification scheme, and do not correspond to any of the families we have introduced. The set of such systems could be more than countable, for example, they could have a power continuum. On the other hand, it is not ruled out that local minima of the moment of inertia could be reached during outward excursions in some triple systems ("false" minima, which do not correspond to real triple encounters). Including these minima could lead to a false classification of the trajectory families. Consequently, the question of the classification of the trajectory families is not as simple as it may seem, and requires a special analysis, which will be the subject for a future study.

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