

Differences in the Parameters of Radio Pulsars with Short and Long Periods

A. A. Loginov and I. F. Malov*

*Pushchino State Natural Sciences Institute, Pushchino, Moscow region, Russia
Pushchino Radio Astronomical Observatory, Astro Space Center, P.N. Lebedev Physical Institute,
Russian Academy of Sciences, Pushchino, Moscow region, Russia*

Received March 12, 2014; in final form, March 17, 2014

Abstract—A comparative analysis of various parameters of pulsars with short ($P < 0.1$ s) and long ($P > 0.1$ s) periods is carried out. There is no correlation between the radio and gamma-ray luminosities of the pulsars and their surface magnetic fields, but there is a correlation between the X-ray luminosity and the surface magnetic field. A dependence of the X-ray and gamma-ray luminosities on the magnetic field at the light cylinder is also found. This result provides evidence for the formation of hard, non-thermal emission at the periphery of the magnetosphere. An appreciable positive correlation between the luminosity and the rate of rotational energy loss by the neutron star is observed, supporting the idea that all radio pulsars have the same basic source of energy. The efficiency of the transformation of rotational energy into radiation is significantly higher in long-period pulsars. The dependence of the pulse width on the pulsar period is steeper for pulsars with short periods than for those with long periods. The results obtained support earlier assertions that there are differences in the processes generating the emission in pulsars with $P < 0.1$ s and those with $P > 0.1$ s.

DOI: 10.1134/S1063772914100072

1. INTRODUCTION

The principal-components method and modern observational data were used in [1] to confirm based on a larger sample the conclusion of [2] that two clusters of radio pulsars can be distinguished in phase spaces constructed for various parameters of the pulsars from the catalog [3]. The method applied is a formal mathematical procedure, but can be used to identify inhomogeneity of the studied objects in the spaces of the principal components. The observed differences between pulsars with periods of $P < 0.1$ s and $P \sim 1$ s had already been noted earlier (see, e.g., [4, Chapter IV]).

The additional observational material that has been accumulated can be used to analyze the indicated differences between the two groups of pulsars in detail. This is the subject of the current study.

More than 2300 radio pulsars are currently known [3]. We selected for our analysis characteristics from the catalog [3] related to physical descriptions of the pulsars (their periods, period derivatives, profile widths, radio luminosities, rate of rotational energy loss, surface magnetic fields, and magnetic fields at the light cylinders). We also used the estimated X-ray and gamma-ray luminosities of [5, 6]. More

than 100 pulsars have been detected in these hard energy bands. As in [1], we excluded pulsars in binary systems, as well as those located in globular clusters, since their observed characteristics could be distorted by the influence of nearby companions.

2. RESULTS

Figure 1 presents the distribution of the period derivatives of the pulsars, and Fig. 2 the distribution of their periods. These two distributions display an obvious bimodality, indicating that pulsars with periods less than tens of milliseconds will never become “normal,” since this would require a time exceeding the age of the Universe for period derivatives of order 10^{-19} . For example, for a pulsar with $P = 10$ ms and $dP/dt = 10^{-19}$, the time required to acquire a period of 1 s is 300 billion years.

This again emphasizes that objects located in the lower left corner of the $dP/dt - P$ diagram (Fig. 3) represent a separate group of pulsars. It turns out that the radio luminosities of the pulsars L_r do not depend on the surface magnetic fields B_s for both groups. The straight lines obtained from formal least-squares fits to the data are given by the equations

$$\log L_r = (0.200 \pm 0.147) \log B_s + (26.073 \pm 1.815) \quad (1)$$

*E-mail: malov@prao.ru

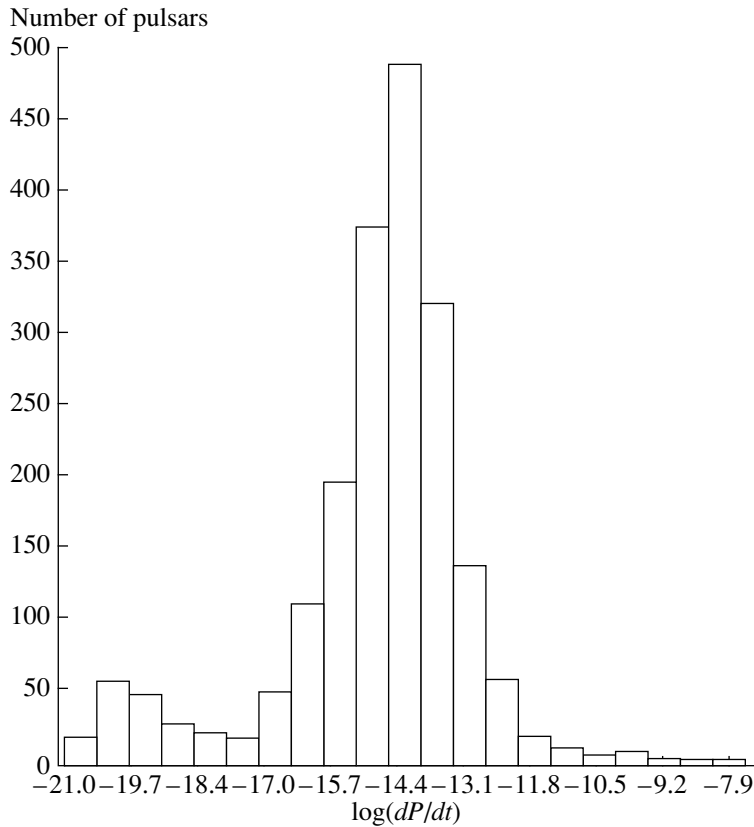


Fig. 1. Distribution of the period derivatives of the radio pulsars.

$$\begin{aligned}
 & (0.1014 \leq P \leq 8.5098 \text{ s}), \\
 \log L_r &= (-0.040 \pm 0.551) \log B_s \quad (2) \\
 & + (29.409 \pm 4.975) \\
 & (0.0016 \leq P \leq 0.0598 \text{ s}).
 \end{aligned}$$

The correlation coefficients for the relations (1) and (2) are equal to $K = 0.112$ and -0.028 , respectively, consistent with a random distribution with probability $p \simeq 0.9$. This result can be explained by the fact that the main radiation mechanism in long-period pulsars is believed to be curvature radiation. In this case, the radiated power depends only on the structure of the magnetic field (its radius of curvature), and not on the field strength:

$$p_{cr} = \frac{2e^2 c \gamma^4}{3\rho^2}, \quad (3)$$

where p_{cr} is the radiated power, e the electron charge, c the speed of light, γ the Lorentz factor of the radiating electrons, and ρ the radius of curvature of the external magnetic field. The radiation of short-period pulsars is generated at large distances from the surface of the neutron star (near the light cylinder), so that it should not depend on the parameters at the surface, regardless of the radiation mechanism.

Here and below, we consider the “luminosities” $S_{400} d^2$ in mJy kpc² from [3] recalculated into radio luminosities L_r in erg/s using the formulas obtained in [7]; S_{400} is the flux density at 400 MHz and d the distance to the pulsar.

Figure 4 presents the dependence of the X-ray luminosity on the surface magnetic field. The corresponding relations can be written

$$\begin{aligned}
 \log L_x &= (0.857 \pm 0.261) \log B_s \quad (4) \\
 & + (23.101 \pm 2.790), \\
 & K = 0.769 \\
 & (0.0016 \leq P \leq 0.0914 \text{ s}),
 \end{aligned}$$

$$\begin{aligned}
 \log L_x &= (1.936 \pm 1.024) \log B_s \quad (5) \\
 & + (6.951 \pm 13.274), \\
 & K = 0.606 \\
 & (0.1014 \leq P \leq 5.5404 \text{ s}).
 \end{aligned}$$

Figure 4 and relations (4), (5) show that there is a correlation between $\log L_x$ and B_s . This can be explained by the contribution of thermal X-ray emission from the neutron-star surface, whose heating could be related to the magnetic field, since the thermal

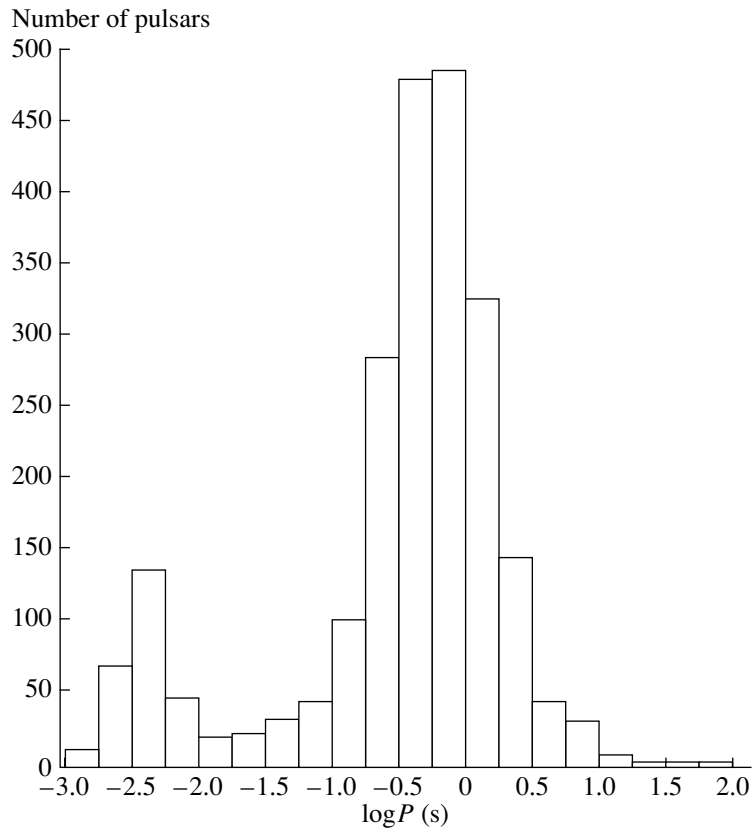


Fig. 2. Period distribution of the radio pulsars.

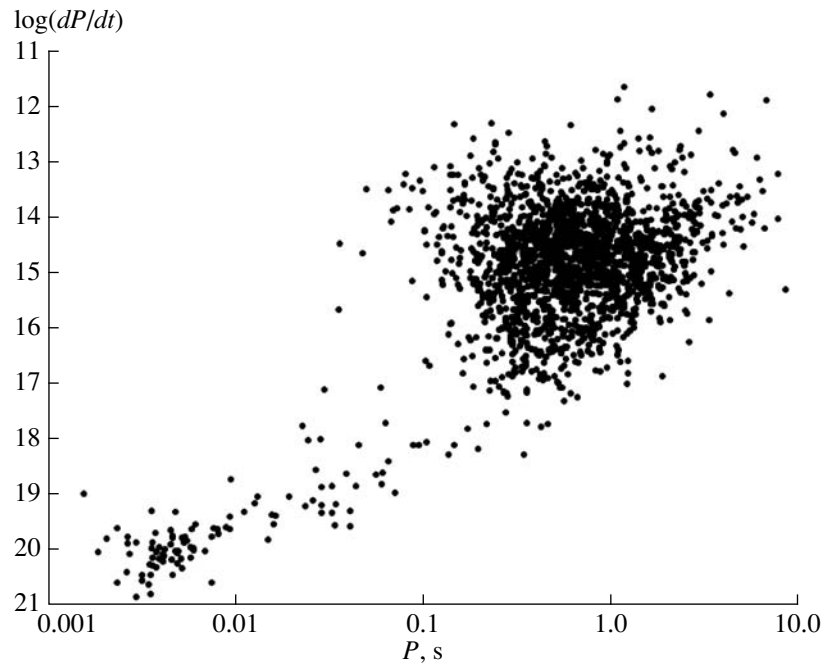


Fig. 3. $dP/dt - P$ diagram [3].

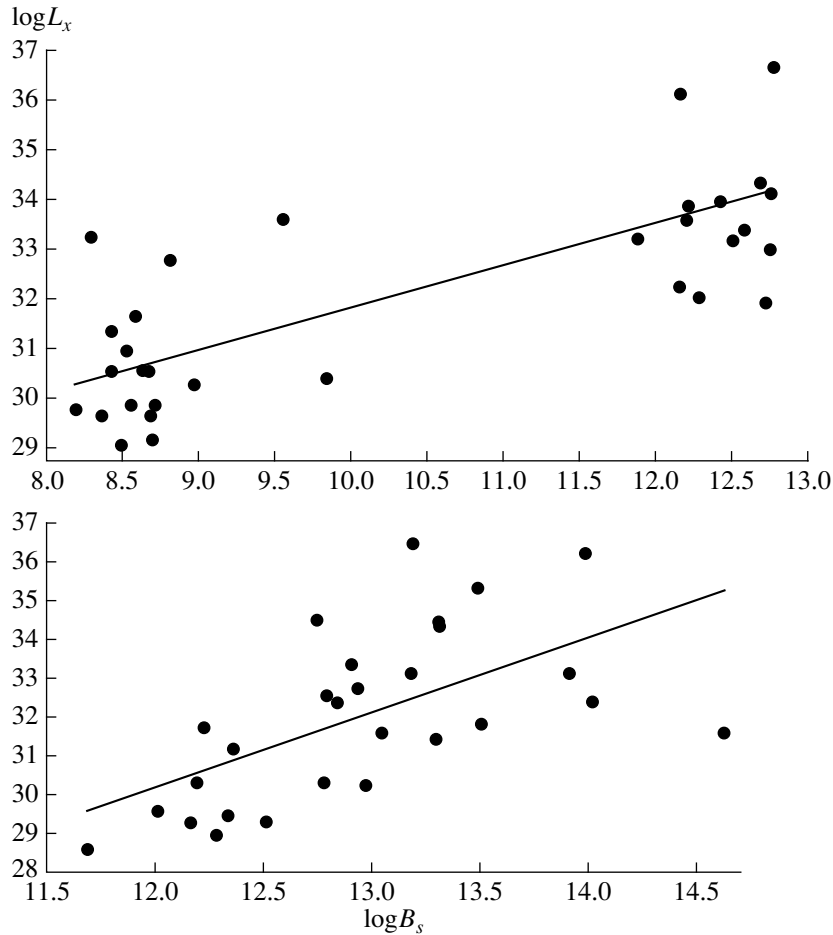


Fig. 4. Dependence of the X-ray luminosity of the radio pulsars on the surface magnetic fields for 33 objects with $0.0016 \leq P \leq 0.0914$ s (upper) and 28 objects with $0.1014 \leq P \leq 5.5404$ s (lower).

conductivity depends on the magnetic field. This process must be investigated in more detail separately. We did not find a significant correlation between the gamma-ray luminosity and the surface magnetic field for either group of pulsars. Figures 5–7 show the dependences of the radio, X-ray, and gamma-ray luminosities on the magnetic field at the light cylinder. The radio luminosities of the pulsars with $P > 0.1$ s do not depend on this magnetic for the same reason they do not depend on B_s , while a dependence is observed for the short-period pulsars:

$$\begin{aligned} \log L_r &= (0.296 \pm 0.082) \log B_{lc} & (6) \\ &+ (28.100 \pm 0.138), \\ &K = 0.287 \\ &(0.1014 \leq P \leq 8.5098 \text{ s}), \end{aligned}$$

$$\begin{aligned} \log L_r &= (0.345 \pm 0.266) \log B_{lc} & (7) \\ &+ (27.749 \pm 1.024), \\ &K = 0.442 \\ &(0.0016 \leq P \leq 0.0649 \text{ s}). \end{aligned}$$

The fitted dependences of the X-ray and gamma-ray luminosities on B_{lc} are

$$\begin{aligned} \log L_x &= (2.079 \pm 1.088) \log B_{lc} & (8) \\ &+ (21.784 \pm 5.443), \\ &K = 0.573 \\ &(0.0016 \leq P \leq 0.0914 \text{ s}), \end{aligned}$$

$$\begin{aligned} \log L_x &= (1.393 \pm 0.643) \log B_{lc} & (9) \\ &+ (27.395 \pm 2.238), \\ &K = 0.658 \\ &(0.1014 \leq P \leq 5.5403 \text{ s}), \end{aligned}$$

$$\begin{aligned} \log L_\gamma &= (0.488 \pm 0.636) \log B_{lc} & (10) \\ &+ (31.745 \pm 3.023), \\ &K = 0.235 \\ &(0.0016 \leq P \leq 0.0988 \text{ s}), \end{aligned}$$

$$\begin{aligned} \log L_\gamma &= (0.808 \pm 0.527) \log B_{lc} & (11) \\ &+ (31.698 \pm 1.974), \end{aligned}$$

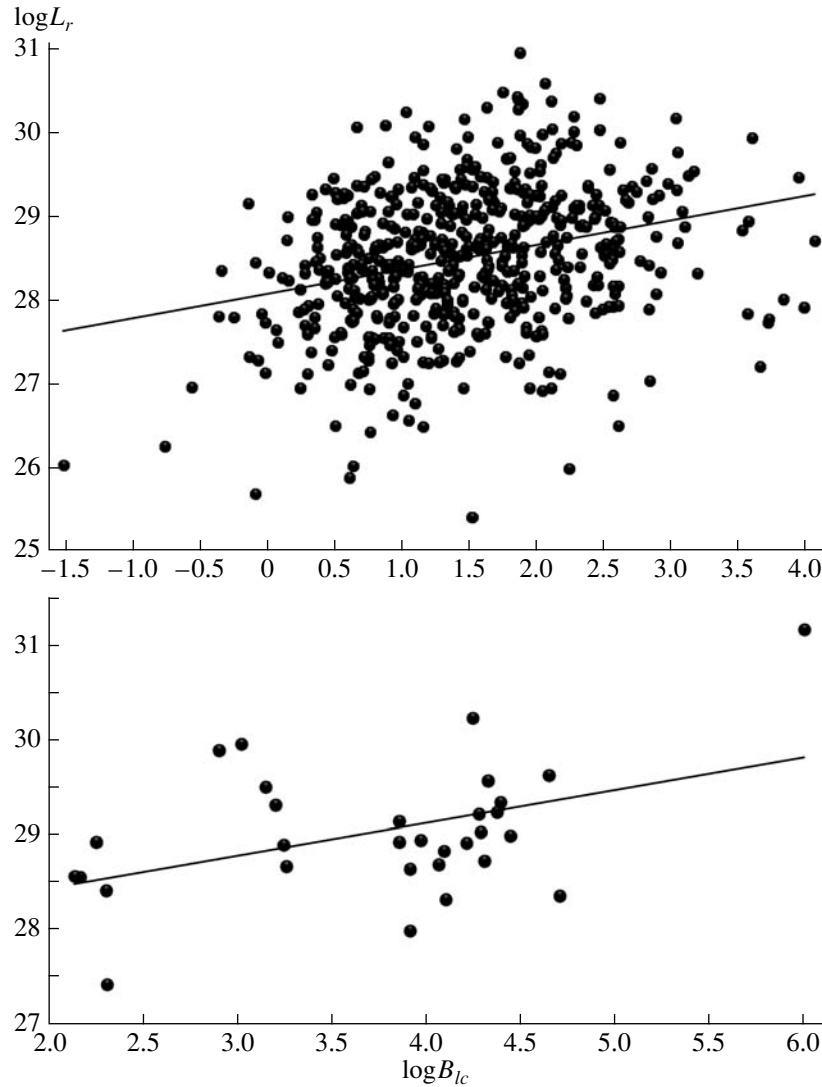


Fig. 5. Dependence of the radio luminosity on the magnetic field at the light cylinder for pulsars with $0.1014 \leq P \leq 8.5098$ s (upper; 561 pulsars) and with $0.0016 \leq P \leq 0.0649$ s (lower; 31 pulsars).

$$K = 0.510$$

$$(0.1024 \leq P \leq 0.4137 \text{ s}).$$

These dependences apparently testify to the generation of emission in these wavebands near the light cylinder; the large dispersion could be due to the range of inclinations of the magnetic moments to the rotational axes of the neutron stars, giving rise to magnetospheres of different extents in different pulsars (see, e.g., [8]). An $L(B_{lc})$ is expected, since appreciable pitch angles ψ can appear for the electrons radiating near the light cylinder [9], so that the synchrotron mechanism, for which the radiated power grows with the magnetic field, becomes dominant [10]:

$$P_{cr} = \frac{2e^4 B^2 \sin^2 \psi}{3m^2 c^3} \gamma^2. \quad (12)$$

It is of interest to investigate how the luminosity is related to the rate of rotational energy loss:

$$\frac{dE}{dt} = \frac{4\pi I \frac{dP}{dt}}{P^3}. \quad (13)$$

Figures 8–10 show that significant correlations between the luminosity and dE/dt , supporting the usual assumption that rotational energy is the basic source of energy for the observed radiation of radio pulsars in all wavebands. Moreover, it follows from these figures that the basic source of energy is the same for pulsars with long and short periods.

The fitted $L(dE/dt)$ dependences in the three wavebands are

$$\log L_r = (0.948 \pm 0.220) \log(dE/dt) \quad (14)$$

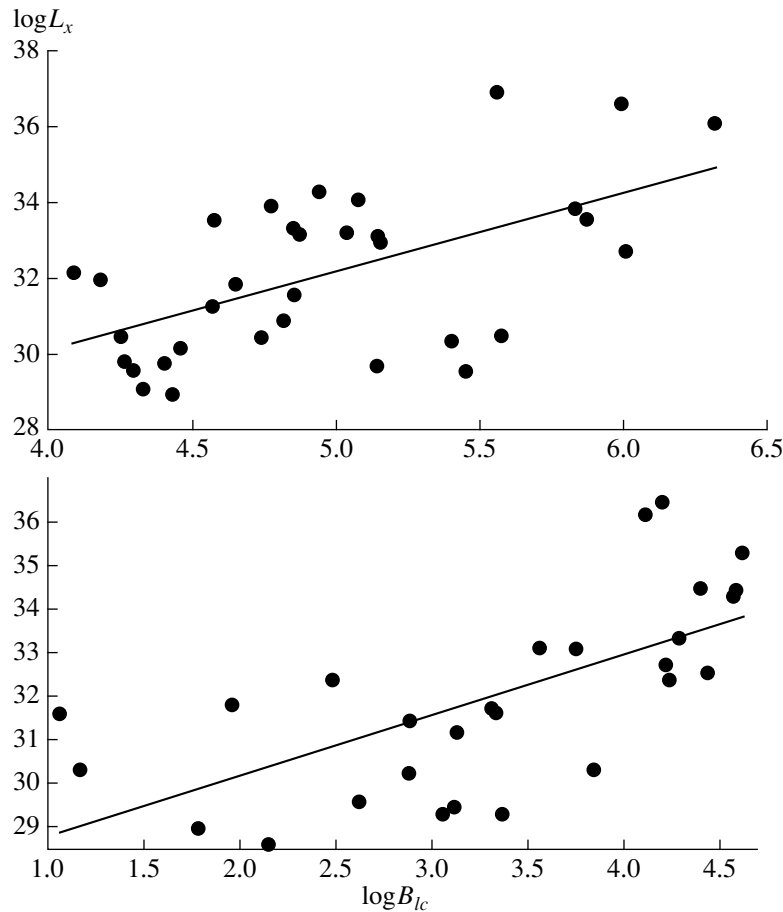


Fig. 6. Dependence of the X-ray luminosity on the magnetic field at the light cylinder for 33 pulsars with $P < 0.1$ s (upper) and 28 pulsars with $P > 0.1$ s (lower).

$$\begin{aligned}
 & - (2.850 \pm 7.287), \\
 & K = 0.573 \\
 & (0.0016 \leq P \leq 0.0649 \text{ s}),
 \end{aligned}$$

$$\begin{aligned}
 \log L_r &= (0.220 \pm 0.058) \log(dE/dt) & (15) \\
 & + (21.439 \pm 1.876), \\
 & K = 0.300 \\
 & (0.1014 \leq P \leq 8.5098 \text{ s}),
 \end{aligned}$$

$$\begin{aligned}
 \log L_x &= (1.146 \pm 0.194) \log(dE/dt) & (16) \\
 & - (8.577 \pm 6.898), \\
 & K = 0.838 \\
 & (0.0016 \leq P \leq 5.5404 \text{ s}),
 \end{aligned}$$

$$\begin{aligned}
 \log L_\gamma &= (0.657 \pm 0.134) \log(dE/dt) & (17) \\
 & + (11.051 \pm 4.727), \\
 & K = 0.728 \\
 & (0.0016 \leq P \leq 0.4137 \text{ s}).
 \end{aligned}$$

Figures 11–13 present the dependences of the efficiency of the transformation of rotational energy into radiation in the various wavebands on the pulsar period:

$$\eta = \frac{L}{dE/dt}. \quad (18)$$

Note that the gamma-ray luminosities taken from [6] were calculated assuming isotropic emission. For some objects, the derived efficiency of transforming dE/dt into L_γ exceeded unity [6, Fig. 9]. This could mean that it is necessary to take into account the directivity of the gamma-ray emission, or to revise the estimated distance to the source. Since it is not currently possible to analyze these factors, we excluded pulsars for which $L_\gamma > dE/dt$ when constructing the dependences of the energy-transformation efficiency on the period. The energy-transformation efficiency was essentially independent on the period in pulsars with short periods. In contrast, the energy-transformation efficiency for all three wavebands grows with increasing period in

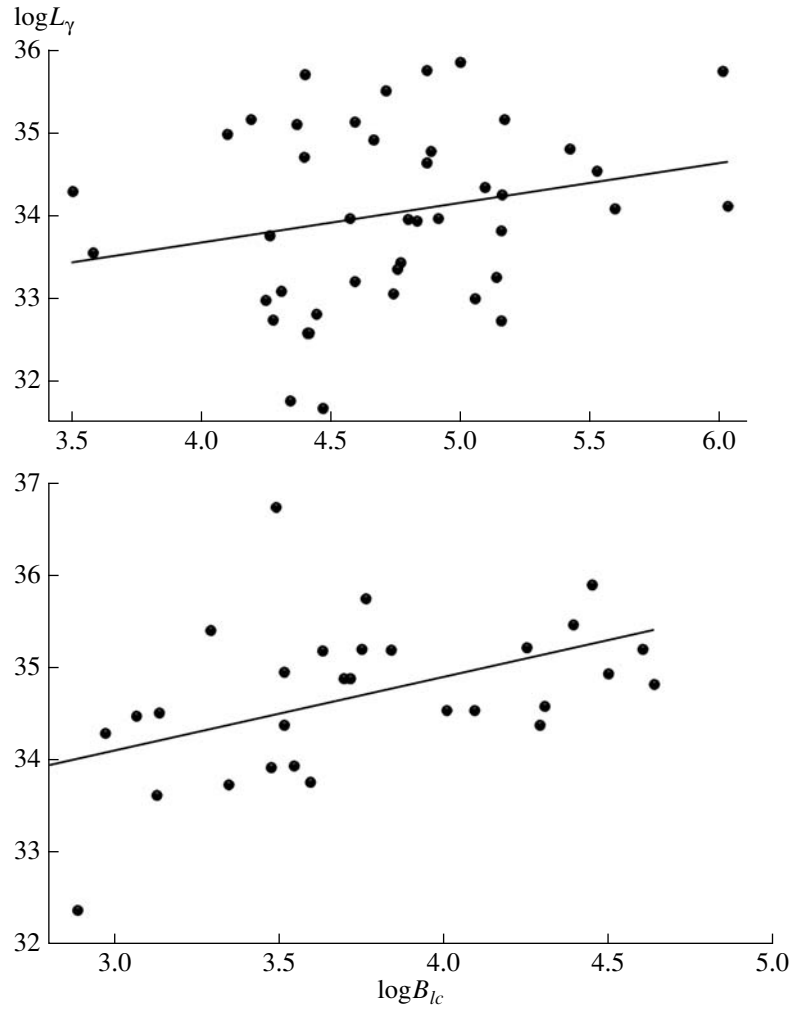


Fig. 7. Dependence of the gamma-ray luminosity on the magnetic field at the light cylinder for 30 pulsars with $P < 0.1$ s (upper) and 43 pulsars with $P > 0.1$ s (lower).

long-period pulsars:

$$\log \eta_r = (0.0247 \pm 0.487) \log P - (4.510 \pm 1.001),$$

$$K = 0.019$$

(31 objects with $0.0015 \leq P \leq 0.0649$ s),

$$\log \eta_r = (2.065 \pm 0.279) \log P - (3.352 \pm 0.098),$$

$$K = 0.524$$

($0.1014 \leq P \leq 8.5098$ s),

$$\log \eta_x = (0.245 \pm 0.600) \log P - (3.132 \pm 1.220),$$

$$K = 0.147$$

(33 objects with $0.0016 \leq P \leq 0.0914$ s),

$$\log \eta_x = (1.041 \pm 1.233) \log P - (2.538 \pm 0.8589),$$

$$K = 0.322$$

($0.1014 \leq P \leq 5.5404$ s),

$$\log \eta_\gamma = (0.142 \pm 0.987) \log P + (0.984 \pm 2.52),$$

$$K = 0.086$$

(15 objects with $0.0016 \leq P \leq 0.0058$ s),

$$\log \eta_\gamma = (0.588 \pm 1.033) \log P + (0.795 \pm 1.065),$$

$$K = 0.233.$$

To explain this difference, we must consider formula (3). As was shown in [7], the radio luminosities

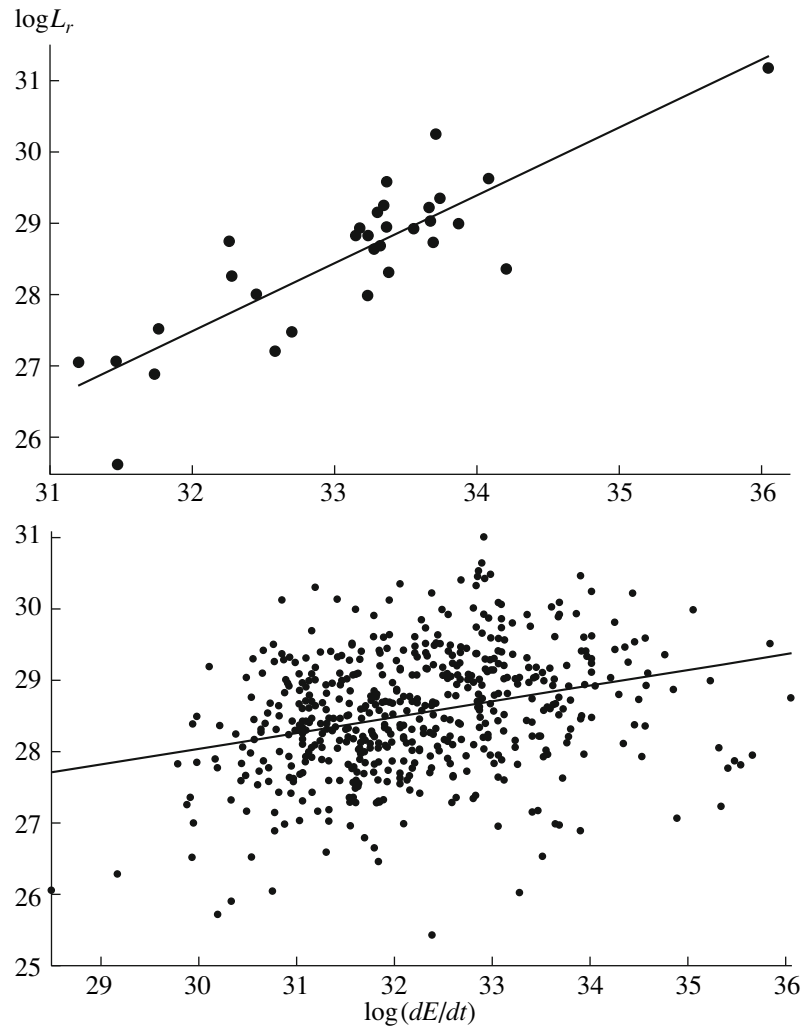


Fig. 8. Dependence of the radio luminosity on the rate of rotational energy loss for 31 pulsars with $P < 0.1$ s (upper) and 561 pulsars with $P > 0.1$ s (lower).

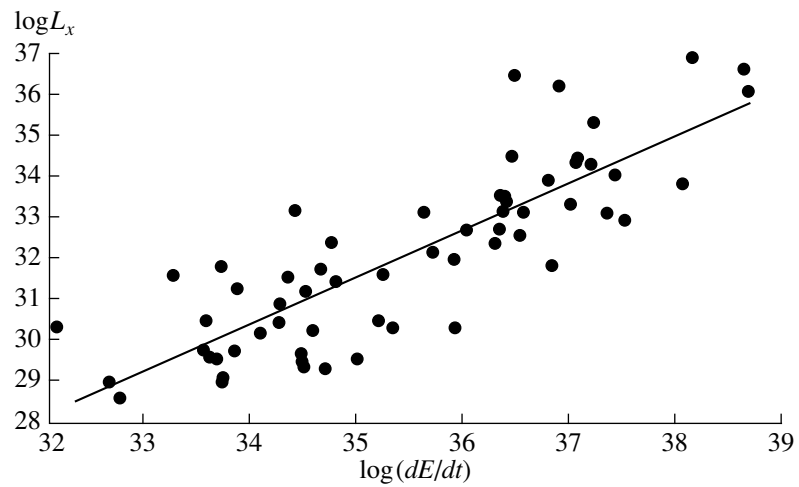


Fig. 9. Dependence of X-ray luminosity on the rate of rotational energy loss for 61 pulsars from both groups.

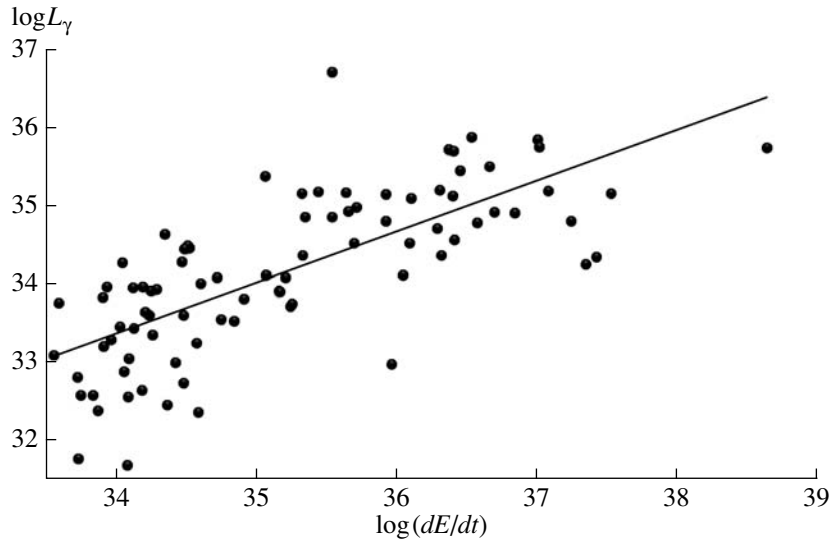


Fig. 10. Dependence of gamma-ray luminosity on the rate of rotational energy loss for 86 pulsars from both groups.

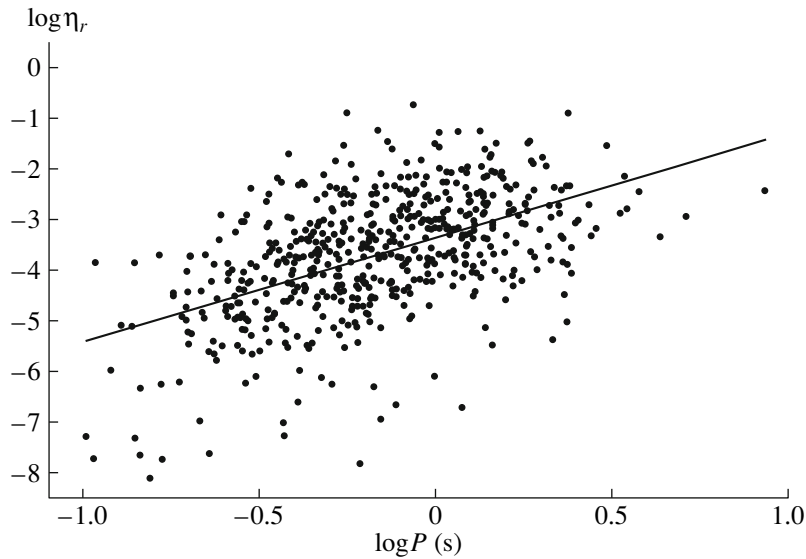


Fig. 11. Period dependence of the efficiency of the transformation of rotational energy into radio emission for 561 pulsars with $0.1014 \leq P \leq 8.5098$ s.

of short-period pulsars do not depend on the pulsar period, while L_r decreases approximately inversely proportional to P for long-period pulsars. Therefore, the ratio of L_r to dE/dt in these objects should grow as P^3 , assuming a magneto-dipolar model ($P \frac{dP}{dt} = \text{const}$). In order for η to be independent of P in short-period pulsars, dP/dt should be proportional to P^3 . This dependence can be traced for objects in the lower-left corner of the $dP/dt - P$ diagram (Fig. 3), and could be provided by a regime that is close to the propeller regime [11], for which $dP/dt \propto P^{7/3}$.

We also emphasize that the mean energy-transformation efficiency in the radio is approximately an order of magnitude higher for long-period than for short-period pulsars ($\langle \log \eta \rangle = -3.5$ and -4.5 , respectively).

The dependence of the width of the observed pulses on the period is important for our understanding of the radiation mechanism. Corresponding plots are shown in Fig. 14 for both groups of pulsars. As expected, the tangent of the inclination in the dependence $\log W_{10}(\log P)$ is equal to 0.5 for the main group of pulsars. For a dipolar field, the angular

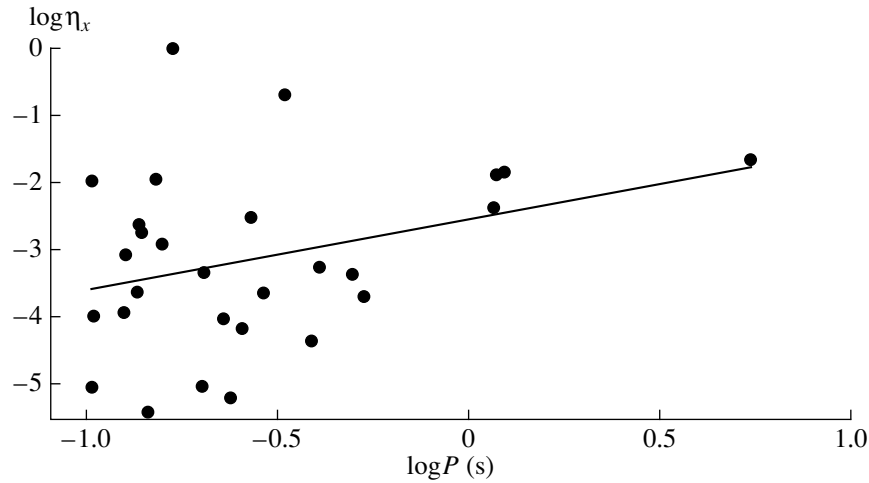


Fig. 12. Period dependence of the efficiency of the transformation of rotational energy into X-ray emission for 28 pulsars with $0.1014 \leq P \leq 5.5404$ s.

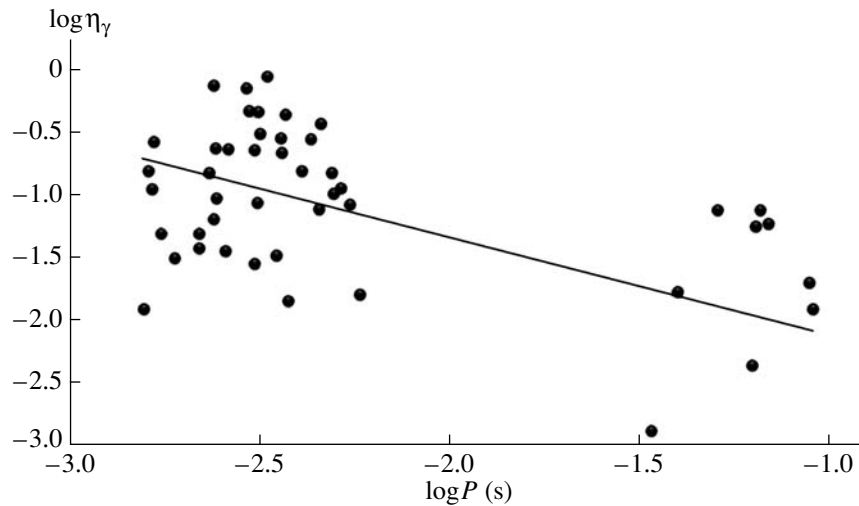


Fig. 13. Period dependence of the efficiency of the transformation of rotational energy into gamma-ray emission.

width of the radiation cone is approximately equal to

$$\theta = \left(\frac{2\pi r}{cP} \right)^{1/2}, \quad (25)$$

where r is the distance from the center of the neutron star. The width measured in units of time (W_{10} is given in milliseconds in Fig. 14) is equal to θP and is proportional to $P^{1/2}$. The radiation of pulsars with short periods is generated near the light cylinder, where relativistic effects become appreciable. In this case, it is predicted that the profile width (in units of

time) should be proportional to the period [12]:

$$W = \frac{AP}{2\pi} (1 - \beta^2)^{3/2}, \quad (26)$$

where $\beta = r/r_{lc}$, $A^2 = 2^{1/(2+\alpha)} - 1$ and α is the spectral index of the pulsar radiation.

The corresponding equations for $W_{10}(P)$ have the form

$$\log W_{10} = (0.960 \pm 0.196) \log P + (2.203 \pm 0.396), \quad (27)$$

$$K = 0.764 \text{ for } 69 \text{ objects with } 0.0016 \leq P \leq 0.0962 \text{ s,}$$

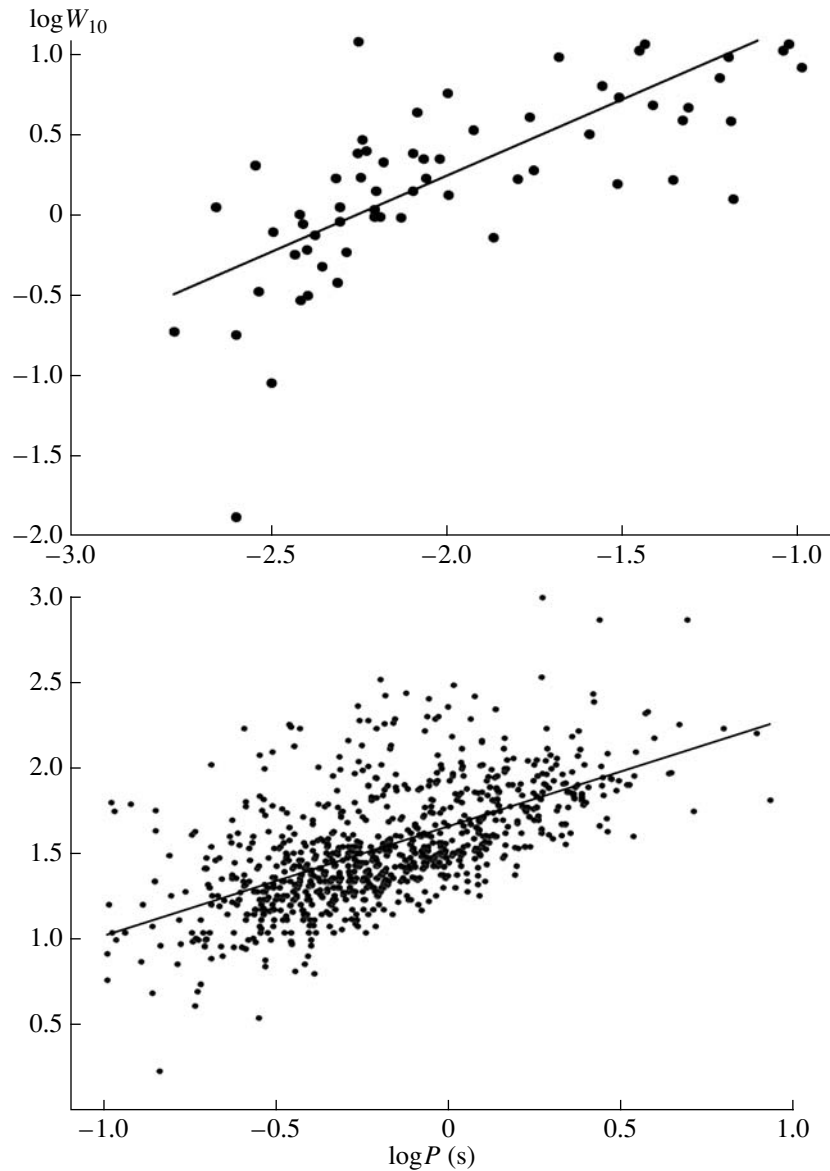


Fig. 14. Period dependence of the pulse width at the 10% level for pulsars with $P < 0.1$ s (upper) and with $P > 0.1$ s (lower).

$$\log W_{10} = (0.638 \pm 0.056) \log P + (1.666 \pm 0.021),$$

$K = 0.607$ for 854 objects with $0.1014 \leq P \leq 8.5098$ s.

3. CONCLUSIONS

1. We have carried out a comparative analysis of the dependences between various parameters (luminosities in various wavebands, the magnetic fields at the surface of the neutron star and near the light cylinder, the efficiency of the transformation of rotational energy into radiation, the pulse width) for two groups of pulsars, with periods $P < 0.1$ s and $P > 0.1$ s.

2. We have obtained linear least-squares fits to the various dependences plotted on logarithmic scales.

3. The radio and gamma-ray luminosities are not correlated with the surface magnetic fields of the neutron stars.

4. The X-ray luminosity is correlated with the surface magnetic field. This correlation is probably related to heating of the surface by inwardly directed positron flows.

5. The derived dependence of the X-ray and gamma-ray luminosities on the magnetic field at the light cylinder provides evidence for the generation of non-thermal radiation in these wavebands at the periphery of the magnetosphere by the synchrotron mechanism.

6. The luminosities in the various wavebands are significantly correlated with the rate of rotational energy loss.

7. The efficiency of the transformation of rotational energy into radiation does not depend on the period in pulsars with $P < 0.1$ s, and increases with increasing period in long-period pulsars.

8. The energy-transformation efficiency in the radio is an order of magnitude higher in long-period than in short-period pulsars.

9. The energy-transformation efficiency increases appreciably in the X-ray and gamma-ray.

10. The pulse widths of short-period pulsars increase more rapidly with period than those of pulsars with $P > 0.1$ s.

The division of pulsars in the space of their principal components into two clusters characterizing short- and long-period objects found in [1] has been confirmed by a detailed investigation of observed and calculated parameters for two groups of pulsars, with $P < 0.1$ s and $P > 0.1$ s. The radio emission of long-period pulsars is generated at distances substantially smaller than the radius of the light cylinder. Here, the main role is played by curvature radiation, which depends on the structure of the magnetic field, but not its strength. Therefore, we did not expect to find a dependence of the radio luminosity on the surface magnetic field. This group does display a correlation between the X-ray luminosity and B_s , apparently associated with thermal emission from the surface. The significant correlation between the gamma-ray luminosity and B_{lc} (in the absence of a correlation with B_s) provides evidence that this part of the spectrum is generated at the periphery of the magnetosphere (possibly in the outer gap, where the charge density is zero). In short-period pulsars, the energy is generated close to the light cylinder [4, Chapter IV, Section 6], making natural the absence of correlations between their luminosities and B_s and the presence of such correlations with B_{lc} .

Comparisons of a number of other parameters that are important for studies of pulsars (polarization, geometrical, and temporal) will be carried out in a separate study.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project 12-02-00661) and the Basic Research Program of the Presidium of the Russian Academy of Sciences “Non-stationary Phenomena in Objects of the Universe.” We thank M.V. Popov for a number of valuable comments.

REFERENCES

1. A. A. Loginov and I. F. Malov, *Astron. Rep.* **57**, 1004 (2013).
2. I. F. Malov and O. I. Malov, *Astron. Rep.* **41**, 56 (1997).
3. R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs, *Astron. J.* **129**, 1993 (2005).
4. I. F. Malov, *Radiopulsars* (Nauka, Moscow, 2004) [in Russian].
5. A. A. Abdo, M. Ajello, A. Allafort, L. Baldini, J. Ballet, G. Barbiellini, M. G. Baring, D. Bastieri, A. Belfiore, R. Bellazzini, B. Bhat-tacharyya, E. Bissaldi, E. D. Bloom, E. Bonamente, E. Bottacini, N. J. Brandt, J. Bregeon, M. Brigida, P. Bruel, R. Buehler, M. Burgay, T. H. Burnett, G. Busetto, S. Buson, G. A. Calian-dro, R. A. Cameron, F. Camilo, P. A. Caraveo, J. M. Casandjian, C. Cecchi, Ö. Çelik, E. Charles, S. Chaty, R. C. G. Chaves, A. Chekhtman, A. W. Chen, J. Chiang, G. Chiaro, S. Ciprini, R. Claus, I. Cognard, J. Cohen-Tanugi, L. R. Comin-sky, J. Conrad, S. Cutini, F. D’Ammando, A. de An-gelis, M. E. DeCesar, A. De Luca, P. R. den Hartog, F. de Palma, C. D. Dermer, G. Desvignes, S. W. Dig-gel, L. Di Venere, P. S. Drell, A. Drlica-Wagner, R. Dubois, D. Dumora, C. M. Espinoza, L. Falletti, C. Favuzzi, E. C. Ferrara, W. B. Focke, A. Franck-owiak, P. C. C. Freire, S. Funk, P. Fusco, F. Gargano, D. Gasparrini, S. Germani, N. Giglietto, P. Giommi, F. Giordano, M. Giroletti, T. Glanzman, G. God-frey, E. V. Gotthelf, I. A. Grenier, M.-H. Grondin, J. E. Grove, L. Guillemot, S. Guiriec, D. Hadasch, Y. Hanabata, A. K. Harding, M. Hayashida, E. Hays, J. Hessels, J. Hewitt, A. B. Hill, D. Horan, X. Hou, R. E. Hughes, M. S. Jackson, G. H., Janssen, T. Jögler, G. Jóhannesson, R. P. Johnson, A. S. John-son, T. J. Johnson, W. N. Johnson, S. John-son, T. Kamae, J. Kataoka, M. Keith, M. Kerr, J. Knödseder, M. Kramer, M. Kuss, J. Lande, S. Larsson, L. Latronico, M. Lemoine-Goumard, F. Longo, F. Loparco, M. N. Lovellette, P. Lubrano, A. G. Lyne, R. N. Manchester, M. Marelli, F. Mas-saro, M. Mayer, M. N. Mazziotta, J. E. McEnery, M. A. McLaughlin, J. Mehault, P. F. Michelson, R. P. Mignani, W. Mitthumsiri, T. Mizuno, A. A. Moi-seev, M. E. Monzani, A. Morselli, I. V. Moskalenko, S. Murgia, T. Nakamori, R. Nemmen, E. Nuss, M. Ohno, T. Ohsugi, M. Orienti, E. Orlando, J. F. Ormes, D. Paneque, J. H. Panetta, D. Par-ent, J. S. Perkins, M. Pesce-Rollins, M. Pierbat-tista, F. Piron, G. Pivato, H. J. Pletsch, T. A. Porter, A. Possenti, S. Rainò, R. Rando, S. M. Ransom, P. S. Ray, M. Razzano, N. Rea, A. Reimer, O. Reimer, N. Renault, T. Reposeur, S. Ritz, R. W. Romani, M. Roth, R. Rousseau, J. Roy, J. Ruan, A. Sar-tori, P. M. Saz Parkinson, J. D. Scargle, A. Schulz, C. Sgrò, R. Shannon, E. J. Siskind, D. A. Smith, G. Spandre, P. Spinelli, B. W. Stappers, A. W. Strong, D. J. Suson, H. Takahashi, J. G. Thayer, J. B. Thayer, G. Theureau, D. J. Thompson, S. E. Thorsett, L. Tibaldo, O. Tibolla, M. Tinivella, D. F. Torres,

- G. Tosti, E. Troja, Y. Uchiyama, T. L. Usher, J. Vandenbroucke, V. Vasileiou, C. Venter, G. Vianello, V. Vitale, N. Wang, P. Weltevrede, B. I. Winer, M. T. Wolf, D. L. Wood, K. S. Wood, M. Wood, and Z. Yang, *Astrophys. J. Suppl. Ser.* **208**, 17 (2013).
6. W. Becker and J. Trumper, *Astron. Astrophys.* **326**, 682 (1997).
 7. I. F. Malov and O. I. Malov, *Astron. Rep.* **50**, 483 (2006).
 8. I. F. Malov, *Astron. Rep.* **58**, 139 (2014).
 9. I. F. Malov and G. Z. Machabeli, *Astron. Rep.* **46**, 684 (2002).
 10. G. Bekefi, *Radiation Processes in Plasmas* (Wiley, New York, 1966; Mir, Moscow, 1971).
 11. A. F. Illarionov and R. A. Sunyaev, *Astron. Astrophys.* **39**, 185 (1975).
 12. V. V. Zheleznyakov, *Astrophys. Space Sci.* **13**, 87 (1971).

Translated by D. Gabuzda