ISSN 1063-7710, Acoustical Physics, 2017, Vol. 63, No. 6, pp. 744–750. © Pleiades Publishing, Ltd., 2017.
Original Russian Text © S.A. Titov, V.M. Levin, Yu.S. Petronyuk, 2017, published in Akusticheskii Zhurnal, 2017, Vol. 63, No. 6, pp. 692–699.

ACOUSTIC SIGNAL PROCESSING.
COMPUTER SIMULATION

Processing an Acoustic Microscope's Spatiotemporal Signal to Determine the Parameters of an Isotropic Layer

S. A. Titov[a, b] *, V. M. Levin[a], and Yu. S. Petronyuk[a, b]
[a]*Institute of Biochemical Physics, Russian Academy of Sciences, Moscow, 119334 Russia*
[b]*Scientific and Technological Center of Unique Instrumentation, Russian Academy of Sciences, Moscow, 117342 Russia*
**e-mail: sergetitov@mail.ru*
Received October 24, 2016

Abstract—This paper presents a method for measuring the thickness and velocities of body waves and the density of an isotropic layer by a pulse scanning acoustic microscope. The method is based on recording the microscope signal as a function of the displacement magnitude of the focused ultrasonic transducer along its axis perpendicular to the sample surface and on the decomposition of the recorded 2D spatiotemporal signal into the spectrum of plane pulse waves. The velocities of the longitudinal and transverse waves and the layer's thickness are calculated from the relative delays of the components of the spectrum of plane waves reflected from the surfaces of the layer and the density is computed by the amplitudes of these components. An experimental investigation of a test sample in the form of a glass plate carried out in the 50-MHz range shows that the error in measuring the thickness and velocities of body waves does not exceed 1% and the density measurement error does not exceed 10%.

Keywords: acoustic microscope, mechanical scanning, spatiotemporal signal, velocity of body waves, density, layer thickness

DOI: 10.1134/S106377101706015X

1. INTRODUCTION

An acoustic microscope uses a high-frequency focused ultrasonic echo-pulse transducer [1]. Acoustic images are generated in the microscope by the mechanical scanning of the transducer in the plane parallel to the surface of the sample. The acoustic images make it possible to visualize the spatial distribution of the acoustic heterogeneities by the investigated volume. At the same time, in practice, in the investigated objects, there are often regions in the form of homogeneous layers, the determination of the unknown thicknesses and acoustic parameters of which is an important task.

It is possible to determine the velocities of the longitudinal and transverse waves in the material of the layer, if its thickness is known, by the relative delay of the pulses reflected from the boundaries of the layer and received by the microscope [2]. It is also possible to determine the layer's thickness if the values of the speed of sound in it are known. However, focused ultrasonic beams are used in the acoustic microscope; thus, the shape of the reflected pulses depends on the distance between the transducer and the sample and on the parameters of the layer itself, which leads to a less accurate measurement of the delay. The method based on the digital correction of the shape of the first pulse makes it possible to increase the measurement accuracy [3]. However, this method does not make it possible to simultaneously determine the velocity and thickness of the layer.

The velocity of longitudinal waves and the layer's thickness can be determined by measuring the difference between the positions of the transducer when it is displaced perpendicular to the surface of the sample in the region of the maximum echo-pulse values and the upper and lower boundaries of the layer [4–6]. However, the relationships that make it possible to find the velocity and thickness by the displacement of the transducer and relative delay of the pulses are obtained in a paraxial approximation, which leads to significant errors in the study of layers with high values of the speed of sound. At the same time, it is often difficult to accurately measure the position of the signal's maxima because of the low signal-to-noise ratio for signals reflected from the lower surface of the layer. This method is also inapplicable for determining the velocity of a transverse wave, since in this case the paraxial approximation is not valid.

In this paper, a method is proposed to measure the velocities of the longitudinal and transverse waves in the material layer and its thickness, as well as the density. The method is based on recording the microscope's response as a function of time and the displacement magnitude of the transducer along its axis in a direction perpendicular to the surface of the sample and processing the recorded spatiotemporal signal by decomposing it into the spectrum of responses of pulse plane waves.

2. THEORETICAL ANALYSIS OF THE METHOD

In the considered method, the focused broadband transducer *1* and isotropic sample *2* with thickness d are placed in an immersion liquid with a known velocity of ultrasonic waves C_W (Fig. 1). The transducer can move perpendicular to the sample plane along the z axis. When the transducer is excited with a pulse $v_p(t)$, several responses are observed in the received signal. The response D is generated by the wave reflected from the upper surface of the sample. Responses L and T are generated by the longitudinal (solid lines) and transverse (dashed lines) waves that propagate in the layer and are reflected from the bottom surface. The response of the mixed mode LT is generated by combinations of longitudinal and transverse waves propagating in the layer in opposite directions.

Assuming that the measuring system is linear, is spatially and temporally invariant, its output signal as a displacement function of the focus z and time t_0 can be represented as a superposition of the responses of plane harmonic waves [7]:

$$v(z,t_0) = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} H(k_z,\omega)R(k_z,\omega) \times \exp(i2k_z z - i\omega t_0)\,dk_z d\omega, \tag{1}$$

where ω is the frequency and k_z is the wave vector component in the liquid related to the angle of incidence of the wave θ by the relation $k_z = \omega C_W^{-1}\cos\theta$. The function $R(k_z, \omega)$ denotes the reflection coefficient of plane harmonic waves from the liquid–sample interface, and $H(k_z, \omega)$ is the effective transfer function of the microscope that depends on the properties of the transducer and the characteristics of the transmit-receive path. The factor $\exp(2ik_z z)$ takes into account the phase advance acquired by a plane wave when propagating in an immersion liquid from the focal plane of the transducer to the sample and back. By introducing the notation for the double scan coordinate $\tilde{z} = 2z$, the signal $v(\tilde{z},t)$ can be expressed as a 2D inverse Fourier transform $\mathcal{F}^{-1}_{k,\omega}\{\}$ with respect to the variables (k_z, ω) of the product of the transfer function and the reflection coefficient.

The delay of the reflected pulses is determined mainly by the distance $\tilde{z}$ traveled by the wave in the liquid. Therefore, it seems rational to compensate this delay by considering the signal as a function of time $t = t_0 - \tilde{z}C_W^{-1}$.

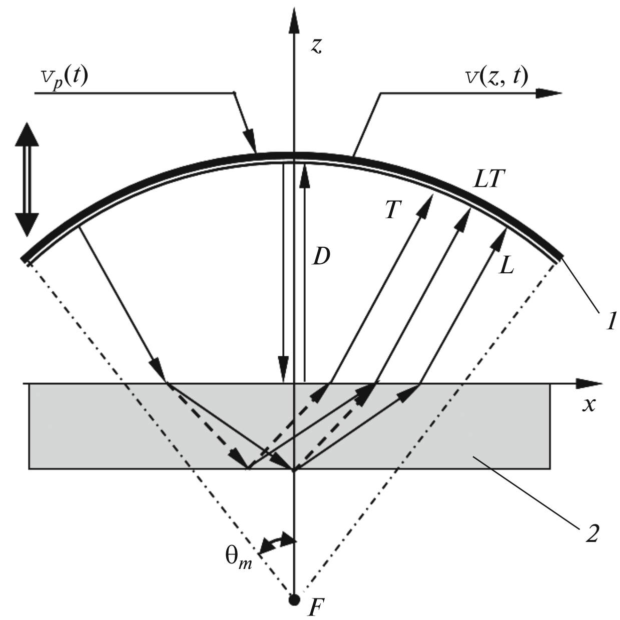

Fig. 1. Measurement design: (*1*) focused transducer and (*2*) test sample.

Assume that the duration of ultrasonic pulses is sufficiently short so that the responses reflected from the upper and lower boundaries of the layer are not superimposed by the delay time. Then the reflection coefficient and the recorded spatiotemporal signal can be represented as a sum of the components corresponding to the waves $\gamma = D, L, LT, T$:

$$R(k_z,\omega) = \sum R_\gamma(k_z,\omega), \tag{2}$$

$$v(\tilde{z},t) = \sum v_\gamma(\tilde{z},t). \tag{3}$$

The term $R_D(k_z, \omega)$ is equal to the reflection coefficient of a plane harmonic wave from the interface between the liquid with a half-space, the acoustic parameters of which coincide with the layer parameters. The analytical expression for the reflection coefficient is well known [7]. While passing the layer, the plane harmonic waves acquire a phase delay that depends on the vertical component of the wave vector in the material:

$$\beta_{L(T)} = \sqrt{\omega^2 C_{L(T)}^{-2} - \omega^2 C_W^{-2} + k_z^2}, \tag{4}$$

where C_L and C_T are the velocities of the longitudinal and transverse waves in the material, respectively.

Neglecting the ultrasound absorption in the layer, the components of the coefficient $R_\gamma(k_z, \omega)$ $\gamma = L, LT, T$ for the waves that passed through the layer can be written as

$$\begin{aligned} R_L(k_z,\omega) &= T_{LL}W_{LL}T_{LL}^*\exp(i2d\beta_L), \\ R_{LT}(k_z,\omega) &= T_{LL}W_{LT}T_{TL}^*\exp(id(\beta_L + \beta_T)), \\ R_T(k_z,\omega) &= T_{LT}W_{TT}T_{TL}^*\exp(i2d\beta_T), \end{aligned} \tag{5}$$

where T_{LL}, T_{TL}, T_{LT} and T^*_{LL}, T^*_{TL}, T^*_{TL} are the transmission or transformation coefficients of the wave modes on the upper surface of the layer when propagating downward and upward, respectively, and W_{LL}, W_{TT}, and W_{LT} are the reflection or transformation coefficients on the lower surface [8]. The maximum aperture angle of the transducer θ_m is usually selected to be smaller than the critical angle for the longitudinal wave in the investigated materials. In this case, for the angles of incidence $\theta < \theta_m$, all the coefficients T and W are independent of frequency and are real.

Thus, taking into account the introduced assumptions, the longitudinal wave response is written as

$$v_L(\tilde{z},t) = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} H(k_z,\omega) T_{LL} W_{LL} T^*_{LL} \times \exp\left\{ i\left(k_z - \frac{\omega}{C_W}\right)\tilde{z} + 2d\beta_L - i\omega t\right\} dk_z d\omega. \quad (6)$$

The responses of the components $\gamma = LT$ and T are represented by similar relations. By substituting the variable

$$k = k_z - \omega C_W^{-1} = \omega C_W^{-1}(\cos\theta - 1) \quad (7)$$

this equation is reduced to the inverse Fourier transform with respect to the variables (k, ω). Thus, the 2D spectral density of the signal is

$$V_L(k,\omega) = F_{\tilde{z},t}\{v_L(\tilde{z},t)\} = H\left(k + \omega C_W^{-1}, \omega\right) \times T_{LL} W_{LL} T^*_{LL} \exp\left(i2d\sqrt{\omega^2 C_L^{-2} + 2k\omega C_W^{-1} + k^2}\right). \quad (8)$$

For the other components of the signal $v_\gamma(\tilde{z},t)$ and their spectra $V_\gamma(k,\omega)$, similar equations can be written.

The recorded signal $v(z, t)$ in the spectrum of plane pulse waves can be decomposed by integrating it in the spatiotemporal region along the line $t = \tau + s\tilde{z}$:

$$W(s,\tau) = \Re\{v(\tilde{z},t)\} = \int_{-\infty}^{\infty} v(\tilde{z}, \tau + s\tilde{z}) d\tilde{z}. \quad (9)$$

The integration of the values of the spatiotemporal signal along the line leads to the accumulation of the response of the plane wave, the propagation direction of which is specified by the slope coefficient s and to the suppression of the responses of the plane waves propagating in other directions. The time parameter τ specifies the delay in the response of the plane waves propagating in one direction. From a mathematical point of view, this transformation can be interpreted as the Radon transformation widely used in tomography and seismology [9]. It was also used in measuring layer parameters based on the recording and processing of the ultrasonic array signals [10]. The spectrum of plane waves $W(s, \tau)$ can be represented as the inverse Fourier transform with respect to the frequency ω of the values of the spectral density of the signal $V(k,\omega)$, taken in the region (k, ω) along the straight line $k = \omega s$ [9, 10]:

$$W(s,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega s, \omega)\exp(-i\omega)\, d\omega. \quad (10)$$

Thus, the parameter s is associated with the propagation angle θ and the vertical slowness vector component s_z of the plane wave in the liquid by the following equations:

$$s = C_W^{-1}(\cos\theta - 1) = s_z - C_W^{-1}. \quad (11)$$

Based on Eq. (10), the spectral decomposition of a wave reflected from the upper surface of the layer is as follows:

$$W_D(s,\tau) = F_\omega^{-1}\left\{R_D H\left(\omega\left(s + C_W^{-1}\right), \omega\right)\right\}. \quad (12)$$

Similarly, the response spectrum formed by the reflection of the longitudinal wave from the lower surface of the layer can be written as

$$W_L(s,\tau) = F_\omega^{-1}\left\{T_{LL} W_{LL} T^*_{LL} H\left(\omega\left(s + C_W^{-1}\right), \omega\right) \times \exp\left(i2d\omega\sqrt{C_L^{-2} + 2sC_W^{-1} + s^2}\right)\right\}. \quad (13)$$

Thus, for each s the spectrum $W_L(s, \tau)$ is a weakened and delayed copy of $W_D(s, \tau)$:

$$W_L(s,\tau) = \xi_L(s) W_D(s,\tau) * \delta(\tau - \tau_L), \quad (14)$$

where δ is the delta function, and the asterisk denotes a convolution with respect to the time variable. The coefficient

$$\xi_L(s) = \frac{T_{LL} W_{LL} T^*_{LL}}{R_D} \quad (15)$$

is equal to the ratio of the amplitudes of the plane waves reflected from the lower and upper surfaces of the layer, and the quantity

$$\tau_L(s) = 2d\sqrt{C_L^{-2} + 2sC_W^{-1} + s^2} \quad (16)$$

determines their relative time shift. For the longitudinal-transverse mode of the spectrum $W_{LT}(s, \tau)$, there is a similar equation, where the amplitude coefficient and the relative delay are, respectively,

$$\xi_{LT}(s) = \frac{T_{LL} W_{LL} T^*_{TL}}{R_D}, \quad (17)$$

$$\tau_{LT}(s) = \tau_L(s)/2 + d\sqrt{C_T^{-2} + 2sC_W^{-1} + s^2}. \quad (18)$$

In the proposed method, the spectrum of plane waves $W(s, \tau)$ is calculated, individual responses $W_\gamma(s, \tau)$ are selected in this spectrum, and relative delays $\tau_\gamma(s)$ and amplitude coefficients $\xi_\gamma(s)$ as functions of the parameter s are found by the measured spatiotemporal signal $v(\tilde{z},t)$. Since the form of the spectral responses is the same for a fixed value of s, it is easy to find the relative delays and amplitudes. Next, using nonlinear estimation methods, the thickness and speed of the ultrasound are determined based on the obtained dependences $\tau_\gamma(s)$, and based on the dependences $\xi_\gamma(s)$, it is possible to estimate the density if the values of the velocity in the material are known.

3. EXPERIMENTAL

A focused broadband ultrasonic transducer with a nominal central frequency of the piezoelectric element of 50 MHz was used in the experiment. Water at the temperature of 22 ± 1°C was used as the immersion liquid. The focal length and the half aperture angle of the transducer were $F = 13$ mm and $\theta_m = 11°$, respectively. Excitation of acoustic pulses and reception of signals reflected from the sample was carried out by an ultrasonic data acquisition system with the continuous motion of the transducer along the z axis. The received analog signals were digitized with a 12-bit resolution at a sampling rate of 500 MHz.

Figure 2 shows a spatiotemporal signal $v(z, t)$ written for a glass plate as a halftone image. The plate thickness measured with a set of end measures and a dial gauge with a scale interval of 1 μm was 1.155 ± 0.003 mm. The longitudinal wave velocity measured in the echo-pulse mode using a flat contact transducer (15 MHz) was 5800 ± 30 m/s. In the recorded data, the response $v_D(z, t)$ reflected from the upper surface of the plate and the responses $v_L(z, t)$ and $v_{LT}(z, t)$ formed by the reflections of the waves from the lower boundary of the layer are distinguished. Since for the represented data the delay $2zC_W^{-1}$, obtained by a normally incident wave in the immersion liquid was subtracted, these responses have a predominantly horizontal orientation in the figure.

It can be seen that the signal $v_D(z, t)$ reaches its maximum when the focus is on the surface of the sample $z \approx 0$, and the phase of the signal changes quite sharply by approximately 90° as it passes through this focal region. In addition, when the focus moves away from the surface in the region $z > 0$, an additional response, which is ahead of the main pulse, is observed. This response is generated by a boundary wave with the source at the edge of the focused transducer [11].

Figure 3 shows the amplitude of the 2D spectrum of this response $V_D(k, \omega)$. Since the data are presented depending on the parameter k, the spectrum is in the range of its negative values limited approximately by the straight lines OA and OB. Along the vertical line OA, there are spatiotemporal signal spectrum components normally incident on the surface of the sample. The straight line OB corresponds to components with an angle of incidence of $\theta \approx 10°$. This value is close to the aperture angle of the used transducer determined by its design features. The phase of the spectrum $V_D(k, \omega)$ (not shown in the figure) is a smooth function in the region where the amplitude has a significant value.

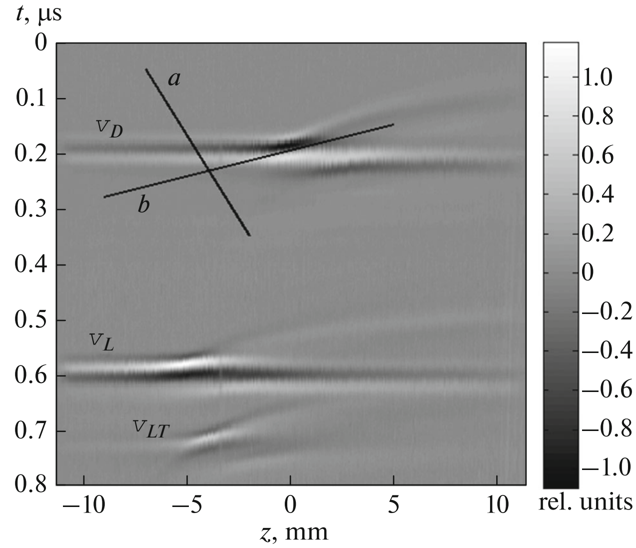

Fig. 2. Signal $v(z, t)$ recorded for glass plate. Amplitude of responses v_L, v_{LT} is tripled.

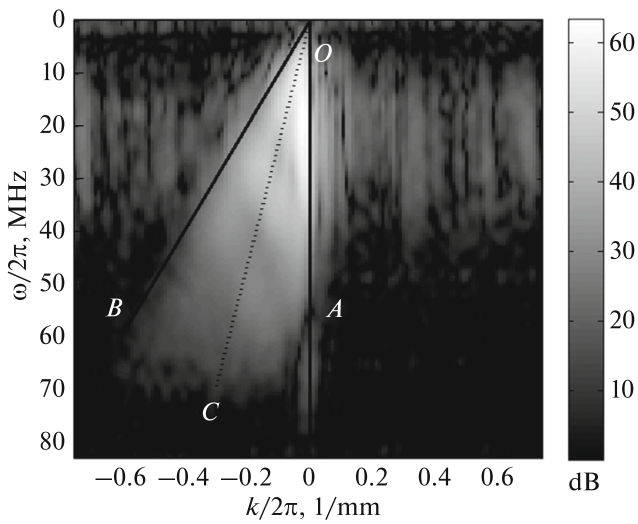

Fig. 3. Spectral density $|V_D(k, \omega)|$.

The signal $v_L(z, t)$ has a structure similar to the considered response $v_D(z, t)$ (Fig. 2). However, it has a large delay due to the propagation of waves through the layer, and its maximum is achieved at a negative value of z, when the converging ultrasonic beam is focused on the lower surface of the layer. Similarly, the response of the longitudinal-transverse mode $v_{LT}(z, t)$ has a maximum at negative z. However, since normal-incidence waves do not participate in its formation, the response amplitude decreases rapidly when moving away from the maximum point.

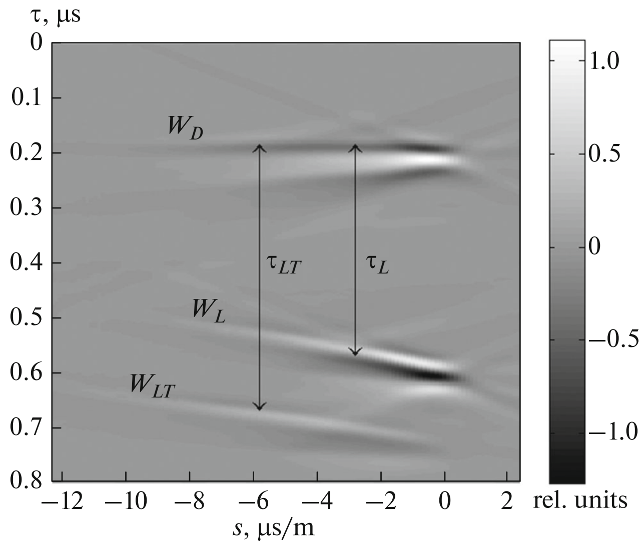

Fig. 4. Spectrum of plane waves $W(s, \tau)$. Amplitude of responses W_L, W_{LT} is tripled.

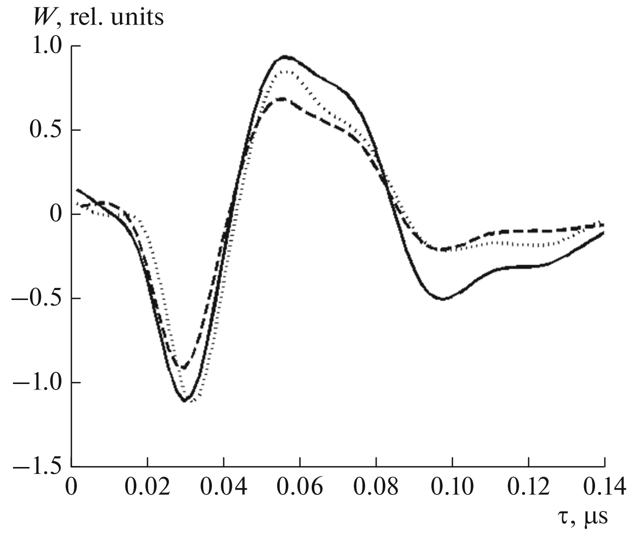

Fig. 5. Responses W_D, W_L (···), and W_{LT} (---) for $s = -4$ μs/m. Pulses W_L and W_{LT} are tripled and inverted. Their delays are reduced by 0.356 and 0.5 μs, respectively.

The result of the decomposition of the recorded signal into the plane wave spectrum $W(s, \tau)$ by directed summation in accordance with (9) is shown in Fig. 4. For $s > 0$, the straight line along which the signal is integrated (for example, line *a*) intersects the alternating response v_D, which causes the result to be small. For $s \approx 0$, the straight line is located approximately parallel to the constant phase lines, which leads to a significant integration result. For $s < 0$, the integration yields a near-zero result in regions remote from the focus. In the focus region, the amplitude and phase of the signal undergo sharp changes. Constant phase lines are inclined. The integration of the signal along these lines (for example, *b*) gives a significant response $W_D(s, \tau)$ with the delay τ approximately equal to the delay of the response $v_D(0, t)$, while remaining constant at different slope angles determined by the parameter *s*.

The generation features of the response $W(s, \tau)$ can also be illustrated by the consideration in the region (k, ω) (Fig. 3). Based on (10), the time spectrum $W(s, \tau)$ is determined by the values of the spectrum of the measured spatiotemporal signal $V_D(k, \omega)$ that lie on the *OC* lines and that satisfy the equation $k = s\omega$. Thus, the experimental response $W(s, \tau)$ is in the region of negative values of the parameter *s*, which agrees with theoretical representation (11).

Responses W_D, W_L, and W_{LT} are determined by the transfer function of the measuring system *H*. Therefore, for one value of the parameter *s*, they have the same shape but different amplitudes and delays (Fig. 5). The amplitudes of the responses W_D and W_L take the maximum values in the case of the normal incidence of the plane waves' spectral components on the surface of the sample ($s = 0$). Transverse waves are involved in the W_{LT} response generation. Thus, its amplitude is zero at normal incidence, and the maximum is shifted toward smaller values of parameter *s*.

The dependences $\tau_L(s)$ and $\tau_{LT}(s)$ were measured in order to determine thickness *d* and the values of the velocities C_L and C_T by the obtained spectra. The delays were determined by the position of the extrema of the correlation functions using the time variable of the W_D response with the W_L and W_{LT} responses. First, the parameters *d* and C_L were nonlinearly estimated using model equation (16). Then, using the obtained values of these quantities, the velocity of the transverse wave C_T was found from Eq. (18). Multiple recording of the signal $v(z, t)$ with subsequent processing by the described method made it possible to obtain the statistical data given in Table 1. It can be seen that the obtained values are in agreement with the reference data and the results of independent measurements, and the accuracy of the measurements of the thickness and velocities of the body waves can be estimated by the proposed method at a level of up to 1%.

It follows from Eq. (14) that the amplitude of the spectral components of plane waves is determined not only by the amplitude of the system response but also by the coefficients of reflection, transmission, and mode transformation at the boundaries of the layer. Figure 6 shows the relative amplitudes of the responses of the longitudinal mode $\xi_L(s)$ and the longitudinal-transverse mode $\xi_{LT}(s)$ measured by the $W(s, \tau)$ spectrum (Fig. 4). If the ultrasonic absorption in the layer can be neglected, these amplitudes depend on *d*, C_L, C_T, and C_W, as well as on the layer material's density ρ and the immersion liquid (water). Assuming that *d*, C_L, and C_T were found earlier by delays and the prop-

Table 1. Measurement results

Parameter	Measured values	Known and reference data
Thickness d, mm	1.15 ± 0.01	1.155 ± 0.003
Longitudinal wave velocity C_L, m/s	5830 ± 45	5800 ± 30 (measured by echo-pulse method) 5770 glass, sheet [12]
Transverse wave velocity C_T, m/s	3450 ± 25	3430 glass, sheet [12]
Density ρ, g/cm^3	2.6 ± 0.2	2.51 glass, sheet [12]

erties of water are known, it is possible to determine the density ρ by the dependences $\xi_L(s)$ and $\xi_{LT}(s)$.

The minimum value of the following form was sought in order to determine the density:

$$\int\left[\left(\xi_L-\xi_L^*\right)^2+\left(\xi_{LT}-\xi_{LT}^*\right)^2\right]ds; \qquad (19)$$

this shows the proximity of the experimental and model data. The model values $\xi_L^*(s, \rho)$ and $\xi_{LT}^*(s, \rho)$ were calculated by the well-known analytical expressions for the coefficients of reflection, transmission, and transformation of the modes at the boundaries of the half-spaces [8]. Residual (19) was calculated in the range of $-9 < s < -1$ μs/m, in which the experimental data are the least distorted. The relative amplitude coefficients ξ_L^* and ξ_{LT}^* calculated for the found density value ρ are shown in Fig. 6. The relative error in estimating the density is about 8% (see Table 1), and its average corresponds to the published value.

4. MEASUREMENT ERROR ANALYSIS

The following should be singled out among the factors affecting the accuracy of the method: the boundedness of the recording interval of the initial spatiotemporal signal, the error in the transducer positioning, the variations of immersion liquid parameters, the magnitude of the angular aperture of the focused transducer, the interference in the form of radioelectronic noise, and the parasitic rereflections of ultrasonic waves.

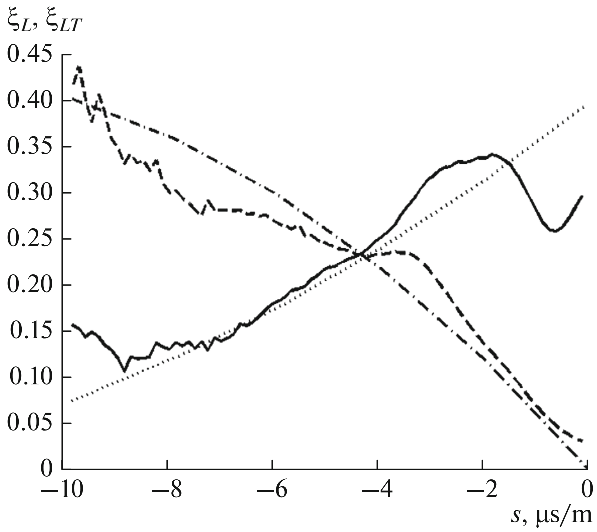

Fig. 6. Relative response amplitudes: measured ξ_L (–), ξ_{LT} (---) and calculated ξ_L^*(···), ξ_{LT}^*(-·-·).

The signal detection range is limited by $z_m \approx F\cos(\theta_m)$ when the transducer is displaced toward the sample. Then, for small s and τ that are approximately equal to the delays of the responses v_D and v_L, the integration line can intersect the boundary of the recording interval of the signal $z = z_m$ ($z = -z_m$) beyond which there is no signal. The distortions in the integration result caused by an abrupt signal step at the boundaries take place at $|s| < T/2z_m$, where T is the response time. For the experimental data (Fig. 2), the estimate of the size of this region is $|s| < 1.5$ μs/m. In this region, there is a strong deviation of the amplitude coefficient ξ_L from the expected one (Fig. 6). Therefore, it was excluded when finding the parameters of the layer. It should be noted that the response v_{LT} is localized. The amplitude coefficient ξ_{LT} is small for small s. Thus, the distortions of the considered type are uncharacteristic for it.

It is possible to use the approximation of model equation (16) to quantitatively estimate the error in determining the speed of sound and the thickness of the investigated layer. For small values of the parameter s, the approximate dependence

$$\tau_L(s) \approx \frac{2d}{C_L}\left(1+sC_W^{-1}C_L^2\right) = a + bs \qquad (20)$$

is linear, which is confirmed by the experimental observations (Fig. 4). The desired speed C_L and thickness d are solutions of the system of equations

$$\begin{cases} a = 2dC_L^{-1}, \\ b = 2dC_LC_W^{-1}, \end{cases} \qquad (21)$$

where the coefficients a and b are determined from the experimental dependence of $\tau_L(s)$. The solution of system (21) is as follows

$$C_L = \sqrt{bC_Wa^{-1}}, \quad d = \sqrt{C_Wab}/2. \qquad (22)$$

Let the speed of sound in water be known with an error ΔC_W and the error in determining the scanning coordinate increase linearly with a displacement from the focus: $\Delta z = \eta z$. Then the variations in the speed of sound in the water lead to a time error in the recording of a signal linearly dependent on the spatial coordinate:

$$\Delta t = \frac{2z}{C_W} - \frac{2z}{C_W - \Delta C_W} \approx \frac{2z\Delta C_W}{C_W^2}; \quad (23)$$

and, consequently, to a shift of the spectrum of plane waves with respect to the parameter s by the quantity $\Delta C_W C_W^{-2}$. The stretching of the measured spatiotemporal signal by a factor of $(1+\eta)$ with respect to the z coordinate leads to a contraction of the spectrum by a factor of $(1-\eta)$. Thus, taking the errors into account, model Eq. (20) becomes

$$\tau_L = a + b\left(s(1-\eta) - \frac{\Delta C}{C_W^2}\right). \quad (24)$$

Errors in determining coefficients a and b (21) can be estimated as follows:

$$\Delta a = b\frac{\Delta C_W}{C_W^2}, \quad \Delta b = b\eta. \quad (25)$$

Then the relative error in determining the velocity δC can be expressed in terms of the partial derivatives of Eq. (22) with respect to the parameters a and b:

$$\delta C = \frac{1}{C_L}\left(\left|\frac{\partial C_L}{\partial a}\right|\Delta a + \left|\frac{\partial C_L}{\partial b}\right|\Delta b\right). \quad (26)$$

Taking into account relations (21), the error δC is

$$\delta C = \frac{1}{2}\left(\frac{C_L^2}{C_W^2}\frac{\Delta C_W}{C_W} + \eta\right). \quad (27)$$

Similarly, it can be shown that the relative error in determining the layer thickness has the same value: $\delta d = \delta C$.

The obtained equations make it possible to numerically estimate the measurement errors. For example, the error of the mechanical scanner is 10 μm when the transducer is displaced from the focal plane by 10 mm, and the relative errors in measuring the velocity and thickness will be 0.1%. At the same time, the deviation of the speed of sound in water, $\Delta C_W = 3$ m/s, and the ratio of the speeds, $C_L/C_W \sim 3$, ensure a measurement error of $\delta d = \delta C \sim 1\%$. Thus, the measurement result is very sensitive to a change in the speed of sound in the immersion liquid. Given that the temperature coefficient of the speed of sound in water in the temperature range of 20–30°C is approximately 2 m/(s °C) [1], during the measurements it is necessary to stabilize and measure the temperature of the immersion medium.

CONCLUSIONS

A method was developed to measure the parameters of an isotropic layer by the spatiotemporal signal of a pulse acoustic microscope recorded as a function of the time and magnitude of the displacement of the focused ultrasonic element of the microscope perpendicular to the sample surface. The decomposition of the data by the pulse plane waves used in the work makes it possible to measure their delays in the layer and relative amplitudes depending on the direction of propagation without knowing the transfer function of the ultrasonic element. The velocities of the longitudinal and transverse waves, as well as the layer thickness, are calculated from the delays of the plane waves' spectral components, and the density is computed from the amplitudes of these components. The performed analysis of the measurement error showed that the boundedness of the spatial range of the signal recording distorts the plane waves' spectrum in the region of small angles of incidence, which it is expedient to exclude when determining the required parameters. It was also found that the measurement error is strongly influenced by the variation of the speed of sound in the immersion medium. The results of the theoretical substantiation of the method were confirmed experimentally by examining the test sample using an acoustic microscope with a central frequency of 50 MHz.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation, project no. 15-12-00057.

REFERENCES

1. G. A. D. Brigg and O. V. Kolosov, *Acoustic Microscopy,* Sec. Ed., (Oxford Univ., New York, 2010).
2. V. M. Levin, A. A. Goryunov, Yu. S. Petronyuk, and K. V. Zakutailov, in *Proc. 2011 Int. Cong. on Ultrasonics ICU-2011,* (Gdansk, Poland, 2011), p. 124.
3. Yu. S. Petronyuk, V. M. Levin, and S. A. Titov, *Physics Procedia* **70** 626–630 (2015).
4. V. Hänel and B. Kleffner, in *Proc. Int. Symp. Acoustical Imaging,* (Santa Barbara, CA, USA, 1998).
5. V. Hänel, *J. Appl. Phys.* **84** (2), 668–670 (1998).
6. J. Chen, X. Bai, K. Yang, and B.-F. Ju, *Ultrasonics* **56**, 505–511 (2015).
7. S. A. Titov, R. G. Maev, and A. N. Bogatchenkov, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* **50** (8), 1046–1056 (2003).
8. L. M. Brekhovskikh and O. A. Godin, *Acoustics of Layered Media* (Nauka, Moscow, 1989) [in Russian].
9. J. F. Claerbout, *Imaging the Earth's Interior* (Blackwell Sci. Publ., Cambridge, MA, USA, 1985; Nedra, Moscow, 1989).
10. S. A. Titov and R. G. Maev, *Acoust. Phys.* **59** (5), 600–607 (2013).
11. J. Zhang, P. Guy, J. C. Baboux, and Y. Jayet, *J. Appl. Phys.* **86** (5), 2825–2835 (1999).
12. A. S. Birks, R. E. Green, and P. McIntire, *Ultrasonic Testing* (Nondestructive Testing) Handbook 2 nd ed., Vol. 7. (Amer. Soc. NondestructiveTesting, Columbus, OH, 1991).

Translated by O. Pismenov