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CLASSICAL PROBLEMS OF LINEAR ACOUSTICS AND WAVE THEORY

On High-Frequency Scattering by a Strip at Nearly Grazing Incidence[1]

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Abstract—The problem of diffraction by an absolutely soft segment is considered in the high-frequency approximation. The asymptotic field decomposition is obtained, which makes it possible to trace the transition from classical asymptotics valid for grazing incidence to geometrical optics asymptotics, which describes scattering at a finite (not small) angle.

Keywords: diffraction, absolutely soft strip, high-frequency asymptotics, nearly grazing incidence

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INTRODUCTION

The problem of diffraction by a strip is classical. A large number of publications are devoted to it. The results obtained prior to the 1960s are presented in [1, 2] and other books. Among the approaches used, three groups are distinguished: methods based on separation of variables in elliptic coordinates, which leads to the Mathew equation; methods in which the problem is reduced to an integral equation of convolution in an interval; and asymptotic approaches of diffraction theory. All the approaches and representations for solutions derived by them have limitations on the domain of their applicability where computations are effective. Investigations are being continued (see, e.g., [3–6]).

In high-frequency diffraction, i.e., for a strip whose halfwidth p and wavenumber k form parameter $kp \gg 1$, two cases are naturally distinguished. In the first, the incident wave travels at a finite (nonzero) angle ϑ_0 to the strip plane. Here, the total effective cross section Σ has the asymptotics [7]

$$\frac{\Sigma}{4p} = \sin\vartheta_0 + (kp)^{-5/2}\sigma + O\left((kp)^{-7/2}\right), \tag{1}$$

where

$$\sigma = \frac{\sin^2\vartheta_0}{16\sqrt{\pi}}\left\{\frac{\cos\left[2kp(1+\cos\vartheta_0)+\pi/4\right]}{(1+\cos\vartheta_0)^3} + \frac{\cos\left[2kp(1-\cos\vartheta_0)+\pi/4\right]}{(1-\cos\vartheta_0)^3}\right\}. \tag{2}$$

In the second case, the incidence is strictly grazing; i.e., the visible cross section of the obstacle is null, and for Σ the following asymptotics occur [8]:

$$\frac{\Sigma}{4p} = \sqrt{\frac{2}{\pi kp}}\left\{1 - \frac{1}{16kp} + \ldots\right\}. \tag{3}$$

We limited (1) and (3) only to two leading order terms. Corrections up to orders $O\left((kp)^{-4}\right)$ for nongrazing incidence and up to $O\left((kp)^{-5}\right)$ for grazing incidence can be found in [2] (formulas (4.113) and (4.119), respectively).

In this paper, we apply the asymptotic procedure developed in [9, 10] and derive an approximate formula for the total scattering cross section Σ, which makes it possible to trace the transition from finite angles described by asymptotics (1) to grazing incidence when asymptotics (3) is valid.

FORMULATION OF THE PROBLEM AND ELLIPTIC COORDINATES

Let us consider the stationary diffraction problem for an acoustically soft strip with a width $2p$. We take the time dependence in the form $e^{-i\omega t}$. Let $kp \gg 1$, where $k = \omega/c$ is the wavenumber of the incident plane wave:

$$u^{(i)} = \exp(ikx\cos\vartheta_0 + iky\sin\vartheta_0). \tag{4}$$

We consider the angle ϑ_0, measured from the strip plane to be so small that the quantity $\alpha = \sqrt{kp}\vartheta_0$ remains bounded when $kp \to +\infty$. Our task is to find the current on the surface of the strip and the amplitude of the far field in the direction of forward scattering. This will allow the total scattering cross section to also be calculated.

[1] The article was translated by the authors.

Let us represent the total field $u(x, y)$ as the sum of the even and the odd parts with respect to coordinate y:

$$u(x,y) = u_e(x,|y|) + i\,\mathrm{sign}(y)u_0(x,|y|). \quad (5)$$

Then the problem is reduced to that in the half-plane $y \geq 0$. Here, the odd part of the incident wave is equal to zero at $y = 0$ and does not excite diffracted field. Thus,

$$u = u_e + i\,\mathrm{sign}(y)\exp(ikx\cos\vartheta_0)\sin(ky\sin\vartheta_0). \quad (6)$$

The even part of the field satisfies the Helmholtz equation and the boundary condition

$$u_e(x,0) = 0, \quad -p < x < p. \quad (7)$$

The Meixner conditions should be satisfied at the edges of the strip, and the radiation condition for the diffracted part of the field $u_e^{(s)}$ at infinity.

Let us introduce elliptic coordinates (η, ξ) so that

$$x = p\xi\eta, \quad y = p\sqrt{\xi^2-1}\sqrt{1-\eta^2}. \quad (8)$$

The Helmholtz equation in (η, ξ) coordinates takes the form

$$\sqrt{\xi^2-1}\frac{\partial}{\partial\xi}\left(\sqrt{\xi^2-1}\frac{\partial u}{\partial\xi}\right) + \sqrt{1-\eta^2}\frac{\partial}{\partial\eta}\left(\sqrt{1-\eta^2}\frac{\partial u}{\partial\eta}\right) + (kp)^2\left(\xi^2-\eta^2\right)u = 0. \quad (9)$$

The elliptic coordinates are convenient, because the surface of the strip is given by the condition $\xi = 1$, i.e., the strip is considered the limiting case of an elliptic cylinder.

ASYMPTOTIC PROCEDURE

First, we consider a small vicinity of the surface (the boundary layer), in which we introduce the stretched coordinate τ by the formula

$$\xi = 1 + \frac{\tau}{2kp} \quad (10)$$

in the parabolic equation method, we extract the quick factor

$$u_e^{(s)} = e^{ikp\eta}U(\eta,\tau). \quad (11)$$

Let us substitute representation (11) into Helmholtz equation (9) and collect terms according to the powers of asymptotically large parameter kp. Here, we assume that differentiation of U with respect to η and τ does not change the asymptotic order, then the resulting equation can be presented as follows:

$$kpL_0U + L_1U = 0, \quad (12)$$

where

$$L_0 = 4\tau\frac{\partial^2}{\partial\tau^2} + 2\frac{\partial}{\partial\tau} + 2i\left(1-\eta^2\right)\frac{\partial}{\partial\eta} + \tau - i\eta, \quad (13)$$

$$L_1 = \tau^2\frac{\partial^2}{\partial\tau^2} + \tau\frac{\partial}{\partial\tau} + \left(1-\eta^2\right)\frac{\partial^2}{\partial\eta^2} - \eta\frac{\partial}{\partial\eta} + \frac{\tau^2}{4}. \quad (14)$$

We seek solution U of Eq. (12) in the form of the asymptotic series

$$U = \sum_{\ell=0}^{\infty}\frac{U_j(\eta,\tau)}{(kp)^j}. \quad (15)$$

It is possible to show that the problems appearing for U_j are solvable for any j; however, we confine ourselves to constructing the higher order approximation U_0, for which the parabolic equation

$$L_0U_0 = 0. \quad (16)$$

takes place. This makes it possible to separate the variables. Let us seek the solution in the form of the integral

$$U_0 = \int C_0(t)Q(\eta;t)F(\tau;t)dt. \quad (17)$$

Here, t is the variable separation parameter, and the integration path should be chosen such that it is possible to apply differential operator L_0 below the integration sign. For functions Q and F, we obtain the ordinary differential equations

$$2i(1-\eta^2)Q' - i\eta Q = 4tQ \quad (18)$$

and

$$4\tau F'' + 2F' + \tau F = -4tF. \quad (19)$$

Equation (18) can be solved in elementary functions:

$$Q = \frac{1}{\sqrt[4]{1-\eta^2}}\left(\frac{1-\eta}{1+\eta}\right)^{it}. \quad (20)$$

Equation (19) is reduced to the Whittaker equation by extracting the multiplier $\tau^{-1/4}$. Choosing the solution that satisfies the radiation condition for $\tau \to +\infty$, we obtain

$$F = \frac{1}{\sqrt[4]{\tau}}W_{it,1/4}(-i\tau), \quad (21)$$

where W is the Whittaker function [11].

Thus,

$$U_0 = \frac{1}{\sqrt[4]{\tau}\sqrt[4]{1-\eta^2}}\int C_0(t)\left(\frac{1-\eta}{1+\eta}\right)^{it}W_{it,1/4}(-i\tau)dt. \quad (22)$$

Such an amplitude C_0 should be found that U_0 satisfies the boundary condition. The incident field at $y = 0$ can be written as

$$u_e^{(i)}\Big|_{y=0} = \exp\left(ikp\eta\cos\vartheta_0\right) = e^{ikp\eta}e^{-i\eta\alpha^2/2}\left\{1 + \frac{i\alpha^4\eta}{24kp} + O\left((kp)^{-2}\right)\right\}. \quad (23)$$

Here, we expanded the cosine of small angle ϑ_0 in a series and used the parameter $\alpha = \sqrt{kp}\vartheta_0$ introduced above. For the leading order, we obtain the boundary condition

$$U_0(\eta,0) = -e^{-i\eta\alpha^2/2}. \quad (24)$$

To obtain the representation for the diffracted field on the surface, i.e., for $\tau = 0$, we use the decomposition for function W of the small argument [11]:

$$W_{it,-1/4}(z) = \frac{\sqrt{\pi}}{\Gamma\left(\frac{3}{4} - it\right)} z^{1/4} - \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1}{4} - it\right)} z^{3/4} + O(z^{5/4}). \quad (25)$$

Then, to determine C_0, we obtain the integral equation

$$\int \left(\frac{1-\eta}{1+\eta}\right)^{it} \frac{1}{\Gamma\left(\frac{3}{4} - it\right)} C_0(t)dt = -\frac{e^{i\pi/8}}{\sqrt{\pi}} \sqrt[4]{1-\eta^2} e^{-i\eta\alpha^2/2}, \quad \eta \in [-1, 1]. \quad (26)$$

To solve this equation, we use the result from [12], where the integral transforms

$$Tc \equiv \int_{-\infty}^{+\infty} \left(\frac{1-\eta}{1+\eta}\right)^{it} c(t)dt = \sigma(\eta), \quad (27)$$

$$T^{-1}\sigma = \frac{1}{\pi} \int_{-1}^{1} \left(\frac{1+\eta}{1-\eta}\right)^{it} \frac{\sigma(\eta)}{1-\eta^2} d\eta. \quad (28)$$

are presented. By choosing the integration path in (22) along the real axis, we obtain

$$C_0 = -\Gamma\left(\frac{3}{4} - it\right) \frac{e^{i\pi/8}}{\pi^{3/2}} \int_{-1}^{1} \left(\frac{1+\eta}{1-\eta}\right)^{it} \frac{e^{-i\alpha^2\eta/2}}{(1-\eta^2)^{3/4}} d\eta. \quad (29)$$

This integral is expressed via the Whittaker function M:

$$C_0 = -\frac{1}{\sqrt{2}\pi^2\sqrt{\alpha}} \times \Gamma\left(\frac{1}{4} + it\right) \Gamma\left(\frac{1}{4} - it\right) \Gamma\left(\frac{3}{4} - it\right) M_{it,-1/4}(i\alpha^2). \quad (30)$$

It is possible to check that the integrated expression in (22) decreases exponentially at $t \to \pm\infty$, which validates the possibility of applying operator L_0 below the integral sign. It is also evident from decomposition (25) that the solution remains bounded for any τ. The behavior at $\eta \to \pm 1$ is determined by the poles of the integrand nearest the real axis, which coincide with the poles of the Gamma function $\Gamma\left(\frac{1}{4} \pm it\right)$ and are at points $t = \pm i/4$. Therefore, the solution is finite for any $\eta \in [-1, 1]$. However, the pole of the Gamma function $\Gamma\left(\frac{3}{4} - it\right)$ causes divergence of the derivative with respect to η as $O\left(1/\sqrt{1-\eta}\right)$. Thus, the assumption that $L_1 U$ is a correction in Eq. (12) is correct for any $\tau \ll kp$ and $\eta \in [-1, 1-\varepsilon]$, where $\varepsilon \gg (kp)^{-2}$. Presumably, the same conditions determine the domain of applicability of the asymptotics.

To further investigate the far field, it is sufficient to compute the normal derivative of the field on the surface. Using decomposition (25), we obtain

$$\left.\frac{\partial U_0}{\partial y}\right|_{y=0} = \frac{\sqrt{k/p}}{\sqrt{1-\eta^2}} \left.\frac{\partial U_0}{\partial \sqrt{\tau}}\right|_{\tau=0} = \frac{2\sqrt{2k/p}}{\sqrt{\pi\alpha}} \frac{e^{-5i\pi/8}}{(1-\eta^2)^{3/4}} \int_{-\infty}^{+\infty} \frac{M_{it,-1/4}(i\alpha^2)}{e^{\pi t} - ie^{-\pi t}} \left(\frac{1-\eta}{1+\eta}\right)^{it} dt. \quad (31)$$

FAR FIELD

To calculate the far field, we use the Green's formula, which we apply to the scattered field $u^{(s)}$ and the Green's function

$$G(\mathbf{r}; \mathbf{r}_0) = \frac{i}{4} H_0^{(1)}(k|\mathbf{r} - \mathbf{r}_0|) + \frac{i}{4} H_0^{(1)}\left(k\left|\mathbf{r} - \mathbf{r}_0^*\right|\right). \quad (32)$$

Here, $\mathbf{r}_0 = (x_0, y_0)$ are the coordinates of the source and $\mathbf{r}_0^* = (x_0, -y_0)$. Green's function (32) satisfies the Neumann boundary condition at $y = 0$, therefore, Green's formula reduces to

$$u^{(s)}(x_0, y_0) = -\int_{-p}^{p} \frac{\partial u^{(s)}(x, 0)}{\partial y} G(x, 0; x_0, y_0)dx. \quad (33)$$

Let us introduce the far field amplitude Ψ of the scattered field by the formula

$$u^{(s)} \sim \sqrt{\frac{2}{\pi kr}} e^{ikr - i\pi/4} \Psi, \quad r = \sqrt{x^2 + y^2} \to +\infty. \quad (34)$$

Using the asymptotics of the Bessel function of the third kind $H_0^{(1)}$, it is simple matter to find the far field amplitude of the Green's function:

$$\Psi_G = \frac{i}{2} \exp(ikx_0 \cos\vartheta) \cos(ky_0 \sin\vartheta). \quad (35)$$

Then, taking the limit below the sign of integration in (33), we obtain the formula for the far field amplitude of the field scattered by the strip:

$$\Psi(\vartheta) = -\frac{i}{2} \int_{-p}^{p} \frac{\partial u^{(s)}(x, 0)}{\partial y} e^{-ikx\cos\vartheta} dx. \quad (36)$$

In this formula, we consider only small angles ϑ_0, which makes it possible to represent the exponential factor in terms of the η coordinate as

$$e^{-ikx\cos\vartheta} = e^{-ikp\eta + i\beta^2\eta/2} \left(1 - \frac{i\beta^4\eta}{24kp} + \dots\right), \quad (37)$$

where $\beta = \sqrt{kp}\vartheta$.

For the leading order with respect to kp, we obtain

$$\Psi = -\frac{ip}{2} \int_{-1}^{1} \frac{\partial U_0(\eta, 0)}{\partial y} e^{i\beta^2\eta/2} d\eta. \quad (38)$$

We substitute here expression (31) and change the order of integration. Comparing the integral with respect to η with the integral representation for the Whittaker function M, we obtain

$$\Psi_0 = -\frac{e^{i\pi/4}\sqrt{kp}}{\pi\sqrt{\alpha\beta}} \times \int_{-\infty}^{+\infty} \frac{\left|\Gamma\left(\frac{1}{4}-it\right)\right|^2}{e^{\pi t}-ie^{-\pi t}} M_{it,-1/4}(i\alpha^2) M_{it,-1/4}(i\beta^2)dt. \quad (39)$$

This formula can conveniently be rewritten via Coulomb wavefunctions F [11], which are related to the Whittaker functions M by the formula [11]

$$M_{it,\nu}(i\beta^2) = \frac{2\Gamma(2\nu+1)\exp\left(i\frac{\pi}{4}(2\nu+1)+\frac{\pi}{2}t\right)}{\sqrt{\Gamma\left(\nu+\frac{1}{2}+it\right)\Gamma\left(\nu+\frac{1}{2}-it\right)}} F_{\nu-\frac{1}{2}}\left(t,\frac{\beta^2}{2}\right). \quad (40)$$

After simple transformations we obtain

$$\Psi_0 = -4\frac{\sqrt{kp}}{\sqrt{\alpha\beta}} \int_{-\infty}^{+\infty} \frac{F_{-3/4}\left(t,\frac{\alpha^2}{2}\right)F_{-3/4}\left(t,\frac{\beta^2}{2}\right)}{1-ie^{-2\pi t}}dt, \quad (41)$$

which agrees with the result from [9].

Let us find the total scattering cross section Σ, which, according to the optics theorem, is expressed via the far field amplitude in the direction of incidence, i.e.,

$$\Sigma = -\frac{4}{k}\mathrm{Re}\Psi(\vartheta_0,\vartheta_0). \quad (42)$$

Substituting the approximation Ψ_0, into this formula, we compute the real part. Here it is noteworthy that F is a real-valued function. The result is

$$\Sigma_0 = \frac{16p}{\sqrt{kp}\alpha} \int_{-\infty}^{+\infty} \frac{F_{-3/4}^2\left(t,\frac{\alpha^2}{2}\right)}{1+e^{-4\pi t}}dt. \quad (43)$$

ANALYSIS OF THE ASYMPTOTIC FORMULA

For small α, we can use the decomposition [11]

$$F_{-\frac{3}{4}}(t,z) \sim \frac{(2z)^{1/4}}{2\sqrt{\pi}e^{\pi t/2}} \times \left|\Gamma\left(\frac{1}{4}+it\right)\right|\left\{1+4tz+\frac{8t^2-1}{6}z^2+\ldots\right\}, \quad (44)$$

substituting it in (43), it is a simple matter to obtain the following decomposition:

$$\Sigma_0 = \frac{4p}{\pi\sqrt{kp}} \times \left\{I_0 + 4I_1\alpha^2 + \left(\frac{16}{3}I_2 - \frac{1}{6}I_0\right)\alpha^4 + \ldots\right\}, \quad (45)$$

where

$$I_j = \int_{-\infty}^{+\infty} \Gamma\left(\frac{1}{4}+it\right)\Gamma\left(\frac{1}{4}-it\right)\frac{t^j dt}{e^{\pi t}+e^{-3\pi t}}. \quad (46)$$

Calculating the integrals (see Appendix), we obtain

$$\Sigma_0 \approx \frac{8p}{\sqrt{kp}}\frac{1}{\sqrt{2\pi}}\left(1+\frac{\alpha^2}{3}+\frac{\alpha^4}{30}+\ldots\right). \quad (47)$$

Note that the leading order term in (47) coincides with the leading order term of asymptotics (3) obtained by Sheshadri and Wu [8].

To analyze the asymptotic formulas, it is convenient to consider the quantity $S = \frac{\sqrt{kp}}{2p}\Sigma_0$. Figure 1 shows the values of S corresponding to exact expression (43) and approximate formula (47). For large α, formula (43) gives a dependence close to linear, which corresponds to the geometric optics approximation for the scattered field, which is expressed with the leading order term of asymptotics (1), in which for small angles of incidence, sine of ϑ_0 is replaced with ϑ_0.

Let us turn to a more accurate numerical analysis of formula (43). We first consider the correction σ in classical asymptotics (1). The authors of [2] stated that this asymptotics is applicable if $kp\sin\vartheta_0 \gg 1$. However, decomposition (1) loses its asymptotic character for $\vartheta_0 \ll \sqrt{kp}$. We consider such small angles ϑ for which $\alpha = \sqrt{kp}\vartheta_0$ is large. We track only the leading order contribution in formula (2). It comes from the second term and is equal to

$$\sigma \approx \frac{\cos\left[kp\vartheta_0^2+\pi/4\right]}{2\sqrt{\pi}\vartheta_0^4} = (kp)^2\frac{\cos\left[\alpha^2+\pi/4\right]}{2\sqrt{\pi}\alpha^5}. \quad (48)$$

Thus, in the considered range of angles, asymptotics (1) gives

$$\sqrt{kp}\frac{\Sigma}{4p} \approx \alpha + \frac{s}{\alpha^4}, \quad s = \frac{1}{2\sqrt{\pi}}\cos\left(\alpha^2+\frac{\pi}{4}\right). \quad (49)$$

The plots of quantities s_0 and s, where

$$s_0 = \alpha^4\left(\sqrt{kp}\frac{\Sigma_0}{4p} - \alpha\right),$$

(Fig. 2), show that the approximation Σ_0 calculated by formula (43) for large α matches asymptotics (1) not only for the leading order, but also for the principal part of the correction σ.

APPENDIX

CALCULATION OF INTEGRALS I_j

The integrals I_j are introduced in (46). Let us first consider I_0, then use the integral representation for the Beta function [11], whence

$$\Gamma\left(\frac{1}{4}+it\right)\Gamma\left(\frac{1}{4}-it\right) = \sqrt{\pi}B\left(\frac{1}{4}+it,\frac{1}{4}-it\right) = \sqrt{\pi}\int_0^1 x^{it-3/4}(1-x)^{-it-3/4}dx. \quad (50)$$

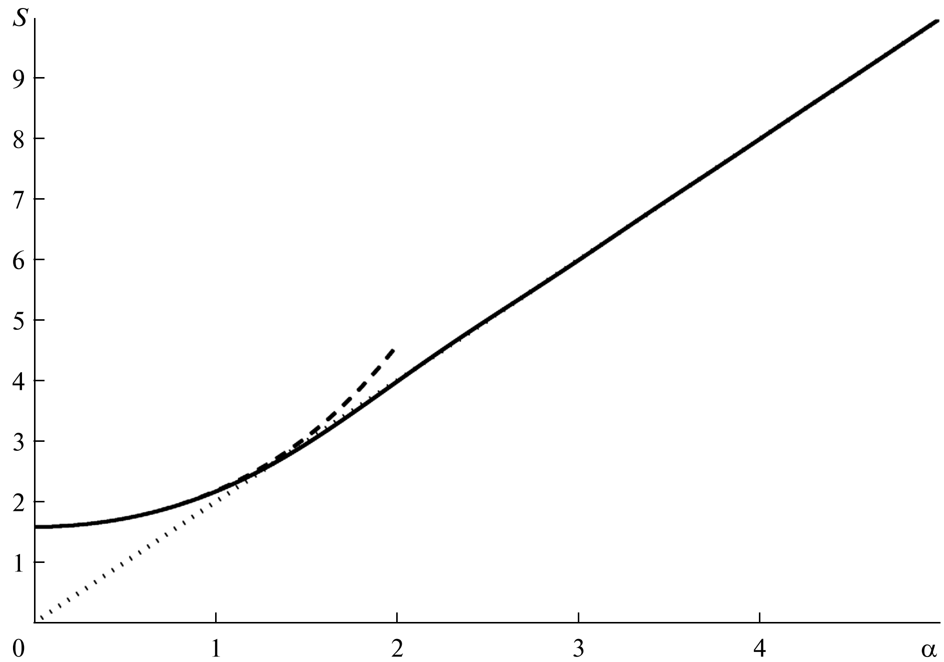

Fig. 1. High-frequency approximations for S. Solid line corresponds to asymptotic formula (43); dashed line, formula (47); dotted line, to geometrical optics approximation.

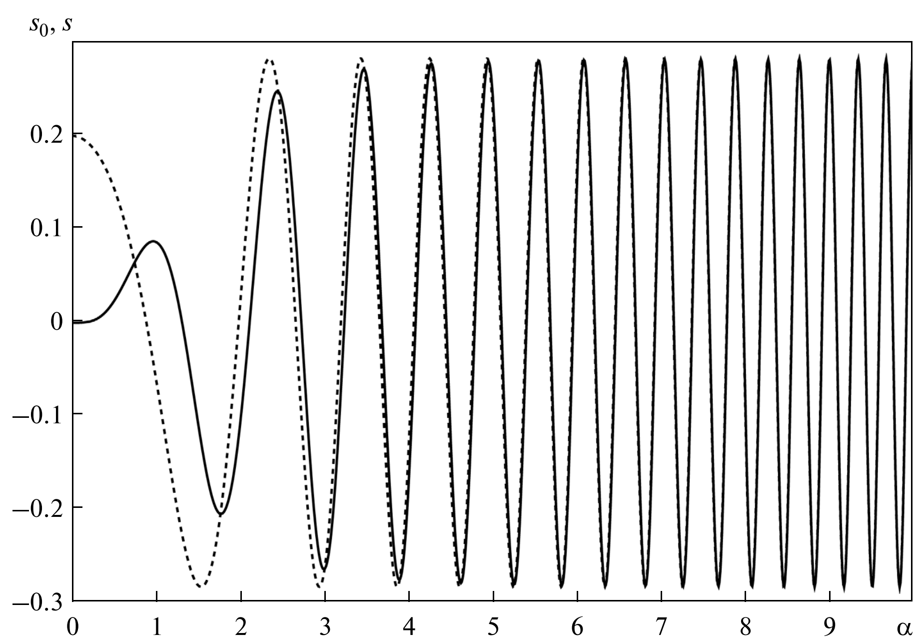

Fig. 2. Correction terms s_0 (solid line) and s (dashed line) and their matching for large values of parameter α.

We change the order of integration and change the variable t to $z = e^{\pi t}$, which yields

$$I_0 = \frac{1}{\sqrt{\pi}} \int_0^1 \int_0^{+\infty} \frac{z^{2+i\xi}}{z^4+1} dz \frac{dx}{x^{3/4}(1-x)^{3/4}}, \quad (51)$$

where

$$\xi = \frac{1}{\pi} \ln\left(\frac{x}{1-x}\right). \quad (52)$$

The integral with respect to z can be calculated; it is equal to $\pi/4/\cos(\pi/4 + i\pi\xi/4)$.

Representing the cosine via complex exponentials, we obtain the elementary integral

$$I_0 = \frac{\sqrt{\pi}}{2}\int_0^1 \frac{dx}{e^{i\pi/4}\sqrt{x}(1-x) + e^{-i\pi/4}x\sqrt{1-x}} = \sqrt{2\pi}. \quad (53)$$

Let us now calculate integral I_1. For this, we consider the following integral:

$$\int_{-\infty}^{+\infty} \Gamma\left(\frac{5}{4} + it\right)\Gamma\left(\frac{1}{4} - it\right)\frac{dt}{e^{\pi t} + e^{-3\pi t}} = \frac{I_0}{4} + iI_1. \quad (54)$$

We represent

$$\Gamma\left(\frac{5}{4} + it\right)\Gamma\left(\frac{1}{4} - it\right) = \frac{\sqrt{\pi}}{2}B\left(\frac{5}{4} + it, \frac{1}{4} - it\right)$$
$$= \frac{\sqrt{\pi}}{2}\int_0^1 x^{it+1/4}(1-x)^{-it-3/4}dx. \quad (55)$$

Derivations similar to those presented above yield

$$\frac{1}{4}I_0 + iI_1 = \frac{\sqrt{\pi}}{4}$$
$$\times \int_0^1 \frac{\sqrt{x}dx}{e^{i\pi/4}(1-x) + e^{-i\pi/4}\sqrt{x}\sqrt{1-x}} = \sqrt{2} + \frac{i}{3}\sqrt{2}, \quad (56)$$

whence

$$I_1 = \frac{\sqrt{2\pi}}{12}. \quad (57)$$

Similarly, we can calculate

$$I_2 = \frac{3}{80}\sqrt{2\pi}, \quad I_3 = \frac{17}{1344}\sqrt{2\pi}$$

etc.

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