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ACOUSTICS OF LIVING SYSTEMS. BIOACOUSTICS

Wave Anisotropy of Shear Viscosity and Elasticity

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Abstract—The paper presents the theory of shear wave propagation in a "soft solid" material possessing anisotropy of elastic and dissipative properties. The theory is developed mainly for understanding the nature of the low-frequency acoustic characteristics of skeletal muscles, which carry important diagnostic information on the functional state of muscles and their pathologies. It is shown that the shear elasticity of muscles is determined by two independent moduli. The dissipative properties are determined by the fourth-rank viscosity tensor, which also has two independent components. The propagation velocity and attenuation of shear waves in muscle depend on the relative orientation of three vectors: the wave vector, the polarization vector, and the direction of muscle fiber. For one of the many experiments where attention was distinctly focused on the vector character of the wave process, it was possible to make a comparison with the theory, estimate the elasticity moduli, and obtain agreement with the angular dependence of the wave propagation velocity predicted by the theory.

Keywords: muscle, soft solids, anisotropy, shear elasticity, viscosity tensor, fiber, polarization vector
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INTRODUCTION

There are a significant number of works devoted to the anisotropic properties of muscle under quasistatic loading (see, e.g., [1, 2]). However, dynamic loads in the form of shear waves have been used relatively recently to determine the viscoelastic characteristics of muscle by ultrasound and MRI-elastography methods [3–13]. These characteristics are informative diagnostic parameters of the state of muscle tissue [5, 8, 10–12, 14, 15]. Structural changes in muscle tissue related to various pathologies, e.g., during myositis or in the aging process, are directly reflected in the mechanical and acoustic characteristics, the measurement of which can yield a deeper understanding of the pathophysiology of muscular diseases and may be used to evaluate therapeutic efficiency [8, 12]. Recent advances in the experimental study of the viscoelastic properties of skeletal muscles in vivo [3–12] are related to the appearance of elastography methods based on the use of shear waves [16–26].

Early studies on the mechanical properties of muscle concentrated mainly on the elastic characteristics, ignoring the viscosity component [14]. However, viscosity is an essential physical characteristic of muscle, reflecting both its structural features and the molecular mechanism of muscle contraction, which is related to the dynamics of actomyosin bridges [14, 15, 27, 28, 31]. The high anisotropy of the mechanical properties is a characteristic feature of skeletal muscle structure differentiating it from the majority of other soft tissues. The study of this anisotropy and the dependence of the corresponding parameters of the normal and pathological states of muscle may become the basis of novel methods for diagnosing and controlling the efficiency of muscle disease therapy.

It is known that muscle fiber consists of approximately 10^4 sequentially connected sarcomeres, each of which in turn contains around 10^6 filaments: thin (the protein actin) and thick (protein myosin). The actomyosin complex is an efficient mechanochemical energy converter of ATP. As one can see from Fig. 1, each individual sarcomere has hexagonal symmetry. Therefore, the shape of a sarcomere does not change when it turns around the axis by an angle of $\pi/3$. However, due to the fact that neighboring sarcomeres are turned relative to each other at a small random angle and there are a large number of sarcomeres in the muscle fiber, the dispersion of rotation turns out to be significant. Therefore, on macroscopic scales comparable to the length of a longitudinal or shear acoustic wave, this form of symmetry disappears. Nevertheless,

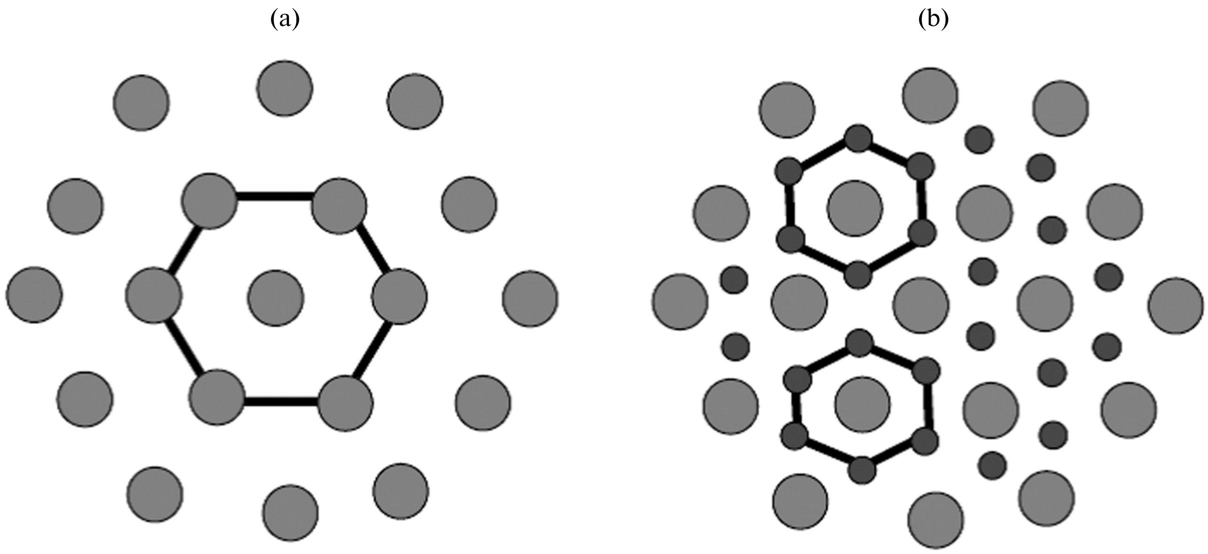

Fig. 1. Location of myosin filaments in cross section of stretched sarcomere (a); section of sarcomere during its contraction—arrays of actin filaments move into myosin arrays (b). Structures have hexagonal symmetry.

in muscle, the anisotropy is retained—the difference in elastic properties in the direction of the fiber axis and in the orthogonal direction. In the plane of the muscle cross section, obviously all directions should be equivalent.

ANISOTROPY OF ELASTIC PROPERTIES OF SKELETAL MUSCLES

The dynamics of anisotropic media, including elastic waves in such media, are described by the system of equations

$$\rho \frac{\partial^2 U_i}{\partial t^2} = \lambda_{iklm} \frac{\partial^2 U_m}{\partial x_k \partial x_l}. \quad (1)$$

Here, U_i is the displacement vector of an element of elastic medium. As is known, the elasticity modulus tensor λ_{iklm} for media with one isolated direction has only five independent components, determined by the moduli A, B, C, D, F [29]:

$$\begin{gathered} \lambda_{xxxx} = \lambda_{yyyy} = A, \;\; \lambda_{xyxy} = B, \;\; \lambda_{xxyy} = A - 2B, \\ \lambda_{xxzz} = \lambda_{yyzz} = C, \;\; \lambda_{xzxz} = \lambda_{yzyz} = D, \;\; \lambda_{zzzz} = F. \end{gathered} \quad (2)$$

Here, the tensor components are written in the Cartesian coordinate system. The z axis is directed along the muscle fiber; the x, y, axes, forming with the z axis a right system of crosscuts, is oriented arbitrarily in the plane orthogonal to the z axis.

Recall that when using tensor notation, twice-repeating subscripts entail summation. Thus, in Eq. (1), summation proceeds by subscripts k, l, m. When replacing an subscripted quantity, e.g., x_m, with one of the Cartesian coordinates x, y, z, summation is not performed. For instance, the expression $\sigma_{mm} = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$ is a spur of matrix σ_{ij}; in contrast, $u_{zz} = u_{zz}$ is a specific element of the matrix.

The dispersion equation following from system of differential equations (1) for monochromatic waves

$$U_i = U_{0i} \exp(-i\omega t + ik_l x_l), \quad (3)$$

has the form

$$\det \left\| \lambda_{iklm} k_k k_l - \rho \omega^2 \delta_{im} \right\| = 0. \quad (4)$$

The solution to system of algebraic equations (4) is three frequency-independent velocities of propagating waves. When the wave vector is directed along the z axis, one of the roots corresponds to a longitudinal wave; the two other roots are equal and pertain to transverse waves:

$$c_l^2 = \frac{F}{\rho}, \;\; c_{t1}^2 = c_{t2}^2 = \frac{D}{\rho}. \quad (5)$$

If the wave vector is directed along the x axis, then all three velocities are different:

$$c_l^2 = \frac{A}{\rho}, \;\; c_{t1}^2 = \frac{B}{\rho}, \;\; c_{t2}^2 = \frac{D}{\rho}. \quad (6)$$

Here, the first transverse wave is polarized along the y axis, and the second, along the z axis. In the general case, for an arbitrary inclination of the wave vector toward the z axis, longitudinal and transverse waves cannot be separated.

However, the anisotropic properties of the medium change radically when the propagation velocities of longitudinal waves significantly exceed the velocities of shear waves. Examples of such media can be soft biological tissues, their phantoms, and certain highly elastic and rubberlike materials. Recently, media for which the shear elasticity moduli are much smaller than the moduli related to the compressibility (change in volume) of a material, have been termed “soft sol-

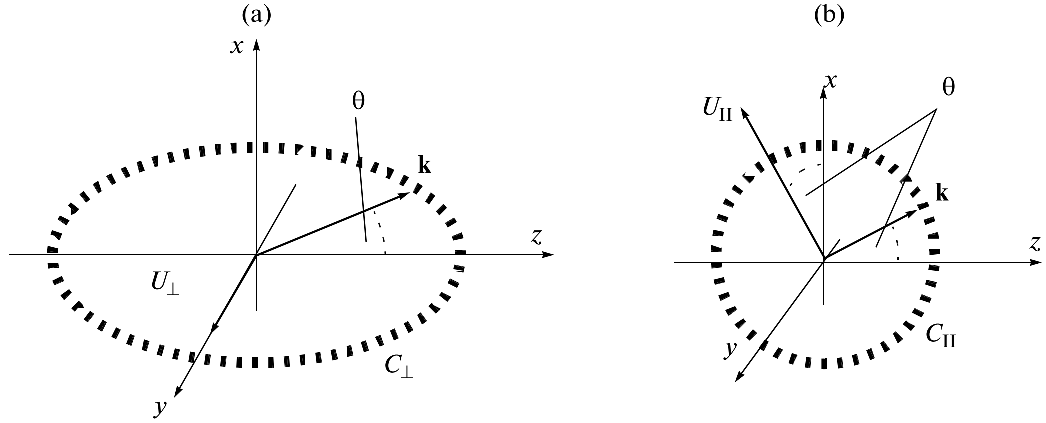

Fig. 2. Wave vector **k** of a shear wave lies in plane of incidence (x, z) and is inclined at angle θ toward fiber axis; (a) polarization vector is orthogonal to plane of incidence; (b) polarization vector lies in plane of incidence.

ids” (see, e.g., [30]). Note that a literal translation of this term into Russian seems strange and the term “rubberlike medium” in the Russian language literature has a somewhat different meaning. A soft solid is a medium whose shear elasticity is small compared to the bulk elasticity. A rubberlike or highly elastic medium has the property of not being destroyed under high deformations.

As we were able to show [31], in soft solid media, new limitations are placed on the tensor components of elasticity moduli. It turns out that moduli A, C, F (2) should be of the same order as the quantity ρc_l^2, and the other two moduli B, D should be much smaller, on the order of ρc_t^2. Since in muscle the velocity of a longitudinal wave $c_l \sim 1500$ m/s and the velocity of a transverse wave is $c_t \sim 1$–100 m/s, obviously $(B, D) \ll (A, C, F)$.

Here, let us consider in more detail the anisotropy of the properties of the dynamic shear elasticity of muscle. In the general case, in anisotropic medium (2), the z axis of the Cartesian coordinates can be conveniently combined with the axis of symmetry (for us, this is the muscle fiber axis). Directions x, y can be chosen arbitrarily. Without limitation of generality, we can conveniently choose the plane (x, z) as the plane of incidence in which wavevector k lies. Vector **k** is inclined toward the z axis at an arbitrary angle θ. Figure 2 illustrates the geometry of the problem.

For this geometry, wave equation (1) takes the form

$$\rho \frac{\partial^2 U_i}{\partial t^2} = \left(\lambda_{ixzm} + \lambda_{izxm}\right) \frac{\partial^2 U_m}{\partial x \partial z} + \lambda_{ixxm} \frac{\partial^2 U_m}{\partial x^2} + \lambda_{izzm} \frac{\partial^2 U_m}{\partial z^2}. \quad (7)$$

Clearly, displacement vector **U** should change according the law of a wave traveling along vector **k**.

$$\mathbf{U} = \mathbf{U}\left(t - \frac{x}{c}\sin\theta - \frac{z}{c}\cos\theta\right). \quad (8)$$

Here, c is the velocity of a propagating wave, which needs to be expressed via the moduli of the medium and the directions of vectors **U** and **k**. Substituting (8) in (7) and integrating twice over time, we bring the wave equation in partial derivatives (7) to a system of algebraic equations:

$$\rho c^2 U_i = \left[\left(\lambda_{ixzm} + \lambda_{izxm}\right)\sin\theta\cos\theta + \lambda_{ixxm}\sin^2\theta + \lambda_{izzm}\cos^2\theta\right] U_m. \quad (9)$$

In Cartesian components, system (9) takes the form

$$\begin{gathered} \left[\rho c^2 - \left(A\sin^2\theta + D\cos^2\theta\right)\right] U_x \\ - \left[(C + D)\sin\theta\cos\theta\right] U_z = 0, \\ \left[\rho c^2 - \left(B\sin^2\theta + D\cos^2\theta\right)\right] U_y = 0, \\ -\left[(C + D)\sin\theta\cos\theta\right] U_x \\ + \left[\rho c^2 - \left(D\sin^2\theta + F\cos^2\theta\right)\right] U_z = 0. \end{gathered} \quad (10)$$

The second equation of this system is independent of the other two equations. It contains only one displacement vector component $U_y = U_\perp$, orthogonal to the plane of incidence and wavevector **k** (see Fig. 2a). As a result, the second equation (10) describes a transverse (shear) wave, the propagation velocity of which is

$$c_\perp = \sqrt{\frac{B}{\rho}\sin^2\theta + \frac{D}{\rho}\cos^2\theta}. \quad (11)$$

The other two equations of system (10) (the first and third) contain both components U_x, U_z of the displacement vector, which lie in the plane of incidence. Equating the determinant of this “truncated” system to zero, we arrive at a quadratic equation for the wave propagation velocity:

$$\left(\rho c^2\right)^2 - \rho c^2\left(A\sin^2\theta + F\cos^2\theta + D\right) + \left(A\sin^2\theta + D\cos^2\theta\right)\left(D\sin^2\theta + F\cos^2\theta\right) - (C+D)^2\sin^2\theta\cos^2\theta = 0. \quad (12)$$

The two roots of Eq. (12) correspond to two waves, which for an arbitrary crystal of the symmetry under consideration are neither purely longitudinal nor purely transverse.

The situation changes fundamentally for a soft solid medium, an example of which is skeletal muscle. Since in this case the propagation velocities of longitudinal and shear waves differ significantly, the longitudinal and transverse component of the wave field cannot themselves be strongly related in a linear approximation. The likelihood of this hypothesis becomes obvious if one considers excitation of the medium by an impact in the form of a pulsed power action. The longitudinal (bulk) wave that arises quickly departs the vicinity of the source, whereas the shear (slow) wave will remain in this vicinity for a long time. The separation of the two waves in the space excludes the possibility of their relationship—a noticeable influence on each other. This means that one of the two roots of Eq. (12) should correspond to the longitudinal wave, and the other, to the transverse.

Let us first consider the longitudinal wave, for which in system (10) we put $U_x = U_{\rm II}\sin\theta$, $U_z = U_{\rm II}\cos\theta$ (see Fig. 2). We obtain two expressions for the velocity:

$$\rho c^2 = A\sin^2\theta + (C+2D)\cos^2\theta, \quad \rho c^2 = (C+2D)\sin^2\theta + F\cos^2\theta. \quad (13)$$

Expressions (13) coincide identically (for any values of angle θ) only if there is the following relationship between the elasticity moduli:

$$A = C + 2D, \quad F = C + 2D. \quad (14)$$

The velocity of the longitudinal wave, determined by formulas (13), is

$$c_l = \sqrt{\frac{A}{\rho}} \quad (15)$$

and does not depend on the angle of incidence.

For a transverse wave we do precisely what we did for thelongitudinal wave, supposing in system (10) $U_x = U_{\rm II}\cos\theta, U_z = -U_{\rm II}\sin\theta$ (see Fig. 2b). We obtain a pair of equations:

$$\rho c^2 = (A - C - D)\sin^2\theta + D\cos^2\theta, \quad \rho c^2 = D\sin^2\theta + (F - D - C)\cos^2\theta. \quad (16)$$

Taking into account relationships (14), Eqs. (16) lead to the following expression for the velocity of the second transverse wave, which also does not depend on direction:

$$c_{\rm II} = \sqrt{\frac{D}{\rho}}. \quad (17)$$

Roots (15) and (17) are obtained directly from dispersion equation (12) if in it we take into account relationship (14) between the elasticity moduli.

Thus, in a "soft"—for shear deformations—medium with one isolated direction, three waves can propagate: one longitudinal (with velocity (15)) and two transverse (with velocities (11) and (17)). Limiting ourselves to considering only the shear (slow) dynamics, it is possible to state the following. With respect to shear waves, muscle can behave like a crystal with enhanced symmetry, for which only the following tensor components of the elasticity moduli differ from zero:

$$\lambda_{xyxy} = B, \quad \lambda_{xzxz} = \lambda_{yzyz} = D. \quad (18)$$

A situation arises similar to the one well known for light waves in uniaxial optical crystals. Ordinary and extraordinary shear waves arise, which differ in their polarization. The dashed lines in Fig. 2 depict the indicatrices of the velocities of shear waves (ellipsis and circle). Clearly, the new symmetry properties established and described above do not give numerical values for the pair of elasticity moduli B, D. These data can be obtained only as a result of experimental measurements. The biophysical models of muscle described in [31] can give a qualitative representation of the behavior of these moduli.

ANISOTROPY OF SHEAR VISCOSITY

To estimate the damping properties of muscle, it is necessary to calculate the energy losses of waves excited in muscle and propagating therein in various directions. This problem requires account of the anisotropy of energy losses, which is related not only to the elastic but also dissipative properties of muscle. Since a corresponding theory had apparently not been developed earlier, we begin by explaining the basics.

As is known, the attenuation rate of energy E in volume V of a continuum is given by the expression [32]

$$\frac{\partial E}{\partial t} = -\frac{1}{2}\int \sigma'_{ik}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)dV. \quad (19)$$

Here, $v_i = \partial U_i/\partial t$ is the velocity vector component and σ'_{ik} is the viscous stress tensor. In the case of isotropic incompressibility of a fluid, it is equal to [32]

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right), \quad (20)$$

where η is the shear viscosity coefficient.

In anisotropic solids, these well-known expressions should be generalized. Here, the viscosity is already not a scalar quantity, but a fourth-rank tensor: η_{iklm}. The symmetry of the viscosity tensor is the same as for the tensor of the elasticity moduli λ_{iklm}. Namely, the form of tensor η_{iklm} does not change when the first and second pairs of subscripts are changed. Neither will the tensor change if the subscripts within the first or second pair change. The viscous stress tensor here has the form [31]

$$\sigma'_{ik} = \eta_{iklm}\frac{\partial}{\partial t}\left(\frac{\partial U_l}{\partial x_m} + \frac{\partial U_m}{\partial x_l}\right). \quad (21)$$

Substituting (21) in (19), we find the attenuation rate of energy E in volume V of an anisotropic solid:

$$\frac{\partial E}{\partial t} = -\frac{1}{2}\eta_{iklm}\int\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)\left(\frac{\partial v_l}{\partial x_m} + \frac{\partial v_m}{\partial x_l}\right)dV. \quad (22)$$

General formula (22) can serve be the basis for subsequent specific conclusions.

Further, so as to avoid cumbersome formulas, we pass from a general consideration to an analysis of the dissipation of shear waves in muscle, for which we use the mentioned fact of enhanced symmetry, which has made it possible to restrict the tensor of elasticity moduli to two independent components (18). For the viscosity tensor, obviously only the following two components will also differ from zero:

$$\eta_{xyxy} = \beta, \quad \eta_{xzxz} = \eta_{yzyz} = \delta. \quad (23)$$

Taking into account symmetry properties (23) and the location of the wave vector in the adopted coordinate system (see Fig. 2), for the energy losses in the volume unit of the medium $e = \partial E/\partial V$ we obtain the following expression from (22):

$$\frac{\partial e}{\partial t} = -2\left[\beta\left(\frac{\partial v_y}{\partial x}\right)^2 + \delta\left(\frac{\partial v_y}{\partial z}\right)^2 + \delta\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2\right]. \quad (24)$$

Bearing in mind dependence (8) of the projections on the coordinates and time, for shear waves polarized in the plane of incidence and orthogonal to this plane, we find, respectively,

$$\frac{\partial e}{\partial t} = -2\rho\frac{\delta}{D}\cos^2(2\theta)\left(\frac{\partial^2 U_{II}}{\partial t^2}\right)^2, \quad (25)$$

$$\frac{\partial e}{\partial t} = -2\rho\frac{\beta\sin^2\theta + \delta\cos^2\theta}{B\sin^2\theta + D\cos^2\theta}\left(\frac{\partial^2 U_{\perp}}{\partial t^2}\right)^2. \quad (26)$$

For traveling monochromatic waves, from formulas (25), (26) we obtain the following attenuation coefficients characterizing the rate of decrease in wave amplitude with distance:

$$\alpha_{II} = \frac{\delta\omega^2}{2c_{II}^3\rho}\cos^2(2\theta), \quad \alpha_{\perp} = \frac{\left(\beta\sin^2\theta + \delta\cos^2\theta\right)\omega^2}{2c_{\perp}^3\rho} \quad (27)$$

In these formulas, the propagation velocities are given by formulas (17) and (11); the second of these velocities itself depends on angle θ.

Analysis of the dependences of the attenuation coefficient on direction is only simple for a shear wave polarized in the plane of incidence (α_{II}). Clearly, absorption reaches a maximum during propagation both along and across muscle fibers. Absorption of this wave turns to zero during propagation at an angle of 45° along the fiber axis. For a wave polarized in the orthogonal plane, here the angular dependence is more complex and depends on all four quantities B, D, β, δ, which, as mentioned above, should be measured experimentally.

Sometimes when conducting measurements, polarization of a shear wave is not taken into account and only its propagation "along fibers" ($\theta = 0$) or "across fibers" ($\theta = \pi/2$) is mentioned. For longitudinal propagation, both attenuation coefficients are equal:

$$\alpha_{II} = \alpha_{\perp} = \frac{\delta\omega^2\rho^{1/2}}{2D^{3/2}}. \quad (28)$$

For transverse propagation, the coefficients are different:

$$\alpha_{II} = \frac{\delta\omega^2\rho^{1/2}}{2D^{3/2}}, \quad \alpha_{\perp} = \frac{\beta\omega^2\rho^{1/2}}{2B^{3/2}}. \quad (29)$$

Thus, the difference in the attenuation coefficients for longitudinal (28) and transverse (29) propagation depend on which of the quantities is larger: $\delta D^{-3/2}$ or $\beta B^{-3/2}$.

COMPARISON WITH EXPERIMENTAL DATA

Recently, experimental data have been published that can be compared with the results of the proposed theory [9, 13, 33].

In [13], a transversally isotropic model for hexagonal crystals was modified for soft solid media similar to what we did in [31]. Two Young's moduli have been introduced for deformations along and across muscle fibers. The limits of their change and the relationship to the bulk shear elasticity moduli have been estimated.

In [9], a list of experiments is discussed that are important for understanding the viscoelastic properties of muscle, among them, shear elastic and dissipative characteristics as a function of muscle tension and oscillation frequency. Corresponding experiments have been performed on volunteers. A large array of data has been obtained, including the distribution patterns of the muscle volume characteristics. However, the details of experiments related to polarization of shear waves are not discussed, which hinders comparison of our theory and these data.

Here we discuss the results of [33], where the mutual orientations of the wave vector, the polarization vector, and muscle fiber have been described quite clearly. In [33] a modern Verasonics experimental sonograph with an open architecture was used to excite an ultrasonic wave. The emitter was a linear array or piezoceramic elements. It was possible to control the beam pattern and ultrasound focusing by feeding electric oscillation with controllable phase delays to the piezoelements. The high radiation intensity

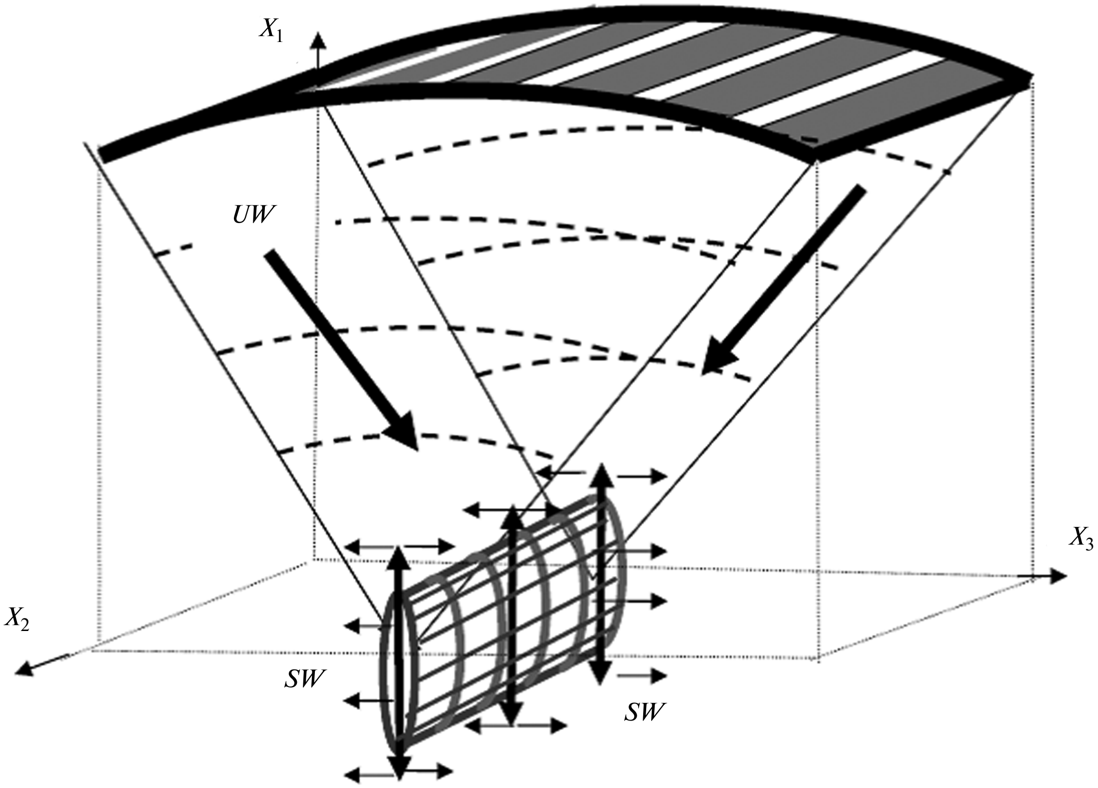

Fig. 3. Diagram of shear wave *SW* excitation by radiative pressure of modulated and focused ultrasound wave.

made it possible to create radiation force at the focus, which excited shear oscillations with the modulation frequency [16, 17]. Figure 3 shows a diagram of the excitation of shear waves via the ultrasound radiation force.

Figure 3 qualitatively depicts how an ultrasonic wave (*UW*) is emitted by the transducer array and is focused into a cylindrical volume elliptical in cross section. The two-sided vertical arrows in the focal region depict the direction of its oscillations under the action of the radiation force. Thus, the directions are as follows: the source is drawn along the X_3 axis, the focal region is drawn along the X_2 axis, shear oscillations of the filament are polarized along the X_1 axis, and the wave vector of the shear wave *SW* traveling from the focal region is directed along the X_3 axis (i.e., parallel to the transducer array).

Figure 4 shows the orientation of the ultrasound array with respect to fibers in experiment [33]. The array is placed on the face of a cubic shape. Muscle fibers are simulated by parallel fishing lines imbedded in a polymer matrix. By rotating the sample it was possible to change angle θ between the fiber axis and the direction of the wave vector of the shear wave.

Comparing Figs. 2 and 4, we see that the polarization vector of the shear wave in this experiment is always directed across the fibers— $U_\perp$ in Fig. 2. As the sample is rotated, angle θ changes between the wave vector and the fiber axis. Our formula (11) is suitable for precisely this case. Supposing in it

$$\sqrt{D/\rho} = 3.75\ \text{m/s}, \quad \sqrt{B/\rho} = 2.85\ \text{m/s},$$

we construct the curve of the dependence of the shear wave velocity on angle θ. This curve is shown in Fig. 5. Clearly, the course of this curve agrees quite well with the measurement data [33]. Thus, in an experiment of this type, it is possible to measure both independent elasticity moduli *B*, *D*, characterizing the shear properties of muscle anisotropy.

CONCLUSIONS

The authors hope that understanding of the physics of wave processes in anisotropic media like soft solids will lead to intelligently constructed novel experiments in phantoms of anisotropic soft tissues, as well as muscles, in vivo and in vitro. The aim of such experiments can be independent measurement of four values: two shear elasticity moduli and two viscosity tensor components, as well as the angular dependences predicted in this work: the propagation velocity and attenuation of all transverse wave modes.

It is of independent interest to measure these characteristics as a function of fiber tension. Obviously, tension should change both the elasticity of muscle and absorption of waves in muscle. In addition, it is important to study dispersion, i.e., the frequency

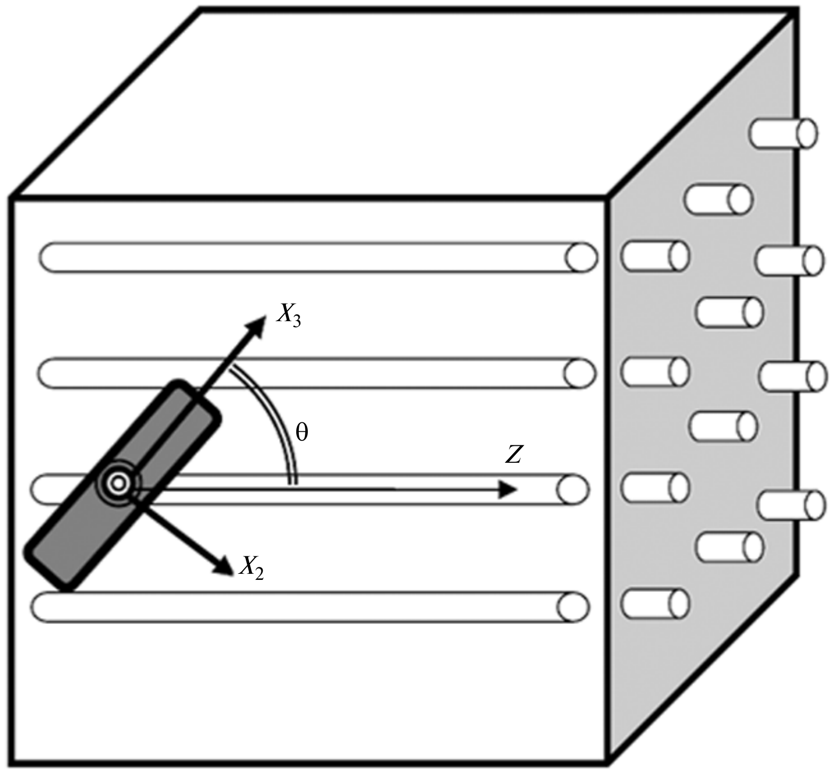

Fig. 4. Orientation of ultrasound source on surface of sample with respect to fibers.

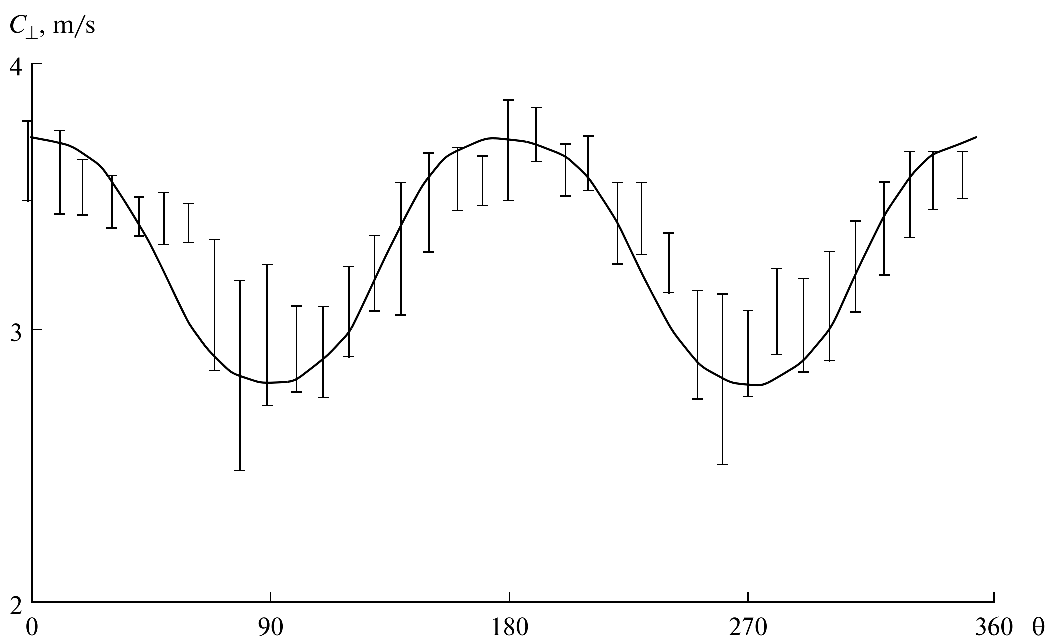

Fig. 5. Velocity of transverse wave polarized orthogonally to plane of incidence as function of angle θ between fiber axis and wave vector. The solid curve is constructed using formula (11); confidence intervals correspond to experimental data [33].

dependence of the parameters. Comprehensively, this information should be useful for diagnostic purposes.

Note that our developed "macroscopic" theory cannot predict the numerical values of the necessary parameters. Therefore, in parallel with this direction, it is necessary to develop a "microscopic" theory based on molecular dynamics models, that is on the motion of myosin bridges and muscle fibers in a surrounding waterlike medium. Our previous paper [31] is an example of such research, which obviously needs to be continued.

This work is a step in elaborating the problems formulated in [31]. The acoustic properties of skeletal muscle are of particular interest for a number of reasons. First, the main functional characteristics of skeletal muscle have a mechanical and, as a result, acoustic nature. Second, in comparison to other soft tissues, skeletal muscle has a starkly pronounced anisotropy closely related to its molecular structure. Third, the main physiological function of skeletal muscle is its contraction, accompanied by a dramatic change in the viscoelastic properties. As well, the characteristic times of elementary molecular processes responsible for contraction and development of muscle strength lie with the range of 10^{-3}–10^{-4} s, i.e., in the frequency range of 1–10 kHz, in which it is convenient to perform acoustic measurements using shear waves. An important direction continuing this work, as well as [31], will be the study of the damping properties of muscle girdles, which protect bones and joints from trauma [34].

We would like to note that acoustic measurements of shear elasticity moduli have been described in several papers published earlier in *Acoustical Physics*. These are primarily the results of the first experiment on generating shear waves in a phantom of muscle tissue by means of radiation force [35], the substantial development of which is described in review [17]. The low velocity of shear waves is one of the causes of strong nonlinearity [36] and may lead to destruction of tissue [37]. Groups of authors have measured the nonlinear moduli of media like soft solids, in particular, with the use of resonators [38].

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