ISSN 1062-7391, Journal of Mining Science, 2022, Vol. 58, No. 3, pp. 347–356. © Pleiades Publishing, Ltd., 2022.
Russian Text © The Author(s), 2022, published in Fiziko-Tekhnicheskie Problemy Razrabotki Poleznykh Iskopaemykh, 2022, No. 3, pp. 3–12.

GEOMECHANICS

Multi-Scale Mathematical Models of Geomedia

A. F. Revuzhenko

*Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences,
Novosibirsk, 630091 Russia
e-mail: revuzhenko@yandex.ru*

Received April 22, 2022
Revised April 28, 2022
Accepted May 6, 2022

Abstract—The author describes the specifics of the multi-scale mathematical modeling of geomedia as a case-study of a two-scale model. The first scale modeling assumes the linearly elastic medium, and the second scale model includes the plastic strains and internal friction. It is shown that in the first approximation, when the micro-scale stress gradients are assumed to be constant, the model acquires elastoplasticity with regard to local bends of structural elements of the geomedium. The solution of the problem on plane S-waves reveals that the waves possess dispersion and their velocity decreases with increasing plastic strains.

Keywords: Geomechanics, mathematical models, role of internal structure, S-waves.

DOI: 10.1134/S1062739122030012

INTRODUCTION

Geomedia are commonly all materials which compose envelopes of the Earth: rocks, soil or granular materials. The generally acknowledged statement says that a geomedium has many scales [1–5]. Multiscaleness is not an exclusive feature of geomedia but rather a general property of most real-life media and processes: plastic deformation and failure of solids, turbulent flows of viscous fluids, etc. [6–9]. As Galileo said, all these phenomena are “written in mathematical language”, and their theoretical research assumes mathematical modeling and analysis. The most mathematical models use the concept of a real straight line. A point of a physical space is three reals, and a time moment is one real.

On the other hand, a real straight line is a single scale object. Assuming the single scale as a length of a step and starting from any point of a straight line, it is possible to reach any other point of this straight line in a finite number of steps (Archimedes’ Axiom) [10].

The mathematical apparatus available for describing physical phenomena should be equivalent to these phenomena. That is, to describe multi-scale phenomena, it makes sense to re-consider the concept of an ordinary, i.e. single-scale, Archimedean real straight line. Some studies addressed that subject [11, 12]. For example, according to [12], in geometry, a straight line, or continuum, can contain more numerous set of point than a set of ordinary real numbers. Among other things, this offers a framework for the geometrical analysis of multi-scale physical phenomena. The present study uses one of alternative descriptions of a multi-scale straight line from [5].

1. As a new scale in [13], an actually infinitesimal number E is introduced. This is a positive number smaller than any value $1/n$ for any natural number n. A manifold

$$X = x + x^{(1)}\mathrm{E} + x^{(2)}\mathrm{E}^2 + x^{(3)}\mathrm{E}^3 + \ldots \qquad (1)$$

forms a multi-scale number line OX. Here, x, $x^{(1)}$, $x^{(2)}$,... are the numbers isomorphic to real numbers, i.e. can be treated as ordinary real numbers. By choosing coordinate axes to be four multi-scale straight lines (1), we obtain a 3D multi-scale space such that all processes in it run in the one-dimensional but multi-scale time. We limit ourselves to the plane-strain deformation and to a single time scale t. In this case, we have five independent variables:

$$x_1,\ \xi_1,\ x_2,\ \xi_2,\ t.$$

Here, x_1, x_2, t are the variables of a real-valued scale; $\xi_1 = x_1^{(1)}\mathrm{E}$, $\xi_2 = x_2^{(1)}\mathrm{E}$ are the micro-level variables in the axes OX_1, OX_2, $X_1 = x_1 + \xi_1$, $X_2 = x_2 + \xi_2$. Let

$$u_1 = u_1(x_1, \xi_1, x_2, \xi_2, t),$$
$$u_2 = u_2(x_1, \xi_1, x_2, \xi_2, t)$$

be the field of displacements and

$$\sigma_{11}, \sigma_{22}, \sigma_{12} = \sigma_{121}(x_1, \xi_1, x_2, \xi_2, t)$$

be the relevant field of stresses. all functions depend on five arguments.

In the classical state of a single-scale space—time, only three arguments appear—(x_1, x_2, t). The appearance of additional arguments ξ_1, ξ_2, i.e. new degrees of freedom, leads to the need to formulate additional equations. This group of equations should describe connections between different scales of a medium.

The mathematical apparatus offers some tools for describing such connections. First of all, these are the conditions of continuity or the conditions of discontinuity of different functions in transition between the scales. For example, the changes in the volume and displacements on different scales are:

$$\varepsilon_x = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}, \quad \varepsilon_\xi = \frac{\partial u_1}{\partial \xi_1} + \frac{\partial u_2}{\partial \xi_2},$$

$$\Gamma_x = \sqrt{\left(\frac{\partial u_1}{\partial x_1} - \frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)^2},$$

$$\Gamma_\xi = \sqrt{\left(\frac{\partial u_1}{\partial \xi_1} - \frac{\partial u_2}{\partial \xi_2}\right)^2 + \left(\frac{\partial u_1}{\partial \xi_2} + \frac{\partial u_2}{\partial \xi_1}\right)^2}.$$

Their differences can be used in closed-type modeling. Furthermore, it is possible to use the invariant differences of rotation operations:

$$\Omega_{x\xi} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) - \frac{1}{2}\left(\frac{\partial u_2}{\partial \xi_1} - \frac{\partial u_1}{\partial \xi_2}\right).$$

It is definite that all laws of conservation and the necessary governing equations should hold true. It is also required to use the condition of consistency: if the behavior of a medium on the microscale totally repeats its behavior on the real scale, then multi-scale mathematical models should transform into the equivalent single-scale models. Evidently, ten equations below:

$$\frac{\partial u_i}{\partial x_j} = \frac{\partial u_i}{\partial \xi_j}, \quad \frac{\partial \sigma_{ik}}{\partial x_j} = \frac{\partial \sigma_{ik}}{\partial \xi_j}, \quad i, j, k = 1, 2,$$

provide sufficient conditions for such transition. They mean that all functions depend not on five but only on three arguments.

In this fashion, transition to a multi-scale space and time opens up new vistas for mathematical modeling of physical processes having the hierarchies of scales.

Let us discuss a model of a geomedium having two scales. Let the medium be linearly elastic on a microscale:

$$\begin{gathered}\frac{\partial\sigma_{11}(x_1,\xi_1,x_2,\xi_2)}{\partial\xi_1}+\frac{\partial\sigma_{12}}{\partial\xi_2}=0,\\ \frac{\partial\sigma_{12}}{\partial\xi_1}+\frac{\partial\sigma_{22}}{\partial\xi_2}=0,\\ \frac{\partial u_1}{\partial\xi_1}=\frac{1}{E}(\sigma_{11}-\nu\sigma_{22}),\\ \frac{\partial u_2}{\partial\xi_2}=\frac{1}{E}(\sigma_{22}-\nu\sigma_{11}),\\ \frac{\partial u_1}{\partial\xi_2}+\frac{\partial u_2}{\partial\xi_1}=\frac{\sigma_{12}(x_1,\xi_1,x_2,\xi_2,t)}{\mu},\end{gathered}\tag{2}$$

where E, ν, μ are the elastic constants. The case when $E\neq 2\mu(1+\nu)$ might be included [14].

Equations (2) hold true at any allowable values of x_1, x_2, ξ_1, ξ_2. These variables play different roles: differentiation is only carried out with respect to the variables ξ_1, ξ_2. The coordinates x_1, x_2 act as parameters. They can be assumed as centers of structural components of the geomedium. The variables ξ_1, ξ_2 at constant x_1, x_2 belong to a specified structural component. Let the interfaces of components locate in the coordinate axes OX_1 OX_2. Furthermore, the medium is assumed to be anisotropic (Fig. 1).

Such anisotropy is of interest in the research of granular media [15, 16], mechanics of crystal lattices [17–21] and nanomaterials [22–26]. In the case under discussion, at the point $A=A^+=A^-$, the normal and shear stresses σ_{11}, σ_{12} are continuous. We denote by l the sizes of a structural component and set that l is an actual infinitesimal number. We extend the function $\sigma_{11}(x_1,\xi_1,x_2,\xi_2)$ to the values $\sigma_{11}(x_1+l,\xi_1,x_2,\xi_2)$. Then the continuity condition at the point A is given:

$$\sigma_{11}(A^+)=\sigma_{11}(A^-)$$
$$\text{or by}\quad \sigma_{11}(x_1+l,-l/2,x_2,\xi_2)=\sigma_{11}(x_1,l/2,x_2,\xi_2).\tag{3}$$

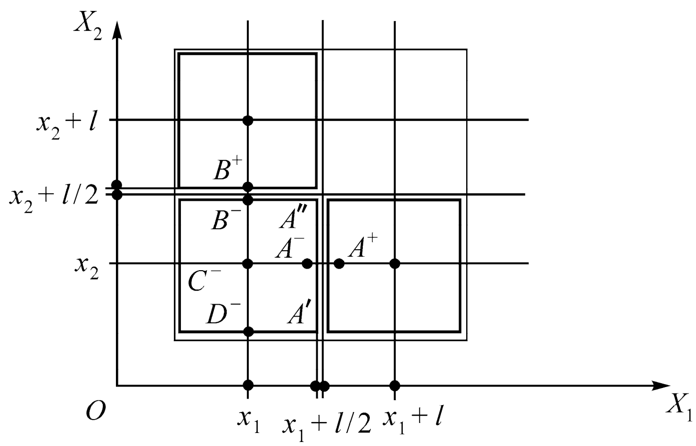

Fig. 1.

Similarly

$$\begin{aligned}\sigma_{22}(x_1, \xi_1, x_2 + l, -l/2) &= \sigma_{22}(x_1, \xi_1, x_2, l/2),\\ \sigma_{12}(x_1 + l, -l/2, x_2, \xi_2) &= \sigma_{12}(x_1, l/2, x_2, \xi_2),\\ \sigma_{12}(x_1, \xi_1, x_2 + l, -l/2) &= \sigma_{12}(x_1, \xi_1, x_2, l/2).\end{aligned} \tag{4}$$

Now we switch to kinematics. At the interfaces of structural components, sliding is possible. Dilatancy is excluded for the simplicity, and the normal displacement can be assumed as continuous therefore:

$$\begin{aligned}u_1(x_1 + l, -l/2, x_2, \xi_2) &= u_1(x_1, l/2, x_2, \xi_2),\\ u_2(x_1, \xi_1, x_2 + l, -l/2) &= u_2(x_1, \xi_1, x_2, l/2).\end{aligned} \tag{5}$$

Discontinuity of the shear displacement is another name of sliding. For the plastic materials, sliding is governed by the shear stresses; for the materials with internal friction—by the ratio of the shear and normal stresses at the relevant interfaces.

This:

$$\begin{gathered}u_2(x_1 + l, -l/2, x_2, \xi_2) - u_2(x_1, l/2, x_2, \xi_2) =\\ = l\, f(\sigma_{12}(x_1, l/2, x_2, \xi_2),\ \sigma_{11}(x_1, l/2, x_2, \xi_2)),\\ u_1(x_1, \xi_1, x_2 + l, l/2) - u_1(x_1, \xi_1, x_2, l/2) =\\ = l\, g(\sigma_{12}(x_1, \xi_1, x_2, l/2),\ \sigma_{22}(x_1, \xi_1, x_2, l/2)).\end{gathered} \tag{6}$$

Equations (6) are constitutive. The functions f and g alongside with the density ρ and elastic constants are assumed to be known. So, we come at the system of thirteen equations: five equations (2) describe deformation of structural components of the medium, six equations (3)–(5) describe the required conditions of continuity and two equations (6) are the constitutive equations for sliding between the structural components. The obtained system is unconventional. The variables x_1, x_2, ξ_1, ξ_2 play different roles in it. Equations (2) hold true at all x_1, x_2, ξ_1, ξ_2 belonging in the domain of deformation, the rest equations hold true only when $|\xi_1|, |\xi_2| = l/2$. This system similarly comprehensively the behavior of the medium both on the macroscale of the variables x_1, x_2 and on the microscale ξ_1, ξ_2. This is an excessive accuracy which complicates analytical description and qualitatively increases the volume of the computations.

2. It is interesting to analyze cases when the microscale can limit to only first approximations with respect to ξ_1, ξ_2.

We choose the linear approximation of the stresses:

$$\begin{aligned}\sigma_{11} &= \sigma_{11}^0 + k_1\xi_1,\\ \sigma_{22} &= \sigma_{22}^0 + k_2\xi_2,\\ \sigma_{12} &= \sigma_{12}^0 + k_3\xi_1 + k_4\xi_2,\end{aligned} \tag{7}$$

where σ_{11}^0, σ_{22}^0, σ_{12}^0, $k_1 - k_4$ are the constant values within a unit volume, i.e. depend only on x_1, x_2, and, moreover, $k_4 = -k_1$, $k_3 = -k_2$.

This stress pattern agrees with the displacements below:

$$u_1(x_1, \xi_1, x_2, \xi_2) = \frac{1}{E}\left[(\sigma_{11}^0 - \nu\sigma_{22}^0)\xi_1 + k_1\frac{\xi_1^2}{2} - \nu k_2\xi_1\xi_2 + \left(\frac{E}{\mu}k_4 + \nu k_1\right)\frac{\xi_2^2}{2}\right] +$$
$$+\frac{\sigma_{12}^0}{\mu}\xi_2 - \Omega\xi_2 + u_1^0(x_1, x_2),$$
$$u_2(x_1, \xi_1, x_2, \xi_2) = \frac{1}{E}\left[(\sigma_{22}^0 - \nu\sigma_{11}^0)\xi_2 + k_2\frac{\xi_2^2}{2} - \nu k_1\xi_1\xi_2 + \left(\nu k_2 + \frac{E}{\mu}k_3\right)\frac{\xi_1^2}{2}\right] +$$
$$+\Omega\xi_1 + u_2^0(x_1, x_2),$$

where Ω, u_1^0, u_2^0 are the constants of integration over ξ_1, ξ_2.

Let us discuss the case of the point contacts of structural components. This means that the connections between structural components are the points A^-, B^-, C^-, D^- (Fig. 1). In the line C^-A^-, $\xi_2 = 0$ and

$$Eu_1 = (\sigma_{11}^0 - \nu\sigma_{22}^0)\xi_1 + k_1\frac{\xi_1^2}{2} + C_1^0,$$
$$Eu_2 = \left(\alpha\frac{E}{\mu}\sigma_{12}^0 + C\right)\xi_1 + \left(\nu k_2 + \frac{E}{\mu}k_3\right)\frac{\xi_1^2}{2} + C_2^0. \tag{8}$$

In the line D^-B^-, $\xi_1 = 0$ and

$$Eu_1 = \left((1-\alpha)\frac{E}{\mu}\sigma_{12}^0 - C\right)\xi_2 + \left(\frac{E}{\mu}k_4 + \nu k_1\right)\frac{\xi_2^2}{2} + C_1^0,$$
$$Eu_2 = (\sigma_{22}^0 - \nu\sigma_{11}^0)\xi_2 + k_2\frac{\xi_2^2}{2} + C_2^0.$$

The equations connect the displacements, stresses and the stress gradients at the contacts, i.e., they are the constitutive equations for the unit volume $A^-B^-C^-D^-$. The right-hand side is introduced with the even and odd degrees of the microscale variables ξ_1, ξ_2. The coefficients of the odd degrees are found in terms of the difference of the displacements and the coefficients of the even degrees—in terms of the relevant sums. For example, it follows from the first equation of (8) that

$$Eu_1(C^-) = -(\sigma_{11}^0 - \nu\sigma_{22}^0)\frac{l}{2} + \frac{k_1}{2}\left(\frac{l}{2}\right)^2 + C_1^0,$$
$$Eu_1(A^-) = (\sigma_{11}^0 - \nu\sigma_{22}^0)\frac{l}{2} + \frac{k_1}{2}\left(\frac{l}{2}\right)^2 + C_1^0. \tag{9}$$

Then

$$\frac{u_1(A^-) - u_1(C^-)}{l} = \frac{1}{E}(\sigma_{22}^0 - \nu\sigma_{22}^0),$$
$$\frac{u_1(A^-) + u_1(C^-)}{2} = \frac{1}{2}\frac{k_1}{2}\left(\frac{l}{2}\right)^2 = \frac{1}{E}\frac{l^2}{8}\frac{\partial\sigma_{11}}{\partial\xi_1} + C_1^0. \tag{10}$$

The analogous equations are incidental to the second pair of the points B^-, D^- and to the displacement u_2. All in all, there are eight equations. After exclusion of stiff transition and rotation from the equations, we have five equations: stresses appear in three equations of the type of (9) and the stress gradients appear in the rest two equations of the type of (10):

$$\frac{u_1(A^-)+u_1(C^-)}{2}-\frac{u_1(B^-)+u_1(D^-)}{2}=2\eta\left(\frac{\partial\sigma_{11}}{\partial\xi_1}-\frac{\partial\sigma_{12}}{\partial\xi_2}\right),$$
$$\frac{u_2(A^-)+u_2(C^-)}{2}-\frac{u_2(B^-)+u_2(D^-)}{2}=2\eta\left(\frac{\partial\sigma_{12}}{\partial\xi_1}-\frac{\partial\sigma_{22}}{\partial\xi_2}\right), \tag{11}$$
$$\eta=\frac{l^2}{32}\left(\frac{1-\nu}{E}+\frac{1}{\mu}\right).$$

Having conditions at the contacts, we can pass to the difference equations on the macroscale. In their first approximations, it is possible to assume that η is the constant of the material and that $l\to 0$. Then the equations of the type of (9) become the differential equations, and equations (11) become the finite equations relative to the displacements:

$$v_i=u_i(A), u_i(C);\quad w_i=u_i(B), u_i(D).$$

The appearance of two additional equations (11) means that the closed system is introduced with two new unknown functions, i.e., instead of one field of displacements $\bar{u}$, we have two vector fields $\bar{v}$ and $\bar{w}$.

Then, as $l\to 0$, with an accuracy to l^2, we arrive at the equations below:

$$\frac{\partial v_1(x_1, 0, x_2, 0)}{\partial\xi_1}=\frac{1}{E}[\sigma_{11}-\nu\sigma_{22}],$$
$$\frac{\partial w_2}{\partial\xi_2}=\frac{1}{E}[\sigma_{22}-\nu\sigma_{11}],\quad \frac{\partial v_2}{\partial\xi_1}+\frac{\partial w_1}{\partial\xi_2}=\frac{\sigma_{12}}{\mu},$$
$$v_1-w_1=2\eta\left(\frac{\partial\sigma_{11}}{\partial\xi_1}-\frac{\partial\sigma_{12}}{\partial\xi_2}\right),$$
$$v_2-w_2=2\eta\left(\frac{\partial\sigma_{12}}{\partial\xi_1}-\frac{\partial\sigma_{22}}{\partial\xi_2}\right).$$

Let us take the stress continuity conditions (3). Thence, with an accuracy to l^2, it follows that

$$\frac{\partial\sigma_{11}(x_1, 0, x_2, 0)}{\partial\xi_1}=\frac{\partial\sigma_{11}(x_1, 0, x_2, 0)}{\partial x_1},$$
$$\frac{\partial\sigma_{22}(x_1, 0, x_2, 0)}{\partial\xi_2}=\frac{\partial\sigma_{22}(x_1, 0, x_2, 0)}{\partial x_2},$$
$$\frac{\partial\sigma_{12}(x_1, 0, x_2, 0)}{\partial\xi_1}=\frac{\partial\sigma_{12}(x_1, 0, x_2, 0)}{\partial x_1}, \tag{12}$$
$$\frac{\partial\sigma_{12}(x_1, 0, x_2, 0)}{\partial\xi_2}=\frac{\partial\sigma_{12}(x_1, 0, x_2, 0)}{\partial x_2}.$$

For the displacements, we have

$$\frac{\partial u_1(x_1, 0, x_2, 0)}{\partial \xi_1} = \frac{\partial u_1(x_1, 0, x_2, 0)}{\partial x_1},$$
$$\frac{\partial u_2(x_1, 0, x_2, 0)}{\partial \xi_2} = \frac{\partial u_2(x_1, 0, x_2, 0)}{\partial x_2}, \tag{13}$$
$$\frac{\partial u_2}{\partial x_1} - \frac{\partial u_2}{\partial \xi_1} = f, \quad \frac{\partial u_1}{\partial x_2} - \frac{\partial u_1}{\partial \xi_2} = g.$$

The eight equations (12), (13) show how the microscale stresses and displacements govern the stress–strain behavior of the medium on the macroscale. Insertion of (12) and (13) in the equations of equilibrium and in the constitutive equations lead to the closed-type system of equations on the macroscale:

$$\frac{\partial \sigma_{11}(x_1, 0, x_2, 0)}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = 0, \quad \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = 0,$$
$$\frac{\partial v_1}{\partial x_1} = \frac{1}{E}[\sigma_{11} - \nu\sigma_{22}], \quad \frac{\partial w_2}{\partial x_2} = \frac{1}{E}[\sigma_{22} - \nu\sigma_{12}],$$
$$\frac{\partial v_2}{\partial x_1} + \frac{\partial w_1}{\partial x_2} = \frac{\sigma_{12}}{\mu} + f(\sigma_{12}, \sigma_{11}) + g(\sigma_{12}, \sigma_{22}),$$
$$v_1 - w_1 = 2\eta\left(\frac{\partial \sigma_{11}}{\partial x_1} - \frac{\partial \sigma_{12}}{\partial x_2}\right), \quad v_2 - w_2 = 2\eta\left(\frac{\partial \sigma_{12}}{\partial x_1} - \frac{\partial \sigma_{22}}{\partial x_2}\right).$$

In this manner, on the macroscale, we come to the equations including local bends. In a special case of these equations, $f \equiv g \equiv 0$ in [27, 28], the formulations of the boundary problems, the unicity theorem and the numerical solutions of some quasi-static problems are analyzed.

Solving of the dynamic problems needs taking into account inertia forces. We confine ourselves to an approximation when the inertia components are only included on the macroscale. The closed system of equations reduces toe the form:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = \rho\frac{\partial^2 V_1}{\partial t^2}, \quad \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = \rho\frac{\partial^2 V_2}{\partial t^2},$$
$$\frac{\partial V_1}{\partial x_1} = \frac{1}{E}[\sigma_{11} - \nu\sigma_{22}] - \eta\frac{\partial}{\partial x_1}\left(\frac{\partial \sigma_{11}}{\partial x_1} - \frac{\partial \sigma_{12}}{\partial x_2}\right),$$
$$\frac{\partial V_2}{\partial x_2} = \frac{1}{E}[\sigma_{22} - \nu\sigma_{11}] + \eta\frac{\partial}{\partial x_2}\left(\frac{\partial \sigma_{12}}{\partial x_1} - \frac{\partial \sigma_{22}}{\partial x_2}\right), \tag{14}$$
$$\frac{\partial V_1}{\partial x_2} + \frac{\partial V_2}{\partial x_1} = \frac{\sigma_{12}}{\mu} + f(\sigma_{12}, \sigma_{11}) + g(\sigma_{12}, \sigma_{22}) - \eta\frac{\partial}{\partial x_1}\left(\frac{\partial \sigma_{12}}{\partial x_1} - \frac{\partial \sigma_{22}}{\partial x_2}\right) + \eta\frac{\partial}{\partial x_2}\left(\frac{\partial \sigma_{11}}{\partial x_1} - \frac{\partial \sigma_{12}}{\partial x_2}\right),$$

where

$$V_1 = \frac{v_1 + w_1}{2}, \quad V_2 = \frac{v_2 + w_2}{2}$$

are the components of the average displacement of the unit volume. Accordingly, average strains are connected with both the stresses and the second space derivatives of the stresses. Therefore, model (14) can be assumed as to be the gradient-type model [29–33].

In the problem on propagation of plane S-waves in a geomedium possessing structure, let $f = 0$, $g(\sigma_{12}, \sigma_{22}) = \sigma_{12} / G$, $G = \text{const}$ and $\partial / \partial x_1 \equiv 0$. In this case, system (14) reduces to a single wave equation

$$\frac{\partial^2 \sigma_{12}}{\partial x_2^2} = \frac{1}{C^2} \frac{\partial^2 \sigma_{12}}{\partial t^2} - \rho\eta \frac{\partial^4 \sigma_{12}}{\partial t^2 \partial x_2^2},$$

where

$$C = \sqrt{\frac{\mu G}{\rho(\mu + G)}}.$$

In case of the harmonic wave

$$\sigma_{12} = \exp(i\omega t - ikx_2),$$

we have the dispersion equation below:

$$\frac{1}{\omega^2} = \frac{1}{C^2} \frac{1}{k^2} + \rho\eta .$$

At $\eta = 0$ the wave have the velocity equal to C. With the decreasing G, i.e. with the increasing part of the plastic strains, the wave velocity lowers. When $\eta \neq 0$ dispersion of S-waves appears.

It is worthy of mentioning that equations (14) are based on assumption (7) of taking into account only the linear stress distribution within a unit volume. In case of inclusion of square members and members of higher degrees, it is possible to arrive at more complex mathematical models.

CONCLUSIONS

The mathematical model of a geomedium, which includes two scales, leads to a system of nonconventional equations. In the first approximation, the equations reduce to the description of the elastoplastic medium with respect to local bends.

As the role of plastic strains increase, the velocity of S-waves decreases. The local bends of the structural components of the medium lead to dispersion of S-waves.

FUNDING

The study was supported with the Federal Research Project, ID 121052500138-4 (Section I), and by the Russian Foundation for Basic Research, project no. 20-05-00184 (Section II).

REFERENCES

1. Sadovsky, M.A., Natural Lumpiness of Rocks, *Doklady AN SSSR*, 1979, vol. 247, no. 4, pp. 829–831.
2. Revuzhenko, A.F., Stazhevskii, S.B., and Shemyakin, E.I., Mechanism of Deformation of a Granular Material under High Shear, *Soviet Mining*, 1974, vol. 10, no. 3, pp. 374–377.
3. Sadosvky, M.A., Bolkhovitinov, L.G., and Pisarenko, V.F., Discreteness of Rocks, *Fizika Zemli*, 1982, no. 12, pp. 3–18.
4. Kocharyan, G.G., *Geomekhanika razlomov* (Geomechanics of Faults), Moscow: Geos, 2016.
5. Vikulin, A.V. and Ivanchin, A.G., Modern Concept of Block Hierarchy in the Structure of Geomedium and Its Implications in Geosciences, *Journal of Mining Science*, 2013, vol. 49, no. 3, pp. 395–408.
6. Grinyaev, Yu.V., Likhachev, V.A., and Panin, V.E., *Strukturnye urovni deformatsii tverdykh tel* (Structural Levels in Deformation of Solids), Novosibirsk: Nauka, 1985.

7. Panin, V.E. (Ed.), *Fizicheskaya mezomekhanika i komp'yuternoe konstruirovanie materialov* (Physical Mesomechanics and Computer-Aided Material Construction), Novosibirsk: Nauka, 1985.
8. Landau, L.D. and Livshits, E.M., *Gidrodinamika* (Fluid Dynamics), Moscow: Nauka, 1988.
9. Zaslavskii, G.M. and Sagdeev, R.Z., *Vvedenie v nelineinuyu fiziku: ot mayatnika do turbulentnosti i khaosa* (The Introduction into Nonlinear Physics: From Pendulum to Turbulence and Chaos), Moscow: Nauka, Fizmatlit, 1988.
10. Nemytsky, V.V., Sludskaya, M.I., and Cherkasov, A.N., *Kurs matematicheskogo analiza* (Mathematical Analysis Course), vol. 1, Moscow–Leningrad: Gos. izd. tekh.-teoret. lit., 1940.
11. Davis, M., *Applied Nonstandard Analysis*, Dover Publications, 2005.
12. Albeverio, S., Fenstad, J.E., Høegh-Krohn, R., and Lindstrøm, T., *Nonstandard Methods in Stochastic Analysis and Mathematical Physics*, Dover Publications, 2009.
13. Revuzhenko, A.F., *Matematicheskii analiz funktsii nearkhimedovoi peremennoi. Spetsializirovannyi matematicheskii apparat dlya opisaniya strukturnykh urovnei geosredy* (Mathematical Analysis of Functions of Non-Archimedean Variable: Application-Specific Mathematical Tool for Description of Structural Levels in a Geomedium), Novosibirsk: Nauka, 2012.
14. Lavrikov, S.V., Stress–Strain Analysis of Softening Block Rock Mass in the Vicinity of a Roadway, *Fiz. Mezomekh.*, 2010, vol. 13, no. 4, pp. 53–63.
15. Pavlov, I.S., Elastic Waves in Two-Dimensional Granular Medium, *Probl. Prochn. Plastichn.*, 2005, iss. 67, pp. 119–131.
16. Pavlov, I.S. and Potapov, A.I., 2D Model of Granular Medium, *Izv. RAN. MTT*, 2007, no. 2, pp. 110–121.
17. Povstenko, Y., Fractional Nonlocal Elasticity and Solutions for Straight Screw and Edge Dislocations, *Phys. Mesomech*., 2020, vol. 23, no. 6, pp. 547–555.
18. Makarov, P.V., Bakeev, R.A., and Smolin, I.Yu., Modeling of Localized Inelastic Deformation at the Mesoscale with Account for the Local Lattice Curvature in the Framework of the Asymmetric Cosserat Theory, *Phys. Mesomech*., 2019, vol. 22, no. 5, pp. 392–401.
19. Rys, M. and Petryk, H., Gradient Crystal Plasticity Models with a Natural Length Scale in the Hardening Law, *Int. J. Plasticity*, 2018, vol. 111, pp. 168–187.
20. Pouriayevali, H. and Xu, B.-X., Decomposition of Dislocation Densities at Grain Boundary in a Finite Deformation Gradient Crystal–Plasticity Framework, *Int. J. Plasticity*, 2017, vol. 96, pp. 36–55.
21. Erofeev, V.I. and Pavlov, I.S., Parametric Identification of Crystals Having a Cubic Lattice with Negative Poisson's Ratios, *J. Applied Mechanics and Technical Physics*, 2015, vol. 56, no. 6, pp. 1015–1022.
22. Zenkour, A.M. and Radwan, A.F., A Nonlocal Strain Gradient Theory for Porous Functionally Graded Curved Nanobeams under Different Boundary Conditions, *Phys. Mesomech*., 2020, vol. 23, no. 6, pp. 601–615.
23. Chih-Ping Wu and Jung-Jen Yu, A Review of Mechanical Analyses of Rectangular Nanobeams and Single-, Double-, and Multi-Walled Carbon Nanotubes Using Eringen's Nonlocal Elasticity Theory, *J. Arch. Appl. Mech*., 2019, vol. 89, pp. 1761–1792.
24. Sedighi, H.M. and Yaghootian, A., Dynamic Instability of Vibrating Carbon Nanotubes near Small Layers of Graphite Sheets Based on Nonlocal Continuum Elasticity, *J. Applied Mechanics and Technical Physics*, 2016, vol. 57, no. 1, pp. 90–100.
25. Pavlov, I.S. and Lazarev, V.A., Nonlinear Elastic Waves in 2D Nanocrystal Medium, *Vestn. Nauch.-Tekh. Razv. Nats. Tekhnol. Gruppa*, 2008, no. 4(8), pp. 45–53.
26. Loboda, O.S. and Kravtsov, A.M., Scale Effect on 3D Nanocrystal Elasticity Model, *Izv. RAN MTT*, 2005, no. 4, pp. 27–41.
27. Revuzhenko, A.F., Three-Dimensional Model of a Structured Linearly Elastic Body, *Physical Mesomechanics*, 2022, vol. 25, pp. 33–41.

28. Lavrikov, S.V. and Revuzhenko, A.F., Model of Linear Elasticity Theory with a Structural Parameter and Stress Concentration Analysis in Solids under Deformation, *AIP Conf. Proc*., 2018, 2051 (1), 020167. DOI: 10.1063/1.5083410.
29. Lewandowski, M.J. and Stupkiewicz, S., Size Effects in Wedge Indentation Predicted by a Gradient-Enhanced Crystal–Plasticity Model, *Int. J. Plasticity*, 2017, vol. 98, pp. 54–78.
30. Liu, D. and Dunstan, D.J., Material Length Scale of Strain Gradient Plasticity: A Physical Interpretation, *Int. J. Plasticity*, 2017, vol. 98, pp. 156–174.
31. Habib Pouriayevali and Bai-Xiang Xu, A Study of Gradient Strengthening Based on a Finite-Deformation Gradient Crystal–Plasticity Model, *Continuum Mech. Thermodyn*., 2017, vol. 29, pp. 1389–1412.
32. Dabiao Liu and Dunstan, D.J., Material Length Scale of Strain Gradient Plasticity: A Physical Interpretation, *Int. J. Plasticity*, 2017, vol. 98, pp. 156–174.
33. Aifantis, E.C., Internal Length Gradient (ILG) Material Mechanics Scales and Disciplines, *J. Adv. Appl. Mech*., 2016, vol. 49, pp. 1–110.