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## GEOMECHANICS

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# Probabilistic Assessment of Rock Slope Stability in Open Pit Mine Chaarat Using the Generalized Hoek–Brown Criterion

K. Kang<sup>a\*</sup>, I. K. Fomenko<sup>b\*\*</sup>, J. Wang<sup>c\*\*\*</sup>, and O. V. Nikolskaya<sup>d\*\*\*\*</sup>

<sup>a</sup>*School of Environment and Civil Engineering, Jiangnan University,  
Wuxi, 214122 P.R. China*

*\*e-mail: kevinkang8@mail.ru*

<sup>b</sup>*Faculty of Hydrogeology, Russian State Geological Prospecting University,  
Moscow, 117997 Russia*

*\*\*e-mail: ifolga@gmail.com*

<sup>c</sup>*Institute of Mineral Resources, Chinese Academy of Geological Sciences,  
Beijing, 100037 P.R. China*

*\*\*\*e-mail: wangjiawei0824@163.com*

<sup>d</sup>*Institute of Geomechanics and Subsoil Development, National Academy of Sciences of the Kyrgyz Republic,  
Bishkek, 720055 Kyrgyz Republic*

*\*\*\*\*e-mail: nikol-48@mail.ru*

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**Abstract**—The slope stability evaluation using the generalized Hoek–Brown criterion and regarding the scale effect has been implemented in terms of the Chaarat gold project. Furthermore, the probabilistic assessment and sensitivity analysis are performed. Slope failure probabilities are determined, and the slope stability factors are obtained as functions of the slope height and angle. The slope stability estimation based on classified approach considering the scale effect, including GSI rating and probabilistic analysis is tested in rock slopes. Slope stability is mainly governed by variability of the Geological Strength Index related with the scale effect.

**Keywords:** Slope, rock mass, slope stability, Hoek–Brown criterion, scale effect, risk analysis.

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## INTRODUCTION

The mining industry is a priority sector for the development of the Kyrgyz economy, while the gold mining takes the lead. Mountaintop gold fields of Kyrgyzstan are located in zones of tectonic disturbances, mainly faults and their junctures, which causes instability of open pit walls and underground openings at the stage of construction and operation [1]. The economic efficiency of mining mineral deposits is directly related to the risks arising during their development. The main factor in optimizing the “risk–economic efficiency” system is coordination of mineral mining parameters with engineering and geological conditions. The choice of optimum parameters for bench face angles and overall slope angle of the pit wall has a decisive influence on mine development performance and mining safety. The problems of pit wall stability providing accident-free mining during long-term development of deposits must be solved by present-day calculation methodology [2].

The Hoek–Brown failure criterion based on the scale effect is widely used when calculating the slope stability in rock mass [3]. This criterion was developed stage-by-stage over four decades drawing on the results of empirical data generalization [4]. Its classic version is known as the generalized Hoek–Brown criterion [5]. This particular version is used in the present paper when assessing the slope stability of the Chaarat gold field.

The use of probabilistic analysis in the quantitative assessment of slope stability allows characterizing the risk of landslide occurrence and activation in the rock mass more reasonably [6–9]. The point of this analysis for rock mass is to obtain the probability distribution function of slope stability factor depending on the changeable parameters included in the Hoek–Brown failure criterion. The sensitivity assessment is close to probabilistic analysis. However, instead of the problem of landslide development probability in a rock mass caused by the variability of landslide formation factors, a solution on the degree of stability factor dependence on the regularity of changes in certain parameters is given [6]. Sensitivity analysis makes it possible to predict the distribution of slope stability factor depending on the changes in one or more landslide formation factors [10].

## 1. CHARACTERISTICS OF THE STUDY OBJECT

The Chaarat gold project is located in the Jalal-Abad region on the right side of the river Sandalash at a height of 2400–2600 m above sea level [1]. This largest gold field in Kyrgyzstan is confined to the zone of the Karator and Sandalash overthrusts feathering the Talas-Fergana fault. The rocks were formed under the influence of dislocation metamorphism and crushed into folds. The entire area is divided by numerous tectonic fractures with different orientations relative to the future side of the open-pit mine. The studied area within the open pit is composed mainly of red silicified sandstones. The rock mass is highly fractured with crushing zones. The average distance between fractures is 10–15 cm (there are up to 70 differently oriented fractures per meter run) [11]. Rock quality designation (RQD) does not exceed 20–30%.

Assessment of the stability of adjacent rock mass is a crucial task in the open-pit development of mountaintop deposits. The main problem is accounting of the scale effect in the calculations.

## 2. SCALE EFFECT

The rock mass is not a random set of soils and has its own internal structure. All elements of the geological body are interconnected and interdependent. When interacting with the structure, the body behaves as an entity. According to [12], the properties of a rock mass are not the sum of the properties of its individual parts, but are the product of these parts. In other words, the strength characteristics of a block-structured rock mass depend not only on the properties of rocks in the sample, but also on structural features of the geological environment (block size, fracturing intensity), i.e., on scale effect.

The scale effect has been known for a long time [13]. It is found that the strength of anthracite samples decreases significantly, as linear dimensions increase [14]. The scale effect in experiments with glass is described in [15]. A characteristic feature is determined in [16]: the scale effect is significantly manifested in the deformation of materials prone to brittle failure, and is less pronounced in testing materials subject to viscous failure.

The following approaches are currently used to assess the scale effect [11]:

- *Empirical approach.* The scale effect is traditionally assessed in terms of structural weakening coefficient:  $\lambda = K_{sa} / K_{samp}$  ( $K_{samp}$ ,  $K_{sa}$  are the strength in the sample and in the studied area of rocky ground, respectively) [17]. The value of  $\lambda$  is not constant and depends on the degree of fragmentation, strength of structural blocks, orientation of the weakening surfaces of the rock mass to the direction of normal stresses, etc. In [18], the dependence of uniaxial compressive strength of rocky ground on the size of  $d$  is proposed:  $\sigma_{cd} = \sigma_c^{50} (50 / d)^{0.18}$  ( $\sigma_c^{50}$  is uniaxial compressive strength of rocky ground 50 mm in diameter);

- *Backward calculation method.* In a few studies, coefficient  $\lambda$  is determined by analyzing the rock mass failure under precisely known circumstances. In this case, coefficient  $\lambda$  requires a more thorough assessment, since the method takes into account not only the structural features of the geological environment, but also reflects all logical imperfections, especially in terms of functional interdependence of the parameters included in the final formula [13];

- *Analytical approach* to describe the difference of strength characteristics of a system from the properties of its structural elements is based on statistical strength theories [13];

- *Classified approach* is based on ratings, such as geological environment quality assessment, RMR,  $Q$ -rating, GSI [5, 11, 19, 20].

A new method for assessing slope stability in rock mass based on the classified approach including GSI and probabilistic analysis is tested in this paper.

### 3. SLOPE STABILITY MODELING IN ROCK MASS

The slope morphology and physical and mechanical characteristics of the ground composing the rock mass are the main input parameters for assessing the stability of the slopes of designed open pit. A two-dimensional slope model with inclination angles of 45, 60 and 75° was used to assess the stability of rock slope (Fig. 1).

The generalized Hoek–Brown criterion is an example of nonlinear shear strength criterion developed for rock mass. Its original version was presented in 1980 for the design of underground openings [4]. In 1988 the criterion was expanded to apply to slope stability problems, and in 2002 an updated version including an improvement in the correlation between the model parameters and GSI was developed [5].

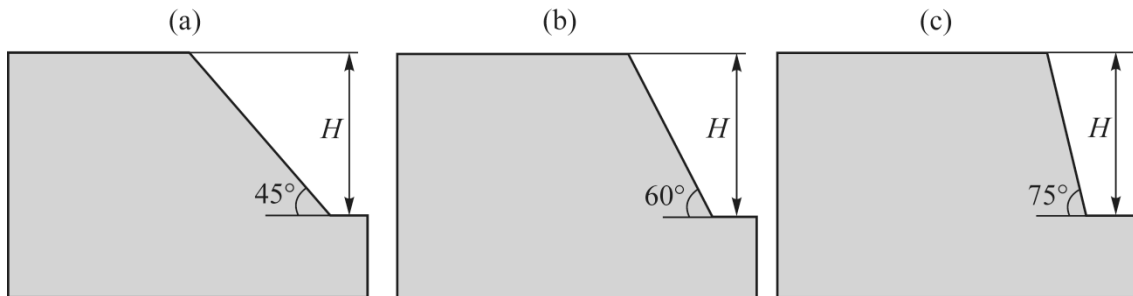
The main idea of the Hoek–Brown criterion is scale effect accounting in the transition from properties in a rock sample to properties in a rock mass. The generalized Hoek–Brown criterion is determined as:

$$\sigma_1 = \sigma_3 + \sigma_c \left( m_b \frac{\sigma_3}{\sigma_c} + s \right)^a,$$

where  $\sigma_1$ ,  $\sigma_3$  are effective principal stresses;  $\sigma_c$  is uniaxial compressive strength;  $s$ ,  $a$ ,  $m_b$  are the Hoek–Brown criterion parameters:

$$m_b = m_i \exp\left(\frac{\text{GSI}-100}{28-14D}\right), \quad s = \exp\left(\frac{\text{GSI}-100}{9-3D}\right), \quad a = \frac{1}{2} + \frac{1}{6}(e^{-\text{GSI}/15} - e^{-20/3}),$$

$m_i$  is the parameter of the lithological type of ground, for example, andesite, marl, quartzite, etc.; GSI is geological strength index depending on rock mass structure: undisturbed, block, flysch, etc. (0–100);  $D$  is the factor of anthropogenic disturbance of the rock mass (0–1);  $e$  is the base of natural logarithm [5].



**Fig. 1.** Models to assess rock mass stability at slope inclination angles of (a) 45°, (b) 60°, (c) 75°;  $H$  is the slope height.

**Table 1.** Physical and mechanical parameters for quantitative assessment of slope stability

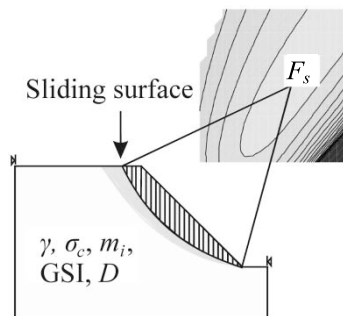
Rock mass parameter	Mean value $\mu$	Standard deviation $\sigma$	Coefficient of variation, %	Type of distribution function	Values
Uniaxial compressive strength $\sigma_c$ , MPa	200	30	15	Normal	110–290
Parameter of the lithological type of rock $m_i$	17	1.30	7		13–21
GSI	35	5	14.3		20–50
Disturbance factor $D$	0.85	0.05	6		0.7–1.0
Unit weight $\gamma$ , kN/m <sup>3</sup>	25.70	0.30	1		24.8–26.6

Based on the limiting envelope of the Hoek–Brown criterion, it is possible to obtain the equivalent parameters of Mohr–Coulomb strength (specific cohesion  $c'$  and internal friction angle  $\phi'$ ) by selecting a linear approximation in the studied stress range.

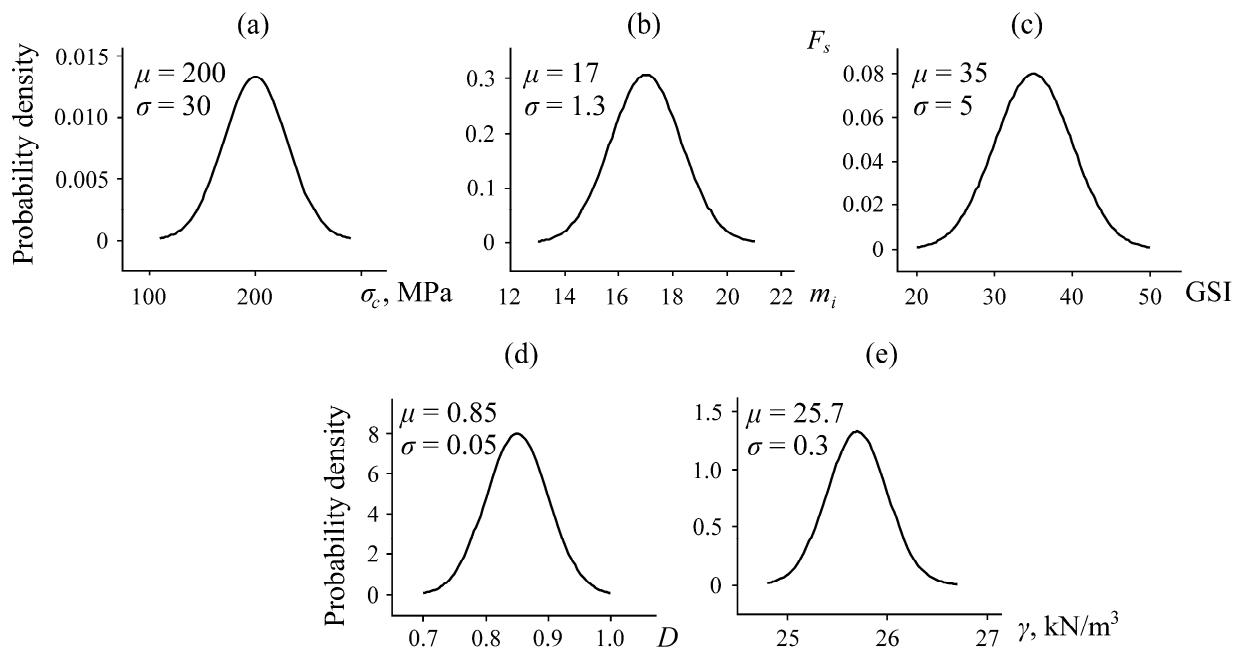
The physical and mechanical parameters of the Hoek–Brown criterion for hard rocks used in the calculations are given in Table 1 [11]. The parameter  $D$  does not depend on the type, structure and on the physical and mechanical properties of rock mass in question, it represents a reflection of man-caused impact. For the open pit construction, the disturbance factor is assumed equal to 0.7–1.0 depending on the mining method. The geomechanical design diagram used to assess the slope stability in the SLIDE 2 software environment is shown in Fig. 2. The slope stability was calculated by the standard method of limit equilibrium, i.e., the Bishop method based on moment equilibrium accounting [21].

#### 4. QUANTITATIVE ASSESSMENT OF SLOPE STABILITY BASED ON PROBABILISTIC ANALYSIS

The probabilistic analysis is not frequently used when quantifying the slope stability. Note that the formation of physico-mechanical rock parameters is influenced by different random factors, i.e., the landslide process is not deterministic, and strength characteristics of the ground, if considered in the space-time aspect, are compositions of random fields that can be described using probability distribution function. This analysis allows obtaining the probability distribution function of slope stability factor depending on the probability functions of change in the parameters determining the Hoek–Brown failure criterion ( $\sigma_c$ ,  $m_i$ , GSI,  $D$ ) and unit weight  $\gamma$ . The fundamental difference between probabilistic analysis and deterministic calculation is using not fixed parameters of ground properties in the form of standard or design values, but their distribution functions. The probabilistic distribution functions of strength characteristics of rock mass are determined. The diagrams of distribution density function characterizing the variability of the physical and mechanical properties of silicified sandstones are shown in Fig. 3.



**Fig. 2.** Geomechanical design diagram used to analyze the slope stability in the SLIDE 2 software environment:  $F_s$  is slope safety factor.

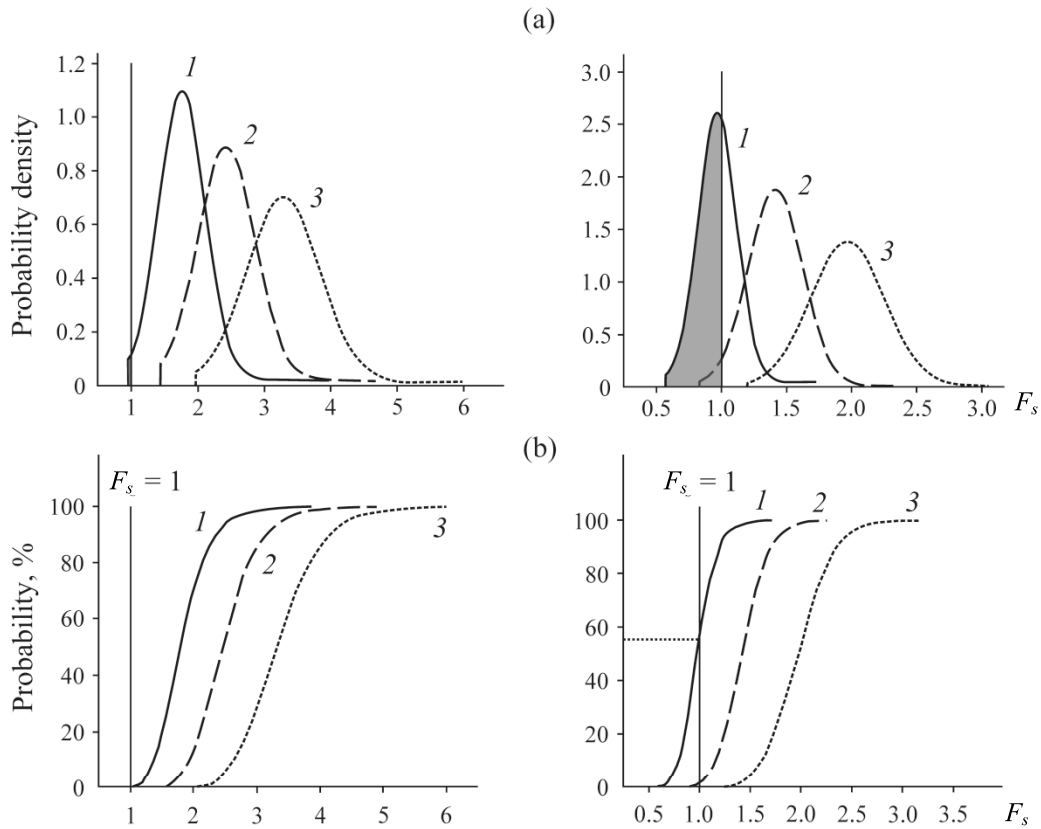


**Fig. 3.** Distribution functions of the factors accounted in slope stability calculation: (a) uniaxial compressive strength; (b) parameter of the lithological type of rock; (c) GSI; (d) factor of rock mass anthropogenic disturbance; (e) unit weight.

Then, the probabilistic change in  $F_s$  was calculated on the basis of distribution functions of the Hoek–Brown exponents using the Monte Carlo method. When performing a probabilistic assessment of slope stability, 2000 calculations accounting for different combinations of exponents of rock mass properties were made. Each analysis was carried out by the Bishop method. A probabilistic quantitative assessment of slope stability was obtained (Table 2). In contrast to deterministic estimates, the mean values of slope stability are characterized, as well as probability ( $F_s < 1$ ) in distribution of value  $F_s$  in a calculation sample.

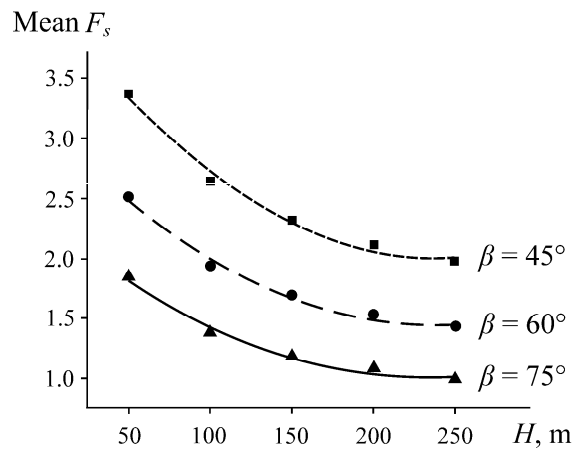
**Table 2.** Results of probabilistic assessment of slope stability

Slope height, m	Slope angle, deg	Mean $F_s$	Slope failure probability ( $F_s < 1$ )
50	45	3.37	0
	60	2.51	0
	75	1.84	0.001
100	45	2.65	0
	60	1.94	0
	75	1.38	0.051
150	45	2.32	0
	60	1.69	0.001
	75	1.18	0.201
200	45	2.12	0
	60	1.53	0.004
	75	1.07	0.387
250	45	1.98	0
	60	1.43	0.012
	75	0.99	0.565



**Fig. 4.** (a) Distribution of probability densities and (b) cumulative probability curves for slope height  $H=50$  and 250 m: 1— $\beta = 75^\circ$ ; 2— $\beta = 60^\circ$ ; 3— $\beta = 45^\circ$ . Vertical line designates slope failure probability ( $F_s=1$ ).

The results of this analysis are shown in Figs. 4 and 5. Finding the distribution function of slope stability representing an estimate of slope failure probability is an additional possibility in using an approach based on variable rock properties [22]. The greater the slope height, the higher the probability of slope failure (Fig. 4). At a slope height of less than 200 m, regardless of the steepness adopted in the calculation (from 45 to 75°), the probability of slope failure is less than 0.5%. At a greater slope height, the probability of slope failure for a steepness of 75° is 0.565%. At a height of <250 m, all slopes with a steepness of <45° are stable (slope failure probability is 0).



**Fig. 5.** Dependences of mean  $F_s$  on slope height.

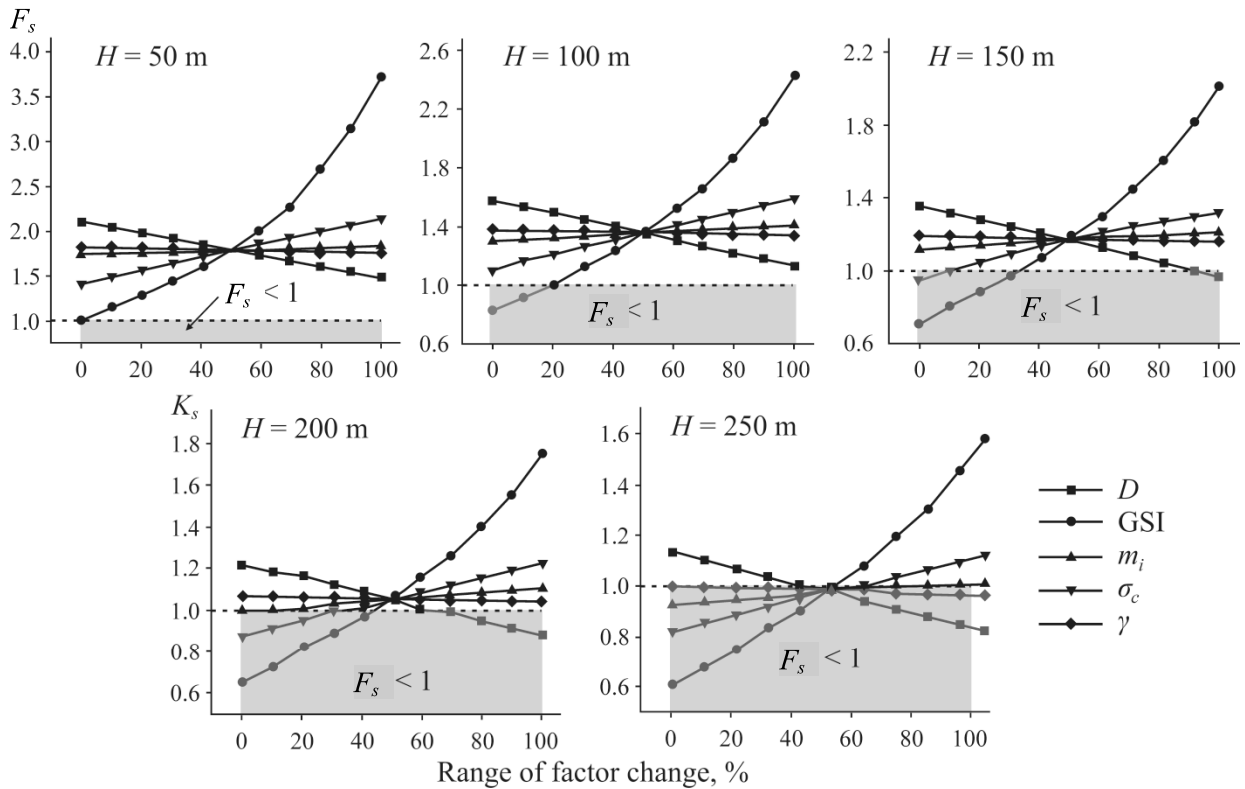


Fig. 6. Slope stability ( $\beta = 75^\circ$ ) sensitivity to change in landslide development factors.

## 5. QUANTITATIVE ASSESSMENT OF SLOPE STABILITY BASED ON SENSITIVITY ANALYSIS

The sensitivity analysis is close to probabilistic assessment. However, instead of the problem of slope stability probability on the predicted variability of landslide formation factors, the problem of the degree of slope stability factor dependence on the regularity of certain parameters distribution is solved. Thus, the sensitivity analysis makes it possible to predict the value of slope stability factor depending on the change in one or several factors of landslide formation.

The sensitivity analysis was carried out with regard to the change in slope height and steepness (Fig. 5) [23, 24]. For convenience, all parameters are distributed by the values from 0 to 100%. Zero values correspond to the lowest factor exponents, and 100% values—to the highest ones. For example, uniaxial compressive strength values of 110 MPa correspond to zero values. The obtained results of the sensitivity analysis (for a slope steepness of  $75^\circ$ ) are shown in Fig. 6. The greatest influence on slope stability is exerted by the variability of geological strength index (GSI) and anthropogenic disturbance factor  $D$ .

## CONCLUSIONS

A new procedure for assessing the slope stability in rock mass is developed and tested. It is shown that the greatest influence on slope stability is exerted by the variability of geological strength index, which is associated with the scale effect.

Quantitative assessment of geological and man-induced risks during the field mining requires an analysis of landslide development probability. The deterministic approaches to obtain slope stability estimates fail to determine the actual level of hazard completely, since they do not allow finding the relationship between the obtained  $K_s$  and landslide development probability, i.e., to identify the

conditions at which  $F_s < 1$ . One of the ways to solve this problem is the use of probabilistic analysis, which means obtaining the probability distribution function of the slope stability factor depending on the probabilistic functions of changes in the physical and mechanical characteristics of rock mass composing the slope. This analysis allows characterizing the hazard of landslide activation more reasonably. The analysis of different factors in slope sensitivity to stability failure is an important addition to quantitative assessment of the slope condition.

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