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ROCK FAILURE

Integral Criterion for Determination of Tensile Strength and Fracture Toughness of Rocks

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Abstract—The method of determining strength characteristics of rocks subjected to tension is tested; the method is based on processing of data from fracture tests of specimens with axial holes of different diameters subjected to loading along diameter. The test data of specimens of rocks and simulating media in the form of cores with axial holes and fractured along diameter are processed based on the integral strength criterion of Novozhilov. The comparison shows good agreement between the fracture toughness and tensile strength values obtained using the proposed method and in standard measurements.

Keywords: Fracture, Brazilian tests, tensile strength, fracture toughness, cumulative strength criterion.

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INTRODUCTION

Investigation of rock masses starts with determination of characteristics of rocks. To this effect, samples of rocks are subjected to various stress tests. The main characteristic is the tensile strength. Although rocks most often experience compression, the tensile strength is important as the compression in rock mass is nonuniform, and there are tension zones, for instance, around underground excavations. The practice shows that failure of underground structures initiates in such zones.

Uniaxial uniform tension testing of rocks is a very laborious process. It is necessary to take many measurements as strength of rocks is variable within a mine field. In such conditions, direct tension tests were unsuitable for rocks and were replaced by indirect testing. Among the indirect tests, the most wide-spread method is diametral compression of core disks, known as the Brazilian test initially meant for concrete. Maximal tension stresses in the core center, in the line of the force application are close to the uniaxial tensile strength of the core material for many rocks [1, 2] though the field in the disk center is biaxial. The method has been many times tested with diverse natural and artificial materials, and is amply described in literature with various recommendations on improvement of testing quality and reliability [3] as the further analysis only embraces samples in which failure originated in the middle, along the line of symmetry.

The attempts to extending the Brazilian test method to ring or disk samples with holes were made in [4–6]. Improvement of the test quality due to the introduction of a stress raiser in the form of a circular hole in the center of a disk is simple and allows considerable stress concentration. Such modification essentially reduces destructive force and guaranties failure initiation in the center of samples. The decrease in the applied destructive forces by the stress raiser pushes the application limits of the method and enables testing soft rocks with plastic properties (gypsum, shale, limestone). Figure 1 illustrates efficiency of the circular stress raiser with gypsum and plexiglass samples subjected to diametral compression.

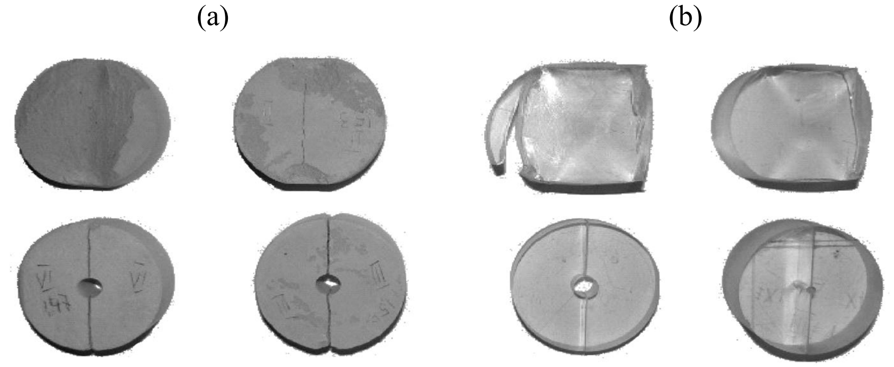


Fig. 1. Samples of (a) artificial gypsum and (b) plexiglass with and without central hole after the Brazilian test.

The above mentioned studies highlight advantages of this approach over the standard Brazilian test but offer no explanation of high strength values calculated at high stress concentration points using the elastic solution and local fracture criterion. The destructive stresses evaluated from the elastic solution and with the local fracture criteria at such points exceed much the uniaxial tensile strength of the medium. A characteristic of such tests is the nonuniform stress field. For this reason, the use of local fracture criteria neglecting structure of the medium is doubtful.

The nonlocal approach to the failure analysis in the nonuniform stress fields [7–12] made it possible to calculate strength values in different stress fields. The comparison of the calculated and test values proved accuracy and correctness of the approach and procedure [13–16].

The estimation procedure of fracture toughness and tensile strength based on the gradient criterion of strength and using the test data of core samples with axial holes of different diameters was proposed in [13], which allowed aligning the calculated and measured values obtained in uniaxial tension of wax and gypsum. The experimental data collected afterwards demonstrated improper application of the gradient approach to calculating strength characteristics of some rocks. For many rocks, the structure parameter δ introduced in the nonlocal fracture model greatly exceeds the parameter L_e characterizing the stress concentration zone in the gradient criterion. The most-preferred integral strength criterion accounts for the stress distribution along the whole δ rather than in the the stress concentration zone L_e (Fig. 2). This is proved in [14–17]. The method to determine strength characteristics of rocks below in this article is based on the integral fracture criterion.

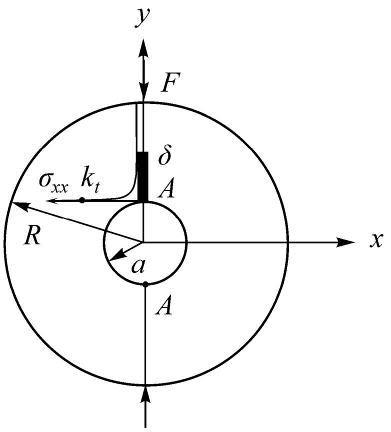


Fig. 2. Disk with central hole for the Brazilian tests.

Table 1. Measured tension strength, stress and critical stress intensity factor

Rock/material	σ_0, MPa	K_{1c}, MPa·m$^{1/2}$	δ, mm	D, mm	a_1, mm	σ_{c1}, MPa	a_2, mm	σ_{c2}, MPa	a_3, mm	σ_{c3}, MPa
Dolerite	25.0	2.06	4.35	37.6	1.70	16.90	—	—	—	—
Gabbro-dolerite	13.4	1.25	5.50	37.6	3.25	7.80	—	—	—	—
Granite	11.2	1.14	6.60	37.6	3.25	7.00	—	—	—	—
Ufaley marble	6.9	0.90	10.80	37.6	3.25	5.11	—	—	—	—
Coarse-grained marble	5.9	0.86	13.40	37.6	2.00	4.70	3.0	4.24	—	—
Marmorized limestone	5.6	—	—	57.0	3.00	3.11	5.0	2.46	—	—
Artifical gypsum	2.5	0.20	4.00	40.0	2.00	1.27	4.0	0.90	—	—
Wax	2.2	0.16	3.40	40.0	2.00	1.14	3.0	0.94	4.0	0.83
Plexiglass	75.0	1.40	0.22	40.0	1.00	19.80	3.0	15.70	4.0	14.30

D is the core diameter; δ is the structural parameter. Average strength $\sigma_{c1}-\sigma_{c3}$ goes with samples with central hole with radius a_1–a_3.

1. TESTING

The samples with and without central holes were made of Ufaley marble, dolerite, granite and gabbro-diorite. The samples of coarse-grained marble, marmorized limestone, as well as model media of gypsum, wax and plexiglass to be subjected to failure in compression had internal holes of various sizes not more than 20% of the disk diameter. The test pattern in Fig. 2 shows the field of tensile stresses at critical points A and the structural parameter δ typical of rocks. The tension strength was determined in the Brazilian tests of all media except for plexiglass samples exposed to failure in direct tension. Gypsum and was samples were tested in direct fracture.

For minimization of scatter in the mechanical characteristics of the test media, samples of each rock were cut from the same slab or core. The artificial samples were manufactured in the same conditions, cured for a long time and then brought to failure along a selected direction. For stable contacts between the samples and loading plates, fluoroplastic gaskets (50 μm) were used.

The strength characteristics and sizes of the test samples are compiled in Table 1. For the samples with holes, the strength σ_{ci} was determined in a gross section, i.e., $\sigma_c = F/(\pi Rt)$ (F is the destructive force; R and t are the radius and thickness of a sample). The critical stress intensity factors K_{1c} for rocks and artificial materials were obtained using the method from [18]. The tests were carried out under the room temperature on UME-10TM testing machine at the cross-beam advance of 0.5 mm/min, which conformed with the loading rate of 1 MPa/s. Each value of strength and fracture toughness fits with a series of tests on not less than 5 samples.

2. STRENGTH ANALYSIS FOR SAMPLES WITH AXIAL HOLE WITH INTEGRAL FRACTURE CRITERION

The strength analysis for samples with holes assumes the small hole radius $a \ll R$ and the hole location in the biaxial stress field preset by the elastic solution of the problem on compression of a disk by concentrated forces. The tensile stress distribution along the load application line at the critical points A is given by:

$$\sigma_x = \frac{\sigma}{2}\left[2 - \frac{2a^2}{r^2} + \frac{12a^4}{r^4}\right], \tag{1}$$

where the stress $\sigma = F/(\pi Rt)$ at the moment of failure reaches the maximum value σ_c.

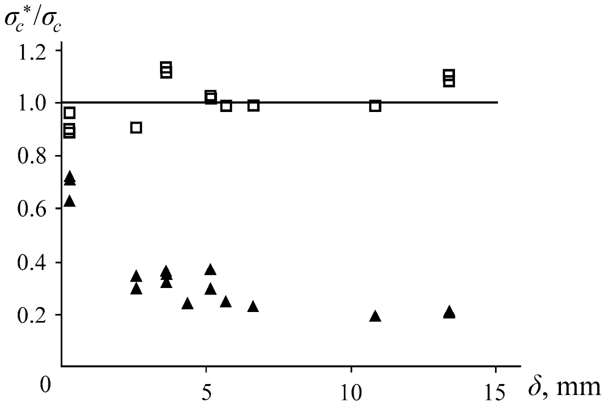


Fig. 3. Relation of calculated and measured destructive stresses in samples with axial hole: □—integral criterion; ▲—local criterion of fracture.

Application of Novoshilov's integral criterion [7]:

$$\sigma_0 = \frac{1}{\delta} \int_a^{a+\delta} \sigma_x(r)dr$$

to the discussed geometry brings the expression for strength of a sample with hole below:

$$\sigma_c = \sigma_0 \left[1 + \frac{5a^3 + 4a^2\delta + a\delta^2}{(a+\delta)^3} \right]^{-1}. \tag{2}$$

Here, σ_0 is the tensile strength in a uniform field; δ is the averaging site determined from the relation [10, 14]:

$$\delta = \frac{2}{\pi} \left(\frac{K_{1c}}{\sigma_0} \right)^2, \tag{3}$$

K_{1c} is the critical stress intensity factor.

Table 2. Calculated destructive stresses for test samples with axial hole

Material	D, mm	σ_0, MPa	a, mm	σ_c, MPa	σ_c^*, MPa	k_t	σ_c^{loc}, MPa	σ_c^* / σ_c	$\sigma_c^{loc} / \sigma_c$
1	2	3	4	5	6	7	8	9	10
Dolerite	37.6	25.00	1.70	16.90	16.82	6.05	4.13	0.99	0.24
Gabbro-dolerite	37.6	13.40	3.25	7.82	7.74	6.96	1.93	0.99	0.25
Granite	37.6	11.20	3.25	7.00	6.92	6.96	1.61	0.99	0.23
Ufaley marble	37.6	6.90	3.25	5.11	5.06	6.96	0.99	0.99	0.19
Coarse-grained marble	37.6	5.93	2.00	4.70	5.08	6.13	0.97	1.08	0.20
Coarse-grained marble	37.6	5.93	3.00	4.24	4.70	6.61	0.90	1.11	0.23
Marmorized limestone	57.0	5.58	3.00	3.11	3.20	6.05	0.92	1.03	0.30
Marmorized limeston	57.0	5.58	5.00	2.46	2.51	6.10	0.92	1.02	0.38
Artificial gypsum	40.0	2.30	2.00	1.27	1.16	6.13	0.38	0.91	0.30
Artificial gypsum	40.0	2.30	4.00	0.90	0.82	7.40	0.31	0.91	0.34
Wax	40.0	2.20	2.00	1.14	1.29	6.13	0.36	1.14	0.29
Wax	40.0	2.20	3.00	0.94	1.07	6.61	0.33	1.13	0.35
Wax	40.0	2.20	4.00	0.80	0.92	7.40	0.30	1.12	0.36
Plexiglass	40.0	75.00	1.00	19.80	17.60	6.00	12.50	0.89	0.63
Plexiglass	40.0	75.00	3.00	15.70	14.20	6.61	11.35	0.90	0.72
Plexiglass	40.0	75.00	4.00	14.30	13.80	7.40	10.14	0.96	0.71

σ_c^*—based on the nonlocal strength criterion; σ_c^{loc}—based on the local criterion.

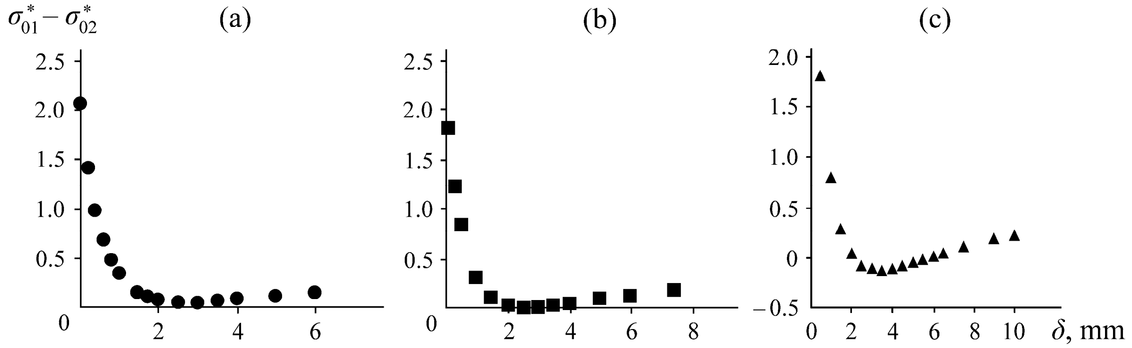


Fig. 4. Closure error of (4) solution versus δ : (a) wax; (b) artificial gypsum; (c) marmorized limestone.

Figure 3 depicts the relation of the calculated and measured destructive stresses for samples with axial hole from table 2. The calculated stresses are obtained using the integral fracture criterion (2). The values obtained using th conventional local fracture criterion are calculated from the formula: $\sigma_c^{loc} = \sigma_0 / k_t(a)$, where $k_t(a)$ is the stress concentration coefficient.

Comparison of columns 9 and 10 in Table 2 shows that the integral approach to the strength analysis of samples with internal holes produces more similar results relative to the test data than the local strength criterion. This inference is visually demonstrated in Fig. 3.

3. DETERMINATION OF TENSILE STRENGTH AND FRACTURE TOUGHNESS

The strength sensitivity to the stress field nonuniformity, which is taken into account in the nonlocal approach, enables determination of tensile strength and fracture toughness of materials in two test series of samples with different geometry. Core material is deficient; for this reason, it is suggested to test the same diameter cores with different diameter holes. The formula for stress distribution in the vicinity of the hole in the line of the future crack (1) is efficient in the range $a/R < 0.2$, and the hole diameters should be selected within this interval.

The test results on samples of coarse-grained marble, marmorized limestone, artificial gypsum and wax with internal holes of two and more sizes were processed using the procedure below.

Solving the system of two equations for samples with different diameter holes:

$$\sigma_0 = \sigma_{c1}\left[1 + \frac{5a_1^{\,3} + 4a_1^{\,2}\delta + a_1\delta^2}{(a_1+\delta)^3}\right], \quad \sigma_0 = \sigma_{c2}\left[1 + \frac{5a_2^{\,3} + 4a_2^{\,2}\delta + a_2\delta^2}{(a_2+\delta)^3}\right] \tag{4}$$

allows estimation of the structural parameter δ. At this stage, the scatter of the test data presents a certain difficulty. Figure 4 shows the dependence of the right-hand sided $(\sigma_{01}^{*} - \sigma_{02}^{*})$ on δ. This closure error of the solution shifts along the ordinate axis with change in the input data inside the confidence interval of the measured strength.

Table 3. Calculated tensile strength and fracture toughness for test materials

Rock/material	σ_0, MPa	K_{1c}, MPa·m$^{1/2}$	δ, mm	σ_0^{*}, MPa	K_{1c}^{*}, MPa·m$^{1/2}$	δ^{*}, mm	σ_0^{*} / σ_0	K_{1c}^{*} / K_{1c}	δ^{*} / δ
Coarse-grained marble	5.93	0.86	13.0	5.94	0.71	9.00	1.00	0.82	0.70
Marmorized limestone	5.58	—	—	5.06	0.49	5.00	0.91	—	—
Artificial gypsum	2.50	0.20	4.0	2.35	0.16	3.00	0.94	0.80	0.74
Wax	2.20	0.16	3.4	2.14	0.14	2.88	0.97	1.00	0.85
Plexiglass	75.00	1.40	0.2	77.00	1.67	0.30	1.03	1.19	1.36

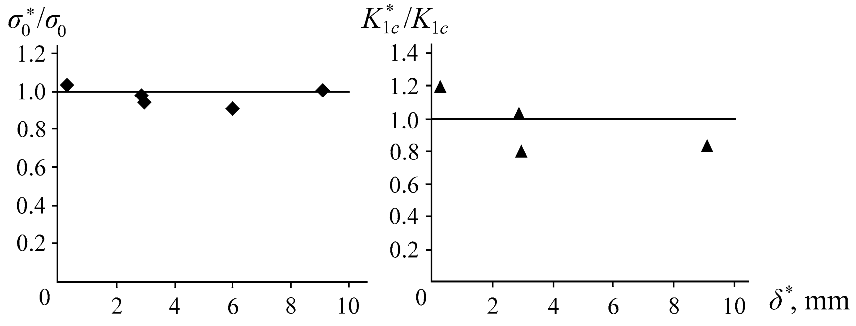


Fig. 5. Relation of the calculated (with asterisk) and measured strength characteristics.

The solution of (4) with minimal closure error complies with δ found from the condition $d(\sigma_{01}^* - \sigma_{02}^*)/d\delta = 0$. The value σ_0^* is found as the arithmetic average of the calculated tensile strength σ_{0i}^* for δ determined by the above method. The test data processed using this procedure are presented in Table 3 and in Fig. 5.

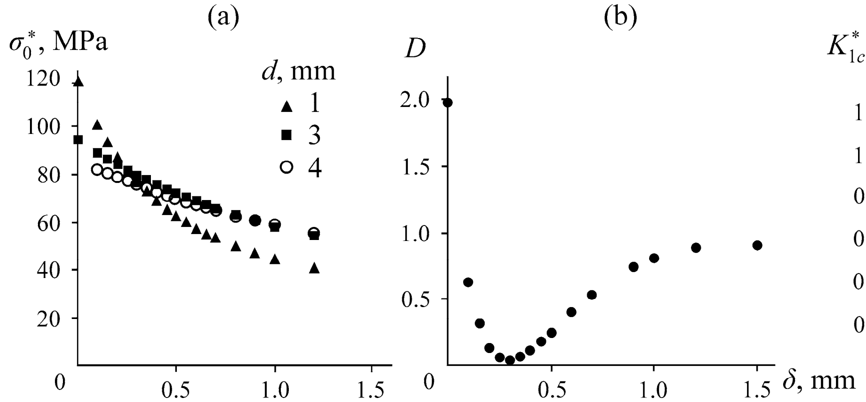
The test results obtained in destruction of plexiglass and wax samples with three diameters of internal hole were processed by minimization of function based on dispersion:

$$D = \sum_{i=1}^{n} \frac{(\sigma_{0i}^* - \bar{\sigma}_0^*)^2}{\bar{\sigma}_0^*}.$$

For wax, both procedures gave the same result in terms of δ. For plexiglass, the calculated strength of which is related with δ in Fig. 6a, minimization of the functional D unambiguously determines δ (Fig. 6b). The strength characteristics calculated for plexiglass using this procedure are given in Table 3 and in Fig. 5, too.

The proposed method for simultaneous determination of tensile strength and fracture toughness offers the calculation accuracy of 20%. The structural parameter δ varies in a wide range. For some rocks listed in Table 1, both solid samples and samples with central hole were subjected to the Brazilian tests. These are strong rocks, and the tensile strength values from the Brazilian tests are similar to the fracture strength. From the test results of solid samples and samples with central hole, the fracture toughness of these rocks was determined (Table 4).

Figure 7 illustrates the calculated versus measured fracture toughness. The calculated stress intensity factors are mostly similar to the measured values. Even the worst result for coarse-grained marble differs from the measurement by less than 20%. The comparison of the results on fracture toughness of marmorized limestone in Tables 3 and 4 demonstrates effectuality of the proposed method.



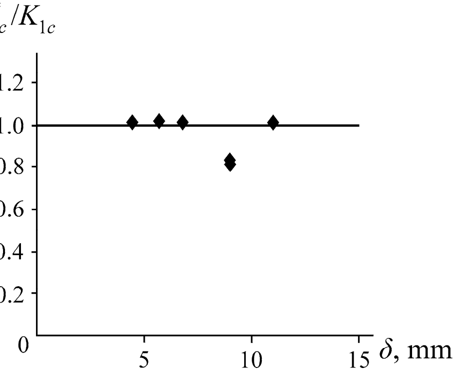


Fig. 6. (a) Calculated strength versus δ for plexiglass samples with various diameters d of internal holes and (b) determination of δ from minimization of the functional D.

Fig. 7. Calculated K_{1c}^* to measured K_{1c} stress intensity factors depending on the structural parameter.

Table 4. Calculated fracture toughness of rocks

Rock	σ_0, MPa	K_{1c}, MPa·m$^{1/2}$	δ, mm	D, mm	a_1, mm	σ_{c1}, MPa	δ^*, mm	K_{1c}, MPa·m$^{1/2}$
Dolerite	25.00	2.06	4.35	37.6	1.70	16.90	4.4	2.08
Gabbro-dolerite	13.40	1.25	5.50	37.6	3.25	7.00	5.7	1.27
Granite	11.20	1.14	6.60	37.6	3.25	7.00	6.8	1.15
Ufaley marble	6.90	0.90	10.80	37.6	3.25	5.11	11.0	0.91
Coarse-grained marble	5.93	0.86	13.00	37.6	2.00	4.70	9.0	0.70
Coarse-grained marble	5.93	0.86	13.00	37.6	3.00	4.24	9.0	0.71
Marmorized limestone	5.58	—	—	57.0	3.00	3.11	4.8	0.48
Marmorized limestone	5.58	—	—	57.0	5.00	2.46	5.0	0.49

CONCLUSIONS

In failure of samples of rocks and similar solid materials in nonuniform stress fields, especially under stress concentration, the conventional local strength criteria produce wrong results. The proposed method of determining tensile strength and fracture toughness of rocks based on the test data on disks with axial holes of different diameters using the integral fracture criterion offers the calculation accuracy not less than 20%.

From the test results on disks with axial holes, the critical stress intensity factor is calculated for rocks of different hardness. The calculated and measured values of fracture toughness show good agreement. This allows recommending this method for determination of fracture toughness in brittle rocks.

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