ISSN 1062-7391, Journal of Mining Science, 2016, Vol. 52, No. 5, pp. 956–964. © Pleiades Publishing, Ltd., 2016.

MINERAL MINING TECHNOLOGY

Optimal Extraction Sequence Modeling for Open Pit Mining Operation Considering the Dynamic Cutoff Grade[1]

E. Moosavi[a*] **and J. Gholamnejad**[b]

[a]*Department of Mining Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran*
e-mail: se.moosavi@yahoo.com
[b]*Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran*

Received September 8, 2015

Abstract— The cutoff grade problem is an important research challenge and vital optimization task in the yearly operational planning of open pit mines due to its combinatorial nature. Because of it's influenced by the economic parameters, the capacities of stages in the mining operation, mining sequence, and grade distribution of the deposit. Essentially, it asserts that the dynamic cutoff grade at any given period is a function of the ore availability and the needs of the mill at that period. Consequently, cutoff grades strategy and extraction sequence should be considered, simultaneously. Due to its goal, various attempts have been made to develop a computerized procedure for the extraction sequence of open pit mine. None of the resulting approaches appear to enjoy wide acceptance because of it's the numerous associated variables. A new model is proposed to overcome this shortcoming. This model solves the problem in the three steps: 1) the actual economic loss associated with each type of processing for each block, 2) the probabilities distribution and average grade for each type processing is computed from independent realization, and 3) each block with its expected economic loss is developed as a binary integer programming model. Using this model, the optimum extraction sequences in each period are identified based on the optimum processing decisions. A case study is presented to illustrate the applicability of the model developed. Results showed that the extraction sequences obtained using the suggested model will be realistic and practical. This model allows for the solution of very large problem in reasonable time with very high solution quality in terms of optimal net present value.

Keywords: Dynamic cutoff grade, open pit mine, binary integer programming, processing decisions, economic loss.

DOI: 10.1134/S1062739116041465

INTRODUCTION

In mineral resource industries, considerable economic gains can be achieved by optimal cutoff grades calculation in each period, which maximizes the net present value (NPV) of a mining project. So, one of the most complicated problems relating to the extraction sequence of mining blocks in the open pit mining operation are choosing the cutoff grades. Cutoff grade directly affects the economic feasibility of mining operation during the project life. The cutoff grade strategy, which results in higher overall NPV for a given project, starts via high cutoff grades. Higher cutoff grade leads to higher average grades per ton of ore in the initial periods of the mining sequence; consequently, higher average grades are realized depending upon the grade distribution of the deposit [1]. In each mining sequence, the cutoff grade states the quantity mined, processed, and finally, the product produced in the refinery for marketing [2–3]. The success of the project is significantly dependent on the selection of cutoff grade. Therefore, cutoff grade optimization is essential in term of life mine.

Some studies have dealt with the concept of cutoff grade developing methods and algorithms with different focuses and merits. As a result of these studies, a number of cutoff grade algorithms have considered a given constant cutoff grade with long mine life. On the other hand, based on the optimum cutoff grade algorithms, several variable cutoff grade procedures have been elaborated. At

[1]The article is published in the original.

first, the economic cutoff grade parameter was defined by Mortimer [4]. In fact this parameter is equivalent to breakeven grade criteria, which does not lead to NPV maximization. One of the best observations for optimization of cutoff grade is Lane's theory [2, 5]. This theory leads into construction a function of the maximization the NPV of cash flow, but is also able to provide various constraints of capacities (mine, mill, and refinery) in the mining operation. Other researchers have worked in this regard, such as Shinkuma and Nishiyama [6], Cairns and Shinkuma [7], Ataee and Osanloo [8−9], Rashidinejad et al. [10], Gholamnejad [11] as well. Halls and John [12] defines dual cutoff grades as a minimum grade that covers all estimated costs (mining, processing, refining and marketing) and has reasonable profit. In next decade, Taylor [13] made a difference between the planning and operation cutoff grades, that they may not always be the same. In fact he states that, "maximum present value and constant cutoff grades are incompatible". Taylor [14] also highlighted the need for establishment of stockpile through a real example. The result of these studies indicate that, the optimization of cutoff grade is due to limiting capacities of any mining, milling, and refining stages but not the mining sequence. These methods have assumed the mining sequence to be known in advance, however the mining sequence is certainly influenced by the cutoff grade choice. Therefore, various attempts have been made to develop a computerized procedure for optimization of cutoff grade considering mining sequence; such as, relying on Penalization (Dantzig-Wolf) [15−16], Lagrangian relaxation [17−21], 4D- Network relaxation [22−23], Dynamic programming [24], and Genetic algorithm [25]. Unfortunately, none of these attempts appears to enjoy wide acceptance. The common theme of these methods is the large size and inherent difficulty of the model.

To overcome this shortcoming, Zhang [26] propose a methodology based on a combination of Genetic algorithm and Topological sort to reduce the problem size, but the quality of final solution is not guaranteed. Boland et al. [27] uses several strategies to reduce the computational time. Gleixner [28] extends the work of Boland et al. [27] by developing a type of aggregation and also presents ideas for applying Lagrangian relaxation. They did not consider the grade blending constraint into account. Nevertheless it is unclear, because these models cannot guarantee qualitative satisfies of the grade blending in each period. Extensive surveys of different operation research techniques and modeling issues are provided by Newman et al. [29] and Espinoza et al. [30].

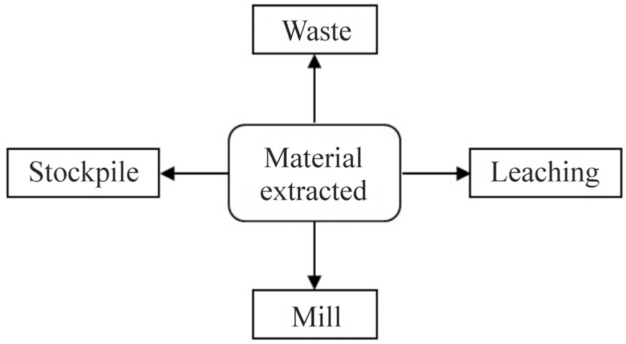
The metal recovery, mining and processing costs depend on the processing decisions. It, in turn, determines the block classification (waste, heap leach, dump leach, mill, etc.) and, therefore, the block economic value during the mine life. Ignoring the effect of these changes in the mining sequence on the optimum cutoff grades would lead to unrealistic mine design analysis. We apply a new binary integer programming model for solving the extraction sequence problem. This paper presents the effect of the cutoff grade strategy on the mining sequence. The formulation is based on economic loss assessment of a block with each alternative of processing decisions and probabilities distribution of the orebody grades. Economic loss assessment of a block and probabilities are integrated with physical constraints in a binary integer programming model. The ultimate goal of the presented model is setting the best mining sequence while optimizing cutoff grades in each period of mine life, simultaneously.

1. CUTOFF GRADE OPTIMIZATION NECESSITY IN OPEN PIT MINES

The cutoff grade analysis, as usual, replies to the following questions:

- What material in a deposit is worth mining and processing, or if not so, it should be considered waste;
- How should it be processed once it is mined.

One cutoff grade, dictated by management, fails to allow the flexibility needed to maximize profits in different situations [31]. In fact, the cutoff grade is calculated for the purposes of sending material that are extracted from the mine. The figure shows the decision criteria based on the cutoff grade.



Decision criteria based on the cutoff grade.

Cutoff grade optimization is an interactive process, including metal price, mining, milling cost, the capacity of processing plant, mining capacity in the mining operation, mining sequence, grade distribution of the deposit, and resulting cash flow. Processing decisions of block play a principal role in change of mining sequence. The effect of these changes will be enormous on the cutoff grades mining operation which changes due to the declining effect of net present value during the mine life. Even though, the fact is, that optimization of cutoff grades should be simultaneously considered in mining sequence [15]. Therefore, cutoff grade optimization is one of the most important topics in mining operation, because:

- Cutoff grade optimization is used for the decision of processing types (mill/leaching/stockpile/waste) to determine which decision dictates the maximum economic profit;
- Cutoff grade optimization can improve mining sequence during mine life.

2. FORMULATION OF MINING SEQUENCE CONSIDERING THE DYNAMIC CUTOFF GRADE

2.1. Definitions and Assumptions

In the mining sequence problem, the extraction of each block in each period is dependent upon the definition of cutoff grade. The cutoff grade dictates the block classification. Obviously, the choice of type in choosing the different types of processing is directly related to the selection of block classification in mining sequence, and is effected by special recovery, costs and special block economic value, as well. Consequently, the misclassification of a block results in a wrong block economic value calculation. In this regard, Richmond [32] defines a function as “economic loss”, which is used to distinguish between ore and waste block. The actual economic loss associated with each type of processing d (d=1, 2,…, D ordered from lowest grade to highest grade) is the potential value less than the recovered value, which can be calculated using the following equation

$$L_{ijk}^{d} = \left[\left(P - C_{sel}\right)\cdot \bar{\alpha}_{ijk} \cdot R^{d} - C_{p}^{d} - C_{m}^{d}\right] - \left[\left(P - C_{sel}\right)\cdot \bar{\alpha}_{ijk} \cdot R^{d'} - C_{p}^{d'} - C_{m}^{d'}\right], \tag{1}$$

where P—unit selling price of the metal; C_{sel}—unit selling cost of the metal; ijk: is the block identification number; $\bar{\alpha}_{ijk}$—average grade of block ijk; R^{d}—total metal recovery of material if processed as type d; C_{p}^{d}—unit processing cost of the material if processed as type d; C_{m}^{d}—unit mining cost of the material if processed as type d; d—correct processing type for block ijk; d'—chosen processing type for block ijk.

Taking from the Eq. (1), if the correct processing type for block ijk is chosen, then economic loss is zero. In practice, the ore block is unknown and it will be represented by a cumulative distribution function. A more realistic method is to use the conditional simulation techniques, which allows the generation of a number of equally probable realizations of block grades. Taken this into consideration the expected economic loss (EEL) for each alternative d is calculated using the probabilities distribution and average grade for each type processing, generated from independent realization as described in the following equation:

$$EEL_{ijk}^{d} = \sum_{d=1}^{D} \left[P_{ijk}^{d} \middle| O \right] \cdot L_{ijk}^{d}, \tag{2}$$

where $P_{ijk}^{d} | O$ –probabilities distribution of block ijk if processed as type d.

The optimal processing type for block ijk is that d for which the EEL is minimized, i.e.:

$$L(opt)_{ijk} = Min \quad [EEL_{ijk}^{d}]. \tag{3}$$

2.1. Formulation as an Integer Problem

In light of the definitions and assumptions described above, the mathematical programming model of the mining sequence in term of integer decision variables imposes in which period the particular block is extracted and its destination is determined correctly. In fact, this model can simultaneously optimize the block extraction sequence and cutoff grades strategy. As mentioned earlier, the objective function of the model can be represented mathematically as the following:

$$Minimize \quad Z = \sum_{ijk \in \Gamma} \sum_{t}^{T} \frac{L(opt)_{ijk}^{t} \cdot b_{ijk}^{t}}{(1+r)^{t}}. \tag{4}$$

The objective function in Equation (4) does not explicitly maximize NPV, rather optimizes feasible extraction sequencing and ensures a desired cutoff grade. Subsequently, destination of the block extracted is quite straightforward. The reason is that feasible extraction sequences and the amount of ore having the desired quality to be sent to the mill need to prioritize. So, the mentioned objective function indirectly leads to a maximum NPV that is optimal. Otherwise, the generated NPV would only be optimal in the theory but not in mining practice. However, economic loss for the suitable ore block which has the desired properties has been integrated in the present model, to maximize the chances of delivering to the mill, the amount and quality of ore required during mining operation. EEL minimization and feasible sequences; result in maximum NPV. On the other hand, the model suggests a unique cutoff grade policy and looks in to the risk of maintaining the cash flows to maximize NPV based on the possible variations in production from mine, processes during the life of operation. The proposed model in Equation (4) contains a series of constraints as follow:

$$\sum_{ijk \in \Gamma} \left(\overline{\alpha}_{ijk} - U_{\overline{\alpha}}^{t} \right) \cdot Q_{ijk}^{o} \cdot b_{ijk}^{t} \le 0 \qquad \text{for all } t; \tag{5}$$

$$\sum_{ijk \in \Gamma} \left(\overline{\alpha}_{ijk} - L_{\overline{\alpha}}^{t} \right) \cdot Q_{ijk}^{o} \cdot b_{ijk}^{t} \ge 0 \qquad \text{for all } t; \tag{6}$$

$$\sum_{ijk \in \Gamma} \left(Q_{ijk}^{o} \cdot b_{ijk}^{t} \right) \le U_{o}^{t} \qquad \text{for all } t; \tag{8}$$

$$\sum_{ijk \in \Gamma} \left(Q_{ijk}^{o} \cdot b_{ijk}^{t} \right) \ge L_{o}^{t} \qquad \text{for all } t; \tag{9}$$

$$\sum_{ijk \in \Gamma} \left(Q_{ijk}^{o} + Q_{ijk}^{w} \right) \cdot b_{ijk}^{t} \le U_{w\&o}^{t} \qquad \text{for all } t; \tag{10}$$

$$\sum_{ijk \in \Gamma} \left(Q_{ijk}^{o} + Q_{ijk}^{w} \right) \cdot b_{ijk}^{t} \ge L_{w\&o}^{t} \qquad \text{for all } t; \tag{11}$$

$$b_k^t - \sum_{y}^{Y} \sum_{r=1}^{t} b_y^r \le 0 \qquad \text{for all } t, k, \tag{12}$$

where $L(opt)_{ijk}^t$—the optimal processing type for block *ijk* in period *t*; *T*—the total number of scheduling periods; *t*— is the scheduling periods index, *t=1,2,…,T*; Γ—the total number of blocks to be scheduled' *r*—the discount rate in each period; b_{ijk}^t—the binary variable equal to:

$$\begin{cases} 1 & \text{if block ijk is extracted at period t} \\ 0 & \text{Otherwise} \end{cases};$$

Q_{ijk}^o—the ore tonnage in block *ijk*; Q_{ijk}^w—the waste tonnage in block *ijk*; $\overline{\alpha}_{ijk}$—the average grade of block *ijk*; $U_{\overline{\alpha}}^t$—the upper bound average grade of material sent to the mill in period *t*; $L_{\overline{\alpha}}^t$—the lower bound average grade of material sent to the mill in period *t*; U_o^t—the upper bound total tons of ore processed in period *t*; L_o^t–the lower bound total tons of ore processed in period *t*; $U_{w\&o}^t$–the upper bound total amount of material (waste and ore) to be mined in period *t*; $L_{w\&o}^t$–the lower bound total amount of material (waste and ore) to be mined in period *t*; *Y*—the total number of blocks overlaying block *k*; *K*—the index of a block considered for extraction in period *t*; *y*—the counter for the *Y* overlaying blocks.

Constraints (5), (6) limit the average grade of the material sent to the mill to a certain value. Constraints (7) enforce that a block is removed in one period only. Constraints (8), (9) ensure that the milling capacities hold. These upper and lower bounds are necessary to secure a smooth feed of ore. Constraints (10), (11) relate the actual available equipment capacity for each period. These upper and lower bounds are total amount of material (ore and waste) to be mined in period. Constraint (12) is the wall slope restriction on the basis of Y constraints for each block per period.

The proposed model provides a tool for evaluating alternative policies as part of feasibility studies at the long-term production scheduling stage. Consequently, the resultant cutoff grade policy facilitates a risk-quantified decision making process involving major investments for sustainable utilization of mineral resources.

The mathematical programming method is defined as follow:

- The implementation of the model begins with a three dimensional block model.
- The ultimate pit limits and pushback design is completed.
- The parameters including mining and processing stages capacities, mining costs and processing type costs, and current metal price through whole extraction periods are known at the beginning of each extraction period.
- The generations of a number of equally probable realizations of the block model were obtained using sequential Gaussian simulation.
- Providing the economic loss matrix of each block for each processing type, and subsequently, calculating the EEL for each alternative *d*.

Developing a mixed integer programming model for optimal selection of the blocks in each period on the specified criteria.

2. APPLICATION OF PROPOSED MODEL IN GOLD ORE

In this section we have implemented and tested the proposed model on real mine block to ensure optimality of the block extraction sequence and cutoff grades strategy optimization process. A gold mine sends its mineralized products to the four destinations: waste dump, dump leach, heap leach or mill. The characteristics of these four classes are listed in Table 1. At first one hundred equally probable realizations of the orebody gold grades were generated using Sequential Gaussian Simulation. The results of the simulation for a given block are shown in Table 2.

Table 1. The characteristics of each mineralized block class for gold ore deposit

Explanation	Unit	Waste	Dump leach	Heap leach	Mill
Processing type (destination)	No.	1	2	3	4
Grade range	gr/ton	0–0.49	0.5–0.99	1-1.99	2
Average grade	gr/ton	0.4	0.7	1.5	4.5
Metal recovery	%	0	45	70	95
Selling cost	\$/ gr of gold	0.5	0.5	0.5	0.5
Mining and processing costs	\$/ ton of ore	1.5	3.5	5.7	10.5
Gold price	\$/ gr of gold	10			

Table 2. The results of the simulation for a given block of gold ore deposit

Grade range	0–0.49	0.5–0.99	1–1.99	≥2
Number of realization	35	28	20	17
Corresponding probability, %	35	28	20	17

Table 3. The results of the economic loss of the block classification

Chosen mining destination	Correct mining destination				Expected economic loss
	1	2	3	4	
1	0	1	5.775	31.612	6.8
2	0.29	0	1.362	14.375	2.81
3	1.54	0.538	0	5.887	1.69
4	5.39	3.68	1.237	0	3.16

For example, assume that a correct mining destination for block *ijk* to be such that it should be sent to the dump leach (d=3), but incorrectly it is sent to the waste dump (d'=2), then the economic loss assessment due to this misclassification can be calculated from Equation (1) in the following way:

$$L_{ijk}^{3} = [(10-0.5)\times1.5\times0.7-5.7]-[(10-0.5)\times1.5\times0.45-3.5]=1.362.$$

Table 3 shows the results of the economic loss for the other values of d and d'. If the chosen class of block *ijk* is d=2, then the expected economic loss due to its misclassification can be achieved from Equation (2) as follows:

$$EEL_{ijk}^{2} = 0.29\times0.35+0\times0.28+1.362\times0.2+14.375\times0.17=2.81.$$

According to Table 3, the optimum expected economic loss is 1.69; therefore, the optimum destination of this block is d=3, it means that it is better to send this block to the heap leach.

In the extraction sequence problem the extraction of each block in each period depends on its economic value in that period. On the other hand, due to changes in price and costs during the time, block economic value varies with time; therefore; it may be possible that the optimum block classification in a period be different from the optimum block classification in the other periods. Accordingly, the loss function for block *ijk* in period t can be calculated as Equation (4). Due to the operational requirements, the minimization of the objective function is subjected the available constraints as Eqs. (5) to (12).

Table 4. The optimal cutoff grade policy and corresponding production rates, and NPV of the gold mine

Year	Mill		Heap leach		NPV, $ M
	Cutoff grade, %	Quantity of ore processed, MT	Cutoff grade, %	Quantity of ore processed, MT	
1	2.36	0.073	1.96	0.054	1.794
2	1.97	0.073	1.61	0.064	0.985
3	1.83	0.073	1.52	0.077	0.745
4	1.75	0.073	1.47	0.071	0.581
5	1.63	0.073	1.43	0.070	0.745
6	1.58	0.073	1.37	0.074	0.454
7	1.47	0.073	1.31	0.078	0.376
8	1.42	0.073	1.26	0.086	0.371
9	1.35	0.073	1.24	0.050	0.319
10	1.28	0.073	1.17	0.028	0.253
11	1.19	0.073	1.04	0.017	0.228
12	1.06	0.046	0.99	0.011	0.110

Since the iterative steps of optimization are boring and time consuming, an Excel spreadsheet was developed to facilitate doing the calculations. For solving presented model in the gold mine provided an input file of block model by using Excel software, includes: characteristics of counters each block, tonnage, grade and ore content of each block, the expected economic loss of each block. Decision variables and available constraints related to the type of block in the model are considered. Our goal was to generate a schedule for a 12 years mine life by maintaining the discount rate at 8%.

Knowing the input information, the steps of presented model in the previous section are implemented to develop the optimal cutoff grade policy along with a portfolio of production rates and net present values. Table 4 demonstrates cutoff grades policy generated by simultaneously utilizing mining sequences.

As shown in Table 4, during year 1, heap leach processes ore carrying a metal content between 1.96% and 2.36%. This dictates that material below 1.96% is treated as waste and transported to the waste dumps. However, it generates a portfolio of optimal net present value.

CONCLUSIONS

This paper presents a mathematical model based on the binary integer programming for open pit mines, which can combine mining reasonable operation and cutoff grades strategy into one. In fact, the proposed procedure is to develop a model that generates a practical and feasible schedule considering processing types, while satisfying system constraints and minimizing expected economic loss. Although the proposed model is not set up to directly maximize NPV. It provides a realized NPV, which is optimum under the mining sequencing and cutoff grade strategy considerations. As a matter of fact, that NPV can be increased by imposing the probabilities distribution of blocks; because the suggested model tends to minimize the EEL. This leads to more high-grade blocks are mined for the earlier periods. In this study we used the economic loss as the objective function. Clearly, the economic loss function method is an efficient technique to determine the optimum processing type of the material in each period. This model overcomes the limitation of conventional methods, and the innovation mainly includes as follow:

- To reduce required number of variables and, subsequently, to handle the available variables and constraints in a short time.
- To be capable of taking various types of processing in to account.

The proposed model was applied on a gold ore deposit. The results of the case study indicate that the proposed model provides flexibility at the mine planning stage for evaluation of various alternatives, and ensures the optimum resource utilization coupled with accurate economic decisions with respect to major mining investments.

REFERENCES

1. Dagdelen, K., An Optimization Algorithm for Open Pit Mine Design, *Proc. 24th Conf. Application of Computers and Operations Research in the Mineral Industry*, Quebec, 1993, pp. 157–165.
2. Lane, K.F., The Economic Definition of Ore, Cutoff Grade in Theory and Practice, Mining Journal Books Limited, London, 1988.
3. Mohammad, W.A., Development of Generalized Cutoff Grade Optimization Algorithm for Open Pit Mining Operations, *Journal of Engineering and Applied Science*, 2002, 21(2), pp. 119–127.
4. Mortimer, G.J., Grade Control, *Trans. Inst. Min. Metall*., 1950, 59, pp. 357–99.
5. Lane, K.F., Choosing the Optimum Cutoff Grade, *Colorado School of Mines Quarterly*, 1964, 59(4), pp. 811–829.
6. Shinkuma, T. and Nishiyama, T., The Grade Selection Rule of Metal Mines: An Empirical Study on Copper Mines, *Resource Policy*, 2000, 26, pp. 31–38.
7. Cairns, R.D. and Shinkuma, T., The Choice of Cutoff Grade in Mining, *Resource Policy*, 2003, 29, pp. 75–81.
8. Ataei, M. and Osanloo, M., Methods for Calculation of Optimal Cutoff Grade in Complex Ore Deposits, *Journal of Mining Science*, 2003, 39, pp. 499–507.
9. Ataei, M. and Osanloo, M., Using a Combination of Genetic Algorithm and the Grid Search Method to Determine Optimum Cutoff Grades of Multiple Metal Deposits, *International Journal of Surface Mining, Reclamation and Environment*, 2004, 18(1), pp. 60–78.
10. Rashidinejad, F., Osanloo, M., and Rezai, B., An Environmental Oriented Model for Optimum Cutoff Grades in Open Pit Mining Projects to Minimize Acid Mine Drainage, *Int. J. Environ. Sci. Tech.*, 2008, 5(2), pp. 183–194.
11. Gholamnejad, J., Incorporation of Rehabilitation Cost into the Optimum Cutoff Grade Determination, *Journal of the Southern African Institute of Mining and Metallurgy*, 2009, 108, pp. 89–94.
12. Halls, J.L. and John, L., Determination of Optimum Ore Reserves and Plant Size by Incremental Financial analysis, *Transactions of the Institute of Mining and Metallurgy*, 1969, 78, A20-A26.
13. Taylor, H.K., General Background Theory of Cutoff Grade, *Transactions of the Institute of Mining and Metallurgy*, 1972, A(96), A204-A216.
14. Taylor, H.K., Cutoff Grades—Some Further Reflections, *Transactions of the Institute of Mining and Metallurgy*, 1985, A(81), A160-A179.
15. Johnson, T.B., Optimum Open Pit Mine Production Scheduling, *PhD Thesis*, Operations Research Department, University of California, Berkeley, 1968.

16. Johnson, T.B., Optimum Production Scheduling, *Proc. 8th Int. Symp. Computers and Operations Research*, 1969, pp. 539–562.
17. Dagdelen, K., Optimum Multi-Period Open Pit Mine Production Scheduling, *PhD Thesis*, Colorado School of Mines, Golden, Colorado, 1985.
18. Dagdelen, K. and Johnson, T.B., Optimum Open Pit Mine Production Scheduling by Lagrangian Parameterization, *Proc. 19th Conf. Application of Computers and Operations Research in the Mineral Industry*, 1986, pp. 127–142.
19. Kawahata, K., A New Algorithm to Solve Large Scale Mine Production Scheduling Problems by Using the Lagrangian Relaxation Method, *PhD Thesis*, Colorado School of Mines, 2007.
20. Moosavi, E., Gholamnejad, J., Ataee-pour, M., and Khorram, E., Improvement of Lagrangian Relaxation Performance for the Open Pit Mines Constrained Long-term Production Scheduling Problem, *Journal of Central South University*, 2014, vol. 21, pp. 2848–2856.
21. Moosavi, E., Gholamnejad, J., Ataee-pour, M., and Khorram, E., A Hybrid Augmented Lagrangian Multiplier Method for the Open Pit Mines Long-Term Production Scheduling Problem Optimization, *Journal of Mining Science*, 2014, vol. 50, pp. 1047–1060.
22. Akaike, A., and Dagdelen, K. A., Strategic Production Scheduling Method for an Open Pit Mine, *Proc. 28th Conf. Application of Computers and Operation Research in the Mineral Industry*, 1999, pp. 729–738.
23. Mogi, G., Adachi, T., Akaike, A., and Yamatomi, J., Optimum Production Scale and Scheduling of Open Pit Mines Using Revised 4D Network Relaxation Method, *Proc. 17th Int. Symp. Mine Planning and Equipment Selection*, 2001, pp. 337–344.
24. Wang, Q., Gu, X., and Chu, D., A Dynamic Optimization Method for Determining Cutoff Grades in Underground Mines, *Mineral Resources Management* (Gospodarka Surowcami Mineralnymi), 2008, pp. 133–142.
25. Xiau-wei, Gu, Qing, Wang, Dao-Zhang, Chu, and Bin, Zhang, Dynamic Optimization of Cutoff Grade in Underground Metal Mining, *Journal of Central South University*, 2010, 17, pp. 492–497.
26. Zhang, M., Combination Genetic Algorithms and Topological Sort to Optimize Open-Pit Mine Plans, *Proc. 15th Conf. Mine Planning and Equipment Selection*, Torino, Italy, 2006, pp. 1234–1239.
27. Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A.M., LP-Based Disaggregation Approaches to Solving the Open Pit Mining Production Scheduling Problem with Block Processing Selectivity, *Computer Operation Research*, 2009, 36(4), pp. 1064–1089.
28. Gleixner, A., Solving Large-Scale Open Pit Mining Production Scheduling Problems by Integer programming, *Master's Thesis*, Technische Universität Berlin, 2008.
29. Newman, A., Rubio, E., Caro, R., Weintraub, A., and Eurek, K., A Review of Operation Research in Mine Planning, *Interface*, 2010, 40(3), pp. 222–245.
30. Espinoza, D., Goycoolea, M., Moreno, E., and Newman, A., MineLib: A Library of Open Pit Mining Problems, *Annals of Operations Research*, 2013, 206(1), pp. 93–114.
31. Gershon, M.E., Optimal Mine Production Scheduling: Evaluation of Large Scale Mathematical Programming Approaches, *International Journal of Mining Engineering*, 1983, 1(4), pp. 315–329.
32. Richmond, A.J., Maximum Profitability with Minimum Risk and Effort, *Application of Computers and Operations Research in the Mineral Industry*, 2001, pp. 45–50.