ISSN 1062-7391, Journal of Mining Science, 2016, Vol. 52, No. 5, pp. 878–884. © Pleiades Publishing, Ltd., 2016.
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GEOMECHANICS

Tensile Strength of Rocks by Test Data on Disc-Shaped Specimens with a Hole Drilled through the Disc Center

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Received August 1, 2016

Abstract—The author reports test data on disc-shaped specimens of rocks and model media with a hole drilled through the center of the specimens loaded along the diameter. The test data processing uses non-local fracture criteria. The calculated destructive forces are compared with the measured destructive loads. Based on the tests of specimens with the central through holes, the tensile strength algorithm is presented.

Keywords: Failure, strength, tension, Brazilian specimen, non-local strength criterion.

DOI: 10.1134/S1062739116041334

INTRODUCTION

The analysis of properties of a rock mass starts with the determination of mechanical characteristics of rocks. One of these characteristics is tensile strength. The standard uniaxial uniform tension testing of rocks requires specific equipment and needs laborious manufacturing and fastening of test specimens. It is difficult to apply tension force without the eccentricity. Additional complication is secondary stresses generated by grips. Reasoning from these difficulties and facing the wanted high stiffness of testing machines, researchers and engineers looked for the other load application patterns to obtain the tensile strength characteristic. Direct tension tests appeared of little use in terms of rocks and were replaced by indirect methods. One more reason to search for express-methods of rock testing is variation of strength properties of rocks within a mine field, which requires for many tests to be carried out.

In the most widespread method of tension testing, Brazilian test, a disc-shaped specimen is diametrically compressed to failure. The maximum tension in the center of a disc, in the line of the load application, is close in value to the rupture strength of the test specimen [1]. This method has a few drawbacks and can be improved by introducing a hole in the center of the disk, which is meant to be a stress raiser [2, 3]. The reduction in the applied destructive force due to the introduced stress raiser enables extending the method for weak rocks with plastic properties, such as gypsum, limestone. According to [2], in viewpoint of the stability of crack initiation from a specimen center, symmetry of fragments relative to the force application line and smaller scatter of the destructive efforts, it is preferable to use disc specimens with a hole. Figure 1 shows the specimens of gypsum and organic glass, solid and with a hole, destroyed along the diameter.

(a) (b)

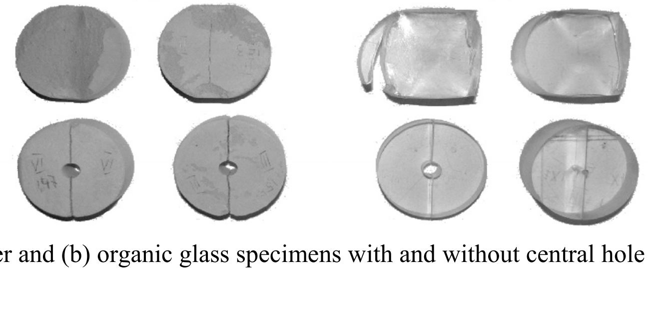

Fig. 1. (a) Pearl filler and (b) organic glass specimens with and without central hole after Brazilian test.

The introduction of a stress raiser represented by a small axial hole allows destruction of specimens without plastic areas to appear under the press plates. Such loading conditions are similar to uniaxial loading as the compression stresses, arising under Brazilian testing of solid specimens, are absent at the points where cracks can initiate.

After processing of test data based on elastic solution and the conventional local fracture criterion, the authors [2] could offer no explanation for high strength of rocks at the points of stress concentration. The stress field under such testing is nonuniform, and the use of the local fracture criterion that neglects structure of a medium is questionable.

Recent descriptions of destruction of structurally nonuniform material use nonlocal fracture criteria [3–10]. The nonlocal fracture criteria allow comparing strength values obtained in tests in the uniform and nouniform fields. One of the known and common nonlocal approaches to failure description is the integrated criterion [3, 4] that connects the average stress in the data smoothing area and the tensile strength in the uniform field. The gradient approach [11] used for processing of data obtained in destruction of cores with a central hole connected the nonlocal strength and the tensile strength of the cores in the uniform field [12–14]. For rock cores with a through central hole drilled ad a distance from the load application axis, the integrated fracture criterion was considered at the moment of fracture initiation [15].

In the framework of this study, the test data obtained on discs with holes are processed using the integrated fracture criterion and the strength gradient. Based on the values of the resultant limit loads, the uniform tensile strengths are determined.

1. TESTS AND RESULTS

The test specimens with the axial hole were made of dolerite, gabbro-diorite, granite and marble, and gypsum and sealing wax to simulate rocks. The tensile strength was determined on solid specimens of rocks using the Brazilian test, and the model gypsum and wax specimens were subjected both to the direct uniaxial tension and to the Brazilian test. Aiming to reduce the scatter of the mechanical properties of rocks, the specimens were cut from the same slab. Destruction of the specimens was carried out along a selected direction. The model medium specimens were manufactured under the same conditions and aged for a long time to minimize the scatter in strength. The stable contact between the specimens and the loading plates was reached using the fluorine plastic gaskets (50 μm). The test diagram is depicted in Fig. 2. All specimens of rocks had a diameter of 37.6 mm and a thickness of 18–20 mm. Diameter of the central hole was 3.4 mm in dolerite and 6.5 mm in gabbro-diorite, granite and marble specimens. Specimens made of gypsum and sealing wax had a diameter of 400 mm, thickness of 20 mm and the central hole diameter of 6 mm.

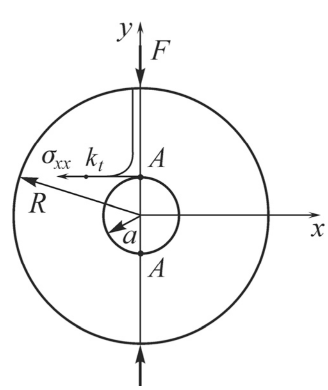

Fig. 2. Brazilian test pattern for disc specimens with the central hole.

Table 1. Average destructive forces on the specimens with and without the central hole, F_c and F_0 in the Brazilian tests

Rock	σ_0, MPa	K_{1c}, MPa·m$^{1/2}$	δ, mm	F_0, N	N	R, mm	a, mm	δ / L_e	F_0 / F_c	k_t
1	2	3	4	5	6	7	8	9	10	11
Dolerite	25	2.06	4.32	2965	1995	18.8	1.7	9.33	1.48	6.05
Gabbro-diorite	13.4	1.26	5.63	1580	923	18.8	3.25	6.35	1.71	6.96
Granite	11.2	1.14	6.6	1325	826	18.8	3.25	7.44	1.6	6.96
Marble	6.9	0.9	10.8	815	608	18.8	3.25	12.2	1.34	6.96
Pearl filler	1.22	0.146	9.12	—	125.8	20	2	16.7	—	6.13
Sealing wax	2.17	0.165	3.58	275.4	153	20	2	6.57	1.8	6.13

K_{1c}—crack resistance; a—inner hole radius; δ—structural parameter determined by (3); L_e—measure of stress field nonuniformity found using (5); R—disc radius; δ/L_e—dimensionless parameter; k_t—stress concentration factor.

The specimens made of gypsum and wax, subjected to direct tension, were flat dumbbells, with the rectangular working area 20 mm thick and 80 mm long. In order to eliminate bending and torque moments during loading, the specimens were subjected to tensile using special grips with spherical elements. For wax, the strength after the direct tension and Brazilian test appeared to be nearly the same. For gypsum, the direct tension strength made 1.22 ± 0.21 MPa, the tension strength after the Brazilian test was 1.88 ± 0.15 MPa, which is an essential difference.

The tests were carried out under the room temperature and at the same bar feed velocity of 0.5 mm/min on the test bench UME-10TM, which corresponded to the loading rate of 1 MPa/s. The test results are presented in Table 1, with the values of crack resistance of the test media, measured beforehand and used in what follows. The crack resistance values were calculated using procedure [16].

The last column of Table 1 contains the stress concentration factor values determined by the elastic solution based on the relation $k_t = \sigma_{\max} / (F_c / \pi R t)$, where R and t—radius and thickness of a specimen. The conventional local fracture criterion, used in this case, produces high values of local strength at "risk" points A in Fig. 2 as follows from Table 1. This fact implies that the use of the local fracture criterion is unjustified. Moreover, Table 1 data show that this relation depends on the test material.

2. MAXIMUM LOAD IN FAILURE OF DISC SPECIMENS WITH THE CENTRAL HOLE

The maximum force to fracture the rock specimens are calculated using the distribution of the tensile stress along the axis y in line of growth of a crack initiated at the "risk" point A (Fig. 2) [11]:

$$\sigma_x = \frac{F}{\pi R t}\left[2 - \frac{2a^2}{y^2} + \frac{12a^4}{y^4}\right], \tag{1}$$

where F—compressive force that is reaches maximum at the moment of fracture.

Let us discuss two variants of the nonlocal fracture criteria. With the integrated fracture criterion [3] $\sigma_0 = \frac{1}{\delta}\int_a^{a+\delta} \sigma_x(r)dr$, given the present geometry, the integration of (1) for finding the destructive force F_c for a specimen with the hole yields:

$$F_c^{(1)} = \pi R t \sigma_0 \left(1 + \frac{5a^3 + 4a^2\delta + a\delta^2}{(a+\delta)^3}\right)^{-1}, \tag{2}$$

where σ_0—tensile strength in the uniform field; δ—data smoothing area found from the relation [8, 17]:

$$\delta = \frac{2}{\pi}\left(\frac{K_{1c}}{\sigma_0}\right)^2, \tag{3}$$

The strength gradient criterion proposed for describing yield initiation in [7] was then applied to brittle failure [13, 14]. This criterion connects the local strength σ_e at the risk point and the tensile by means of the relation $\sigma_e = \sigma_0\left(1+\sqrt{\delta / L_e}\right)$. Regarding the cores with the hole, this approach offers the fracture condition:

$$F_c^{(2)} = \pi R t \sigma_0 \left(\frac{1+\sqrt{\delta / L_e}}{k_t}\right), \tag{4}$$

where $k_t(a)$—stress concentration factor calculated from the elasticity theory.

The values of the stress concentration factor used in this study as function of the ratio of the hole diameter to the core radius are give in [11]. The structural parameter δ of the medium is found from (3), the nonuniformity L_e for the given geometry is calculated as:

$$L_e = \frac{\sigma_e}{|\mathrm{grad}\,\sigma_e|} = \frac{3}{11}a. \tag{5}$$

The limit loads calculated using the formulas (2), (4) using the nonlocal fracture criteria are compiled in Table 2. The calculated values agree with the experimental loads. The last rows in columns 5 and 6 in Table 2, where the ratios of the calculated and experimental destructive loads are given, show the data of the statistic processing.

Figure 3 demonstrates capabilities of the local models by showing the ration of the calculated and experimental destructive loads applied to the specimens with the hole (Table 2). The calculated values are added with the ratios determined using the conventional local fracture criterion from the formula $F_c^{\mathrm{loc}} = \pi R t \sigma_0 / k_t$ (marked as black squares).

Table 2. Calculated limit loads under fracture of specimens with the hole using the integrated fracture criterion $F_c^{(1)}$ and the strength gradient $F_c^{(2)}$ and their ratio to the experimentally measured destructive loads

Rock	F_c, N	$F_c^{(1)}$, N	$F_c^{(2)}$, N	$F_c^{(1)}/F_c$	$F_c^{(2)}/F_c$	δ/L_e
1	2	3	4	5	6	7
Dolerite	19.95	19.85	19.78	0.995	0.991	9.33
Gabbro-diorite	9.23	9.136	8.003	0.99	0.87	6.35
Granite	8.26	8.164	7.084	0.988	0.857	7.44
Marble	6.08	5.98	5.26	0.984	0.865	12.2
Pearl filler	1.258	1.25	1.30	0.99	1.04	16.7
Sealing wax	1.55	1.61	1.60	1.04	1.03	6.57
Average value and confidence interval				1.0 ± 0.03	0.94 ± 0.09	

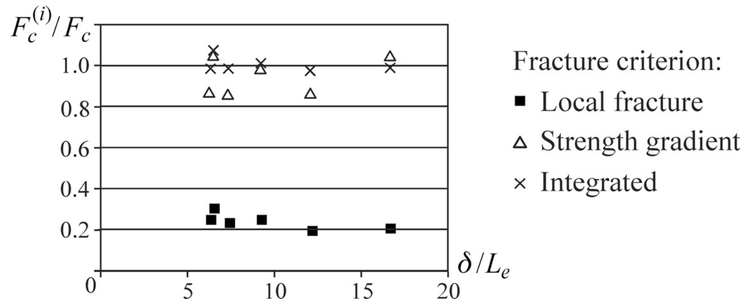

Fig. 3. Ratios of the calculated and experimental destructive forces during fracture of discs with the holes.

It follows from Table 2 and Fig. 3 that the values of the destructive loads found using the nonlocal fracture criteria are closer to the experimental values than the loads determined with the help of the conventional criterion. Based on that, the author makes a conclusion that the use of the integrated fracture criterion (crosses in Fig. 3) is preferable as against the strength gradient.

3. DETERMINATION OF TENSILE STRENGTH BASED ON THE VALUES OF THE DESTRUCTIVE LOAD AND CRACK RESISTANCE

The calculation of the destructive forces using the nonlocal fracture models involved the known values of the structural parameter δ dependent on the strength σ_0 of a medium. Actually, the critical load under which a specimen fails is known, and its value is used to determine the strength of a medium, which is analogous to the uniform tension tests.

Withdrawing δ from (2), (3) and using the integrated criterion, we arrive at the equation:

$$a^3\sigma_t^7 - 6a^3\sigma_c\sigma_t^6 + 3a^2b\sigma_t^5 - 7a^2b\sigma_c\sigma_t^4 + 3ab^2\sigma_t^3 - 4ab^2\sigma_c\sigma_t^2 + b^3\sigma_t - b^3\sigma_c = 0, \quad (6)$$

where σ_t—tensile strength; $b = (2/\pi)K_{1c}^2$; $\sigma_c = F_c/\pi Rt$—strength of a specimen.

The application of the gradient criterion to strength calculation yields the relation below:

$$\sigma_t = k_t\sigma_c - \sqrt{\frac{2}{\pi L_e}}K_{1c}. \quad (7)$$

Table 3 gives the calculated tensile strengths of rocks for the models under analysis and their ratio to the tensile strength obtained in the Brazilian tests of rocks and under direct tension tests of gypsum and wax. The strengths are calculated at the averaged values of the destructive forces for each type of the materials.

Table 3. Tensile strength values (MPa) calculated using the integrated fracture criterion $\sigma_t^{(1)}$ and the strength gradient criterion $\sigma_t^{(2)}$

Rock	$\sigma_t^{(1)}$	$\sigma_t^{(2)}$	σ_0	$\sigma_t^{(1)}/\sigma_0$	$\sigma_t^{(2)}/\sigma_0$
Dolerite	25.5	25.6	25	1.02	1.027
Gabbro-diorite	14.5	20	13.4	1.13	1.52
Granite	12.1	18	11.2	1.08	1.6
Marble	7.0	11.3	6.9	1.01	1.64
Gypsum	1.23	1.0	1.22	1.0	0.81
Sealing wax	1.82	1.92	2.2	0.83	0.87
Average value and confidence interval				1.014 ± 0.104	1.24 ± 0.38

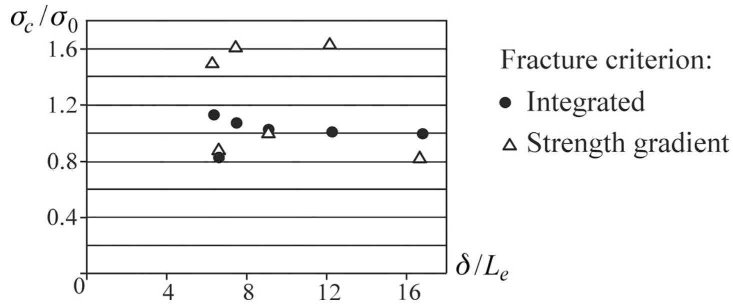

Fig. 4. Ratio of the calculated and measured tensile strengths after Brazilian test for rocks and after direct tension for the model media.

Figure 4 presents the ratios of the calculate tensile strength and the tensile strength values measured in rocks after Brazilian test and in model media after direct tension tests. It is noteworthy that the solution of Eq. (6) strongly depends on the input parameters, i.e. critical load and crack resistance. In order to ensure acceptable accuracy of the calculated strength at an error of 20%, it is necessary to determine the input parameters at an error of 5%. The experimentation has shown that the efficient range of the formula used in the calculations is limited to $a/R < 0.2$.

It follows from Fig. 4 and Table 3 that the use of the integrated fracture criterion in processing the data obtained in the fracture tests of specimens with the central hole offers the best agreement with the tensile strength values for rocks and model media under study. The limit strength of gypsum discs with the hole in the center coincides with the strength of direct tension. This is the confirmation of the applicability of this method of testing to the media that are impossible to subject to the standard Brazilian test.

CONCLUSIONS

When processing the data on indirect fracture test of rocks, when the stress field is nonuniform, it is not always correct to use the conventional local fracture criteria.

The calculated values of the critical load that result in diametrical destruction of specimens with a hole in the center are close to the measured values using the nonlocal fracture criteria.

The calculation of the tensile strength of a medium by the test data of cores with the axial hole using the integrated fracture criterion yields the best agreement with the experimental strength values of the materials.

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