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GEOMECHANICS

# **Rock Deformation around Stopes at Deep Levels**

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**Abstract**—The authors offer an approach to estimating deformation of rock mass around a stope (zonal disintegration) at deep levels. The algorithm of geomechanical condition of rocks is developed, and the ranges of stresses and strains in the zone of influence of a stope are identified.

*Keywords:* Stope, physical phenomena, rock, deep levels, analytical solution, Young modulus, boundary conditions.

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### INTRODUCTION

The concept of deep level mining involves the depth from the ground surface, ground conditions, stresses and strains and physico-mechanical properties of rocks, including rock mass disintegration around underground excavations. Conventionally, deep level rock mass is under compression when all pores and fractures are closed and the rock mass is solid. Fractures and open pores may only be observed on exposures where stress relief takes place.

Analyses of deep level rock mass require additional data on physical behavior of rocks at depth and purpose-oriented testing aimed at finding the role of this behavior. Otherwise, the deeper level geotechnology calculations using methods for shallow depths are of no use. An auxiliary hypothesis is required, based on experience observations, for identifying the trend of the research.

On approach to a relief zone due to drivage, various physical phenomena arise, neglected in the standard formulations of problems of rock mechanics. The relevant generalization of models is necessary. Some of them were discussed in [1]. It is possible to say that a deeper level model does not exist. We do not speak about a mathematical, rather complicated model. Even a satisfactory description of the system of cause & effect is unavailable. Statements on special role of analogies on transition from shallow depth to deeper levels seem premature. We can only speak about a more or less justified representation of deep level processes. When moving from the ground surface depthwards, rock mass undergoes continuous restructuring.

In case of an underground excavation, its drivage creates a transition layer between the excavation and rock mass. Local release of deformation energy takes place in this layer and, as a consequence, stress concentration lowers in some areas of the rock mass. The complex stress state in the vicinity of the excavation initiates phase transformations and other unsteady processes, and, being of practical current interest, calls for search of solutions for the appropriate boundary value problems [2–5]. After analyzing effect of properties of rocks on rock mass deformation in the vicinity of an excavation (after stress relief), it is required to construct an analytical solution for model problems in order to eliminate additional errors of numerical solutions. It is useful to obtain analytical solutions for such problems for testing various computational approaches.

The problems in description of zonal disintegration (ZD) phenomenon [6] and deep level mining in many ways coincide. Some aspects of ZD are discussed in [7] where it is stated that formation of a closed zone around a driven excavation allows for deeper level mining. With the deeper excavations, the protective effect exerted by ZD on stoping is stronger. There are many questions on physics of ZD and detail experimental recording of ZD at a distance of  $\sqrt{2R}$  (*R*—the excavation radius) from the excavation center. For instance, in the vicinity of an excavation, no damage zone was observed in hole drilling in the zone  $R \le r \le \sqrt{2R}$  while the damage was observed in a small zone at  $r = \sqrt{2R}$ .

Based on these data, the authors are going to construct the first (basic) approximation of the process of deep level mineral mining.

### 1. FORMULATION AND SOLUTION OF THE PROBLEM

Let an elastic plane be weakened by a circular hole with a radius R, simulating a deep level excavation. It is assumed that the excavation perimeter is free from external stresses, and the compression P acts in the line x at infinity (Fig. 1).

According to [8], the solution of this problem in the polar coordinates  $(r, \theta)$  is for the stresses:

$$\sigma_{r} = -\frac{P}{2} \left( 1 - \frac{R^{2}}{r^{2}} \right) - \frac{P}{2} \left( 1 - \frac{4R^{2}}{r^{2}} + \frac{3R^{4}}{r^{4}} \right) \cos 2\theta ,$$
  

$$\sigma_{\theta} = -\frac{P}{2} \left( 1 + \frac{R^{2}}{r^{2}} \right) + \frac{P}{2} \left( 1 + \frac{4R^{2}}{r^{2}} + \frac{4R^{4}}{r^{4}} \right) \cos 2\theta ,$$
  

$$\tau_{r\theta} = \frac{P}{2} \left( 1 + \frac{2R^{2}}{r^{2}} - \frac{3R^{4}}{r^{4}} \right) \sin 2\theta ;$$
  
(1)

and for the displacements:

$$V_{r} = \frac{P}{8\mu r} \left\{ (\kappa - 1)r^{2} + 2R^{2} + 2\left[ R^{2}(\kappa + 1) + r^{2} - \frac{R^{4}}{r^{2}} \right] \cos 2\theta \right\},$$

$$V_{\theta} = -\frac{P}{4\mu r} \left\{ R^{2}(\kappa - 1) + r^{2} + \frac{R^{4}}{r^{2}} \right\} \sin 2\theta,$$
(2)
Paisson's ratio:  $\mu = -\frac{E}{r^{2}} = E$ . Now give modulus

where  $\kappa = 3-4\nu$ ,  $\nu$ —Poisson's ratio;  $\mu = \frac{E}{2(1+\nu)}$ , E—Young's modulus.

In the analysis of the behavior of shearing stresses  $\tau_{r\theta}$  from (1) [7], it is shown that at  $r = \sqrt{3}R$  the shearing stresses  $\tau_{r\theta}(r,\theta)$  reach extreme values when the plane with the circular hole is subjected to tension and compression in the line of the axis x. In the tension or compression tests of rock specimens, Lüders bands are observed in the specimens as a result of shearing, and failure goes afterwards. The analogous situation takes place in rock mass surrounding an excavation subjected to compression at infinity.



Fig. 1. Scheme of an underground excavation in polar coordinates.



Fig. 2. Displacement of the excavation contour: (a) component  $V_r$ ; (b) component  $V_{\theta}$ .

Let us consider the displacements  $V_r$  and  $V_{\theta}$  from (2). At the contour r = R:

$$V_r = \frac{PR(\kappa+1)}{8\mu} (1 + 2\cos 2\theta),$$

$$V_{\theta} = -\frac{PR}{4\mu} (\kappa+1)\sin 2\theta,$$
(3)

similarly, at  $r = \sqrt{3}R$ :

$$V_r = \frac{PR}{8\sqrt{3}\mu} [3\kappa - 1 + 2(\kappa + 3.66)\cos 2\theta],$$

$$V_{\theta} = -\frac{PR}{4\sqrt{3}\mu} (\kappa + 2.33)\sin 2\theta.$$
(4)

Figure 2 illustrates displacement of the excavation contour in the form of ovals displaced relative one the other by an angle 450°.

The maximum value:

$$V_r = \frac{3PR}{8\mu}(\kappa+1)$$

is reached at  $\theta = 0$  and 180°, and at  $\theta = 90$  and 270°:

$$V_r = -\frac{PR}{8\mu}(\kappa+1) \,.$$

In the same manner, for  $\theta = 45$  and  $225^{\circ}$ :

$$V_{\theta} = -\frac{PR}{4\mu}(\kappa+1),$$

and for  $\theta = 135$  and  $315^{\circ}$ , respectively:

$$V_{\theta} = \frac{PR}{4\mu} (\kappa + 1) \, .$$

At  $r = \sqrt{3}R$  the qualitative pattern of the displacement component remains but the values are different.

Let us analyzes the extremum of  $V_{\theta}(r, \theta)$  relative to the distance from the excavation center:

$$\frac{dV_{\theta}}{dr} = -\frac{P}{4\mu} \left[ -\frac{R^2}{r^2} (\kappa - 1) + 1 - 3\frac{R^4}{r^4} \right] \sin 2\theta = 0, \qquad (5)$$

whence:

$$r = R_{\sqrt{\frac{\kappa - 1 + \sqrt{(\kappa - 1)^2 + 12}}{2}}},$$
(6)

i.e., at *r* from (6), the displacement  $V_{\theta}$  has an extremum, that is minimum in this case. Of interest is the fact that position of these minimums depends on Poisson's ratio of rocks, which is a transversal/longitudinal strain ratio. When  $0 \le v \le 0.5$ , the minimums have the coordinates in the range:

$$1.315R \le r \le \sqrt{3}R,\tag{7}$$

and r = 1.315R fits with v = 0.5, and  $r = \sqrt{3}R$  fits with v = 0. For  $r = \sqrt{2}R$ , v = 0.375.

As follows from (1) and (2), the stresses do not depend on the elastic properties of rocks while the displacements do. Coordinates of the maximum shearing stresses coincide with the boundary of the range of the minimum  $V_{\theta}(r,\theta)$ . In other words, the domains of the increased values of  $\tau_{r\theta}$  and the decreased values of  $V_{\theta}$  overlap. The contour of the excavation experiences the highest recovery of the elastic strains and, thus, the highest values of shear modulus, and the comparison of the change of the excavation contour can be carried out starting from r = R and finishing at r found from (6).

Now let us address the problem on zonal disintegration. It is assumed that the elastic moduli of rocks may depend on the accumulated plastic strain. The question of the decrease in the elastic modulus versus the accumulated plastic strain was discussed in [8]. We are interested in the influence exerted by this property of rocks on the formation of disintegration zone in rocks. Analysis of influence exerted by any property of rocks on deformation of underground excavations requires construction of analytical solution of a model problem. This papers studies effect of relationship between Young's modulus and high strains on distribution of stresses and strains in the vicinity of a circular cross section excavation with a radius R in the course of destressing at the excavation contour based on the analytical solution.

Figure 3 shows the loading curve for a steel specimen; the slope of the initial straight line is assumed as Young's modulus E. The point M in the curved section of the plot characterizes the plastic properties of the material. For the point M, the specimen material, from the viewpoint of the Hooke law, has Young's modulus equaling the slope of a half-line plotted from the coordinate origin to the point M, i.e. a secant modulus. For example, there is no general sense Young's modulus for cast iron, while the calculations performed for cast iron use the Hooke law with the assumption of a certain average secant modulus value conforming with the range of effective stresses. The similar situation takes place in the vicinity of an underground excavation.

There is a model describing formation of transition layers in the vicinity of an excavation as a new phase domains. It is supposed that the transition layers around an excavation are the changed rock material, which becomes compound, composed of a shell (intact rocks) and a transition ring layer of a new phase, which can be continuously or discontinuously nonuniform. One of the facts to be explained is the perceptive alteration of deformation characteristics of rocks (Young's modulus, strengthening) in the framework of the problem on zonal disintegration of rocks. In our case, the phase transformation is initiated both by the uniaxial tension–compression and by the subsequent relief due to the excavation drivage.



Fig. 3. Steel tension curve.

As for the size and structure of the transition ring layer of rocks around an excavation, the prime source data are offered by the analysis of shearing stresses [7] and the discussion of the shearing displacements in this paper. It is assumable that if the displacements (2) depend on Young's modulus, the ring layer radius  $R^1$  should be found from (6). At the same time, Young's modulus changes from the deeper rock mass value  $E_0$  to the value  $E_1$  at the excavation contour (Fig. 4).

So, as a result of excavation drivage (rock mass destressing), Young's modulus changes jump-wise (curve *1* in Fig. 4). The ring zone radius varies in the range  $1.315R \le R^1 \le \sqrt{3}R$  conditioned by Poisson's ratio.

In case of shearing:

$$\tau_{r\theta} = \mu \gamma, \tag{8}$$

where  $\mu$ —shear modulus;  $\gamma$ —shearing deformation given by [8]:

$$\gamma = \frac{dV_r}{rd\theta} + \frac{dV_\theta}{dr} - \frac{V_\theta}{r}.$$
(9)

From (9), considering (2), it is readily found that the shearing deformation reaches extremum at  $r = \sqrt{3}R$  and is proportional to  $\tau_{r\theta}$  at the constant value of Young's modulus.

Hole drilling from the excavation shows no damage of rocks in the vicinity of the excavation. In this zone, the shearing stresses are small and the shearing modulus is high, and there are no causes for the broken rock to fall in the hole. At  $r = \sqrt{3}R$  the shearing stresses are maximum and the shearing modulus is low, and shearing takes places with rock damage and fall in the hole. With the longer hole drilled, the value of  $\tau_{r\theta}$  gets lower at the same shearing modulus, and damage process stops, so, the situation gets steady. From this viewpoint, the assumption that  $E_1 > E_0$  (Fig. 4) is quite feasible. Slipping and failure are only possible if a free surface is created in this area, for instance, by hole drilling. It is for certain that zonal disintegration will arise around the hole but shearing due to the excavation drivage at  $r = \sqrt{3}R$  can actualize in any weakened zone. That was the case when the zonal disintegration phenomenon was discovered [6].

The decrease in Young's modulus due to high stresses in an intact rock mass and the growth of the modulus owing to destressing at an excavation contour impose on an analytical solution having nothing to do with the properties of rocks. Nevertheless, this solution allows estimating E and harmonizing what is wanted and what is actual. It is assumable that numerical values of  $E_1$  are close to figures quoted in literature as they are obtained in laboratory tests of rock specimens. Determination of mechanical characteristics of rocks based on the transversal and longitudinal wave velocities and in laboratory tests of rock specimens can give the values of  $E_0$  and  $E_1$ . It is necessary to take into account that in a rock mass with an excavation, destressing is stronger in the roof than in the floor. Therefore, the ring zone  $R \le r \le R_1$  can transform in an oval, or can keep but Young's modulus in this zone will grow in the line from the floor to the roof. Poisson's ratio is also important for the analytical solution of this phenomena requires the targeted in situ research.



Fig. 4. The change in Young's modulus: 1-4—variants of passing from  $E_0$  to  $E_1$ .

So, "degradation" of Young's modulus under high stress in an intact rock mass and its growth on the free surface result in reduced displacement. The ring zone  $R \le r \le R_1$  prevents from displacements found from (2) in the zone  $r \ge R_1$ , i.e., there is a bearing reaction that raises stresses for  $r \ge R_1$ , which vanish at infinity. This creates conditions for subsequent ZD. At the same time, the rise of Young's modulus in the vicinity of an excavation results in the increased ultimate stress limit of rocks, which strengthens weakening contour and enables deep level mining. Mainly, this is a qualitative result, and real calculations need additional in situ research.

Efficient mine planning requires, on the one hand, the knowledge on mechanisms of origination and growth of failure and structural changes in rocks, on the other hand, the mathematical theory of in situ data interpretation (methods of solving inverse problems) and technologies ensuing from the said knowledge. This integration is the basis of the new-formulated engineering rock mechanics—a scientific area that enjoys fast development in recent years in view of the diversity of rock types and new opportunities open. A feature of engineering rock mechanics is multidisciplinarity, therefore, it will be filled up with physical and mathematical content aimed at improved efficiency and ecological cleanness.

## CONCLUSIONS

The paper offers the approach to accounting for the effect of large depth on stress-strain state redistribution in rocks around an excavation owing to local change of Young's modulus. The authors find maximum shear displacement, nonlinear relationship between coordinates and Poisson's ratio, and their correlation with location of maximum shearing stress zone. This algorithm as a first approximation can be used to find general laws of change of rock mass properties due to large depths, including zonal disintegration of rocks around an underground excavation.

The ground control method is discussed from the viewpoint of the described analytical solution as applied to deep level mining.

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