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Development of Sliding Regimes in Faults and Slow Strain Waves

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Abstract—The paper investigates a model simulating crustal fault dynamics and strain wave generation in a fault block geological medium, the parameters determining sliding regimes in faults, and the physics of transitions between different deformation regimes. The model comprises the most important mechanisms responsible for the interaction of fault walls: friction, geometric irregularities (roughness and asperities on the fault surface), and external load, which govern sliding along the fault. The results of field and laboratory studies of deformation migration on the macro/mesoscale are consistent with the concept of localized deformation propagation in the form of solitary waves (kinks, solitons) and autowaves. The conditions are defined which make possible the transition from the model simulating solitary waves in a conservative medium with low "friction" (soliton-like behavior of the system) toward the model of solitary waves in an active medium with diffusion (autowave-like behavior of the system). Two possible deformation regimes of the fault block structure in the high-friction limit are considered. The fault wall displacement is stopped due to this friction, but the adjacent blocks move relative to each other in the core of the fault. It is shown that in the high-friction limit a perturbed sine-Gordon equation applied for fault dynamics modeling is reduced to a reaction-diffusion equation, whereas the system goes from the soliton regime to the autowave regime. In the case of high friction and a lack of energy supply to the fault from an external source, the transfer of localized deformation is changed by a diffusive dissipation of stress.

Keywords: fault dynamics, stick-slip, slow strain waves, solitons, autowaves, sine-Gordon equation

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1. INTRODUCTION

The problem of deformation and dynamics of crustal faults is in the identification of processes and parameters governing sliding regimes in faults and in the understanding of the physics of transitions between different deformation regimes.

Deformation phases and sliding regimes on faults are currently classified as interseismic, preseismic, coseismic, postseismic slip, as well as episodic tremor and slip [1, 2]. Physical mechanisms and signs of transitions between deformation regimes were extensively investigated in experimental and theoretical papers (see, for example, [3–8] and works of references). It is universally accepted that the transition from creep to unstable sliding along a fault, often accompanied by a tectonic earthquake, is caused by geometrical irregularities of the fault walls, decreased friction in certain segments of the fault core, and anomalies of pore pressure [9]. Seismic slip can be also initiated by slow strain waves excited in the crust and lithosphere [7, 8, 10].

One of the most important and still unsolved aspects of the fault dynamics problem is the onset of frictional motion or sliding along a fault. The transition from static to dynamic friction, which is often thought to initiate sliding, is an empirical ratio that gives no elucidation but presents a convenient heuristic device.

Numerous stick-slip experiments performed on rock specimens and various materials report that dynamic slip—the final stage of every stick-slip cycle—is always preceded by localized deformation propagation in the form of a slip wave traveling along the block contact [11]. Laboratory experiments demonstrate [12] that slip starts after the passage of three types of wave fronts, which are visible at the contact of two blocks. In so doing, the predominant mechanism of contact failure is the slow front propagation with the velocity from 40 to 80 m/s. One block does not move relative the other one, i.e. the contact is not weakened, until the slow front has crossed the contact surface of the blocks. Stick-slip experiments disclosed solitary fracture fronts propagating with the constant velocity about 30–60 m/s [13]. The existence of slow strain waves with the properties of solitons was proved by laboratory experiments [14].

The discovered effects of unstable sliding at the contact of rock blocks [15] are reproduced using perturbed sine-Gordon equations [16]. The calculation results are in good qualitative and quantitative agreement with the results of later laboratory measurements of displacement and slip velocity [6, 17].

The idea that slow tectonic deformation and the related anomalies of geophysical fields also propagate in the form of solitary waves has a wealth of direct and indirect evidence [18, 19].

The observed behavior of spatiotemporal migration of modern deformations in fault zones [20] and dynamics of seismic activity [21] allows an assumption on the autowave pattern of deformation in the fault block geological medium. Slow autowave perturbations in the form of localized plastic deformation fronts are found to propagate in compression of specimens made of various rocks [22].

Thus, the study of migration at the macro- and mesolevels inevitably leads to the concepts of localized deformation propagation in the form of solitary waves (kinks, solitons) and autowaves.

Mathematical models of solitary waves and autowave processes in fault block geomedia can be conventionally divided into two types: conservative (for a medium with dispersion) and dissipative (for a medium with diffusion).

Conservative models corresponding to the classical sine-Gordon equation are intensively used to model fault dynamics and crustal block motion generating strain waves of a soliton type and to interpret the observed seismic and strain-induced effects [18, 23–26].

Dissipative models are used to describe unstable sliding in the excitable block medium with elastic bonding [27, 28], slow autowave deformation processes in the excitable geomedium [20], and seismic migration in the excitable hierarchical block medium [21]. Mathematical models of such processes are reduced to FitzHugh–Nagumo and Kolmogorov–Petrovsky–Piskunov–Fischer reaction-diffusion equations and describe the wave front dynamics.

The solutions of conservative models corresponding to the classical sine-Gordon equation are kinks and solitons. Dissipative models described by reaction-diffusion equations give solutions in the form of autowaves.

The solutions of these models have drastically different features. Thus, the ability of solitons to retain their velocity, shape, and amplitude is related to the lack of dissipation in the medium, though it can carry solitons of various velocity and amplitude. Autowaves, in contrast, propagate in the medium with dissipation and retain their velocity, shape, and amplitude due to supply of external energy. All autowaves in the active medium are similar and their characteristics depend solely on the medium parameters.

It is necessary to clarify the mechanism and conditions of the transition from a dispersive (soliton or quasi-soliton) behavior of the medium to a diffusive (autowave) behavior, which is needed for the understanding of the transition between different deformation regimes on faults.

2. MATHEMATICAL MODEL OF TRANSITION OF THE SLIDING REGIME ON A FAULT AND GENERATION OF STRAIN WAVES (FRONTS) OF A SOLITON TYPE

A model of formation and transition of the sliding regime on a fault and generation of strain waves was earlier presented for the fault block geological medium [29]. This model of solitary waves in the fault corresponds to the perturbed sine-Gordon equation and, in contrast to conservative [18, 23–25] or dissipative [20, 21, 27, 28] block models generating strain waves, takes account of both inertia and dissipation, which is more realistic in describing the deformation regime in the fault block system.

The model takes account of the three important mechanisms of interaction of fault walls: friction, geometrical irregularities (roughness and asperities), and external load, which, in this or that time interval, control unstable sliding along the fault. The resulting mathematical model corresponds to the perturbed sine-Gordon equation [29]:

$$\frac{\partial^2 U}{\partial \xi^2} - \frac{\partial^2 U}{\partial \eta^2} = \sin U + \alpha \frac{\partial U}{\partial \eta} + \gamma(\xi)\delta(\xi - L)\sin U + \sigma(\eta), \quad (1)$$

$$U = 2\pi \frac{u}{a},\ \xi = \frac{\pi x}{ap},\ \eta = \frac{\pi \omega_0 t}{p},\ p^2 = \frac{a^2 D_{\mathrm{t}}}{4mgh},$$

$$\omega_0^2 = \frac{D_{\mathrm{t}}}{m},\ \alpha \approx \frac{a\mu}{d\Delta\rho_{\mathrm{s}}(gh)^{1/2}},\ \gamma = \frac{H}{L}.$$

Here u is the displacement of blocks periodically placed along the fault, a is the distance between the block centers, D_t is the tangential contact stiffness, $m = 4\pi r^3\rho_s/3$ is the block mass, ρ_s is the block material density, r is the radius (size) of blocks, h is the distance between the adjacent block layers, g is the gravitational acceleration, μ is the viscosity of an interlayer between the blocks, d is the contact size between the blocks, Δ is the interlayer thickness, α and γ are the parameters of friction and roughness, H and L are the height and distance between the asperities normalized to ap/π, $\delta(\xi)$ is the Dirac delta function, and $\sigma(\eta)$ is the function representing the external action on the contact between the fault walls.

In the right-hand side of Eq. (1), the first term corresponds to the restoring force in response to shear along the sinusoidal surface of the fault walls, the second term stands for the frictional force proportional to the velocity of relative displacement, and the third one for the corrections to point irregularities placed with the special period apL/π.

Numerical experiments based on Eq. (1) disclose that strain-induced effects related to decreased friction ($\alpha << 1$) at the contact between rough fault walls can induce solitary strain waves with velocity V_α, referred to as waves of fault activation [29]. These waves are strain localized at the mesolevel ε (see Eq. (2)) and propagating along the fault with dimensionless velocity β related to V_α by ratio (3) and governing the sliding regime on the fault:

$$\varepsilon = \frac{\partial U}{\partial \xi} = \pm\frac{1}{(1-\beta^2)^{1/2}}\operatorname{sech}\left(\frac{\xi-\beta\eta}{(1-\beta^2)^{1/2}}\right), \qquad (2)$$

$$\beta = \frac{2r}{\alpha}\left(\frac{\pi r\rho_s}{3D_t}\right)^{1/2} V_\alpha. \qquad (3)$$

The calculated block velocity profile v on the fault wall surface is shaped to a soliton $v(x, t) = v_{max}\operatorname{sech}(x - V_\alpha t)$ moving along the fault with velocity V_α (Fig. 1). At low V_α, the velocity v is insignificant, resulting in stable slow sliding (creep). At V_α of about 1–10 m/s, a soliton-like profile of the fault wall velocity $v \sim 0.1$–1.0 m/s and a stepwise profile of displacement (kink) $u(x, t)$ are observed (Fig. 1).

Thus, the passage of a solitary wave (2), as in the previous experiments [12], weakens the contact, which, under constant external loading, leads to displacement of the fault walls—dynamic slip. These waves are in nature similar to sliding waves observed in numerous laboratory stick-slip experiments at the contact between rock blocks before their relative displacement [11].

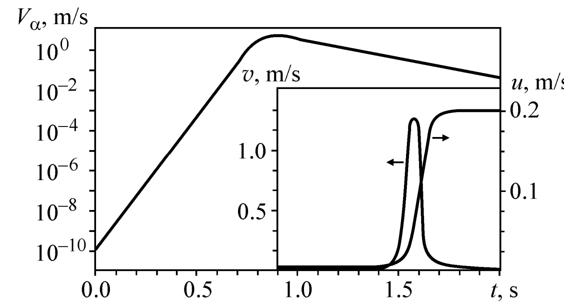

Fig. 1. Evolution of activation wave velocity V_α, displacement u, and slip velocity v on a fault.

At specified fault parameters, a solitary wave changes to the stationary regime with the velocity $V_\alpha \sim 10^{-4}$–10^{-1} m/s or 30 km/year–10 km/day corresponding to strain waves [18]. Specific combinations of roughness and friction parameters cause the fault to change the "excited" state to stable sliding with the velocity $V_\alpha = 10^{-9}$ m/s $\approx$ 3 cm/year, i.e. the system switches to the "locked fault" regime [29].

Hence it follows that an introduction, into the classical sine-Gordon equation, of perturbations in the form of friction and roughness allows an investigation of the influence of rheological and geometrical characteristics of faults on the deformation regime (dynamics of faulting and its special features).

However, results of numerical modeling based on Eq. (1) do not fully elucidate the physics of transitions between different deformation regimes, for example, the transition from the unstable state of preseismic or episodic slow slip to stable aseismic creep or the locked fault regime. Within the mathematical description of the system, this corresponds to the transition from hyperbolic perturbed sine-Gordon Eq. (1) to parabolic equations of reaction-diffusion and diffusion types or from the dispersive (soliton) to diffusive (autowave) behavior of the system.

We are to find the conditions under which the transition occurs from the model of solitary waves in a conservative medium with low "friction" to the model of solitary waves in an active medium with diffusion. The understanding of transition of the system from one deformation regime to the other is evidently related to the analysis of equations stemming from Eq. (1) in this or that limiting case.

3. DEFORMATION MODELS OF THE FAULT BLOCK STRUCTURE IN THE HIGH-FRICTION LIMIT

Let us consider two possible deformation regimes of a fault block structure in the high-friction limit,

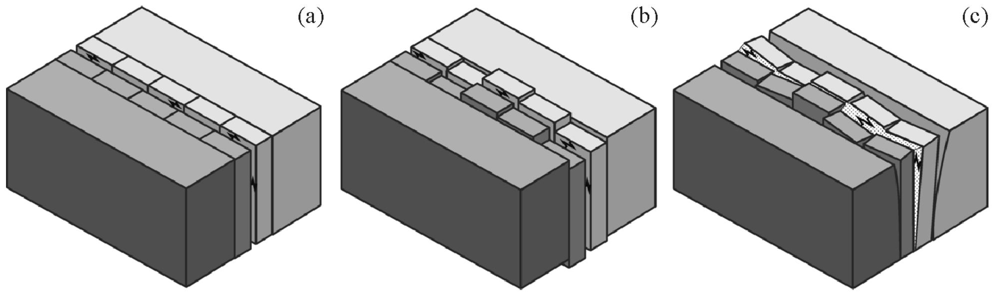

Fig. 2. Schematic of the structure and position of blocks in the fault core: periodic position of the blocks along the fault walls (a); vertical oscillations of separate blocks (b); pendular oscillations (c).

when the amplitude decreases rapidly and the vibration period increases. The physical model of such a structure can be represented as a set of blocks periodically placed along both walls of the fault (Fig. 2a). In the most general case, separate blocks can even execute vertical oscillations (Fig. 2b) or pendular oscillations when their lower parts are fixed and the upper ones are out of equilibrium (Fig. 2c). High friction prevents a displacement of one fault wall relative to the other, but neighboring blocks in the fault core rub against each other and against blocks of the opposite wall. This behavior of the blocks is quite realistic when it is considered that, due to different block contact stiffnesses or different effective viscosities of the block interlayers, some blocks of the fault core can be compressed and some can be unloaded. The block interaction destroys the contact surface structure, and dynamic slip can occur in due course, with blocks moving along the fault.

3.1. Transition to a Reaction-Diffusion Model

We assume that the fault wall surface completely lacks asperities ($\gamma = 0$) and friction (dimensionless viscous damping) in the system is high ($\alpha > 1$). Then the blocks cannot oscillate along the fault near the equilibrium position, they are in aperiodic motion without leaving their places. In this case in Eq. (1), the term with the first time derivative corresponding to dissipative losses significantly exceeds the term with the second time derivative (inertial term), which can be neglected. Perturbed sine-Gordon Eq. (1) in the high-friction limit ($\alpha > 1$) or in the diffusive regime goes over into the equation

$$\frac{\partial^2 U}{\partial \xi^2} = \sin U + \alpha \frac{\partial U}{\partial \eta} + \sigma(\eta). \qquad (4)$$

Equation (4) coincides in structure with equations describing, for example, autowaves in active media with energy dissipation and supply [30–32] or excitation waves in reaction-diffusion systems [33].

It is known from theoretical physics that weak damping ($\alpha << 1$) of solitons propagating in a medium with “friction” can be compensated by energy supplied to the soliton from an external source. Such stationary solitary waves in a medium with low “friction” hardly differ in their properties from solitons in conservative systems [34]. Under these conditions, evolutionary processes in a nonlinear medium with diffusion, where autowaves are generated, are to be described not only by parabolic reaction-diffusion equations but also by hyperbolic equations [35], among which is perturbed sine-Gordon Eq. (1).

However, with increasing dissipation, i.e. with increasing “friction” in the system, this difference is more and more pronounced. The similarity of solitary waves in active media with diffusion to solitons remains until the critical damping value α_c is exceeded. After this, the system switches from the soliton to autowave regime, with properties of the medium changing sharply. This primarily refers to the reaction of the medium to the interaction of slow solitary strain waves, which can serve as one of the tests determining a medium model (active or conservative). In collision, two autowaves annihilate, i.e. they are either mutually destroyed or transformed into an autowave of another type (static or pulsating autosoliton) [36]. On the contrary, colliding solitons restore their shape and continue to move at the same velocity and in the same directions as before interaction. More complex combined variants are also possible. Thus, from numerical experiments [7, 8] it follows that under certain conditions slow strain autowave fronts exhibit a soliton-like behavior, passing through each other after the collision in a loaded elastoplastic medium.

If the kink and antikink (images of the strain wave fronts) move towards each other at a certain equilibrium velocity V_e, at which the energy loss due to dissipation is equal to the energy supplied to the kink, then according to McLaughlin and Scott [37], the velocity can be expressed in the used designations:

$$V_e = (1 + (4\alpha/\pi\sigma)^2)^{-0.5}.$$

This case corresponds to the classical autowave behavior of the medium. At $\alpha \ll 1$, the fronts pass through each other after elastic collision with conservation (soliton-like behavior). The only effect of such a collision is a phase shift [32]. At sufficiently high damping values $\alpha > 1$, the fronts collide with annihilation (autowave-like behavior).

To determine critical parameters of damping α and source σ separating zones of possible autowaves (destructive collisions of kinks) and solitary waves (nondestructive collisions of kinks) is of particular interest and requires special investigation, since these zones correspond to fundamentally different deformation regimes on faults.

3.2. Transition to the Model of Diffusive Dissipation of Stress

By isolating the external source $\sigma(\eta)$ and giving no account of the "restoring" force of $\sin U$ (fault surface roughness), the system goes into the usual diffusive regime and Eq. (4) takes the form of the classical diffusion equation:

$$\frac{\partial U}{\partial \eta} = \alpha^{-1}\frac{\partial^2 U}{\partial \xi^2}, \tag{5}$$

which, by replacing dimensionless quantities with physical parameters entering initial Eq. (1), can be written in the following way:

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2}, \tag{6}$$

$$\kappa = \frac{a^2}{2\pi}\frac{D_t}{m}\frac{d\Delta\rho_s}{\mu}, \text{ or } \kappa \approx \frac{a^2 d\Delta\rho_s}{2\pi}\frac{\omega_0^2}{\mu}, \tag{7}$$

where κ is the stress diffusion coefficient.

Equations of this type were previously used to model the stress transfer along the lithosphere–asthenosphere contact [38] and to describe the migration of deformation and earthquakes [39–41].

The standard solution of diffusion Eq. (6) has the form

$$u(x,t) = u_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{\kappa t}}\right), \tag{8}$$

where the erfc function is the error integral. Solution (8) describes a bell-shaped asymmetric single pulse, which smears due to diffusion over time and tends to the form of a smoothed Gaussian curve. From Eq. (8) it follows that the average distance to which the perturbation propagates during time t is equal to $x = 2(\kappa t)^{1/2}$. Under a sinusoidal load with the period $T = 2\pi/\omega$, the strain wave velocity V_d is defined as

$$V_d = \sqrt{2\kappa\omega} = 2\sqrt{\frac{\pi\kappa}{T}} \approx \sqrt{\frac{\Delta D_t}{\mu T}} = \sqrt{\frac{D_t}{\delta T}} = a\omega_0\Lambda, \tag{9}$$

where $\delta = \mu/\Delta$ is the specific contact viscosity, and Λ is the dimensionless coefficient.

The wave velocity V_d is largely influenced by the specific viscosity of the interlayer δ, tangential contact stiffness D_t, and the wave period T. Kocharyan refers to fault stiffness as the control parameter of deformation [2]. Interlayers separating blocks of the Earth's crust can present fluid-saturated fractured media with elastic moduli and viscosity much lower than the moduli and viscosity of the block material. The viscosity of contact between the blocks varies in a rather wide range: $\mu = 10^8$–10^{14} Pa s [42, 43]. The dynamic viscosity of the interlayer can even be several orders of magnitude lower [2].

When studying creep in the San Andreas fault, the diffusion coefficient is found to be $\kappa = 0.1$–$1.0\ \text{m}^2/\text{s}$ [44]. According to Johnson and McEvilly [45], the seismic diffusion coefficient is $\kappa_s = D^2/t = DV_E = 1$–$20\ \text{m}^2/\text{s}$ (D and t are the distance and time between two successive earthquakes, and V_E is the fracture velocity). The monograph by Scholz [46] states the coefficient of hydraulic diffusion κ_s in the Earth's crust within 0.01–10 m^2/s.

From calculations by (7) and (9) it follows that, at the parameters $\mu = 10^6$–10^8 Pa s, $\rho_s = 3 \times 10^3\ \text{kg/m}^3$, $D_t \sim 10^6$ N/m, $a = 10^4$ m, $\omega_0 = 10^{-5}\ \text{s}^{-1}$, $\Delta = 10$–10^2 m, $d = 10^4$ m, and $T = 10^8$ s (~3 years), the diffusion coefficient is $\kappa = 5 \times 10^{-2}$–$5 \times 10^{-1}\ \text{m}^2/\text{s}$ and the diffusion wave velocity is $V_d = 8 \times 10^{-5}$–2.5×10^{-4} m/s (2.5–10 km/year), which is comparable with the velocity of slow strain (tectonic) waves of the order of 1–100 km/year [47]. At the interlayer viscosity $\mu \geq 10^{10}$ Pa s and the other fixed parameters, the wave velocity (9) takes on values $V_d \leq 5 \times 10^{-7}$ m/s ($\leq 10^{-2}$ km/year), which almost corresponds to the locked fault regime.

Thus, in the case of high friction, the transfer of localized deformation (kink displacement) is replaced by diffusive dissipation of stress. The main factor regulating the velocity of slow stress migration is the viscosity of the contact zone (interlayer).

4. CONCLUSIONS

From the results of laboratory experiments and field observations, it follows that kinks, solitons, and autowaves can serve, under specified conditions, as adequate dynamic images of strain perturbations on faults (slip waves). The idea is gradually formed that frictional motion along the contact surface of rock blocks or along crustal faults is accompanied by slip waves of various types in creep [48–50]. Such slip waves can exist in the form of solitary waves (slip pulses), periodic waves, or wave fronts. Nevertheless, in respect to physical understanding and mathematical modeling, it remains unclear how slip pulses or solitary strain waves can be excited during long-term stable sliding (creep).

The analysis of model (1) and the reported laboratory data [12, 50] show that with decreasing friction at the block contact (in the fault) the regime can change from slow slip or creep to stick-slip and can be accompanied by the generation of solitary strain waves or slip pulses, which, on passing, weaken the contact and lead to dynamic slip.

The model of solitary strain waves on the fault [29], corresponding to the perturbed sine-Gordon equation, allows for a determination of physical conditions under which the model of solitary waves in a conservative medium with low friction can change to the model of solitary waves in an active medium with diffusion and to models with ordinary diffusion. In the high-friction limit and with no asperities on the fault wall surface, perturbed sine-Gordon Eq. (1) reduces to reaction-diffusion Eq. (4), which means the soliton-to-autowave transition of the system. At high friction, weak external source, and significant roughness of the fault walls, the transfer of localized deformation in the form of kinks and solitons becomes impossible, resulting in the regime of diffusive dissipation of stress corresponding to Eq. (6).

It should be noted that model [16, 29], in contrast to conservative or dissipative models, takes into account both inertia and dissipation, which is characteristic of many natural systems.

It might appear at first glance that the studies of interaction of soliton-like strain waves and dynamics of seismogenic faults are not directly related, but this is not the case. It was earlier suggested [51, 52] that a strong earthquake can be caused by a collision of two large-scale tectonic waves moving towards each other, and it was shown that the kink-antikink collision, modeling such an interaction, can lead to a breather—a stable long-lived pair—which radiates energy and therefore gradually damps [52]. Known also is the solution of the modified Burridge–Knopov model that represents localized fracture (a breather propagating along the fault and damping in a finite fault segment [53]).

In addition, the possibility of interaction [20, 46, 54, 55] and synchronization [56, 57] of faults was studied in relation to static or dynamic stress transfer. According to Scholz [57], interaction and synchronization are of special significance in systems of subparallel faults with fairly close sliding velocities. It is proposed to take the tectonic fault as a fault-generator, the frequency of which is given by the ratio $\omega = v/\Delta u$, where Δu is the earthquake-related slip, and v is the velocity of the "geological" slip. Based on the Kuramoto group synchronization model, it was suggested that faults with comparable slip velocities will synchronize both in frequency and phase over time [57]. For characteristic values $\Delta u \sim 0.1$–1 m and $v \sim 1$–10 cm/year the oscillation frequency of the fault-generator is 5×10^{-10} Hz and falls within the frequency range of strain waves 10^{-10}–10^{-9} Hz [47]. This once again points to the wave character of fault interaction.

Direct geodetic measurements were used to find that deformation is transferred from fault to fault in the form of an "interfault" wave at a velocity of 20–30 km/year or more [20]. Strain autowaves are formed during the interaction of anomalous local stress and strain fields. Constant energy supply due to energy distributed in the geological environment, for example, tectonic stresses inside the Earth, as well as due to regional and global geodynamic processes, ensures the generation and maintenance of strain autowaves. Therefore, the direct statistical dependence is valid between the average migration velocity of earthquake epicenters (the deformation transfer velocity inside the Earth) and the velocity of tectonic plates [58], i.e. sliding velocity along faults and their fragments.

The formation of sliding regimes on faults is directly related to the presence of slow dynamics in the geological environment, i.e. wave processes significantly slower than a seismic wave. Slow dynamics of deformable fault zones includes transfer of localized deformation in the form of solitary waves and autowaves as well as formation of strain waves of various types and wave fronts of different scales. Slow dynamics is determined by the interaction of crystal blocks and their time synchronization. Physical modeling confirmed that slow dynamics in the fault zone depends on its internal fault block structure and the level of accumulated stresses [59].

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