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Ductile Failure Prediction of U-Notched Bainitic Functionally Graded Steel Specimens Using the Equivalent Material Concept Combined with the Averaged Strain Energy Density Criterion

H. Salavati[1]* **and H. Mohammadi**[2]

[1] *Department of Mechanical Engineering, Shahid Bahonar University of Kerman, Kerman, 76169-14111 Iran*
[2] *Laboratori de Càlcul Numèric, Universitat Politècnica de Catalunya, Barcelona, 08034 Spain*
** e-mail: hadi_salavati@uk.ac.ir*

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Abstract—In this paper, the ductile fracture of bainitic functionally graded steel has been studied. Fracture tests was performed on U-notched specimens made of bainitic functionally graded steel under mode I. The averaged strain energy density criterion combined with equivalent material concept was employed to predict the ductile fracture of bainitic functionally graded steel. For this purpose, first, based on equivalent material concept, the mechanical properties of virtual brittle functionally graded steel were obtained. Then the averaged value of strain energy density over a well-defined control volume was calculated by finite element analysis for U-notched virtual brittle functionally graded steel. After that, the fracture loads was obtained based on the averaged strain energy density criterion. The agreement between experimental fracture loads and theoretical predictions was good.

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1. INTRODUCTION

Prediction of fracture in presence of stress concentrator parts such as notch is a significant issue in the field of structural integrity. The stress field in the vicinity of crack [1, 2] and notch tip [3] has been studied in various contributions. In addition, three dimensional effects on stress field have been investigated [4]. Different researchers proposed some approaches to predict the fracture of specimens with notches [5–13]. A recent approach [14] based on the averaged strain energy density (SED) in a structural control volume has attracted serious attention. This criterion was originally proposed for prediction of fracture of brittle and quasi-brittle materials and it was applied to different types of notch and different fracture modes [15, 16]. Recently, by using the equivalent material concept (EMC) [17], application of the averaged strain energy density criterion has been extended for prediction of ductile fracture [18]. As the equivalent material concept suggests, ductile material having valid fracture toughness is equated with a virtual brittle material having the same elastic modulus and the same fracture toughness, but a different tensile strength. The tensile strength of the equivalent brittle material can be determined by assuming the same values of the tensile strain energy density required for both ductile and virtual brittle materials for crack initiation [17, 18].

Recently, the averaged strain energy density criterion was employed to predict the fracture of notched specimens made of different types of materials such as polyurethane [19], titanium alloys [20], tungsten-copper functionally graded material (FGM) [21, 22] and functionally graded steels [23, 24].

To predict the fracture of notched specimens made of functionally graded materials by the averaged strain energy density criterion, unlike homogeneous materials for which the outer boundary of the control volume is a circular arc, the strain energy density is averaged over a well-defined control volume which its outer boundary is an oval arc [21–24]. In these works, the notch depth was in brittle or semi-brittle region. Therefore, the strain energy density approach could predict

Table 1. Chemical composition of original ferritic and austenitic steels [23]

	C, %	Ni, %	Cr, %	Mo, %	Cu, %	Si, %	Mn, %	S, %	P, %
AISI1020 $\dot{\gamma}$	0.01	9.58	16.69	1.89	0.43	0.53	1.5	0.04	0.04
316L $\dot{\alpha}$	0.11	0.07	0.12	0.02	0.29	0.19	0.63	0.08	0.01

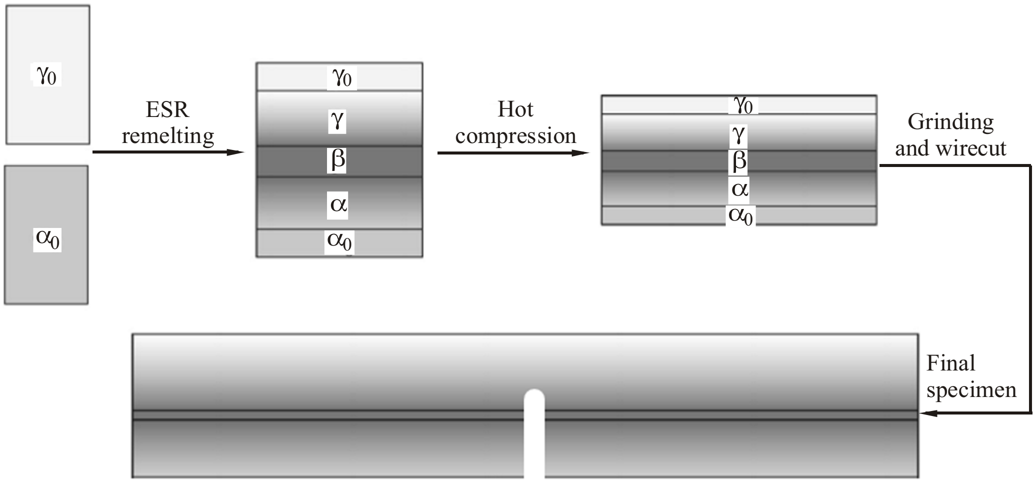

Fig. 1. The fabrication scheme of the U-notched FGS specimens.

the critical fracture load precisely and it was not necessary to use the equivalent material concept criterion.

In this paper, ductile fracture of bainitic functionally graded steel (FGS) is studied. In the experimental program, bainitic functionally graded steel was produced by electroslag remelting (ESR). The experiments was carried out on U-notched specimens under mode I loading condition. In the theoretical section, equivalent material concept combined with the averaged strain energy density criterion in functionally graded materials was employed to predict the ductile fracture of bainitic functionally graded steel. The accuracy of the predictions was acceptable.

2. EXPERIMENTAL PROCEDURE

Fracture experiments were carried out on bainitic functionally graded steel. The fabrication procedure of this material has been explained in elsewhere [23]. The chemical composition of the two materials is summarised in Table 1. Figure 1 shows the fabrication scheme of the functionally graded steel.

Figure 2 shows the Vickers microhardness profile of the αβγ composite.

The αβγ specimens drawn from the ingots were 90 mm in length, 18 mm in width and 9 mm in thickness. The geometry is in agreement with ASTM E1820 for the crack arrester configuration. A U-notch was drawn from ferritic steel side of each specimen with a notch root radius of 1 mm. Three different notch depths of 9, 11 and 14 mm were considered in the experiments.

To measure the critical fracture load, three-point bending load was used. The span length between two supports was set to be equal to 72 mm. The load was applied normally to the interface layers at the notch bisector line in order to obtain mode I loading condition (Fig. 3). The tests were performed by a ZWICK 1494 testing machine under load displacement control with constant displacement-rate of 1 mm/min. The load-displacement curves were recorded and used to obtain the critical fracture load.

3. MECHANICAL PROPERTIES

The mechanical properties of single phase steels present in the considered functionally graded steels

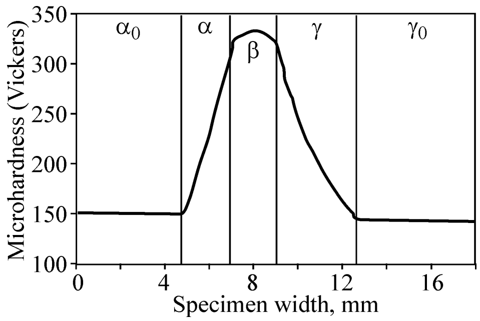

Fig. 2. Vickers microhardness profile versus width in αβγ functionally graded steel.

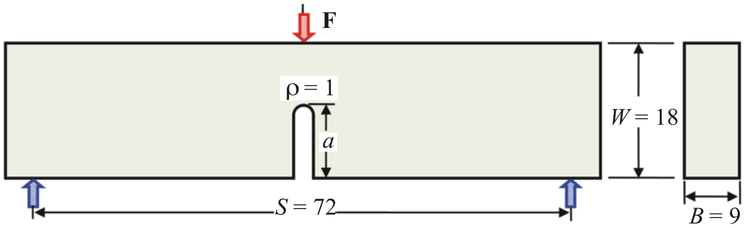

Fig. 3. Notched beam specimen geometries (dimensions are in mm).

are summarized in Table 2 [24, 25]. As is stated in [23], the thickness of the bainitic layer is approximately equal to 2 mm. In addition, the thickness of α and γ graded regions are 2.5 and 3.5 mm, respectively. Moreover, the thickness of the original ferritic $\dot{\alpha}$ and the original austenitic layers $\dot{\gamma}$ are 4.5 and 5.5 mm, respectively. The mechanical properties (ultimate tensile strength and plain strain fracture toughness) of α and γ graded regions can be described by using exponential function as follow:

$$\sigma_{\mathrm{ut}}(x)_{(\alpha)} = (\sigma_{\mathrm{ut}})_{\alpha_0} e^{\frac{x-4.5}{2.5}\ln\frac{(\sigma_{\mathrm{ut}})_\beta}{(\sigma_{\mathrm{ut}})_{\alpha_0}}}, \quad (1)$$

$$\sigma_{\mathrm{ut}}(x)_{(\gamma)} = (\sigma_{\mathrm{ut}})_\beta e^{\frac{x-9}{3.5}\ln\frac{(\sigma_{\mathrm{ut}})_{\gamma_0}}{(\sigma_{\mathrm{ut}})_\beta}}, \quad (2)$$

$$K_{\mathrm{Ic}}(x)_{(\alpha)} = (K_{\mathrm{Ic}})_{\alpha_0} e^{\frac{x-4.5}{2.5}\ln\frac{(K_{\mathrm{Ic}})_\beta}{(K_{\mathrm{Ic}})_{\alpha_0}}}, \quad (3)$$

$$K_{\mathrm{Ic}}(x)_{(\gamma)} = (K_{\mathrm{Ic}})_\beta e^{\frac{x-9}{3.5}\ln\frac{(K_{\mathrm{Ic}})_{\gamma_0}}{(K_{\mathrm{Ic}})_\beta}}, \quad (4)$$

where $(\sigma_{\mathrm{ut}})_{\alpha_0}, (\sigma_{\mathrm{ut}})_\beta$ and $(\sigma_{\mathrm{ut}})_{\gamma_0}$ are the ultimate tensile strength corresponding to α_0, β and γ_0, respectively, and $(K_{\mathrm{Ic}})_{\alpha_0}, (K_{\mathrm{Ic}})_\beta$ and $(K_{\mathrm{Ic}})_{\gamma_0}$ are the plane strain fracture toughness corresponding to α_0, β and γ_0, respectively, as shown in Table 2.

4. FRACTURE CRITERION BASED ON AVERAGED STRAIN ENERGY DENSITY (ASED) IN FGMS AND EQUIVALENT MATERIAL CONCEPT

According to the averaged strain energy density criterion, brittle fracture occurs when the averaged value of the strain energy density over a well-defined control volume reaches a critical value W_{c} [14]:

$$W_{\mathrm{c}} = \frac{\sigma_{\mathrm{ut}}^2}{2E}. \quad (5)$$

The critical length R_{c} can be evaluated as follow under plane strain conditions [26]:

$$R_{\mathrm{c}} = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{\mathrm{Ic}}}{\sigma_{\mathrm{ut}}}\right)^2, \quad (6)$$

where K_{Ic} is the fracture toughness, σ_{ut} is the ultimate tensile stress and ν is the Poisson's ratio.

For an embedded crack or notch in a FGM specimen, it is assumed that the properties of the material in which the crack or notch tip is placed plays a key role in initiation of fracture of the specimen. Accordingly, the criterion states that fracture initiates when the averaged value of strain energy density over a well-defined control volume reaches the corresponding to the notch tip value of W_{c}. In a nonhomogeneous medium with a smooth unidirectional variation of mechanical properties in the direction x (along the notch depth), the value of W_{c} can be determined as follow [23]:

$$W_{\mathrm{c}} = \frac{\sigma_{\mathrm{ut}}^2(a)}{2E(a)}, \quad (7)$$

where a is the notch depth.

Unlike homogeneous materials, in functionally graded materials due to a gradual change in material properties, the outer boundary of control volume is no longer a circular arc, but rather an oval arc is assumed. This notion has been used in a number of previous studies on fracture of notched specimens made of func-

Table 2. Mechanical properties of single phase steels present in the considered functionally graded steels [25]

	Yield stress σ_y, MPa	Ultimate tensile strength σ_{ut}, MPa	Fracture toughness K_{Ic}, MPa m$^{0.5}$	Poisson's ratio ν	Elasticity modulus E, GPa	Area under stress-strain curve, MPa	$\sigma_{\mathrm{ut\,EMC}}$, MPa
Ferrite	245	425	45.72	0.33	207	71.4	5437
Bainite	1025	1125	82.08	0.33	207	120.2	7054
Austenite	200	480	107.77	0.33	207	155.6	8026

tionally graded materials [21, 22, 27]. For a U-notched functionally graded material specimen with a material variation in the direction x under mode I, the outer boundary can be determined by a numerical approach using the following equations. For more details please see Ref. [23]:

$$\begin{aligned} x &= a - \rho/2 + (R_c(x) + \rho/2)\cos\theta, \\ y &= (R_c(x) + \rho/2)\sin\theta, \\ R_c(x) &= \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{Ic}(x)}{\sigma_{ut}(x)}\right)^2, \end{aligned} \tag{8}$$

where x and y are the coordinates of a point on the outer boundary, θ is the corresponding angle to that point, a is notch depth, and $R_c(x)$ is the critical length as a function of coordinate x.

The mentioned criterion has been used in different investigations to predict the fracture of notched specimens made of functionally graded materials [21–23]. However, in this study, as the notch root has been located in a very ductile region, the mentioned criterion cannot give reasonably accurate results. So, the mentioned criterion is modified by using equivalent material concept [17, 28]. As is proposed by Torabi [17], a ductile material having valid fracture toughness is equated with a virtual brittle material having the same elastic modulus and the same fracture toughness, but different tensile strength. The tensile strength of the equivalent brittle material can be determined by assuming the same values of the tensile strain energy density required for both ductile and virtual brittle materials for crack initiation:

$$\frac{\sigma_{\text{ut EMC}}^2}{2E} = (\text{SED})_{\text{ductile}}. \tag{9}$$

In Eq. (9), $\sigma_{\text{ut EMC}}$ is the ultimate tensile strength of the virtual brittle material, E is the elasticity modulus, and $(\text{SED})_{\text{ductile}}$ is the area under stress-strain curve of the ductile material until necking.

In this paper, the bainitic functionally graded steel has been modelled as a virtual brittle one. To this end, the ultimate tensile strength of single phase steels in the virtual brittle functionally graded steel was calculated by using the values of the area under stress-strain curves provided in Table 2. The values of the ultimate tensile strength of single phase steels in the virtual brittle functionally graded steel $\sigma_{\text{ut EMC}}$ have been summarized in Table 2. In the virtual brittle functionally graded steel, the ultimate tensile strength of α and γ graded regions can be described by Eqs. (1) and (2), respectively, by replacing σ_{ut} with $\sigma_{\text{ut EMC}}$. After describing the virtual ultimate tensile strengths of each point, the control volume in the virtual brittle functionally graded steel can be determined by Eq. (8) by replacing σ_{ut} with $\sigma_{\text{ut EMC}}$.

5. APPLICATION OF EMC-ASED TO PREDICT THE FRACTURE LOADS OF U-NOTCHED FUNCTIONALLY GRADED STEEL SPECIMENS AND RESULTS

In order to obtain the averaged value of strain energy density over the control volume, some finite element analyses were carried out by using ABAQUS software version 6.11. It should be noted that the Young's modulus ($E = 207$ GPa) and the Poisson's ratio ($\nu = 0.33$) have been assumed to be constant along the specimen width. All the finite element analyses

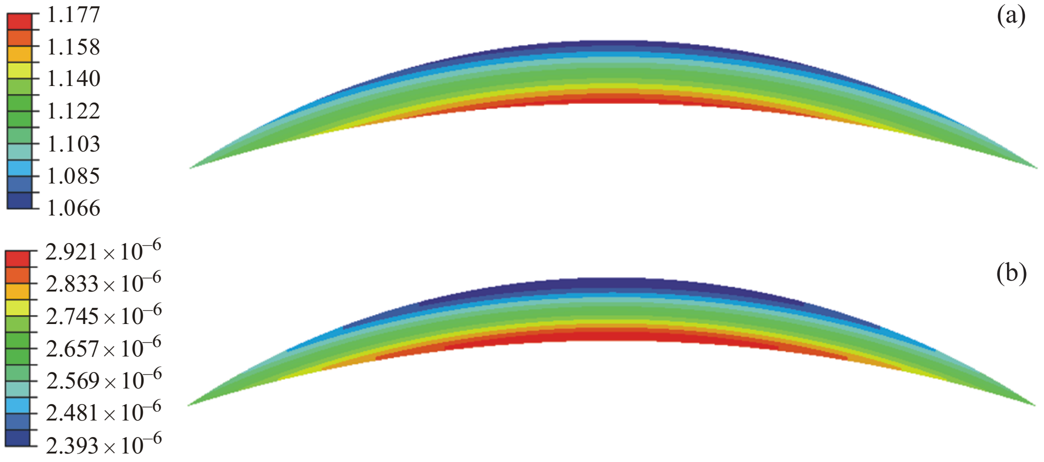

Fig. 4. Contour lines of maximum principal stress (a) and strain energy density for notch depth a = 14 mm (b).

Table 3. Theoretical values of SED and fracture load F_{th} together with experimental results F_{exp} (the SED in this table has been evaluated by applying $F = 1$ N in finite element models)

a, mm	$\sigma_{\mathrm{ut\,EMC}}$, MPa	E, GPa	$W_{\mathrm{C\,EMC}}$, MJ/m^3	SED, J/m^3	F_{th}, KN	F_{exp}, KN	$F_{\mathrm{exp}}/F_{\mathrm{th}}$
8	7054	207	120.2	0.172	26.4		
9	7054		120.2	0.238	22.5	24.9	1.11
10	7319		129.4	0.343	19.4		
11	7594		139.3	0.505	16.6	15.3	0.92
12	7879		150.0	0.819	13.5		
13	8026		155.6	1.391	10.6		
14	8026		155.6	2.523	7.9	7.08	0.90
15	8026		155.6	5.697	5.2		

were carried out under plane strain conditions and linear elastic hypothesis. Eight-node elements were used in analyses.

By applying an arbitrary load in ABAQUS the fracture load can be obtained as follow:

$$\frac{F_{\mathrm{ap}}}{F_{\mathrm{th}}} = \sqrt{\frac{\mathrm{SED}}{W_{\mathrm{c}}}}, \qquad (10)$$

where F_{ap} is the applied load in the numerical model, F_{th} is the theoretical fracture load, SED is the averaged value of strain energy density over the control volume and W_{c} is the critical value of strain energy density corresponding to the notch tip.

The notch depths from 8 to 15 mm have been considered in numerical analyses in order to assess the effect of notch depth on fracture load. Figure 4 shows the contour lines of maximum principal stress and strain energy density obtained from finite element analyses.

Table 3 summarizes the theoretical predictions and experimental results. As can be seen from the table, the agreement is satisfactory. The experimental and theoretical results have been depicted in Fig. 5. As can be seen from Fig. 5, the experimental and theoretical results follow the same trend.

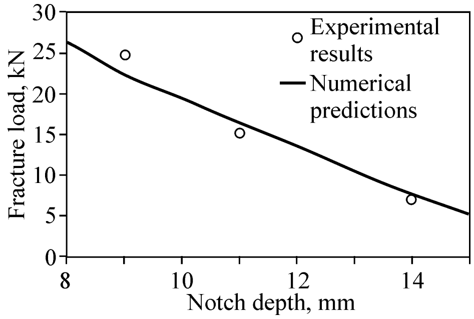

Fig. 5. Comparison between experimental results and numerical predictions.

6. CONCLUSIONS

In this paper, equivalent material concept in conjunction with the averaged strain energy density criterion in functionally graded materials was employed to predict the ductile fracture of bainitic functionally graded material. The accuracy of the predictions was reasonable.

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