ISSN 1029-9599, Physical Mesomechanics, 2015, Vol. 18, No. 1, pp. 75–80. © Pleiades Publishing, Ltd., 2015.
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Coefficient of Friction between a Rigid Conical Indenter and a Model Elastomer: Influence of Local Frictional Heating

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Received July 14, 2014

Abstract—We investigate the coefficient of friction between a rigid cone and an elastomer with account of local heating due to frictional dissipation. The elastomer is modeled as a simple Kelvin body and an exponential dependency of viscosity on temperature is assumed. We show that the coefficient of friction is a function of only two dimensionless variables depending on the normal force, sliding velocity, the parameter characterizing the temperature dependence as well as shear modulus, viscosity at the ambient temperature and the indenter slope. One of the mentioned dimensionless variables does not depend on velocity and determines uniquely the form of the dependence of the coefficient of friction on velocity. Depending on the value of this controlling variable, the cases of weak and strong influence of temperature effects can be distinguished. In the case of strong dependence, a generalization of the classical "master curve" procedure introduced by Grosch is suggested by using both horizontal and vertical shift factors.

DOI: 10.1134/S1029959915010087

Keywords: sliding friction, elastomer, Kelvin material, frictional heating

1. INTRODUCTION

Friction of elastomers is an important topic for many industrial applications [1]. Greenwood and Tabor [2] have shown as early as 1958 that the friction of elastomers can be attributed to deformation losses in the material volume [2]. In 1963, Grosch supported this idea by showing that the elastomer friction has the same "temperature shifting factors" as the complex modulus [3]. In the following years, the role of rheology [4] and of surface roughness [5, 6] in elastomer friction has been studied in detail. Most works on elastomer friction discuss coefficient of friction, thus implicitly implying the validity of the Amontons law: the force of friction is proportional to the normal load [7]; the coefficient of friction is considered to be a quantity which may depend on velocity but does not depend on the normal load [8, 9]. However, there are many experimental evidences of a strong dependence of the coefficient of friction of elastomers on the normal force. Early experiments illustrating the load dependence of the elastomer friction were carried out by Schallamach [10]. Power-law dependence of the coefficient of friction was also found in polymer-based composites [11]. In a series of recent publications [12–14], the elastomer friction has been studied beyond the regions of validity of the Amontons law, thus providing generalized laws of friction. In these papers, changes of local temperature in microcontacts of rubbing bodies have not been taken into account. However, it is known that the temperature effects may play an important role in the elastomer friction [15, 16]. In the present paper, we investigate the elastomer friction as a function of velocity and normal force with account of local temperature changes in the contact.

To achieve the basic understanding of influence factors, we consider a simplest model: (a) an elastomer is modeled as a simple incompressible Kelvin body, which is completely characterized by its static shear modulus and viscosity, (b) a non-disturbed surface of the elastomer is assumed to be plane and frictionless, (c) we consider only one single contact (“one asperity model”) in the shape of a cone, (d) no adhesion or capillarity effects are taken into account, (e) a simple exponential Arrhenius law is used for the temperature dependence of viscosity, (f) we consider a one-dimensional model. These simple assumptions still result in non-trivial and complicated frictional behavior. We would like to note that there is an evidence coming from recent studies of contact mechanics of both rotationally symmetric profiles [17, 18] and self-affine fractal surfaces [19, 20] that suggest that results obtained with one-dimensional foundations may have a broad area of applicability if the rules of the method of dimensionality reduction (MDR) [21, 22] are applied. Following this method, the elastomer was modeled as a row of independent elements with a small spacing Δx, each element consisting of a spring with normal stiffness $\Delta k = 4G\Delta x$ and a dashpot having the damping constant $\Delta\gamma = 4\eta\Delta x$, where G is the shear modulus, and η is the viscosity of the elastomer.

We start our analysis with an analytic estimation of the coefficient of friction for a material with temperature-independent viscosity, and then generalize it by incorporating the temperature dependence of viscosity. Finally, we discuss a possible extension to rough surfaces.

2. FRICTION BETWEEN A RIGID CONE AND A VISCOELASTIC MEDIUM WITH TEMPERATURE-INDEPENDENT VISCOSITY

In the following, we consider a rigid conical indenter $\tilde{z} = f(r) = r\tan\theta$, where $\tilde{z}$ is the coordinate normal to the contact plane, and r in the in-plane polar radius. The one-dimensional MDR-image of this profile, according to the method of dimensionality reduction, is

$$\tilde{z} = g(x) = \pi/2\,|x|\,\tan\theta = c\,|x|. \quad (1)$$

This profile is now pressed into a viscoelastic foundation to a depth of d and moved tangentially with the velocity v (Fig. 1a) so that its form is described at time t by the equation

$$\tilde{z} = g(x + vt) = g(\tilde{x}). \quad (2)$$

For convenience, we have introduced the coordinate $\tilde{x}$ in the frame of reference that moves with the rigid indenter.

As stated above, we assume that the elastomer is a simple viscoelastic material (Kelvin body), which can be modeled as parallel-connected springs and dampers (Fig. 1b). If the three-dimensional medium is characterized by the shear modulus G and the viscosity η, then the single elements of the viscoelastic foundation must be chosen as parallel-connected springs with stiffness Δk_z and dampers with the damping coefficient $\Delta\gamma$ [18]:

$$\Delta k_z = 4G\Delta x, \;\; \Delta\gamma = 4\eta\Delta x, \quad (3)$$

where Δx is the spatial size of a single element of the viscoelastic foundation (Fig. 1b).

We denote the coordinates of the boundary of the contact area as $\tilde{x} = -a_1$ and $\tilde{x} = a_2$ (Fig. 1a). Vertical displacements u_z in the entire contact area are determined by the purely geometric condition

$$u_z(x, t) = d - g(x + vt) = d - g(\tilde{x}). \quad (4)$$

The vertical velocities are

$$\frac{\partial u_z(x,t)}{\partial t} = -\frac{\partial g(x+vt)}{\partial t} = -vg'(\tilde{x}) \quad (5)$$

and the force acting on one element is

$$f_N(\tilde{x}) = \Delta k_z u_z + \Delta\gamma \dot{u}_z$$
$$= 4[G(d - g(\tilde{x})) - \eta v g'(\tilde{x})]\Delta x. \quad (6)$$

The left boundary of the contact area is determined by the condition $u_z(-a_1) = 0$ and the right boundary by the condition of $f_N(a_2) = 0$. From this, it follows that

$$a_1 = d/c, \;\; a_2 = d/c - v\tau, \quad (7)$$

where we have introduced the relaxation time

$$\tau = \eta/G. \quad (8)$$

We can consider two velocity domains:

$$\text{I: } v < d/(c\tau), \quad (9)$$

$$\text{II: } v > d/(c\tau). \quad (10)$$

In the first domain, the right contact point lies to the right of the tip of the cone. In the second one, it coincides with the tip of the indenter.

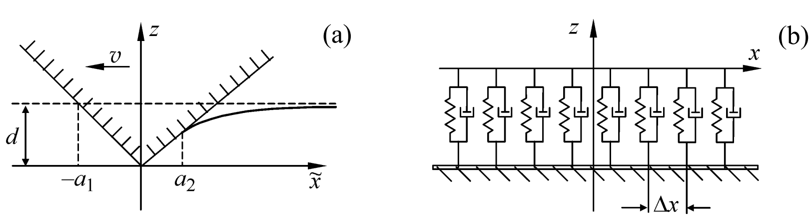

Fig. 1. Contact between an elastomer and rigid conical indenter which is moved tangentially with the velocity v (a); rheological model for a viscoelastic medium (b).

Velocity domain I. The total normal force is

$$F_N = 4\int_{-a_1}^{a_2}\left[G\left(d-g(\tilde{x})\right)-\eta v g'(\tilde{x})\right]\mathrm{d}\tilde{x}$$

$$=\frac{4G}{c}\left[d^2+\frac{1}{2}(cv\tau)^2\right]. \qquad (11)$$

The tangential force is calculated as

$$F_x = -4\int_{-a_1}^{a_2} g'(\tilde{x})\left[G\left(d-g(\tilde{x})\right)-\eta v g'(\tilde{x})\right]\mathrm{d}\tilde{x}$$

$$=4Gc\left[2d(v\tau)-\frac{c}{2}(v\tau)^2\right]. \qquad (12)$$

The resulting coefficient of friction is

$$\mu=\frac{F_x}{F_N}=c\frac{2cv\tau/d-1/2(cv\tau/d)^2}{1+1/2(cv\tau/d)^2}. \qquad (13)$$

Velocity domain II. The normal force is

$$F_N = 4\int_{-a_1}^{0}\left[G\left(d-g(\tilde{x})\right)-\eta v g'(\tilde{x})\right]\mathrm{d}\tilde{x}$$

$$=\frac{4G}{c}\left[\frac{d^2}{2}+cdv\tau\right], \qquad (14)$$

and the coefficient of friction is

$$\mu = c = \text{const}. \qquad (15)$$

If we express the indentation depth as a function of normal force (11) and substitute it into (12), then we obtain a coefficient of friction

$$\mu=\frac{F_x}{F_N}=\begin{cases} c\left[2^{3/2}\psi\sqrt{1-\psi^2}-\psi^2\right], & \psi^2<1/3, \\ c, & \psi^2>1/3, \end{cases} \qquad (16)$$

with

$$\psi^2=\frac{2cGv^2\tau^2}{F_N}=\frac{2cv^2\eta^2}{GF_N}. \qquad (17)$$

This result was first obtained in [18]. The coefficient of friction occurs to be a universal function of the parameter combination ψ and is dependent on the viscosity, shear modulus, velocity, normal force, and surface gradient. For values of ψ larger than the critical value $\psi = 1/\sqrt{3}$, the coefficient of friction remains constant (Fig. 2).

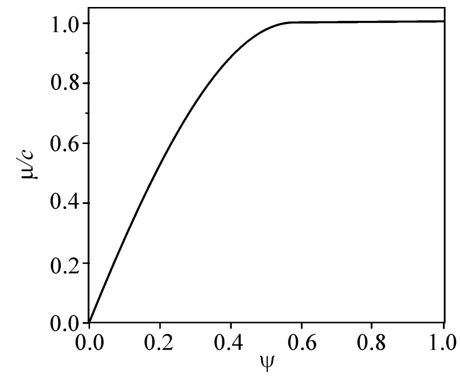

Fig. 2. The coefficient of friction (normalized by the surface gradient) for the conical indenter as a function of the variable $\psi = v\tau\sqrt{2cG/F_N}$.

3. FRICTION BETWEEN A RIGID CONE AND A VISCOELASTIC MEDIUM WITH TEMPERATURE-DEPENDENT VISCOSITY

It was shown in [18] that the heat production in frictional contacts can be easily taken into account in the framework of the method of dimensionality reduction provided the Péclet number is small enough:

$$\mathrm{Pe}=\frac{va}{2\beta}\ll 1, \qquad (18)$$

where $\beta=\lambda/\rho c$ is the thermal diffusivity of the medium, λ is the specific thermal conductivity, ρ is the density, c is the specific heat capacity of the medium, and a is the order of magnitude of the contact size. Under this assumption, the power of the heat production in a single element of the viscoelastic foundation, $\Delta\dot{W}$, was shown to be related to the temperature change ΔT by the equation

$$\Delta\dot{W}=2\lambda\Delta x\Delta T, \qquad (19)$$

where $\Delta T = T - T_0$ and T_0 is the environment temperature far away from the contact point [18]. At this point, it should be noted that this one-dimensional temperature distribution does not coincide with the true three-dimensional temperature distribution in the initial three-dimensional system. The exact three-dimensional temperature distribution can be obtained by an integral transformation described in [18]. However, for the sakes of qualitative estimation, it is eligible to use directly the temperature change in the one-dimensional model. In calculating the rate of the energy production we will assume that the whole dissipation occurs only in dampers and that the whole energy dissipated by dampers is converted into the heat. The dissipation power due to one single viscoelastic element is

$$\Delta\dot{W}=4\eta\Delta x\dot{u}_z^2. \qquad (20)$$

Substituting (20) into (19), we get for the temperature change

$$\Delta T=\frac{2\eta}{\lambda}\dot{u}_z^2. \qquad (21)$$

Let us assume a simplest dependence of the viscosity on temperature according to Arrhenius law

$$\eta(T) = A_0 \exp\left(\frac{U_0}{kT}\right) = A_0 \exp\left(\frac{U_0}{k(T_0 + \Delta T)}\right), \quad (22)$$

where A_0 is a constant, U_0 is the activation energy, and k is the Boltzmann constant. Expanding (22) into a Tailor series, we get

$$\eta(T) \approx A_0 \exp\left(-\frac{U_0}{kT_0} - \frac{U_0}{kT_0^2}\Delta T\right) = \eta_0 \exp(-\alpha\Delta T), \quad (23)$$

where η_0 is the viscosity at the environmental temperature, and α is a constant characterizing the influence of temperature on effective viscosity of the material.

For the conical indenter, the vertical velocity $\dot{u}_z$ for any element of the viscoelastic foundation, which is in contact with the indenter, is equal to $\dot{u}_z = \pm cv$. According to (21), this means that the temperature is constant in the entire contact area:

$$\Delta T = \frac{2\eta}{\lambda} c^2 v^2. \quad (24)$$

Substituting (23) into (24), we get an equation for determining the temperature raise in the contact and, finally, the changed viscosity:

$$\Delta T = \frac{2\eta_0 c^2 v^2}{\lambda} \exp(-\alpha\Delta T). \quad (25)$$

Just as temperature, the viscosity will be constant in the whole contact area. This means that all relations obtained in Section 2 for the case of a constant viscosity remain valid, provided the corrected value of viscosity is used. Introducing notations

$$\xi = \alpha\Delta T, \quad \phi = 2\alpha\eta_0 c^2 v^2 / \lambda, \quad (26)$$

we can rewrite (25) in the form

$$\xi = \phi e^{-\xi}. \quad (27)$$

Solution of this equation with respect to $\xi = \xi(\phi)$ provides the dependence of the temperature raise on loading parameters.

Let us stress again that all equations of the Section 2, including (16), remain valid in the case of temperature depending viscosity, provided the corrected value of viscosity is used in (17). We rewrite (16) with new notations as

$$\mu = \mu(\tilde{\psi}) = \begin{cases} c[2^{3/2}\tilde{\psi}\sqrt{1-\tilde{\psi}^2} - \tilde{\psi}^2], \tilde{\psi}^2 < 1/3, \\ c, \tilde{\psi}^2 > 1/3, \end{cases} \quad (28)$$

with

$$\tilde{\psi}^2 = \frac{2cv^2\eta^2}{GF_N} = \frac{2cv^2\eta_0^2}{GF_N}\exp(-2\xi(\phi)), \quad (29)$$

where $\xi(\phi)$ is the solution of Eq. (27). The quantity ϕ (26) can be written as $\phi = \psi^2\zeta$ with

$$\psi = v\eta_0\sqrt{2c/(GF_N)}, \quad \zeta = \alpha c G F_N / (\lambda\eta_0). \quad (30)$$

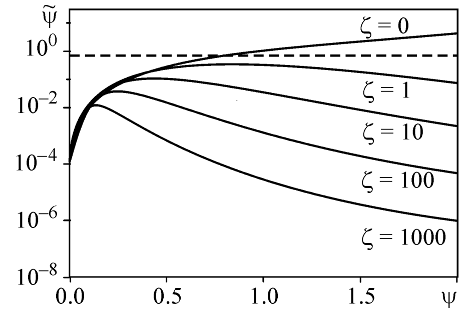

Fig. 3. Dependency of the variable $\tilde{\psi}$ on ψ, Eq. (31).

With this notation, equation (29) can be rewritten as

$$\tilde{\psi}^2 = \psi^2 \exp(-2\xi(\psi^2\zeta)). \quad (31)$$

According to Eq. (28), the coefficient of friction depends only on the variable $\tilde{\psi}$. Note that the variable ζ depends only on the normal force but not on the sliding velocity, while the variable ψ is proportional to the sliding velocity. We therefore can consider ψ as a normalized velocity. Equation (31) shows that for any value of parameter ζ the variable $\tilde{\psi}$ and thus the coefficient of friction are unique functions of only dimensionless velocity ψ. To better understand the character of this dependency, let us first analyze the function (31) (Fig. 3). Depending on the value of the parameter ζ, the following cases can occur: (a) For $\zeta < \zeta_c = 1.1035$, the variable ψ increases over the value $\psi = 1/\sqrt{3}$. This means that the coefficient of friction increases and achieves a constant value. Further increase of the dimensionless velocity ψ will later lead to decreasing of $\tilde{\psi}$ under the critical value and decreasing of the coefficient of friction. (b) At the critical value $\zeta_c = 1.1035$, the variable $\tilde{\psi}$ achieves the critical value and starts decreasing again. Thus the plateau value of the coefficient of friction is achieved for only one value of velocity. (c) For $\zeta > \zeta_c = 1.1035$, the variable $\tilde{\psi}$ never achieves the value of $\psi = 1/\sqrt{3}$, thus the plateau value of the coefficient of friction is not achieved.

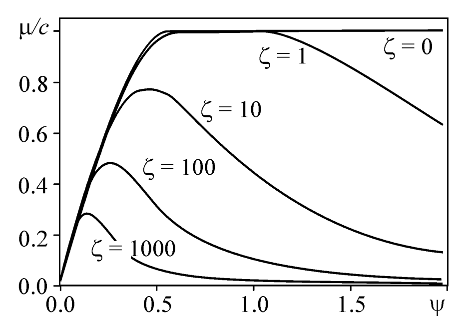

Fig. 4. The coefficient of friction (normalized by the surface gradient) for a conical indenter as a function of the variables $\psi = v\eta_0\sqrt{2c/(GF_N)}$ and $\zeta = \alpha c G F_N/(\lambda\eta_0)$.

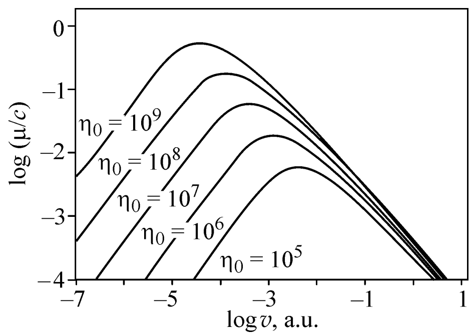

Fig. 5. Logarithm of coefficient of friction as a function of logarithm of velocity has the same form for different viscosities η_0 (which correspond to various temperatures.) The curves plotted for different viscosities can thus be shifted to one single curve (master curve). However, other than by the classical master curve procedure by Grosch, shifting both along the $\ln v$ and $\ln \mu$ axes is necessary.

The corresponding dependencies of the coefficient of friction on the dimensionless velocity for various values of parameter ζ are shown in Fig. 4. One can see that the temperature effect leads to a decrease of the coefficient of friction at large sliding velocities.

For very large ζ only small values of $\tilde{\psi}$ are achieved. Correspondingly, for the coefficient of friction, only the linear term in (28) may be used: $\mu/c \approx 2^{3/2}\tilde{\psi}$ and the coefficient of friction can be written as

$$\frac{\mu}{c} \approx 2^{3/2}\psi\exp(-\xi(\psi^2\zeta))$$

$$= 4v\eta_0\sqrt{\frac{c}{GF_N}}\exp\left(-\xi\left(\frac{2\alpha\eta_0 c^2 v^2}{\lambda}\right)\right). \quad (32)$$

In this case, the coefficient of friction contains viscosity once in the multiplicative prefactor, and secondly, in the argument of the exponential function. This means that if the logarithm of the coefficient of friction will be presented as a function of the logarithm of velocity, then the dependencies for different viscosities (corresponding to different temperatures) will all have the same shape only shifted horizontally and vertically by corresponding shift factors. This shifting property is illustrated in Fig. 5. This may explain the observation that the master curve procedure often does work only if shifting along both axes $\log\mu$ and $\log v$ is realized [23].

4. CONCLUSION

We have shown that the coefficient of friction between a single conical indenter and a Kelvin body with exponential dependency of the viscosity on temperature is a function of only two dimensionless combinations of material and loading parameters: $\psi = v\eta_0\sqrt{2c/(GF_N)}$ and $\zeta = \alpha cGF_N/(\lambda\eta_0)$. The general character of the dependency of the coefficient of friction on the sliding velocity is governed completely by the parameter $\zeta = \alpha cGF_N/(\lambda\eta_0)$ which depends on the viscosity, the shear modulus, the specific thermal conductivity, the normal force and the "Arrhenius factor" α but is independent of the shape of the indenter. Previous studies of friction between elastomers and nominally flat rough surfaces [12] or differently shaped rough surfaces [13] (without account of thermal effects) have shown that the qualitative behavior in these complicated cases is the same as for single asperities: In all cases, the coefficient of friction occurs to be a function of a dimensionless product of powers of loading and material parameters, the exact powers depending on the details of the shape and fractal properties of roughness. We thus expect that the results obtained in the present paper can be used for qualitative understanding of dependencies which will be realized in contacts of rough surfaces. In the region of strong temperature effects, the generalization of the well-known master curve procedure by both horizontal and vertical shifting was substantiated.

ACKNOWLEDGEMENT

This work is supported in part by COST Action MP1303, the Deutsche Forschungsgemeinschaft and the Ministry of Education of the Russian Federation. Andrey Dimaki is thankful for financial support of German Academic Exchange Service (DAAD).

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