

Numerical Simulation of the Receptivity of a Supersonic Boundary Layer to Acoustic Disturbances in Compression and Rarefaction Flows

Corresponding Member of the RAS I. V. Egorov^{a,b,*} and N. V. Palchekovskaya^{a,b,**}

Received January 20, 2021; revised January 20, 2021; accepted January 25, 2021

Abstract—The receptivity of a boundary layer on a flat plate to acoustic disturbances in an incoming supersonic gas flow is studied numerically. Angles of attack are considered, at which shock waves and a rarefaction fanlike spread are generated in the flow field. The peculiarities of the interaction of these structures with acoustic waves have been studied, and regularities in the origin of the boundary layer instability region have been obtained.

Keywords: boundary layer, receptivity, acoustic disturbances, numerical simulation, Navier–Stokes equations

DOI: 10.1134/S1028335821030022

Aircraft (AC) move in the cruise mode, as a rule, at low positive angles of attack. In this case, compression and rarefaction flows are formed on the surface of the aircraft. The phenomena associated with the laminar–turbulent transition (LTT) on the leeward side of the aircraft seem to be extremely important, but at the same time, poorly studied. Studies of the occurrence of turbulence in boundary layers have been relevant for many years. This is due to the fact that prediction of LTT location and transition control are important practical problems. For hypersonic flows, these problems are especially acute, since the transition affects not only the aerodynamic quality of the aircraft, but also leads to a sharp increase in heat fluxes to the streamlined surface. The transition to turbulence is a complex unsteady process that depends on a large number of parameters and develops according to different scenarios [1]. Since this process depends on the spectral composition and the level of free-stream disturbances, full simulation of natural conditions is impossible in wind tunnels, especially at high flow velocities [2]. Therefore, the problems of studying the initial stages of LTT, such as the susceptibility and instability of the flow in a low-perturbed flow, are urgent. In this paper, we studied the process of suscep-

tibility on a plate at different angles of attack in supersonic flow regimes.

FORMULATION OF THE PROBLEM AND METHODS OF NUMERICAL SIMULATION

The simulation was carried out using the numerical solution of the unsteady Navier–Stokes equations in the two-dimensional formulation. This is due to the fact that in this work we considered the second perturbation mode, which is the most unstable at high supersonic flow velocities and has a two-dimensional behavior [3]. The main computational studies were carried out for a perfect gas model. All quantities are included in the conservative notation of the equations in dimensionless form. When the Navier–Stokes equations are rendered dimensionless, the Cartesian coordinates are referred to the characteristic linear size; the time, to the characteristic time; the velocity vector components, to the modulus of the incoming flow velocity vector; the pressure, to the doubled velocity head of the incoming flow, and the other gas-dynamic variables, to their values in the incoming

Table 1

Angle of attack, AoA	Frequency, ω	Wave angle, θ
-5°	3517	5°
0°	1905	0°
5°	949	-5°

^a Moscow Institute of Physics and Technology, Dolgoprudny, Moscow oblast, Russia

^b Central Aerohydrodynamic Institute, Zhukovskii, Moscow oblast, Russia

*e-mail: egorov.ivan.v@mipt.ru

**e-mail: palchekovskaia.nv@mipt.ru

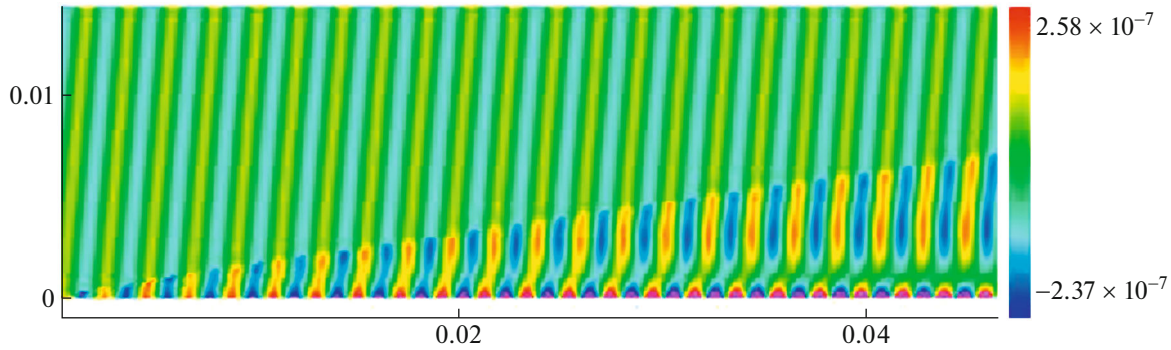


Fig. 1. Pressure disturbance field for $\text{AoA} = -5^\circ$ and a fast acoustic wave near the leading edge of the plate.

flow. With such dimensionlessness, the main similarity parameters appear in the Navier–Stokes equations: the adiabatic exponent, the free-stream Mach number, the Reynolds number, and the Prandtl number. The Navier–Stokes equations thus dimensioned were used for numerical integration. The integration of the Navier–Stokes equations was carried out using the integro-interpolation method (finite volume method). Its application to the Navier–Stokes equations allows one to obtain difference analogs of the laws of conservation. For a monotonic difference scheme, the fluxes at half-integer nodes were calculated based on the solution of the Riemann problem on the decay of an arbitrary discontinuity. When approximating the convective component of the flux vectors at half-integer nodes, the WENO scheme of the third order of accuracy was used. When approximating the diffusion component

of flux vectors on the edge of a unit cell, a difference scheme of the type of central differences of the second order of accuracy is used. As a result of the difference approximation of the Navier–Stokes equations and the corresponding boundary conditions on a certain grid, the integration of nonlinear partial differential equations was reduced to solving a system of nonlinear algebraic equations. A modified Newton–Raphson method was used to solve nonlinear grid equations. The numerical method is implemented on a multiprocessor supercomputer of the cluster type [4].

The susceptibility problem was solved in two stages. First, the steady-state flow field was calculated by the settling method. Then, at the input and upper boundaries of the computational domain, unsteady boundary conditions were set, simulating an acoustic wave with a given frequency and wave vector components, and then the unsteady problem was solved. The calculations were carried out until the time-harmonic perturbation field was established.

The computational grid for all cases had a dimension of 6050×603 nodes and was divided into 96 blocks, each of which was processed on one core of a multiprocessor supercomputer. The adhesion conditions for the velocity and the isothermal condition for the temperature with the temperature factor were set on the plate surface $t_w = 0.3$, which corresponds to a cold wall. On the right output boundary, the condition of extrapolation of dependent variables was set.

The first step was to solve the problem of unperturbed flow around a plate with the free-stream Mach number $M_\infty = 6$, temperature $T_\infty = 80$ K, Reynolds number $\text{Re}_{L,\infty} = 3 \times 10^7$ calculated along the length of the plate. At the input boundary, the Dirichlet conditions were set, and at the upper boundary, the conditions for extrapolating the dependent variables were set. As a result, a stationary flow field with a shock wave was obtained by the method established in time at the angles of attack of the incident flow $\text{AoA} = -5^\circ$ and $\text{AoA} = 0^\circ$ or with a fanlike spread of rarefaction waves at $\text{AoA} = 5^\circ$.

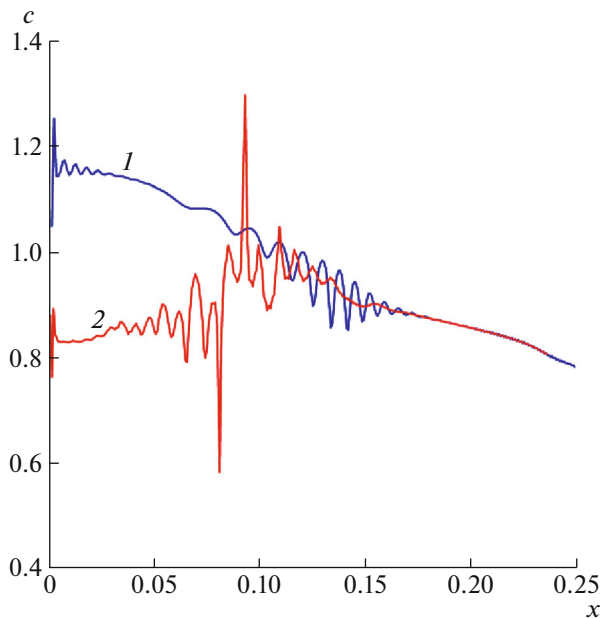


Fig. 2. The phase velocities of the disturbances generated by the (1) fast and (2) slow acoustic waves for $\text{AoA} = -5^\circ$.

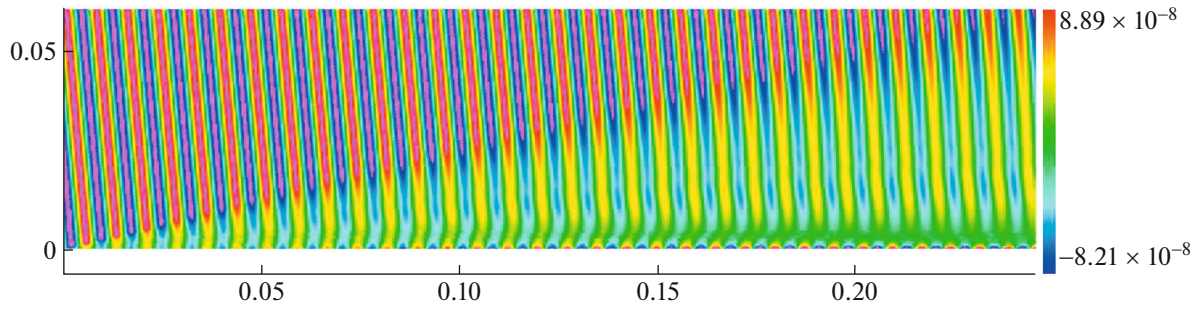


Fig. 3. Pressure disturbance field for $\text{AoA} = 5^\circ$ and a fast acoustic wave near the leading edge of the plate.

At the second stage, perturbations in the form of a monochromatic acoustic wave were imposed on the stationary field obtained [5]. The cases of fast and slow acoustic waves were considered separately. In this work, the amplitude of acoustic disturbances $\varepsilon = 10^{-7}$ was chosen to ensure the linearity of the receptivity process. For each mode, the frequency of the acoustic waves was chosen so that the instability region was within the computational domain and the resulting gain was the same (N , the factor was approximately equal to the same number [3]). The dimensionless frequencies (the frequency is normalized to the ratio of the incident flow velocity to the characteristic linear size) and the angles of incidence of acoustic waves for each angle of attack are presented in Table 1.

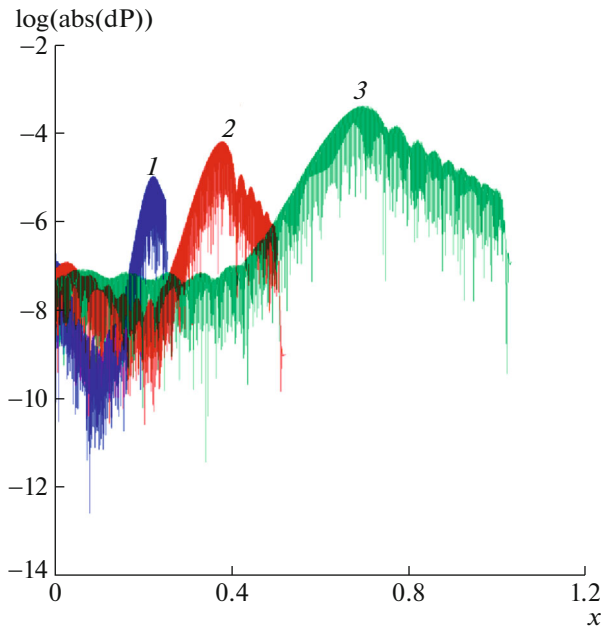


Fig. 4. Distribution of the absolute value of the pressure perturbation along the plate surface for (1) $\text{AoA} = -5^\circ$, (2) $\text{AoA} = 0^\circ$, (3) $\text{AoA} = 5^\circ$ for the case of a slow acoustic wave.

RESULTS

In the analysis of the results obtained, we mainly used the perturbation fields of the velocity, pressure, and temperature, which represent the difference between the fields with acoustic perturbations in the incoming flow and stationary fields without perturbations. In the case of $\text{AoA} = -5^\circ$ and a fast acoustic wave in the incident flow, the phase velocity of perturbations converges to the phase velocity of the mode F (in the terminology of [6]) from the leading edge to $x = 0.03$. This is clearly demonstrated by the pressure perturbation field near the leading edge: at $x \approx 0.03$, the “bridges” that connect the cellular structures in the boundary layer with streaky structures emanating from the shock wave disappear (Fig. 1). Then the mode F pumps up the mode S (according to the terminology of [6]) and, starting from $x = 0.15$, mode S dominates (Fig. 2). In the case of $\text{AoA} = -5^\circ$ and a slow acoustic wave, the phase velocity rapidly converges to the phase velocity of the S mode. Then oscillations are observed, which disappear near $x = 0.15$. In the case of $\text{AoA} = 0^\circ$, the behavior of the phase velocities is similar to the mode $\text{AoA} = -5^\circ$, but the S mode begins to dominate significantly downstream near $x = 0.3$. Figure 3 shows the field of pressure perturbations near the leading edge for the angle of attack $\text{AoA} = 5^\circ$, when the fanlike spread of rarefaction waves influences the process of susceptibility. With $\text{AoA} = 5^\circ$ the behavior of the phase velocities is also similar to the previous regimes, and the S mode dominates, starting from $x = 0.6$.

These results allow us to conclude that the F mode (S mode) is excited by a fast (slow) acoustic wave in a small vicinity of the leading edge of the plate. Further downstream, intermode exchange occurs, as a result of which the dominance of the unstable mode S is observed until the end of the computational domain.

Distributions of pressure perturbations along the plate surface demonstrate the development of perturbations in the flow field. It can be seen that with an

increase in the angle of attack, the instability region shifts significantly downstream (Fig. 4).

ACKNOWLEDGMENTS

The authors are deeply grateful to A.V. Fedorov for discussing this work.

FUNDING

This study was supported by the Russian Science Foundation, project no. 19-79-00184.

REFERENCES

1. M. V. Ustinov, Uch. Zap. TsAGI **44** (1), 3 (2013).
2. R.-S. Lin and M. R. Malik, SAE Tech. Paper No. 952041 (1995).
3. A. Fedorov, Ann. Rev. Fluid Mech. **43**, 79 (2011).
4. I. V. Egorov and A. V. Novikov, Comput. Math. Math. Phys. **56**, 1048 (2016).
5. I. V. Egorov, V. G. Sudakov, and A. V. Fedorov, Izv. Akad. Nauk, Mekh. Zhidk. Gaza **41**, 42 (2006).
6. A. V. Fedorov, J. Fluid Mech. **491**, 101 (2003).