ISSN 1028-3358, Doklady Physics, 2019, Vol. 64, No. 4, pp. 159–164. © Pleiades Publishing, Ltd., 2019.
Russian Text © The Author(s), 2019, published in Doklady Akademii Nauk, 2019, Vol. 485, No. 4.

## MECHANICS

# An Analogy between Electromagnetic and Internal Waves

**V. G. Baydulov[a,b,*] and P. A. Lesovskiy[a]**

Presented by Academician V.F. Zhuravlev August 17, 2018

Received August 23, 2018

**Abstract**—For the symmetry group of internal-wave equations, the mechanical content of invariants and symmetry transformations is determined. The performed comparison makes it possible to construct expressions for analogs of momentum, angular momentum, energy, Lorentz transformations, and other characteristics of special relativity and electro-dynamics. The expressions for the Lagrange function are defined, and the conservation laws are derived. An analogy is drawn both in the case of the absence of sources and currents in the Maxwell equations and in their presence.

**DOI:** 10.1134/S1028335819040013

In the presence of gravity, internal gravity waves arise in a density-stratified fluid. Their properties have been studied in sufficient details by various theoretical methods: the equations describing their dynamics, the dispersion relation, and the kinematics are known. We propose an invariant description of internal waves, at the basis of which is an analogy with electromagnetic waves. The apparatus of the theory of continuous groups enabled us to determine the characteristics of the waves that remain invariant, when the vibration frequency varies, to construct the law of velocity transformation, and to derive new conservation laws.

## SYMMETRY GROUP OF INTERNAL-WAVE EQUATIONS

An important significance in wave processes proceeding in the bulk of an inhomogeneous incompressible fluid [1] is played by the unperturbed distribution of the stratifying component $s_0(z)$, which determines the profiles of the density ρ and the frequency of the natural vibrations (buoyancy) $N^2 = -\frac{g}{\rho_0}\frac{d\rho}{dz}$ (the axis $z$ is directed against gravity). In the Boussinesq approximation, the linearized form of the equations of an incompressible stratified fluid takes the form

$$\operatorname{div}\mathbf{v} = 0, \quad \frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho_0}\nabla P + \mathbf{g}s, \quad \frac{\partial s}{\partial t} = \frac{w}{\Lambda}, \tag{1}$$

where $\mathbf{v} = (u, v, w)$ is the velocity, $P$ and $s$ are the dynamical components of pressure and salinity, respectively; and $\Lambda = g/N^2$ is the stratification scale.

Set (1) admits an infinite-dimensional symmetry group with the generators

$$X_1 = \partial_t, \quad X_2 = \partial_x, \quad X_3 = \partial_y, \quad X_4 = \partial_z,$$
$$X_5 = \rho g z \partial_P - \partial_s,$$
$$X_6 = y\partial_x - x\partial_y + v\partial_u - u\partial_v, \tag{2}$$
$$X_7 = \mathbf{v}\partial_{\mathbf{v}} + P\partial_P + s\partial_s,$$
$$Z_{\mathbf{w}} = \mathbf{w}(t,\mathbf{r})\partial_{\mathbf{v}} + p(t,\mathbf{r})\partial_P + S(t,\mathbf{r})\partial_s, \quad Z_\pi = \pi(t)\partial_P,$$

where $\mathbf{w}(t,\mathbf{r})$, $p(t,\mathbf{r})$, $S(t,\mathbf{r})$ are the solution of set (1).

Set (2) of operators includes the operators of groups of the space—time shifts $X_1$–$X_4$, of the mutually stipulated shifts $X_5$ of pressure and salinity, of the rotation $X_6$ in the horizontal plane, and of the infinite-dimensional subalgebra of the pressure shifts $Z_\pi$. The set (2) of symmetry generators, which are common for all linear equations, includes the generators of the group of tensions of dependent variables (generator $X_7$) and the infinite-dimensional subgroup (generator $Z_{\mathbf{w}}$), which reflect the principle of the superposition of solutions.

The Fourier time transform converts the set of equations of an ideal linearly stratified fluid ($s(z) = -\frac{z}{\Lambda} + s'$, $s' = \frac{1}{\Lambda}\int w dt$)

[a] Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, 117526 Russia
[b] Bauman Moscow State Technical University, Moscow, 105005 Russia
*e-mail: baydulov@gmail.com

**Fig. 1.** Schlieren-interferometric image of the internal-wave field generated by a vertically vibrating cylinder with the diameter $d = 1$ cm in a fluid with a linear density profile.

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla P - N^2 \int w dt \mathbf{e}_z, \quad \operatorname{div} \mathbf{v} = 0,$$

into the equation of internal waves

$$i\omega \mathbf{v} = -\nabla P - \frac{N^2}{i\omega} w \mathbf{e}_z, \quad \operatorname{div} \mathbf{v} = 0. \tag{3}$$

The symmetry group allowed by set (3) is significantly different from the group of set (1). The generators, which create it, have the following form:

the shears

$$X_1 = \partial_z, \quad X_2 = \partial_x, \quad X_3 = \partial_y;$$

the tensions

$$X_4 = u\partial_u + v\partial_v + w\partial_w + P\partial_P,$$
$$X_5 = x\partial_x + y\partial_y + z\partial_z + P\partial_P;$$

the rotations in the horizontal plane

$$X_6 = y\partial_x - x\partial_y + v\partial_u - u\partial_v;$$

the hyperbolic rotations

$$X_7 = \tilde{c}^2 z\partial_x + x\partial_z + \tilde{c}^2 w\partial_u + u\partial_w,$$

$$X_8 = \tilde{c}^2 z\partial_y + y\partial_z + \tilde{c}^2 w\partial_v + v\partial_w, \tag{4}$$

$$X_9 = 2\tilde{c}^2 z(x\partial_x + y\partial_y) + (x^2 + y^2 + \tilde{c}^2 z^2)\partial_z$$
$$+ \tilde{c}^2[(2xw - 3zu)\partial_u + (2yw - 3zv)\partial_v]$$
$$+ (2(xu + yv) - 3\tilde{c}^2 zw + P)\partial_w - \tilde{c}^2 zP\partial_P;$$

the inversions

$$X_{10} = (x^2 - y^2 + \tilde{c}^2 z^2)\partial_x + 2x(y\partial_y + z\partial_z)$$
$$- (2(yv - \tilde{c}^2 zw) + 3xu + P)\partial_u + (2yu - 3xv)\partial_v$$
$$+ (2zu - 3xw)\partial_w - xP\partial_P;$$

$$X_{11} = 2y(x\partial_x + z\partial_z) + (-x^2 + y^2 + \tilde{c}^2 z^2)\partial_y$$
$$+ (2xv - 3yu)\partial_u - (2(xu - \tilde{c}^2 zw) + 3yv + P)\partial_v \tag{5}$$
$$+ (2zv - 3yw)\partial_w - yP\partial_P,$$

where

$$\tilde{c}^2 = N^2/\omega^2 - 1.$$

In addition to the generators of the groups of spatial shifts, tensions, and rotations in the horizontal plane, set (4) of operators includes the operators $X_7$ and $X_8$ of hyperbolic rotations, as well as those of inversions $(x \to 1/x)$ for each of three independent coordinates $X_9$, $X_{10}$, and $X_{11}$. The Maxwell equations also admit the group of hyperbolic rotations in the space–time generating the Lorentz transformations. The role of the time variable in the equations of internal waves is played by the vertical coordinate, and the speed of light is the ratio of the frequencies of buoyancy and the wave frequency $\tilde{c}$. Thus, the transformations generated by the operators $X_7$, $X_8$ for the field of internal waves can be correlated with the Lorenz transformations of the Maxwell equations [3].

## MECHANICAL CONTENT OF HYPERBOLIC-ROTATION TRANSFORMATIONS

Taking into account the symmetry group of set (3), we can show that the principal characteristics of internal waves are determined by the properties of the pseudo-Euclidean space in which the interval is the invariant of the three-parameter transformation group with the generators $X_6$, $X_7$, and $X_8$:

$$I^2 = \tilde{c}^2 z^2 - x^2 - y^2. \tag{6}$$

The symmetries of the system are clearly manifested themselves in the structure of the field pattern of periodic internal waves, which exist in the form of a wave wedge in the 2D case or a cone in the 3D case propagating at an angle $\vartheta = \arcsin(\omega/N)$ to the horizon in the media with a constant frequency of buoyancy. The typical image of the internal-wave field generated by a horizontal cylinder oscillating with a constant frequency is shown in the photograph (Fig. 1) obtained by schlieren-interferometry [4].

The opening angle of wedge or cone defines the region of existence of causally related events. The variation of the internal-wave frequency, which determines the angular position of the wave cone, is equivalent to a variation of the speed of light or the transition to another inertial frame of reference moving with a new proper velocity in the Maxwell's equations for electromagnetic waves. By the analogy with the electrodynamics, the analogs of the Lorentz transformations and other characteristics of relativistic mechanics can be constructed for flows in a stratified fluid.

We find the type of transformation relating the sets of equations of monochromatic internal waves of different frequencies. To do this, we write two sets of equations of monochromatic internal waves with the frequencies $\omega_1$ and $\omega_2$ preliminarily ruling out the pressure:

$$\begin{aligned} &\frac{\partial u_1}{\partial y_1} = \frac{\partial v_1}{\partial x_1}, \quad \frac{\partial c_1 w_1}{\partial x_1} = -\frac{\partial u_1}{\partial c_1 z_1}, \\ &\frac{\partial c_1 w_1}{\partial y_1} = -\frac{\partial v_1}{\partial c_1 z_1}, \quad \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = -\frac{\partial c_1 w_1}{\partial c_1 z_1}; \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{\partial u_2}{\partial y_2} = \frac{\partial v_2}{\partial x_2}, \quad \frac{\partial c_2 w_2}{\partial x_2} = -\frac{\partial u_2}{\partial c_2 z_2}, \\ &\frac{\partial c_2 w_2}{\partial y_2} = -\frac{\partial v_2}{\partial c_2 z_2}, \quad \frac{\partial u_2}{\partial x_2} + \frac{\partial v_2}{\partial y_2} = -\frac{\partial c_2 w_2}{\partial c_2 z_2}, \end{aligned} \tag{8}$$

where

$$c_j^2 = N^2/\omega_j^2 - 1, \quad j = 1, 2.$$

Sets (7) and (8) differ from each other in the value of the parameter $c_j$. We find the type of transformations connecting them. The replacement of variables

$$c_1 w_1 = w, \quad c_2 w_2 = w', \quad c_1 z_1 = \tau, \quad c_2 z_2 = \tau' \tag{9}$$

leads Eqs. (7) and (8) to the same set of equations in the form of

$$\begin{aligned} &\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \quad \frac{\partial w}{\partial x} = -\frac{\partial u}{\partial \tau}, \\ &\frac{\partial w}{\partial y} = -\frac{\partial v}{\partial \tau}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial \tau}. \end{aligned} \tag{10}$$

At the same time, $c_1$ and $c_2$ are related to each other by the transformations of tension, which have the form $c_2 = c_1 \exp(a)$, where $a$ is the transformation parameter, in the standard parametrization (the zero value of the parameter corresponds to the identity transformation). Transformed sets (10) are the invariants with respect to the Lorentz transformations; then relations (9) give the form of the transformations, which convert sets (7) and (8) into each other

$$x' = x\cosh a + \tau \sinh a, \quad \tau' = \tau \cosh a + x \sinh a, \tag{11}$$

$$u' = u\cosh a + w \sinh a, \quad w' = w\cosh a + u \sinh a, \tag{12}$$

which, with taking into account the chosen parametrization, can be rewritten through the values included in sets (7) and (8)

$$\begin{aligned} x_2 &= \frac{c_2^2 + c_1^2}{2c_1c_2} x_1 + \frac{c_2^2 - c_1^2}{2c_1c_2} c_1 z_1, \\ c_2 z_2 &= \frac{c_2^2 + c_1^2}{2c_1c_2} c_1 z_1 + \frac{c_2^2 - c_1^2}{2c_1c_2} x_1, \end{aligned} \tag{13}$$

$$\begin{aligned} u_2 &= \frac{c_2^2 + c_1^2}{2c_1c_2} u_1 + \frac{c_2^2 - c_1^2}{2c_1c_2} c_1 w_1, \\ c_2 w_2 &= \frac{c_2^2 + c_1^2}{2c_1c_2} c_1 w_1 + \frac{c_2^2 - c_1^2}{2c_1c_2} u_1, \end{aligned} \tag{14}$$

where $c_1$ and $c_2$ depend on the source-vibration frequency.

In this case, the values, which are invariant with respect to the vibration frequency, have the form of intervals in the pseudo-Euclidean space:

$$\begin{aligned} c_2^2 z_2^2 - x_2^2 &= c_1^2 z_1^2 - x_1^2 = inv, \\ c_2^2 w_2^2 - u_2^2 &= c_1^2 w_1^2 - u_1^2 = inv. \end{aligned} \tag{15}$$

Using the invariance of set (10) with respect to the Lorentz transformations and taking into account the correspondence between the stream functions and the electromagnetic-field strengths, we can find the law of transformation for all hydrodynamic functions.

## VELOCITY-TRANSFORMATION LAW

By analogy with the theory of electromagnetic waves, we find the transformation law for the phase velocity of internal waves with the variation of the source vibration frequency for which we construct the correspondence relations connecting the variables of electromagnetic and internal waves (2D and 3D). Since the equations of internal waves are written in the Fourier images, the number of independent variables proves to be reduced, which leads to imposing restrictions on the values included in the Maxwell equations compared with the equations of internal waves.

## CORRESONDENCE BETWEEN 2D EQUATIONS OF INTERNAL WAVES AND THE MAXWELL EQUATIONS IN A VACUUM

We consider the 1D electromagnetic field with the strength vectors in the form of

$$\mathbf{E} = E(x,t)\mathbf{e}_y, \quad \mathbf{H} = H(x,t)\mathbf{e}_z. \tag{16}$$

For electromagnetic field (16), the Maxwell equations are reduced to the set of two equations:

$$\frac{\partial E}{\partial x} = -\frac{1}{c}\frac{\partial H}{\partial t}, \quad \frac{\partial H}{\partial x} = -\frac{1}{c}\frac{\partial E}{\partial t}. \tag{17}$$

For constructing the relationships that determine the relation between the variables of internal and electromagnetic waves, the equations of 2D internal waves in the Fourier space can be reduced to the form

$$\frac{\partial \tilde{c}w}{\partial x} = -\frac{1}{\tilde{c}}\frac{\partial u}{\partial z}, \quad \frac{\partial u}{\partial x} = -\frac{1}{\tilde{c}}\frac{\partial \tilde{c}w}{\partial z}. \tag{18}$$

Comparing Eqs. (17) and (18), we obtain the correspondence between the dependent and independent variables of electromagnetic waves (EMWs) and internal waves (IWs):

*Correspondence 1 between the variables in sets (17) and (18)*

| | | | | |
|---|---|---|---|---|
| EMWs | $t$ | $c$ | $E$ | $H$ |
| IWs | $z$ | $\tilde{c}$ | $\tilde{c}w$ | $u$ |

Expressing the strengths of the electric and magnetic fields through the vector and scalar potentials

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi, \quad \mathbf{H} = \operatorname{rot}\mathbf{A} \tag{19}$$

and assuming by virtue of the calibration condition that the scalar potential is zero, we write the potentials of the electric and magnetic field in the form of

$$\mathbf{A} = A(x,t)\mathbf{e}_y, \quad E = -\frac{1}{c}\frac{\partial A}{\partial t}, \quad H = \frac{\partial A}{\partial x}. \tag{20}$$

Similar expressions for the velocity components of internal waves can also be written using the vector potential:

$$u = \frac{\partial A_\omega}{\partial x}, \quad \tilde{c}w = -\frac{1}{\tilde{c}}\frac{\partial A_\omega}{\partial z}. \tag{21}$$

The use of the potential enables us to write the electromagnetic-field tensor and its analog for the internal waves:

$$F_{ik} = \begin{pmatrix} 0 & 0 & E & 0 \\ 0 & 0 & -H & 0 \\ -E & H & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{22}$$

$$F_{ik} = \begin{pmatrix} 0 & 0 & \tilde{c}w & 0 \\ 0 & 0 & -u & 0 \\ -\tilde{c}w & u & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{23}$$

Knowledge of tensors (23) enables us also to compose the invariant values, which remain unchanged when passing from one vibration frequency to another instead of only writing the form of the Lagrange function for the problems of the theory of internal waves.

The form of these invariants can be easily found on the basis of a 4D-field representation using the asymmetric 4-tensor $F^{ik}$. It is obvious that the following invariant values can be composed from the components of this tensor:

$$F_{ik}F^{ik} = inv, \quad e^{iklm}F_{ik}F_{lm} = inv,$$

where $e^{iklm}$ is an absolutely asymmetric unit tensor. Expressing the components of $F^{ik}$ through the velocity components, we can ascertain that these invariants have in the 3D case the following form:

for the electromagnetic field

$$F_{ik}F^{ik} = \mathbf{E}^2 - \mathbf{H}^2 = inv, \quad \mathbf{EH} = inv, \tag{24}$$

and for the internal-wave field

$$F_{ik}F^{ik} = 2(u^2 - \tilde{c}^2(\omega)w^2) = inv, \quad u^2 - \tilde{c}^2(\omega)w^2 = inv. \tag{25}$$

Pseudo-scalar invariant (24) in the theory of internal waves is always zero, which has an obvious physical meaning: the velocity components of the internal waves remain orthogonal, when the vibration frequency of the source varies.

## 2D MAXWELL EQUATIONS IN A VACUUM

To compare the equations of 3D internal waves with the Maxwell equations, we choose the electromagnetic field in the form

$$\mathbf{H} = H\mathbf{e}_z, \quad \mathbf{E} = (E_x, E_y).$$

In this case, the Maxwell equation in a vacuum are written as

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c}\frac{\partial H}{\partial t}. \tag{26}$$

The following 3D equations of internal waves in the Fourier time images correspond to set (26):

$$\begin{gathered} \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\tilde{c}}\frac{\partial \tilde{c}w}{\partial z}, \\ \frac{\partial \tilde{c}w}{\partial x} = -\frac{1}{\tilde{c}}\frac{\partial u}{\partial z}, \quad \frac{\partial \tilde{c}w}{\partial y} = -\frac{1}{\tilde{c}}\frac{\partial v}{\partial z}. \end{gathered} \tag{27}$$

Comparing sets (26) and (27), we write the correspondence between dependent and independent variables of the electromagnetic and internal waves:

*Correspondence 2 between the variables in sets (26) and (27)*

| | | | | | |
|---|---|---|---|---|---|
| EMWs | $t$ | $c$ | $E_x$ | $E_y$ | $H$ |
| IWs | $z$ | $\tilde{c}$ | $-v$ | $u$ | $\tilde{c}w$ |

Further, in full correspondence to the case of the 1D Maxwell equations in a vacuum, we find the expressions for the velocity components of internal waves through the vector potential:

$$u = -\frac{1}{\tilde{c}}\frac{\partial A_{\omega,y}}{\partial z},$$
$$v = \frac{1}{\tilde{c}}\frac{\partial A_{\omega,x}}{\partial z}, \quad \tilde{c}w = \frac{\partial A_{\omega,y}}{\partial x} - \frac{\partial A_{\omega,x}}{\partial y}, \tag{28}$$

and the analog of the electromagnetic-field tensor for internal waves

$$F_{ik} = \begin{pmatrix} 0 & -v & u & 0 \\ v & 0 & -\tilde{c}w & 0 \\ -u & \tilde{c}w & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad F^{ik} = \begin{pmatrix} 0 & v & -u & 0 \\ -v & 0 & -\tilde{c}w & 0 \\ u & \tilde{c}w & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{29}$$

Then the following values prove to be invariant for the internal waves with respect to the change in the vibration frequency:

$$F_{ik}F^{ik} = -2(u^2 + v^2 - \tilde{c}^2 w^2) = inv,$$
$$v^2 + u^2 - \tilde{c}^2 w^2 = inv. \tag{30}$$

Thus, the comparison of the equations of internal and electromagnetic waves enabled us also to find the law of transformation of flow functions and construct the values that remain unchanged during this transition instead of only to give the geometric interpretation of the flow patterns corresponding to different vibration frequencies.

## ANALOGS OF CHARGES AND CURRENTS IN THE THEORY OF INTERNAL WAVES

For solving the problems on the generation of internal waves and the construction of analytical solutions for the problems of flow around finite-sized bodies in the theory of a stratified fluid, the force and mass sources are often used, which made it possible to solve the problems with finite-sized bodies on the basis of the solution of the problems with point inhomogeneities. Similar approaches previously spurred great development in electromagnetism.

The linearized equations of internal waves in the Fourier time images with mass forces and sources have the form

$$\operatorname{div}\mathbf{v} = m, \quad i\omega\mathbf{v} = -\nabla P - \frac{N^2}{i\omega}w\mathbf{e}_z + \mathbf{f}, \tag{31}$$

where $\mathbf{v} = (u, v, w)$, $m$ and $\mathbf{f}$ are the densities of mass and force sources, respectively.

Ruling out the pressure from set (31), we come to the following set of equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = m, \quad i\omega\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x},$$
$$i\omega\left(\frac{\partial w}{\partial x} + \frac{1}{\tilde{c}^2}\frac{\partial u}{\partial z}\right) = \frac{1}{c_\omega^2}\left(\frac{\partial f_x}{\partial z} - \frac{1}{\tilde{c}}\frac{\partial f_z}{\partial x}\right), \tag{32}$$
$$i\omega\left(\frac{\partial w}{\partial y} + \frac{1}{\tilde{c}^2}\frac{\partial v}{\partial z}\right) = \frac{1}{\tilde{c}^2}\left(\frac{\partial f_y}{\partial z} - \frac{1}{\tilde{c}}\frac{\partial f_z}{\partial y}\right).$$

For constructing the relationships that connect the force and mass sources included in the equations of internal waves with charge and current densities, we write the Maxwell equations in the general form:

$$\operatorname{div}\mathbf{E} = 4\pi\rho, \quad \operatorname{div}\mathbf{H} = 0,$$
$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}. \tag{33}$$

As previously, we consider electromagnetic fields of two types:

$$(1)\ \mathbf{E} = (E_x(x,y,t), E_y(x,y,t)), \quad \mathbf{H} = H(x,y,t)\mathbf{e}_z; \tag{34}$$

$$(2)\ \mathbf{H} = (H_x(x,y,t), H_y(x,y,t)), \quad \mathbf{E} = E(x,y,t)\mathbf{e}_z. \tag{35}$$

The choice of the electromagnetic-field configuration of two types enables us to construct an analogy with the equations of the flows of a stratified fluid in the presence of sources:

*Correspondence 3 between the sources of internal waves and electric currents and charges generating the electromagnetic field in the form of Eqs. (34)*

| EMWs | $t$ | $c$ | $H$ | $E_x$ | $E_y$ | $j_x$ | $j_y$ | $j_z$ | $\rho$ |
|---|---|---|---|---|---|---|---|---|---|
| IWs | $z$ | $\tilde{c}$ | $\tilde{c}w$ | $-v$ | $u$ | $\frac{1}{4\pi i\omega}\left(\frac{\partial f_y}{\partial z} - \frac{\partial f_z}{\partial y}\right)$ | $-\frac{1}{4\pi i\omega}\left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)$ | 0 | $\frac{1}{4\pi i\omega}\left(\frac{\partial f_x}{\partial y} - \frac{\partial f_y}{\partial x}\right)$ |

*Correspondence 4 between the sources of internal waves and electric currents and charges generating the electromagnetic field in the form of Eqs. (35)*

| EMWs | $t$ | $c$ | $E$ | $H_x$ | $H_y$ | $j_x$ | $j_y$ | $j_z$ | $\rho$ |
|---|---|---|---|---|---|---|---|---|---|
| IWs | $z$ | $\tilde{c}$ | $\tilde{c}w$ | $v$ | $-u$ | 0 | 0 | $-\frac{\tilde{c}}{4\pi}m$ | 0 |

Writing the Maxwell equations in the component-by-component form while taking into account the chosen configuration and comparing with set (32), we write the correspondence between the dependent and independent variables of internal and electromagnetic waves.

As can be seen from correspondence 4, when constructing the correspondence between the sources for the field in the form of Eqs. (35), an additional condition arises for the mass forces in the internal-wave equations, that is, rot $\mathbf{f} = 0$, which follows from the known asymmetry of the Maxwell equations associated with the absence of magnetic charges.

In conclusion, it should be noted that the examples presented do not exhaust the possibilities provided by the methods of the theory of continuous groups for analyzing the properties of internal waves, but serve as the basis for further mutual transfer of the methods developed for solving the problems in hydrodynamics and electromagnetism.

## ACKNOWLEDGMENTS

This work was supported by the Russian Academy of Sciences, Division of Energetics, Machine Building, Mechanics, and Control Processes, project no. IV-4-12.

## REFERENCES

1. J. Lighthill, *Waves in Fluids* (Cambridge Univ. Press, Cambridge, 1978).
2. V. G. Baidulov and Yu. D. Chashechkin, Dokl. Phys. **47** (12), 888 (2002).
3. L. D. Landau and E. M. Lifshits, *Hydrodynamics* (Nauka, Moscow, 1988) [in Russian].
4. S. A. Makarov, V. I. Neklyudov, and Yu. D. Chashechkin, Izv. Akad. Nauk, Ser. Fiz. Atm. Okean, **26** (7), 744 (1990).

*Translated by V. Bukhanov*