

Modeling the Angular Momentum Control of Small Spacecraft with a Magnetic Attitude Control System

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Abstract—The problem of modeling the angular momentum control modes of small spacecraft using electromagnetic systems interacting with the Earth’s magnetic field is considered. The electromagnetic system control law has been constructed for various compositions of measurable parameters. A set of scale factors has been formed to investigate the angular momentum control mode of dynamically similar models. Based on a dynamical test stand, we have carried out experimental studies to model the angular motion dynamics of small spacecraft with a magnetic attitude control system.

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INTRODUCTION

Apart from attitude control, the magnetic attitude control systems of small spacecraft creating an external control torque are used to remove the initial rotation upon separation from the booster [1, 10, 11]. In [3] we constructed a mathematical model for the angular motion dynamics of a spacecraft and proposed an approach to determining the parameters of the control law that ensures angular momentum dissipation.

The problem of a ground-based dynamical tryout of both the attitude control and stabilization systems of small spacecraft and the control software is highly topical in designing spacecraft [8]. Test stands

equipped with a small-spacecraft weightlessness system, which allows the uniaxial spacecraft rotation with the simulation of space flight conditions to be modeled under ground-based conditions, are used to solve this problem. In this case, the ground-based test stand tryout of the dynamical spacecraft control processes suggests the existence of advanced mathematical apparatus that ensures a one-to-one correspondence between the test stand and orbital motion modes of small spacecraft. A comparative analysis of the magnetic attitude control system is based on dynamically similar modeling [5, 8]. We considered a mathematical model for the angular motion dynamics of a spacecraft with a magnetic attitude control system and a set

Table 1. Scale factors for modeling the angular motion dynamics of a spacecraft with a magnetic attitude control system

Parameter	Defining formula	Basic scale factors ($K_J = 33$, $K_B = 0.9$, $K_L = 4$)
Orbital angular velocity ω_0	$K_{\omega_0} = K_L^{1/2} K_J^{-1/2} K_B^{1/2}$	0.33
Time t	$K_t = K_{\omega_0}^{-1}$	3.0
Angle φ	$K_\varphi = 1$	1
Angular velocity ω	$K_\omega = K_{\omega_0}$	0.33
Angular momentum K	$K_K = K_J K_{\omega_0}$	11
Torque M	$K_M = K_J K_{\omega_0}^2$	3.6

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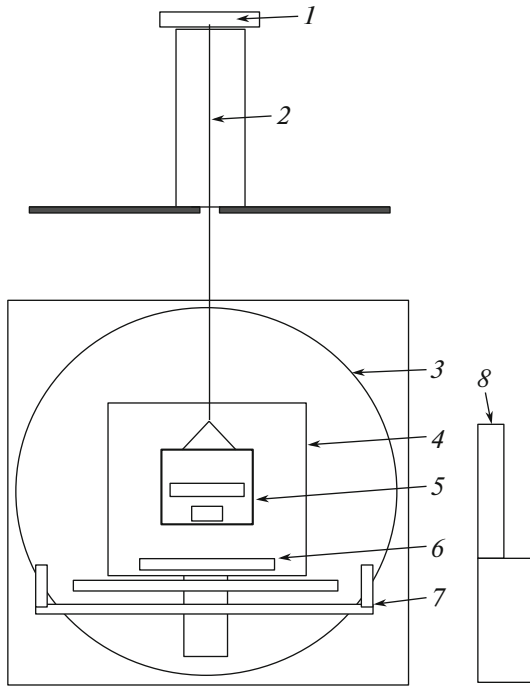


Fig. 1. Scheme of the test stand: (1) system actuator; (2) cable suspension; (3) external magnetic field simulator; (4) aerodynamic braking minimization system; (5) spacecraft mock-up with equipment and a magnetic actuator; (6) optical system; (7) turnable attached equipment system; (8) test stand remote control and control equipment.

of dimensionless parameters (similarity criteria) to investigate the characteristics of the magnetic attitude control system based on dynamically similar modeling in [5]. Based on these previously developed approaches, in this paper we present a mathematical model for the operation of a small spacecraft in the angular momentum dissipation mode verified by a test stand experiment.

MODELING THE DYNAMICAL CHARACTERISTICS OF A SPACECRAFT IN THE ANGULAR MOMENTUM DISSIPATION MODE

The angular motion of a satellite is described by the dynamical equation

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M},$$

where $\mathbf{K} = \mathbf{J} \times \boldsymbol{\omega}$ is the angular momentum of the small spacecraft, \mathbf{J} is its inertia tensor, $\boldsymbol{\omega}$ is the absolute angular velocity of the satellite in the bound reference frame, and \mathbf{M} are the torques acting on the small spacecraft.

The external control torque is $\mathbf{M} = \mathbf{L} \times \mathbf{B}$, where \mathbf{B} is the geomagnetic induction vector and \mathbf{L} is the magnetic moment of the magnetic actuators.

In the angular momentum dissipation mode the control system must produce a torque whose direction is opposite to the direction of the vector \mathbf{K} , i.e.,

$$\mathbf{L} \times \mathbf{B} = -k_K \mathbf{K},$$

where k_K is some proportionality coefficient.

Multiplying both parts of the equality vectorially by \mathbf{B} and solving it for \mathbf{L} , we will find

$$\mathbf{L} = k_K \frac{\mathbf{K} \times \mathbf{B}}{B^2}. \quad (1)$$

Here, we assumed that $\mathbf{L} \cdot \mathbf{B} = 0$, because this condition corresponds to an optimal control of \mathbf{L} with respect to \mathbf{B} in terms of energy consumption.

The condition (1) can be used to develop a control algorithm [2].

To slow down the angular velocity $\boldsymbol{\omega}$, it is necessary that the magnetic moment \mathbf{L} have a component directed oppositely with respect to the product $(\boldsymbol{\omega} \times \mathbf{B})$. The greatest efficiency will be achieved when the vectors \mathbf{L} and $(\boldsymbol{\omega} \times \mathbf{B})$ are antiparallel, i.e.,

$$\mathbf{L} = k_\omega (\boldsymbol{\omega} \times \mathbf{B}) / B^2, \quad (2)$$

where k_ω is the small-spacecraft rotation damping coefficient.

The control law for linear magnetic systems consists in forming the magnetic moment \mathbf{L} from information about the vectors \mathbf{B} and $\boldsymbol{\omega}$ [6].

To implement this algorithm, it is necessary to use information about the geomagnetic induction vector \mathbf{B} from magnetometers and about the angular velocity vector $\boldsymbol{\omega}$ of the small spacecraft from angular velocity sensors.

In the case of fast spacecraft rotation, we can write

$$\dot{\mathbf{B}} = -(\boldsymbol{\omega} \times \mathbf{B}),$$

then

$$\mathbf{L} = (k_\omega \dot{\mathbf{B}}) / B^2. \quad (3)$$

To implement this algorithm, it will suffice to use information about the geomagnetic induction vector \mathbf{B} from magnetometers.

MODELING THE ANGULAR MOMENTUM CONTROL MODES OF A SMALL SPACECRAFT WITH A MAGNETIC ACTUATOR BASED ON DYNAMICAL SIMILARITY

Our comparative analysis of the magnetic attitude control system is based on dynamically similar modeling [5–8]. For a comparative evaluation of the dynamical characteristics when investigating the magnetic orientation, it is appropriate to introduce the scale factors to scale the parameters when modeling with respect to the conditions of the actual spacecraft motion in the orbit of an Earth satellite.

Table 2. Comparison of the parameters for our modeling

Parameter	Full-scale spacecraft	Model spacecraft
Spacecraft moment of inertia J , kg m ²	20	0.6
Earth's magnetic induction B , T	4.5×10^{-5}	5×10^{-5}
Magnetic moment of magnetic coil L , A m ²	7	1.75
Spacecraft angular velocity ω deg s ⁻¹	3.0	9.0
Time t , h	2	0.67
Angular momentum K , N m s	1.75	0.16
Torque M , N m	2.0×10^{-4}	5.5×10^{-5}
Angular velocity control coefficient k_ω , N m s	6.0×10^{-3}	5.3×10^{-4}
Angular momentum control coefficient k_K , s ⁻¹	1.8×10^{-4}	5.5×10^{-4}

We present the results of our dynamically similar modeling of the angular motion of a satellite with a magnetic attitude control system when a small spacecraft with a mass of 150 kg and a moment of inertia $J = 20 \text{ kg m}^2$ moves in a polar orbit (the height of the orbit is 550 km, the orbital inclination is $i = 100^\circ$) at a magnetic moment $L = 7 \text{ A m}^2$. The angular momentum dissipation mode was modeled on a dynamical test stand for a mock-up with a moment of inertia $J = 0.6 \text{ kg m}^2$ at a magnetic moment $L = 1.75 \text{ A m}^2$. The scale factors for modeling the magnetic attitude control system are given in Table 1 [5]. The parameters for our modeling based on the similarity criteria for the full-scale and model spacecraft are compared in Table 2.

EXPERIMENTAL STUDIES OF A SMALL SPACECRAFT WITH A MAGNETIC ATTITUDE CONTROL SYSTEM ON A DYNAMICAL TEST STAND

The test stand with a simulator of the interaction (magnetic actuator) with the Earth's magnetic field made at the Moscow Aviation Institute as a result of the execution of the RFMEFI57414X0103 project within the Federal Goal-Oriented Program "Studies and Designs on Priority Directions of Development of the Science and Technology Complex of Russia for 2014–2020" has unique possibilities for physically modeling the mode of uniaxial spacecraft rotation in weightlessness. The test stand provides the modeling of the angular motion dynamics for a small spacecraft with a magnetic actuator and the spacecraft attitude control and angular momentum dissipation modes. A scheme of the test stand is presented in Fig. 1.

Experimental dynamical stand is shown in Fig. 2.

The modes of motion are tried out using a design (mass-size) mock-up of a small spacecraft equipped with the attitude control and stabilization system under study.

The magnitude and dynamics of the change in magnetic field under actual spacecraft flight conditions differ from the model characteristics. This requires scaling the characteristics of the spacecraft motion dynamics based on dynamically similar modeling.

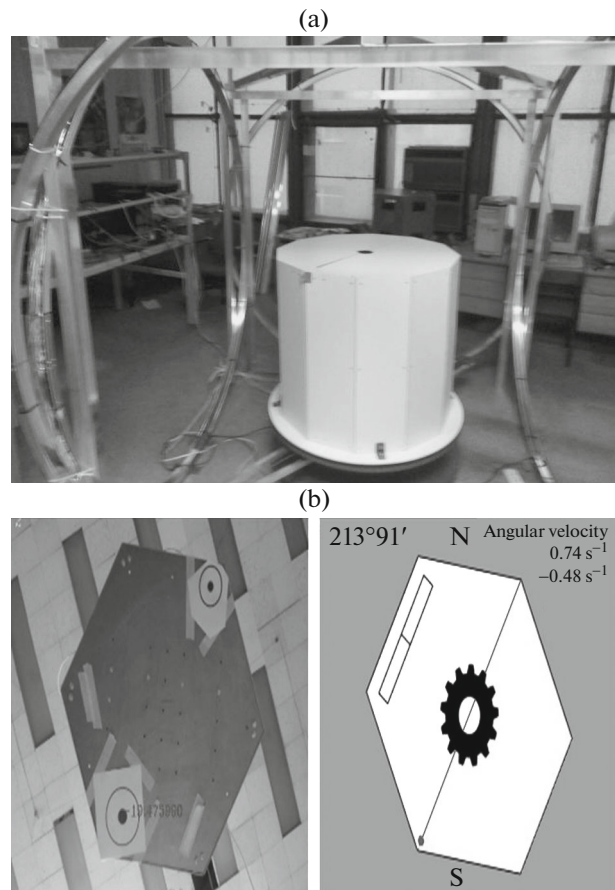


Fig. 2. Experimental dynamical stand: (a) an external view of the test stand for modeling the motion control modes of a small spacecraft with a magnetic actuator; (b) spacecraft mock-up rotation visualization system.

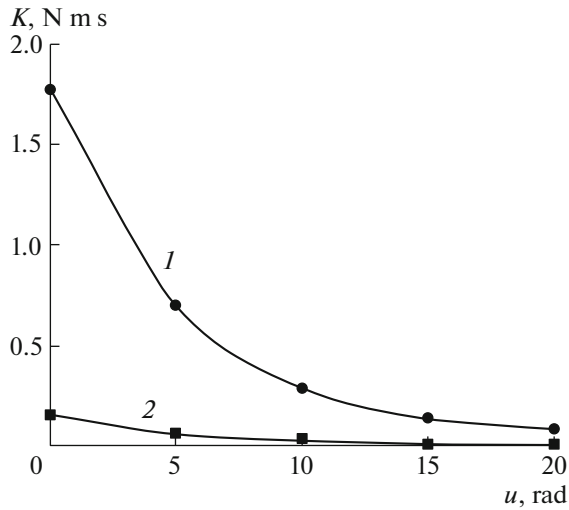


Fig. 3. Change in the angular momentum for the full-scale (1) and model (2) spacecraft.

Figure 3 presents the results of our numerical simulations of the angular momentum dissipation mode using a magnetic actuator for a full-scale small spacecraft when moving in a polar orbit (the height of the orbit is 550 km, the orbital inclination is $i = 100^\circ$) and for a model spacecraft on the dynamical test stand. Figure 3 compares the changes in angular momentum for the full-scale and dynamically similar model spacecraft with latitude.

A comparison of the computational results and the physical modeling (investigation tests on a full-scale object) of the actuator operation confirms that the developed mathematical models are correct and that they are suitable for scaling functional mathematical models.

The developed mathematical models and the techniques for calculating and scaling the magnetic actuators were confirmed by the correspondence between the calculated parameters and experimental tryout results. This allows them to be used for a maximally broad spectrum of design tasks.

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