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MECHANICS

Modeling the Angular Motion Dynamics of Spacecraft with a Magnetic Attitude Control System Based on Experimental Studies and Dynamic Similarity

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Abstract—The problem of spacecraft attitude control using electromagnetic systems interacting with the Earth's magnetic field is considered. A set of dimensionless parameters has been formed to investigate the spacecraft orientation regimes based on dynamically similar models. The results of experimental studies of small spacecraft with a magnetic attitude control system can be extrapolated to the in-orbit spacecraft motion control regimes by using the methods of the dimensional and similarity theory.

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INTRODUCTION

Apart from attitude control, magnetic spacecraft attitude control systems creating an external control torque are used for removing the initial rotation upon separation from the booster, the primary orientation and reorientation in space (observation, communication, and orbit correction sessions), damping the oscillations, and eliminating the "saturation" regimes for rotor orientation. Such engineering solutions gain popularity for a rapidly growing number of small spacecraft being created with the participation of students and staff members of universities [1], because, in contrast to thrusters, they require no propellant consumption. As a consequence, their service life places no constraints on the spacecraft lifetime. They are as good as gravitational systems in reliability and service life [2], but surpass them in efficiency. In addition, they mesh perfectly with flywheels or powered gyroscopes in combined attitude control and stabilization systems.

Studies in the area of modeling the magnetic attitude control systems of small spacecraft are aimed at an experimental confirmation of the theoretical results on full-scale mock-ups and the development of methods for applying dynamically similar models.

STATEMENT OF THE PROBLEM

Dynamically similar models are needed in practice for the test bench ground-based tryout of spacecraft control algorithms. Several types of restrictions imposed by the test bench equipment when investigating the spacecraft motion dynamics can be identified: the mass-size restrictions and the spacecraft weightlessness restrictions. The sizes, mass, and corresponding moments of inertia of the spacecraft being designed can be beyond the capabilities of the ground-based test bench equipment accessible to higher education institutions and small organizations and groups—the designers. The solution can consist in obtaining the necessary data for the design tryout on a reduced small spacecraft mock-up. In addition, for small, mock-up designs the test bench implementation of such characteristics of the external, artificial magnetic field, which together with the magnetic attitude control system will allow one to reduce the duration of bench experiments and/or to bring the conditions close to the orbital ones, is technically feasible. An equally important aspect of the test bench tryout is a similarity in controlling the motion of a weightless spacecraft, which is easier to implement, for example, for uniaxial weightlessness followed by the modeling of triaxial in-orbit spacecraft control.

To construct an algorithm for controlling a spacecraft with magnetic actuators, it is necessary to determine the relationship of the magnetic reference frame to the orbital reference frame, which is specified by the orbital inclination of the satellite to the equatorial plane i and the argument of latitude u. The orientation

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of the bound reference frame relative to the magnetic reference frame is specified by the Euler angles α, β, and γ. It is also necessary to formulate the control law for linear magnetic systems. It consists in forming a magnetic moment $\mathbf{L}$ based on information about the geomagnetic induction vector $\mathbf{B}$ and the angular velocity vector $\boldsymbol{\omega}$ of the satellite [3–6].

MATHEMATICAL MODEL FOR THE ANGULAR MOTION DYNAMICS OF A SPACECRAFT WITH A MAGNETIC ATTITUDE CONTROL SYSTEM

In order to obtain a set of dimensionless parameters (similarity criteria), it is necessary to analyze the influence of the control being formed on the dynamics of the entire system using its mathematical model of motion constructed by applying the laws of change of the angular momentum (dynamical Euler equations) and the kinematic equations.

The angular motion of a satellite is described by the dynamical equation [3]

$$\frac{d\mathbf{K}}{dt} + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}, \tag{1}$$

where $\mathbf{K} = \mathbf{J} \times \boldsymbol{\omega}$ is the angular momentum of the small spacecraft, $\mathbf{J}$ is its inertia tensor, $\boldsymbol{\omega}$ is the absolute angular velocity of the satellite in the bound reference frame, and $\mathbf{M}$ are the torques acting on the spacecraft.

The external control torque is

$$\mathbf{M} = \mathbf{L} \times \mathbf{B}, \tag{2}$$

where $\mathbf{B}$ is the geomagnetic induction vector.

The control magnetic moment $\mathbf{L}$ can be represented as [4]

$$\mathbf{L} = -[k_{\omega}(\mathbf{B} \times \boldsymbol{\omega})]/B^2 \tag{3}$$

where B is the magnitude of the geomagnetic induction vector and k_{ω} is the proportionality coefficient.

The condition (3) can be used for the development of spacecraft control algorithms [5]. A block diagram of the spacecraft attitude control loop and the control law for linear magnetic systems based on information about the geomagnetic induction vector $\mathbf{B}$ and the vector $\boldsymbol{\omega}$ are given in [6].

MODELING THE CONTROL REGIMES FOR A SPACECRAFT WITH A MAGNETIC ATTITUDE CONTROL SYSTEM BASED ON DYNAMIC SIMILARITY

Our comparative analysis of the magnetic attitude control system is based on dynamically similar modeling [7]. Taking the magnetic induction B_0, the moment of inertia of the spacecraft J, and the orbital angular velocity of the spacecraft ω_0 as the basic parameters, we can obtain the following similarity criteria for the spacecraft angular momentum dissipation process:

$$\Pi_1 = \omega_0 t, \quad \Pi_2 = \varphi_i, \quad \Pi_3 = \frac{\omega_i}{\omega_0}, \quad \Pi_4 = \frac{K_i}{J_i \omega_0}, \quad \Pi_5 = \frac{L_i B_0}{J \omega_0^2}, \quad \Pi_6 = \frac{M_i}{J \omega_0^2}, \tag{4}$$

where t is the characteristic time of spacecraft motion; $\varphi_i(\alpha, \beta, \gamma)$ are the Euler rotation angles of the spacecraft; $\omega_i(\omega_x, \omega_y, \omega_z)$ are the components of the spacecraft angular velocity vector; $K_i(K_x, K_y, K_z)$ are the components of the spacecraft angular momentum; $J_i(J_x, J_y, J_z)$ are the components of the spacecraft inertia tensor; $L_i(L_x, L_y, L_z)$ are the magnetic moment components for the current coils; $M_i(M_x, M_y, M_z)$ are the torque components.

Equalities (4) specify the similarity conditions when modeling the magnetic attitude control system under ground-based conditions. For the modeled dynamical process to be similar to the actual one, the dimensionless combinations of parameters for the full-scale and model objects must be equal.

For a comparative evaluation of the dynamical characteristics when investigating the magnetic orientation, it is appropriate to introduce the scale factors to scale the parameters when modeling with respect to the conditions of the actual spacecraft motion in the orbit of an Earth satellite.

As the initial modeling scales we take the following scales of independent dimensions: the moment of inertia K_J, the magnetic induction K_B, and the orbital angular velocity K_{ω_0} (because the frequency of geomagnetic field variations as the spacecraft moves in its orbit is $\omega_B = 2\omega_0$ [3]).

Based on these scale factors, we can construct the scale factors to scale other dynamical parameters for the actual and model conditions. The derived modeling scales are expressed via the initial ones using the similarity conditions (4):

$$\Pi_1 = \omega_0 t, \quad \omega_{0\mathrm{f}} t_{\mathrm{f}} = \omega_{0\mathrm{m}} t_{\mathrm{m}}, \quad K_{\omega_0} K_t = 1, \quad K_t = K_{\omega_0}^{-1};$$

$$\Pi_2 = \varphi_i, \quad \varphi_{i\mathrm{f}} = \varphi_{i\mathrm{m}}, \quad K_{\varphi} = 1, \quad K_{\varphi} = 1;$$

$$\Pi_3 = \frac{\omega_i}{\omega_0}, \quad \frac{\omega_{i\mathrm{f}}}{\omega_{0\mathrm{f}}} = \frac{\omega_{i\mathrm{m}}}{\omega_{0\mathrm{m}}}, \quad \frac{K_{\omega}}{K_{\omega_0}} = 1, \quad K_{\omega} = K_{\omega_0};$$

$$\Pi_4 = \frac{K_i}{J_i \omega_0}, \quad \frac{K_{i\mathrm{f}}}{J_{\mathrm{f}} \omega_{0\mathrm{f}}} = \frac{K_{i\mathrm{m}}}{J_{\mathrm{m}} \omega_{0\mathrm{m}}}, \quad \frac{K_K}{K_J K_{\omega_0}} = 1,$$

$$K_K = K_J K_{\omega_0};$$

$$\Pi_5 = \frac{L_i B_0}{J \omega_0^2}, \quad \frac{L_{i\mathrm{f}} B_{0\mathrm{f}}}{J_{\mathrm{f}} \omega_{0\mathrm{f}}^2} = \frac{L_{i\mathrm{m}} B_{0\mathrm{m}}}{J_{\mathrm{m}} \omega_{0\mathrm{m}}^2}, \quad \frac{K_L K_B}{K_J K_{\omega_0}^2} = 1,$$

$$K_L = K_J K_{\omega_0}^2 K_B^{-1};$$

Table 1. Scale factors for modeling the angular motion dynamics of a spacecraft with a magnetic attitude control system

Parameter	Defining formula	Basic scale factors	Derived scale factors
Moment of inertia	K_J	33	
Magnetic induction	K_B	0.9	
Orbital angular velocity	$K_{\omega 0}$	0.33	
Time	$K_t = K_{\omega_0}^{-1}$		3
Angle	$K_\varphi = 1$		1
Angular velocity	$K_\omega = K_{\omega_0}$		0.33
Angular momentum	$K_K = K_J K_{\omega_0}$		11.0
Magnetic moment	$K_L = K_J K_{\omega_0}^2 K_B^{-1}$		4.0
Torque	$K_M = K_J K_{\omega_0}^2$		3.6

$$\Pi_6 = \frac{M_i}{J\omega_0^2}, \quad \frac{M_{if}}{J_\mathrm{f}\omega_{0\mathrm{f}}^2} = \frac{M_{i\mathrm{m}}}{J_\mathrm{m}\omega_{0\mathrm{m}}^2}, \quad \frac{K_M}{K_J K_{\omega_0}^2} = 1,$$

$$K_M = K_J K_{\omega_0}^2.$$

The indices “m” and “f” denote the parameters of the model and the full-scale spacecraft.

The scale factor K_J characterizes the relationship between the moments of inertia for the full-scale and model spacecraft.

The scale factor K_B simulates the magnetic induction of the geomagnetic field under the model conditions.

The scale factor K_{ω_0} defines the dynamism relationship for the processes under the actual and model conditions (determined by the dynamics of magnetic induction variations).

The scale factor K_t characterizes the time of motion of the full-scale and model spacecraft.

The scale factor K_φ models the spacecraft rotation angles.

The scale factor K_ω characterizes the spacecraft angular velocity.

The scale factor K_K models the angular momentum of the full-scale and model spacecraft.

The scale factor K_L models the relationship between the magnetic moments of the current coils of the full-scale and model spacecraft.

The scale factor K_M models the control regimes for the control torque.

The modeling of the spacecraft control regimes should be considered separately as the test bench ground-based modeling and the modeling of the in-orbit spacecraft motion. The test bench on which a weightless mock-up of a small spacecraft with a magnetic actuator is mounted has been designed and made at the Moscow Aviation Institute. At a full-scale spacecraft mass of 150 kg the mockup mass does not exceed 20 kg, while the moments of inertia are, respectively, 20 and 0.6 kg m^2. On the test bench it is possible to specify the spacecraft rotation along the suspension axis, to produce the current-coil magnetic moment, and to determine the angles and angular velocity of the spacecraft mock-up by the noncontact optical measurement method. Knowing the Earth’s magnetic field at the place of the bench experiment and the magnetic coil parameters, the mathematical model for the angular motion of a spacecraft can be verified. Various spacecraft control regimes can be tried out on the test bench. The in-orbit dynamics of the spatial angular motion of a spacecraft with a magnetic attitude control system is subsequently modeled using the proposed similarity criteria and modeling scale factors.

The scale factors for modeling the magnetic attitude control system are given in Table 1.

The proposed criteria, scale factors, and modeling methods can be used when the geometric scaling of spacecraft under test bench tryout conditions for mock-ups with mass-size parameters differing from the full-scale spacecraft is needed.

CONCLUSIONS

We have considered a mathematical model for the angular motion dynamics of a spacecraft with a magnetic attitude control system. A set of dimensionless parameters (similarity criteria) has been formed to investigate the characteristics of the magnetic attitude control system based on dynamically similar modeling. We have proposed the approaches to an experimental (test bench) tryout of magnetic actuators based on the application of dynamically similar models.

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