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GEOPHYSICS

Excitation of Under-Ice Seiches of a Sea Port of the Sea of Okhotsk

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Abstract—Multiannual observations (2009–2017) by means of autonomous seiche recorders have been conducted for marine seiches with one second discreteness in the boot basin for harbor fleet of the settlement of Okhotskoe (Sakhalin Island) at the depth of ~2 m beneath ice. The spectral analysis of the data showed the presence of several significant peaks at periods from 2 to 15 s for the moments of strong swell at the sea. These peaks are caused by wave processes, which are generated by nonlinear transformation of the under-ice rippling. Numerical modeling of the reaction of the dynamic system (water area described by Duffing equation) is carried out depending on the parameters of the experimental observations. It is shown (including the Poincaré map) that the amplitude of the external influence greatly affects the transition of the system to chaos.

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It is known that shallow water areas exhibit nonlinear effects during propagation and transformation of swell waves. Nonlinearities can locally be weak or can be characterized by significant cross-spectral energy transformation, if the shallow area is wider in comparison with the distance of the nonlinear interaction [1]. It is also known that nonlinear Korteweg–de Vries equations for finite amplitude waves in shallow areas above the flat seafloor exhibit significant transfer of interspectral energies because of the almost resonance square interactions [2].

Numerous studies are known on free liquid oscillations in a limited basin (seiches), e.g., [3–5]. Significant attention has been paid to study of the main modes of eigen bay oscillations [6] and relations of changes in the period of main oscillation mode of the water area with a value of the external excitation force [7]. Nonetheless, studies [8] showed that the seiche periods beneath the ice are distinct from those of open water seiches.

The results of our observations below the fast ice of the Sea of Okhotsk near the settlement of Okhotskoe (Sakhalin Island) showed that under-ice swell can transfer energy to short seiches with the formation of a mode structure depending on the relief and the presence of walls. There were scenarios when the energetic spectra of the level oscillation within 2–15 s exhibit several significant peaks.

The ice cover can largely affect the seiche movements, suppress them, and hamper their generation. Publication [9] provides the critical amplitude of marine waves of different periods: 9 and 28 cm for waves and swell with periods of 5–10 and 15 s, respectively. Thus, seiches of large amplitude can effectively destroy the ice cover and facilitate the formation of polynyas [4] and thus can be dangerous for staff working on ice. This explains the interest in study of under-ice seiches.

Study of the behavior of chaotic marine dynamic systems (in particular, our water basin) is necessary for practical targets and those effects that can result from the complex dynamics, e.g., the solution of the task of dynamics of a vessel or an oil platform in the presence of a seiche [10]. Thus, nonlinear analysis allows comprehensive understanding of the situation and elaboration of conditions for avoiding catastrophe.

Since 2009, the Institute of Marine Geology and Geophysics, Far East Branch, Russian Academy of Sciences, has been studying seiches in the coastal zone near the settlement of Okhotskoe (southeastern coast of Sakhalin Island) using AWR autonomous seiche recorders, which record the data with one second discreteness. Figure 1 shows the area of the whole-year seiche observations.

The observation period yielded a qualitative time series of sea level oscillations in the summer and winter periods. The absence of noise (significant wind waves and swell) in the records indicate the period of ice-covered sea. The results of further spectral analysis of series were compared with model calculations.

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Spectral analysis revealed periodic energy rises in the level oscillations in the range of periods from 2 to 15 s (Fig. 2). By value, the energetic peaks significantly exceed the confidence interval; they also do not occur on divisible periods and, thus, cannot be harmonics of the main period. A similar pattern is observed in [1].

The analysis of observation time series of different years showed that this spectrum is typical only of the ice-covered sea. The observed energy peaks corresponding to the level fluctuations in the range of periods from 2 to 15 seconds are manifested only for situations when the sea swell of large amplitude (Fig.2, the spectra of 3, 4, 6). In the absence of ice (e.g., Fig. 2, spectra 1 and 2), for cases where the fast ice has not yet formed or with a weak swell under the ice (Fig.2, spectrum 5) there are no peaks in the spectra.

We suggested that with the arrival of waves swell with great energy to the place of sensors installation in the boot basin of the settlement of Okhotskoe, as a result of the nonlinear transformation of the excitation of seiches of several modes that are shorter than the swell. To determine their period, we used a mathematical apparatus [8], according to which the periods of modes are determined by equation for seiches under fast ice:

$$T_n = \frac{2(L - 2\sqrt{\tilde{T}/g\rho_w})}{n\sqrt{gH_{\text{red}}(\omega_n)}}, \qquad (1)$$

where $n = 1, 2, 3, \ldots$ is the oscillation mode, L is the length of the basin, ρ_w is the density of seawater (~1025 kg m^{-2}), ω_n is the frequency of the n mode under the ice, $\tilde{T} = \tilde{k}h$ is the coefficient of proportionality depending on the physical properties of the ice, and $\tilde{k}$ (106 N/m^2) is the coefficient of ice compression (extension). Equation (1) uses the reduced depth of the basin:

$$H_{\text{red}} = H[1 - F(aH)], \qquad (2)$$

where H is the depth of the basin, $F(\alpha H)$ is the function depending on the depth of the basin and parameter $\alpha = \sqrt{\omega/2A}$, which, in turn, depends on the wave frequency. Here, A (10^{-2} m^2/s) is the coefficient of vertical turbulent exchange.

Using equations (1), (2), we calculated the periods of modes of longitudinal seiches in the boot basin of the settlement of Okhotskoe for various depths, which varies with the tide, and the thickness of fast ice. The results showed that the measured oscillation periods are most similar to calculated periods for the depth of 2 m and the ice thickness of 2 m, which is typical of the boot basin during winter. The calculated modes of transverse seiches of the boot basin showed that their periods are significantly distinct from the measured ones and thus they were removed from consideration.

Chaotic oscillations can occur in nonlinear dynamic systems (in particular, our water basin). Let the dynamics of the nonlinear system be represented by the Duffing difference equation, which describes the second order system with nonlinear oscillations and external periodical excitation [11]:

$$\ddot{x} + k\dot{x} + \omega_0^2 x + \alpha x^3 = F\cos\omega t, \qquad (3)$$

where F and ω are the amplitudes of frequencies of the external periodic excitation (swell, period T), ω_0 is the frequency proper of the oscillator (boot basin, period T_0), k is the coefficient of attenuation, and α is the coefficient of nonlinearity.

Using (3), we conducted numerical modeling of this system (interaction of the ice-covered water basin with arriving waves) with numerical solution of the equation in our program PUAN [12], which significantly accelerated the calculation of the phase portrait and the Poincaré map for larger amount of points. The phase portraits, modes of oscillations, and Poincaré maps were calculated for measured periods of swell waves of various amplitudes, coefficients of attenuation and nonlinearity, and seiche periods. The scenario for $F = 20$, $T = 15.3$, $T_0 = 15.3$, $k = 0.05$, and $\alpha = 6.6$ is shown in Fig. 3.

The study of the reaction of the dynamic system described by the Duffing equation, depending on the experimental parameters of the equation, showed that the amplitude of the external excitation has the greatest excitation on chaos transfer. The modeling results showed that, with increasing $F > 0.5$ for $T = 15.3$, $T_0 = 15.3$ (typical periods of swell and the first mode of seiche oscillations), the system transits from periodic to chaotic, which is evident from the Poincaré map.

Under excitation of seiches of shorter periods, in comparison with the period of the arriving swell, the system also exhibits chaotic oscillations. The image of the Poincaré map significantly depends on the period and amplitude of the external wave, and, to a lesser degree, the attenuation coefficients and nonlinearity affect.

Thus, the swell penetrating farther beneath the ice facilitate generation of chaotic oscillations on the periods of eigen oscillations of the water basin boot basin of the settlement of Okhotskoe. Under decreasing swell amplitude, the chaos in oscillations decreases and transits to periodic oscillations.

Thus, it is established that a strong swell penetrates the ice over a greater distance and, due to nonlinear effects, excites under-ice seiches in the boot basin of the fishing plant of the settlement of Okhotskoe and generates several modes of seiches on nondivisible periods.

Taking into account the periods of under-ice waves relative to the open water and conclusions of the model for eigen oscillations in the water area of the boot basin, the analysis showed that seiches in this area are characterized by a significantly shorter (15.3 s) period of the first mode in comparison with model calculation (60.9 s for the first mode). At the same time, there is a good agreement of the periods between the fourth mode of the calculated seiches and the first mode of the observed seiches. The modes of higher orders correspond only to each four from those calculated, which has no explanation yet.

The peak of the energetic spectrum of a 15.3-s period corresponds to the period of the arriving swell, and it is likely that the swell excites the seiche with the same period, because the model calculation has a mode of oscillation with a closer period.

The model calculation showed that the generation of the longitudinal seiche mode is described well by the equation of V.N. Zyryanov. The lowermost seiche mode corresponds to the swell period. The calculation results for the periods of transverse oscillations in the boot basin were far from those observed.

We modeled the reaction of the water mass of the boot basin water area: a dynamic system described by the Duffing equation depending on the experimental parameters of the equation, in particular, the periods of external force and eigen periods of oscillations of the water basin. It is shown (including using the Poincaré map) that an increase in the amplitude of an external excitation of more than 0.5 greatly affects the transition of the system to chaotic oscillations.

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