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OPTICAL MODELS AND DATABASES

Determination of the Average Latitudinal Temperature by Linear Transformation of Astronomical Insolation

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Abstract—Time series of multiyear average temperature, obtained at 927 Northern Hemisphere weather stations for the period 1955–2014, are compared with the known data on astronomical insolation for the same time interval and location. It is shown that the annually averaged astronomical insolation as a function of latitude, subject to linear transformation, should be considered as the average latitudinal temperature. This result is justified using a regression of the compared data and by grouping meteorological stations. Our estimates of the increment of the average latitudinal temperature in the period 1985–2014 as compared with those in 1955–1984, as well as the contribution to the temperature variations from the components determined by astronomical insolation and by stochastic processes in the geosphere, do not contradict the well-known estimates, which verifies the introduced linear transformation of astronomical insolation.

Keywords: astronomical insolation, surface temperature, latitudinally average temperature

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INTRODUCTION

A current trend in climatology remains the study of the transformation of the climate system by solar energy, initiating hydrometeorological processes. This is important in regard to the need to identify the causes and nature of climate change with a view to searching for the opportunities for the population to adapt to them and implement preventive measures.

To date, there are few climatic phenomena for which theoretical support has been found and some generalizations have been obtained. Global and hemispheric average temperatures are accepted as an integral indicator of global climate change [1, 2]. These characteristics are used to study environmental responses to climate trends.

Variations in global and hemispheric temperatures are analyzed on a regular basis by a few scientific groups: the NASA Goddard Institute for Space Studies (GISS) [3, 4], the NOAA National Climatic Data Center (NCDC) [5, 6], and the Met Office Hadley Centre and University of East Anglia (HadCRUT, CRUTEM) [7, 8]. The estimates, provided by these world meteorological centers, are based on the same measurements of surface temperature, accumulated in Global Historical Climatology Network (GHCN), and differ by the processing and analytical techniques used.

The temperature field is continuous; however, it is represented by data from separate weather stations. The calculations of the hemispheric and global average temperatures can be significantly complicated by the inhomogeneity of observations, in particular due to changes in measurement instrumentation and techniques, changes in the methods for calculating the average temperatures, meteorological periods of measurements, environmental conditions at station, and changes in location of weather stations. Calculations of hemispheric and global average temperatures should take into account the density and configuration of the network of weather stations [4, 9]; the uncertainty becomes the largest in regions with a small number of stations. Spatial interpolation of temperature into the nodes of a regular grid was applied to increase the reliability of estimates of distribution of the average temperature over the Earth's surface [10]. The method of climatic anomalies [11, 12], objective analysis [13], and the method of a reference station [14] are often used. In interpolation, the data fields are centered using the time series of anomalies, thus avoiding somewhat the problem of nonstationary real fields of meteorological elements [15].

The method of climatic anomalies is based on simple nonweighted averaging of anomalies from separate stations in each grid cell 5° latitude × 5° longitude in size. The anomalies are calculated as deviations of initial data from a norm, determined for the period 1961–1990 [11, 16]. This method is applied in compiling the CRUTEM database. The regions without observations are assumed to have a temperature anomaly equaling the hemispheric average anomaly. The NCDC data on anomalies in surface temperature are averaged on a similar grid, but the interval 1901–2000 is used as a basis period [6].

In the GISS model the hemispheric average temperatures for land are obtained by averaging data on all anomalies from ground-based stations in 80 cells with nearly equal areas [12], with the same steps applying for the ocean surface. The interval 1951–1980 is used as a basis period. Blank grid cells are filled using linear spatial interpolation of anomalies from one or a few stations located within 1200 km because the temperature anomalies are usually large-scale, especially at middle and high latitudes [4].

Interpolation according to the method of objective analysis [13] is based on linear Kolmogorov–Wiener interpolation formulas. Data are interpolated into points of a regular grid, specified in Cartesian coordinates, according to the condition of a minimum standard deviation by varying the set of weighting coefficients characterizing the statistical structure of the field.

Results of systematic instrumental observations of climate elements are available for a short period of time. However, it is known from experience that the fundamental basis for climate formation is the geographic distribution of fluxes of incoming solar radiation. This phenomenology can be taken as a basis for calculating global and hemispheric characteristics of the temperature field, as well as average latitudinal temperatures [17], which can then be used to calculate the hemispheric and global average temperatures.

H. W. Dove [18] was first to calculate the average latitudinal air temperatures, average temperatures for hemispheres and the entire globe. For this, he used about 1100 stations, though certain regions on the globe remained unaccounted for. A classical method for calculating the average latitudinal temperature through averaging of isotherms determined from maps at points located at the corresponding latitude [19] gives very approximate estimates, especially in a mountain terrain and over ocean surfaces. In work [20] it is suggested to calculate the average latitudinal values from data on temperatures at the nodes of a regular grid on a 1000-hPa surface to eliminate the effect of the underlying surface.

A promising approach is described in [21–24], where the authors compared solar radiation incident on the Earth and surface temperature. Insolation for the interval from 3000 BC to 3000 AD was calculated using astronomical ephemerides [24], which are mainly determined by the structure of the Solar System. The authors of work [23] carried out a piecewise polynomial approximation of astronomical insolation in different latitude intervals, and then performed a regression with data on surface temperature. They note that such a modification of insolation made it possible to reflect the effect of the underlying surface, greenhouse effect, and modes of heat exchange. On the whole, for the Earth the measured surface temperature and modified insolation coincided [23], their correlation coefficient as a function of latitude from 90° N to 90° S was 0.942. As a result of their transformations, the temperature components, determined by astronomical insolation and caused by geospheric stochasticity, turn out to be mixed. It is noteworthy that a piecewise polynomial approximation of insolation by segments of different-order polynomials introduces some arbitrariness and renders the result devoid of the required generality and complicates the construction of computational algorithms.

The methods used to calculate the global, hemispheric, and average latitudinal temperatures do not ensure a unique solution; they have a number of assumptions and give similar, but not identical, results. Nonuniform distribution of stations prohibits obtaining correct unambiguous estimates on data averaging. Uncertainty associated with completeness and with method of accounting for the temperature data over seas and oceans still awaits its resolution. Moreover, it is unclear how position of the interval on the time axis and its length influence the result.

In this work, the astronomical insolation (input effect) and surface temperature (output result) are compared with the purpose of determining their functional dependence which would support the empirical views on the climatic processes considered. It is expedient to derive an analytical formula for the relationship between insolation and temperature, considered to be direct and logical [25], and to suggest a new approach to calculating the average latitudinal average temperature as a basis for calculating the hemispheric and global average temperatures. In this work, we will show the validity of our statement (*): the annual average astronomical insolation as a function of latitude, subject to the linear transformation, should be considered as an average value of multiyear average temperatures at weather stations at the corresponding latitudes for the time period considered.

This statement is justified by comparing the theoretical distribution of astronomical insolation with measurements of surface temperature; regression and grouping of weather stations are applied.

DATA DESCRIPTION

Average values of astronomical insolation for tropical year are presented in a database at the website [26]. The calculations were performed using the method described in [27]; its author used refined distances from the Earth to the Sun, declinations, and ecliptic longitude of the Sun taken from the NASA DE-406 model [24]. The calculated values of insolation were normalized to 1412 W/m^2, which corresponds to the solar constant at the perigee point and presumably does not change.

The posed problem was solved by averaging the annual average insolation from the initial database over time within the 60-year period 1955–2014 for latitudes from 0 to 90° N with a step of 5°. Then, these values were interpolated along the latitude using a cubic spline so that we could calculate the annual average insolation for the location of each weather station. Two consecutive 30-year periods were used to estimate the temperature increments.

We denote the insolation values through s_n, where $n = 1, 2, ..., N$ is the conventional weather station number, and $N = 927$ is the quantity of weather stations. The astronomical insolation obtained is a theoretical calculated value and, as defined, depends on latitude; it can be calculated for any location of the observation point.

The time series of monthly average surface temperature, measured over 1955–2014 at 927 Northern hemisphere weather stations, are provided by the University of East Anglia [7]. We selected stations at which the total period when there were no data did not exceed 3% of the number of monthly averages considered. The multiyear average temperature t_n at each weather station was obtained by averaging the monthly average values from March 1955 to February 2017 in order to correspond to intervals of averaging for insolation over the tropical year.

TRANSFORMATION OF INSOLATION INTO TEMPERATURE

We chose the linear transformation for representing the relation between estimates of astronomical insolation s_n and surface temperature $\hat{t}_n$ in the form

$$\hat{t}_n = As_n - B, \tag{1}$$

where the unknown constants A and B are to be determined.

The annual average insolation s_n in (1) is, in its nature, a function of latitude. Since A and B are constants, the $\hat{t}_n$ value also depends only on latitude. Since the observed temperature is in degrees Celsius, the dimensions of the quantities $\hat{t}_n$, A, and B are the same, considering that s_n is a normalized dimensionless parameter.

In accordance with the statement (*), the physical meaning of $\hat{t}_n$ should be defined as the arithmetic mean for long-term average temperatures t_n. We will apply the least squares method for this. The theoretical annual average astronomical insolation s_n will be used as a regression curve for temperature observations at weather stations with the corresponding latitudinal locations.

Natural and weighted data were used to determine the effect the nonuniform distribution of weather stations over the Earth's surface has on the $\hat{t}_n$ estimate; we implemented few variants of minimizing the standard deviation:

$$\begin{aligned} \delta &= \left(\frac{1}{N}\sum_n (t_n - \hat{t}_n)^2\right)^{1/2}, \\ \delta_g &= \left(\frac{1}{G}\sum_n (t_n - \hat{t}_n)^2 / g_n\right)^{1/2}. \end{aligned} \tag{2}$$

Here, G is the number of cells of the rectangular grid; g_n is the weighting function defined as the number of weather stations with any number n within any cell of this grid. The g_n value is constant within the cell.

We will consider certain properties of this weighting. If, e.g., each cell contains the same number of weather stations, even with different t_n, the weighting function g_n for all n inside the cell will be equal to a constant, and $G \times g_n = N$, i.e., the two formulas in (2) will become identical. If the size of the cell decreases, both formulas in (2) will also coincide in the limit. A cell is disregarded in the calculations if none of the weather stations falls within it. This weighting procedure inside a cell leaves the relationships between temperatures at different weather stations unchanged and adjusts the contribution of each cell in accordance with the number of stations inside it.

Different sizes of grid cell G were used to minimize δ_g (2). Cell size 5° × 5° was usually used to estimate global quantities, and the size 2.5° × 2.5° is more efficient for regional calculations. The A and B values in (1) determined for these grids, as well as δ are given in Tables 1 and 2; the weighted moments δ_g were much smaller than δ, but are intermediate and not presented.

Table 1. Transformation of insolation using weights (2.5° × 2.5° grid)

Interval, years	A	B	min(δ), °C	μ, °C	R^2, %
1955–2014	227.654	39.673	4.18	0.096 ± 0.269	84.6
1955–1984	229.538	40.475	4.21	0.098 ± 0.272	84.6
1985–2014	225.769	38.872	4.15	0.094 ± 0.267	84.6

In Tables 1–3 the coefficients A and B are calculated for astronomical insolation, normalized by the solar constant at the perigee point; for the average differences μ the two-sided intervals are presented at the 0.95 confidence probability.

Table 2. Transformation of insolation using weights (5° × 5° grid)

Interval, years	A	B	min(δ), °C	μ, °C	R^2, %
1955–2014	228.243	39.993	4.19	0.287 ± 0.270	84.6
1955–1984	230.136	40.796	4.22	0.288 ± 0.272	84.7
1985–2014	226.351	39.190	4.16	0.285 ± 0.268	84.5

Table 3. Transformation of insolation without weights

Interval, years	A	B	min(δ), °C	μ, °C	R^2, %
1955–2014	225.030	39.005	4.18	0.000 ± 0.270	84.6
1955–1984	226.990	39.821	4.21	0.000 ± 0.272	84.7
1985–2014	223.069	38.189	4.15	0.000 ± 0.268	84.6

Table 4. Increment of latitudinal average temperature (°C) at weather stations in 1985–2014 as compared to 1955–1984. Entries in the first row are calculated without weights

Cell size, deg	$\langle \widehat{te}_n - \widehat{to}_n \rangle$	$\min(\widehat{te}_n - \widehat{to}_n)$	$\max(\widehat{te}_n - \widehat{to}_n)$
–	0.775	0.479	1.135
2.5 × 2.5	0.779	0.495	1.125
5 × 5	0.779	0.493	1.126

We determined the average differences between long-term average temperatures at weather stations and transformed insolation:

$$\mu = \frac{1}{N}\sum_n (t_n - \hat{t}_n). \quad (3)$$

For these averages (3), we calculated the confidence interval at 0.95 confidence probability (Tables 1–3).

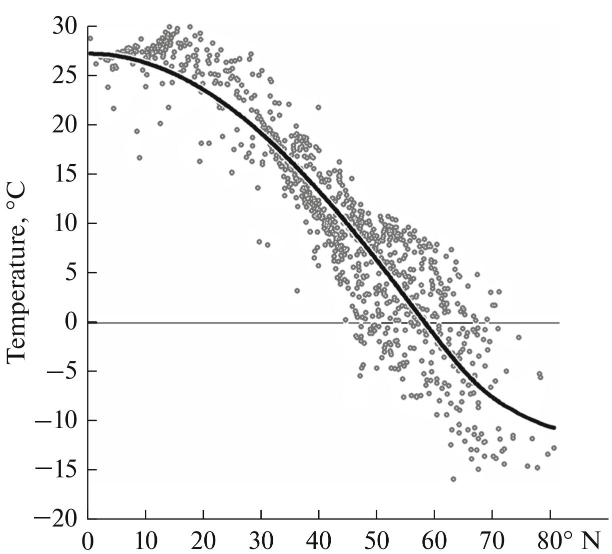

Fig. 1. Estimate of the latitudinal average temperature. Circles show the distribution of the long-term average temperature t_n over latitude at 927 Northern hemisphere weather stations over the period from 1955 to 2014; the black line shows the estimate of the latitudinal average temperature $\hat{t}_n$ obtained by transforming the astronomical insolation s_n; no weighting functions were used.

The relative contribution of $\hat{t}_n$ to the total temperature variations was estimated using the determination coefficient R^2.

The calculations are also performed for two halved time intervals. Similar to $\hat{t}_n$, we calculated the quantities $\widehat{to}_n$ for the interval 1955–1984 and $\widehat{te}_n$ for 1985–2014. As in the case of the total interval, moments in (2) were minimized. Different estimates of the difference of these quantities, i.e., $\widehat{te}_n - \widehat{to}_n$, at each station with the number n are given in Table 4.

RESULTS AND DISCUSSION

Results of minimizing second-order moments δ and δ_g (2) differ insignificantly (Tables 1–3). The same is true for the determination coefficients R^2 calculated for situations displayed in Figs. 1 and 2. The coefficients R^2 show that in 1955–2014, as in halved periods of time, the astronomical insolation was responsible for about 85% of temperature variations, and stochastic geospheric processes, for about 15% of variations.

Using the well-known values of the average temperature of Earth's surface layer (288 K) and its effective temperature (249 K) [28], the authors of work [23] calculated the relationship between factors of insolation and greenhouse effects in formation of Earth's thermal regime (86.46 and 13.54%, respectively). These values do not contradict the determination coefficients calculated in this work.

Analysis of Fig. 1 shows that nonuniform positions of points prohibit a direct averaging of t_n. It is noteworthy that, based on the possible symmetry of the cloud of points around the quantity $\hat{t}_n$, we see which temperatures are lack to improve its estimate; this can favor optimization of the network of weather stations. On the other hand, the $\hat{t}_n$ value obtained from theoretical considerations for astronomical insolation s_n through a regression of measured temperatures t_n is defined within the hemisphere as a continuous monotonic function of latitude and is the average value for t_n because average differences μ (3) turn out to be zero for data calculated without weights (Table 3) and close

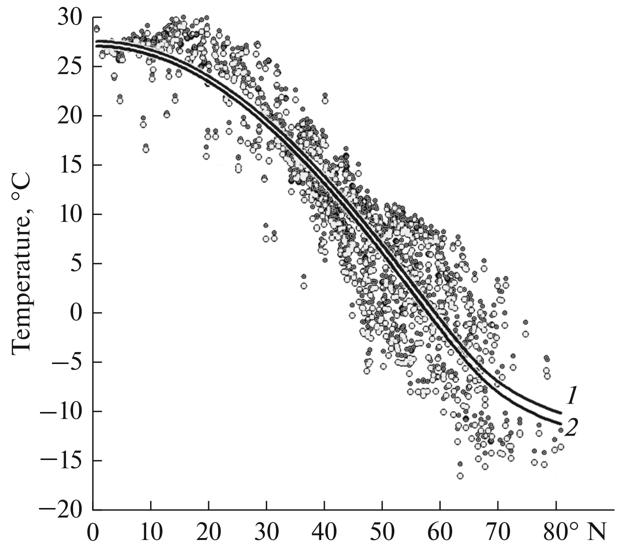

Fig. 2. Increment of the latitudinal average temperature. Curve *1* and closed circles for long-term average temperatures correspond to period from 1985 to 2014, curve *2* and open circles correspond to period from 1955 to 1984; no weighting functions were used.

to zero in other cases (Tables 1 and 2) with acceptable confidence intervals. It can be seen that the average differences μ converge to zero with decreasing sizes of grid cells.

Results of regression and weighted regression for μ and δ differ insignificantly. This is not surprising because, in accordance with the moment δ and determination coefficient R^2 (Tables 1–3), insolation has a predominant effect on temperature as compared to stochastic processes in the surface layer. Choice of $\hat{t}_n$, characterizing the entire hemisphere, already restricts the possibilities for a substantial change of the moment δ for any method of its minimization.

Change of the coefficients of the linear transformation *A* and *B* for different periods of time leads to a marked relative alteration of the functions $\widehat{to}_n$ and $\widehat{te}_n$ (see Fig. 2). Estimates of differences of these quantities diverge for different methods of δ minimization by no more than hundredths of a degree Celsius, making it possible to use $\widehat{to}_n$ and $\widehat{te}_n$ for estimating the temperature increment. Warming estimates (see Table 4) correspond to well-known data on changes in the global temperature [29]. Also in that work, and in a number of other works, it is noted that a rapid warming is characteristic for high latitudes. In Fig. 2, it can be seen that the latitudinal average temperatures obtained for different periods diverge maximally at latitudes poleward of 65° N.

Thus, in the period 1955–2014, the well-known model of astronomical insolation s_n [21–23], latitudinal average temperature $\hat{t}_n$ (1), and observations of temperature t_n [7] well agree in the sense of statement (*). In other words, the result of insolation transformation (1) found in this work should be considered as a latitudinal average temperature. In the region of acceptable confidence intervals for average differences (3), there are solid grounds to believe that this estimate is also valid in expanding the observation network.

CONCLUSIONS

In the framework of the accepted astronomical insolation model, based on temperature observations from 927 Northern Hemisphere weather stations in period from 1955 to 2014 and estimates obtained, we can state that the latitudinal average temperature should be estimated by applying a linear transformation of the astronomical insolation, with the average differences of the transformed astronomical insolation and long-term average temperatures over all latitudes being close to zero at a small 95% confidence interval. The latitudinal average temperature thus determined well corresponds to experience, is quasi-stable, and can be considered as a climate element.

Contributions of two components, namely, deterministic astronomical insolation and stochastic processes in the geosphere, to the temperature variations are calculated. The increment of latitudinal average temperature in 1985–2014 as compared to 1955–1984 is estimated. The results of the calculations do not contradict the well-known estimates, thus verifying our linear transformation. The analytical interrelation found, immanent to climate system, should be the subject of further studies.

The linear transformation of astronomical insolation into temperature contains constants determined by minimizing the deviations of actual long-term average temperatures from the transformed theoretical distribution of astronomical insolation over latitudes. Our estimates of the latitudinal average temperature make it possible to determine and refine the temperature in any zone, at any latitude, as additional observations become available.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

REFERENCES

1. *The Second Assessment Report of Roshydromet on Climate Change and Its Consequences on the Territory of the Russian Federation. General summary* (Roshydromet, Moscow, 2014) [in Russian].

2. G. V. Gruza and E. Ya. Ran'kova, *Observed and Expected Climate Changes in Russia: Air Temperature* (VNIIGMI-MCD, Obninsk, 2012) [in Russian]

3. GISS Surface Temperature Analysis. http://data.giss.nasa.gov/gistemp/. Cited December 1, 2018.

4. J. Hansen, R. Ruedy, M. Sato, and K. Lo, "Global surface temperature change," Rev. Geophys. **48** (2010). https://doi.org/10.1029/2010RG000345

5. Global Average Anomalies. www.ncdc.noaa.gov/oa/climate/research/anomalies/index.htm. Cited December 1, 2018.

6. T. M. Smith, R. Reynolds, T. C. Peterson, and J. Lawrimore, "Improvements to NOAA's historical merged land–ocean surface temperature analysis (1880–2006)," J. Climatol. **21**, 2283–2296 (2008).

7. Archive of the University of East Anglia. http://www.metoffice.gov.uk, http://www.cru.uea.ac.uk. Cited June 1, 2017.

8. P. D. Jones, D. H. Lister, T. J. Osborn, C. Harpham, M. Salmon, and C. P. Morice, "Hemispheric and large-scale land-surface air temperature variations: An extensive revision and an update to 2010," J. Geophys. Res. **117** (D5) (2012). https://doi.org/10.1029/2011JD017139

9. G. V. Gruza, E. Ya. Ran'kova, L. K. Kleshchenko, and V. D. Smirnov, "On spatial averaging in climate monitoring problems," Problemy Ekologich. Monit. Modelir. Ekosistem. **XXV**, 42–71 (2013).

10. R. S. Vose, D. Wuertz, T. C. Peterson, and P. D. Jones, "An intercomparison of trends in surface air temperature analyses at the global, hemispheric and grid-box scale," Geophys. Rev. Lett. **32** (2005). https://doi.org/10.1029/2005GL023502

11. P. Jones, M. New, D. E. Parker, S. Martin, and I. G. Rigor, "Surface air temperature and its changes over the past 150 years," Rev. Geophys. **37** (1999). https://doi.org/10.1029/1999RG900002

12. P. D. Jones and T. M. L. Wigley, "Estimation of global temperature trends: What's important and what isn't," Clim. Change **100**, 59–69 (2010).

13. L. S. Gandin, *Objective Analysis of Meteorological Fields* (Gidrometeoizdat, Leningrad, 1965) [in Russian].

14. J. E. Hansen and S. Lebedeff, "Global trends of measured surface air temperature," J. Geophys. Res. **92**, 13345–13372 (1987).

15. V. G. Dmitriev, "About a feature of optimal (and other linear) interpolation of hydrometeorological processes and fields," Navigatsiya Gidrografiya, No. 1, 99–101 (1995).

16. P. D. Jones and A. Moberg, "Hemispheric and large-scale surface air temperature variations: An extensive revision and an update to 2001," J. Clim. **16**, 206–223 (2003).

17. M. I. Budyko, "About the origin of ice epochs," Meteorol. Gidrol., No. 11, 17–19 (1968).

18. H. W. Dove, *Verbreitung der Wärme auf der Oberfläche der Erde: Erläutert durch Isothermen, Thermische Isanomalien und Temperaturcurven* (Dietrich Reimer, Berlin, 1852).

19. S. P. Khromov and M. A. Petrosyants, *Meteorology and Hydrology* (Moscow State University, Moscow, 1994) [in Russian].

20. V. P. Zakusilov, T. N. Zadorozhnaya, and R. A. Belkin, "Peculiarities of the longitudinal distribution of climate air temperature parameters in different latitudes in the Northern hemisphere," in *Proc. of the Intern. Conf. "Science–Society Interaction: Problems and Prospects", 2018.* P. 26–30.

21. V. M. Fedorov and P. B. Grebennikov, "Calculation of long-term averages of surface air temperature based on insolation data," Izv. Atmos. Ocean. Phys. **53** (8), 757–768 (2017).

22. V. M. Fedorov, "The correlation analysis of the Earth's insolation and anomalies in surface air temperatures," Uch. Zapiski RGGMU. No. 45, 151–166 (2016).

23. V. M. Fedorov and P. B. Grebennikov "Calculation of long term averages of surface air temperature based on insolation data," Geofiz. Proc. Biosfera **16** (1), 5–24 (2017). https://doi.org/10.21455/GPB2017.1-1

24. The JPL HORIZONS on-line solar system data and ephemeris computation service provides access to key solar system data and flexible production of highly accurate ephemerides for solar system objects. http://ssd.jpl.nasa.gov. Cited December 1, 2018.

25. A.S. Monin, *Earth Rotation and Climate* (Gidrometeoizdat, Leningrad, 1972) [in Russian].

26. Solar radiation and climate. http://www.solar-climate.com/sc/bd01.htm. Cited December 1, 2018.

27. V. M. Fedorov, *Earth's Insolation and Modern Climate Change* (Fizmatlit, Moscow, 2018) [in Russian].

28. S. M. Semenov, "Greenhouse effect: Discovery, concept development, role in formation of global climate and its human-induced changes," Fund. Pprikl. Klimatologiya **2**, 103–126 (2015).

29. *Climate Change 2013: Physical Scientific Basis,* ed. by T.F. Stoker, D. TSin', Plattner, M.B. Tignor, S.K. Allen, D. Boshung, A. Nauels, Yu. Sya, V. Beks, and P.M. Midglej (Cambridge University Press, Cambridge; New York, 2013).

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