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MATHEMATICAL PHYSICS

Mathematical Simulation of Satellite Motion with an Aerodynamic Attitude Control System Influenced by Active Damping Torques

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Abstract—The dynamics of a satellite moving in a central Newtonian force field in a circular orbit under the influence of aerodynamic and active damping torques depending on projections of the satellite's angular velocity is studied. A method for determining all equilibrium positions (equilibrium orientations) of the satellite in the orbital coordinate system given the values of aerodynamic torque, damping coefficients, and the principal central moments of inertia is proposed. In the case when the axes of the coordinate system attached to the satellite coincide with the axes of the orbital coordinate system, necessary and sufficient conditions for the asymptotic stability of the corresponding zero equilibrium position are obtained using the Routh–Hurwitz criterion. The domains with satisfied asymptotic stability conditions for the zero equilibrium position are analyzed depending on various dimensionless parameters of the problem. The damping of spatial oscillations of the satellite is numerically studied for various values of aerodynamic torque and damping coefficients.

Keywords: satellite, circular orbit, aerodynamic torque, active damping torque, equilibrium positions, stability, numerical methods

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INTRODUCTION

The study of a satellite moving in a central Newtonian force field in a circular orbit under the influence of gravitational and aerodynamic torques is of considerable practical interest as motivated by the creation of attitude control systems for artificial satellites orbiting the Earth. The principle underlying the operation of gravitational attitude control systems is that, in a central Newtonian force field, a satellite with different principal central moments of inertia moving in a circular orbit has 24 equilibrium positions, of which four are stable [1–3]. The dynamics of satellites with gravitational attitude control systems were considered in detail in [4]. An important property of gravitational attitude control systems is that they can sustain operations in their orbits for a long time without consumption of power and (or) working mass. For orbits ranging from 250 to 700 km in altitude, a significant effect on the satellite orientation is exerted not only by gravitational torque, but also by aerodynamic torque. Accordingly, it is necessary to investigate their joint influence on a satellite moving relative to the center of mass; specifically, its equilibrium positions existing in this case and the conditions for their stability have to be studied. An overview of the main results concerning the study of satellite motion with an aerodynamic attitude control system can be found in [4–6].

In this work, primary attention is given to the dynamics of the rotational motion of a satellite influenced by aerodynamic and active damping torques depending on projections of the satellite's angular velocity. Such active damping can be achieved using angular velocity sensors and attitude control actuators. It is assumed that the center of pressure of aerodynamic forces is on one of the principal central axes of inertia of the satellite.

1. EQUATIONS OF MOTION

Consider a satellite (rigid body) moving relative to the center of mass in a circular orbit under the influence of gravitational and aerodynamic torques. Additionally, the satellite is influenced by active damping torques depending on projections of the satellite's angular velocity.

To write the equations of motion, we introduce two right-handed rectangular coordinate systems with the origin placed at the satellite's center of mass O. In the orbital coordinate system $OXYZ$, the OZ axis is directed along the radius vector connecting the centers of mass of the Earth and the satellite, and the OX axis is directed along the linear velocity of the satellite's center of mass O. Then the OY axis is directed along the normal to the orbital plane. In the coordinate system $Oxyz$ attached to the satellite, the Ox, Oy, and Oz axes are the principal central axes of inertia of the satellite.

The orientation of the coordinate system $Oxyz$ with respect to the orbital coordinate system is determined by the aircraft angles α (pitch), β (yaw), and γ (roll). The direction cosines of the Ox, Oy, and Oz axes in the orbital coordinate system are expressed in terms of the aircraft angles with the help of the relations (see [4])

$$\begin{gathered} a_{11} = \cos(x, X) = \cos\alpha\cos\beta, \quad a_{21} = \cos(x, Y) = \sin\beta, \\ a_{12} = \cos(y, X) = \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma, \quad a_{22} = \cos(y, Y) = \cos\beta\cos\gamma, \\ a_{13} = \cos(z, X) = \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma, \quad a_{23} = \cos(z, Y) = -\cos\beta\sin\gamma, \\ a_{31} = \cos(x, Z) = -\sin\alpha\cos\beta, \\ a_{32} = \cos(y, Z) = \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma, \\ a_{33} = \cos(z, Z) = \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma. \end{gathered} \tag{1}$$

For small satellite oscillations, the pitch angle corresponds to rotation around the OY axis; the yaw angle, to rotation around the OZ axis; and the roll angle, to rotation around the OX axis.

To derive the equations of motion, we make the following assumptions [4].

(i) The action of the atmosphere on the satellite is reduced to the drag force applied to the center of pressure and directed against the velocity of the satellite's center of mass relative to the air. The center of pressure is located on the Ox axis of the satellite. This assumption holds sufficiently accurately for satellites of nearly spherical shapes.

(ii) The influence of the atmosphere on the translational motion of the satellite is negligibly small.

(iii) The entrainment of the Earth's rotating atmosphere is neglected.

Suppose that, in addition to the aerodynamic torque, the satellite experiences active damping torques with the projections of their total vector onto the Ox, Oy, and Oz axes given by $M_x = \bar{k}_1 p_1$, $M_y = \bar{k}_2(q_1 - \omega_0)$, and $M_z = \bar{k}_3 r_1$, respectively. Here, $\bar{k}_1$, $\bar{k}_2$, and $\bar{k}_3$ are damping coefficients; p_1, q_1, and r_1 are the projections of the satellite's angular velocity onto the Ox, Oy, and Oz axes; and ω_0 is the angular velocity of the center of mass of the satellite moving in a circular orbit. Then the equations of motion of the satellite relative to the center of mass can be written as

$$\begin{gathered} Ap_1' + (C - B)q_1 r_1 - 3\omega_0^2(C - B)a_{32}a_{33} + \bar{k}_1 p_1 = 0, \\ Bq_1' + (A - C)r_1 p_1 - 3\omega_0^2(A - C)a_{33}a_{31} + \omega_0^2 H_1 a_{13} + \bar{k}_2(q_1 - \omega_0) = 0, \\ Cr_1' + (B - A)p_1 q_1 - 3\omega_0^2(B - A)a_{31}a_{32} - \omega_0^2 H_1 a_{12} + \bar{k}_3 r_1 = 0; \end{gathered} \tag{2}$$

$$\begin{gathered} p_1 = (\alpha' + \omega_0)a_{21} + \gamma', \\ q_1 = (\alpha' + \omega_0)a_{22} + \beta'\sin\gamma, \\ r_1 = (\alpha' + \omega_0)a_{23} + \beta'\cos\gamma. \end{gathered} \tag{3}$$

In Eqs. (2) and (3) A, B, and C are the principal central moments of inertia of the satellite; $H_1 = -Qa/\omega_0^2$, where Q is the drag force acting on the satellite; and $(a, 0, 0)$ are the coordinates of the satellite's center of pressure in the coordinate system $Oxyz$. For the satellite to be aerodynamically stable, its center of pressure lies behind its barycenter; therefore, $a < 0$. The prime denoted differentiation with respect to time t.

After introducing the dimensionless parameters $\theta_A = A/B$, $\theta_C = C/B$, $p = p_1/\omega_0$, $q = q_1/\omega_0$, $r = r_1/\omega_0$, $k_1 = \bar{k}_1/\omega_0 B$, $k_2 = \bar{k}_2/\omega_0 B$, $k_3 = \bar{k}_3/\omega_0 B$, and $h_1 = H_1/B$ and passing from the time t to the dimensionless independent variable $\tau = \omega_0 t$, system (2), (3) becomes

$$\begin{gathered}\theta_A\dot{p} + (\theta_C - 1)qr - 3(\theta_C - 1)a_{32}a_{33} + k_1 p = 0,\\ \dot{q} + (\theta_A - \theta_C)rp - 3(\theta_A - \theta_C)a_{33}a_{31} + h_1 a_{13} + k_2(q-1) = 0,\\ \theta_c\dot{r} + (1-\theta_A)pq - 3(1-\theta_A)a_{31}a_{32} - h_1 a_{12} + k_3 r = 0;\end{gathered} \tag{4}$$

$$\begin{gathered}p = (\dot{\alpha}+1)a_{21} + \dot{\gamma},\\ q = (\dot{\alpha}+1)a_{22} + \dot{\beta}\sin\gamma,\\ r = (\dot{\alpha}+1)a_{23} + \dot{\beta}\cos\gamma.\end{gathered} \tag{5}$$

Dotted variables in Eqs. (4) and (5) denote derivatives with respect to τ.

2. EQUILIBRIUM POSITIONS OF THE SATELLITE

Setting $\alpha = \alpha_0 = \text{const}$, $\beta = \beta_0 = \text{const}$, and $\gamma = \gamma_0 = \text{const}$ in (4) and (5) and assuming that $A \neq B \neq C$, we obtain the equations

$$\begin{gathered}a_{22}a_{23} - 3a_{32}a_{33} + \tilde{k}_1 a_{21} = 0,\\ (1-v)(a_{23}a_{21} - 3a_{33}a_{31}) + h(a_{21}a_{32} - a_{22}a_{31}) + \tilde{k}_2(a_{22}-1) = 0,\\ v(a_{21}a_{22} - 3a_{31}a_{32}) - h(a_{23}a_{31} - a_{21}a_{33}) + \tilde{k}_3 a_{23} = 0,\end{gathered} \tag{6}$$

from which we can determine the equilibrium positions of the satellite in the orbital coordinate system. Here, $\tilde{k}_1 = k_1/(\theta_C - 1)$, $\tilde{k}_2 = k_2/(\theta_A - \theta_C)$, $\tilde{k}_3 = k_3/(1-\theta_A)$, $h = h_1/(1-\theta_C)$, and $v = (1-\theta_A)/(1-\theta_C)$.

System (6) can be treated as consisting of three equations with unknowns α_0, β_0, and γ_0. Another way of closing Eqs. (6) is to add three orthogonality conditions for the direction cosines:

$$\begin{gathered}a_{21}^2 + a_{22}^2 + a_{23}^2 = 1,\\ a_{31}^2 + a_{32}^2 + a_{33}^2 = 1,\\ a_{21}a_{31} + a_{22}a_{32} + a_{23}a_{33} = 0.\end{gathered} \tag{7}$$

Equations (6) and (7) form a closed algebraic system of equations for six direction cosines, which determine the equilibrium positions of the satellite. For this system, we set up the following problem: given h, v, $\tilde{k}_1$, $\tilde{k}_2$, and $\tilde{k}_3$, determine nine direction cosines, i.e., all equilibrium positions of the satellite in the orbital coordinate system. After finding six direction cosines a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, and a_{33}, the remaining ones (a_{11}, a_{12}, a_{13}) are determined by the orthogonality conditions.

For a given set of parameter values, the equilibrium positions can be determined numerically by applying numerical methods for solving systems of nonlinear algebraic equations, which are implemented in many modern software packages for engineering computations.

In the special case of $k_1 = k_2 = k_3 = k$, the conditions for the existence of various numbers of satellite equilibrium positions under the action of aerodynamic torque can be investigated analytically by applying an algorithm for constructing Gröbner bases [7] with the use of computer algebra systems. The method for constructing Gröbner bases represents an algorithmic procedure that completely reduces the problem with polynomials in many variables to the consideration of a polynomial in a single variable. In this case a Gröbner basis for polynomials (6) and (7) can be constructed with respect to a lexicographic ordering of variables. In the resulting Gröbner basis, there is a biquadratic polynomial in only one variable a_{22}, whose roots determine all equilibrium positions of the satellite in this special case. The remaining direction cosines of system (6), (7), namely, a_{21}, a_{23}, a_{31}, a_{32}, and a_{33} can be obtained by setting the corresponding polynomials in the Gröbner basis to zero and finding the roots of the resulting algebraic equations. The coefficients of these equations depend on three parameters v, k, h and have cumbersome expressions [8].

In this paper, primary attention is given to the study of conditions for the asymptotic stability of the zero equilibrium $\alpha_0 = 0$, $\beta_0 = 0$, $\gamma_0 = 0$, when the axes of the coordinate system attached to the satellite coincide with axes of the orbital coordinate system.

3. NECESSARY AND SUFFICIENT CONDITIONS FOR ASYMPTOTIC STABILITY OF SATELLITE EQUILIBRIUM POSITIONS

To study necessary and sufficient conditions for the asymptotic stability of equilibrium positions of system (6), (7), we linearize the equations of system (4), (5) in a neighborhood of the satellite equilibrium orientation $\alpha = \alpha_0$, $\beta = \beta_0$, $\gamma = \gamma_0$, satisfying system (6), (7). The angles α, β, and γ are represented in the form $\alpha = \alpha_0 + \bar{\alpha}$, $\beta = \beta_0 + \bar{\beta}$, and $\gamma = \gamma_0 + \bar{\gamma}$, where $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\gamma}$ are small deviations from the equilibrium position of the satellite. Then the equations of system (4), (5) linearized around the equilibrium $\alpha = \alpha_0$, $\beta = \beta_0$, $\gamma = \gamma_0$ can be written as

$$\begin{gathered}
\theta_A\ddot{\bar{\alpha}}\sin\beta_0 + [2(\theta_C - 1)a_{22}a_{23} + k_1 a_{21}]\dot{\bar{\alpha}} + 3(\theta_C - 1)(a_{12}a_{33} + a_{13}a_{32})\bar{\alpha} \\
+ \cos\beta_0[(\theta_A + \theta_C - 1) - 2(\theta_C - 1)\sin^2\gamma_0]\dot{\bar{\beta}} \\
+ \cos\beta_0\{(\theta_C - 1)[(1 + 3\sin^2\alpha_0)\sin\beta_0\sin 2\gamma_0 - \frac{3}{2}\sin 2\alpha_0\cos 2\gamma_0] + k_1\}\bar{\beta} \\
+ \theta_A\ddot{\bar{\gamma}} + k_1\dot{\bar{\gamma}} + (\theta_C - 1)[(a_{23}^2 - a_{22}^2) - 3(a_{33}^2 - a_{32}^2)]\bar{\gamma} = 0, \\
\ddot{\bar{\alpha}}a_{22} + [2(\theta_A - \theta_C)a_{21}a_{23} + k_2 a_{22}]\dot{\bar{\alpha}} + 3(\theta_A - \theta_C)(a_{13}a_{31} + a_{11}a_{33})\bar{\alpha} \\
+ h_1 a_{33}\bar{\alpha} + \ddot{\bar{\beta}}\sin\gamma_0 + [(\theta_A - \theta_C - 1)\sin\beta_0\cos\gamma_0 + k_2\sin\gamma_0]\dot{\bar{\beta}} \\
- \left\{(\theta_A - \theta_C)\left[(1 + 3\sin^2\alpha_0)\cos 2\beta_0\sin\gamma_0 + \frac{3}{2}\sin 2\alpha_0\sin\beta_0\cos\gamma_0\right] + k_2\sin\beta_0\cos\gamma_0\right\}\bar{\beta} \\
+ (\theta_A - \theta_C + 1)\dot{\bar{\gamma}}a_{23} + [(\theta_C - \theta_A)(a_{21}a_{22} - 3a_{31}a_{32}) + k_2 a_{23}]\bar{\gamma} - h_1 a_{23}\cos\alpha_0\bar{\beta} - h_1 a_{12}\bar{\gamma} = 0, \\
\theta_C\ddot{\bar{\alpha}}a_{23} + \cos\beta_0[2(1 - \theta_A)\sin\beta_0\cos\gamma_0 - k_3\sin\gamma_0]\dot{\bar{\alpha}} + 3(1 - \theta_A)(a_{11}a_{32} + a_{12}a_{31})\bar{\alpha} \\
- h_1 a_{32}\bar{\alpha} + \theta_C\ddot{\bar{\beta}}\cos\gamma_0 + [(\theta_C - \theta_A + 1)\sin\beta_0\sin\gamma_0 + k_3\cos\gamma_0]\dot{\bar{\beta}} \\
+ \left\{(1 - \theta_A)\left[(1 + 3\sin^2\alpha_0)\cos 2\beta_0\cos\gamma_0 - \frac{3}{2}\sin 2\alpha_0\sin\beta_0\sin\gamma_0\right] + k_3\sin\beta_0\sin\gamma_0\right\}\bar{\beta} \\
+ h_1 a_{22}\cos\alpha_0\bar{\beta} - (\theta_A + \theta_C - 1)\dot{\bar{\gamma}}a_{22} + [(1 - \theta_A)(a_{21}a_{23} - 3a_{31}a_{33}) - k_3 a_{22}]\bar{\gamma} - h_1 a_{13}\bar{\gamma} = 0.
\end{gathered} \tag{8}$$

Consider small oscillations of the satellite in a neighborhood of the trivial solution

$$\alpha_0 = \beta_0 = \gamma_0 = 0. \tag{9}$$

Then Eqs. (8) yield the system

$$\begin{gathered}
\ddot{\bar{\alpha}} + k_2\dot{\bar{\alpha}} + [3(\theta_A - \theta_C) + h_1]\bar{\alpha} = 0, \\
\theta_C\ddot{\bar{\beta}} + k_3\dot{\bar{\beta}} - (\theta_A + \theta_C - 1)\dot{\bar{\gamma}} + [(1 - \theta_A) + h_1]\bar{\beta} - k_3\bar{\gamma} = 0, \\
\theta_A\ddot{\bar{\gamma}} + (\theta_A + \theta_C - 1)\dot{\bar{\beta}} + k_1\dot{\bar{\gamma}} + k_1\bar{\beta} + 4(1 - \theta_C)\bar{\gamma} = 0.
\end{gathered} \tag{10}$$

The characteristic equation of system (10) represents the product of polynomials of the second and fourth degrees in λ:

$$[\lambda^2 + k_2\lambda + [3(\theta_A - \theta_C) + h_1]](A_0\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4) = 0, \tag{11}$$

where

$$\begin{gathered}
A_0 = \theta_A\theta_C, \quad A_1 = k_1\theta_C + k_3\theta_A, \\
A_2 = k_1k_3 + (\theta_A + \theta_C - 1)^2 + \theta_A(1 - \theta_A) + 4\theta_C(1 - \theta_C) + \theta_A h_1, \\
A_3 = k_1\theta_C + k_3(3 + \theta_A - 3\theta_C) + k_1 h_1, \quad A_4 = k_1k_3 + 4(1 - \theta_C)(1 - \theta_A + h_1).
\end{gathered}$$

The necessary and sufficient conditions for the asymptotic stability of the trivial solution (9) (Routh–Hurwitz stability criterion) have the form

$$\begin{gathered}
k_2 > 0, \quad 3(\theta_A - \theta_C) + h_1 > 0, \\
\Delta_1 = A_1 = k_1\theta_C + k_3\theta_A > 0, \\
\Delta_2 = A_1A_2 - A_0A_3 = k_1^2k_3\theta_C + k_1k_3^2\theta_A + k_3h_1\theta_A^2 \\
+ (1 - \theta_C)[k_1\theta_C(1 - \theta_A + 3\theta_C) + k_3\theta_A(1 - \theta_A)] > 0,
\end{gathered}$$

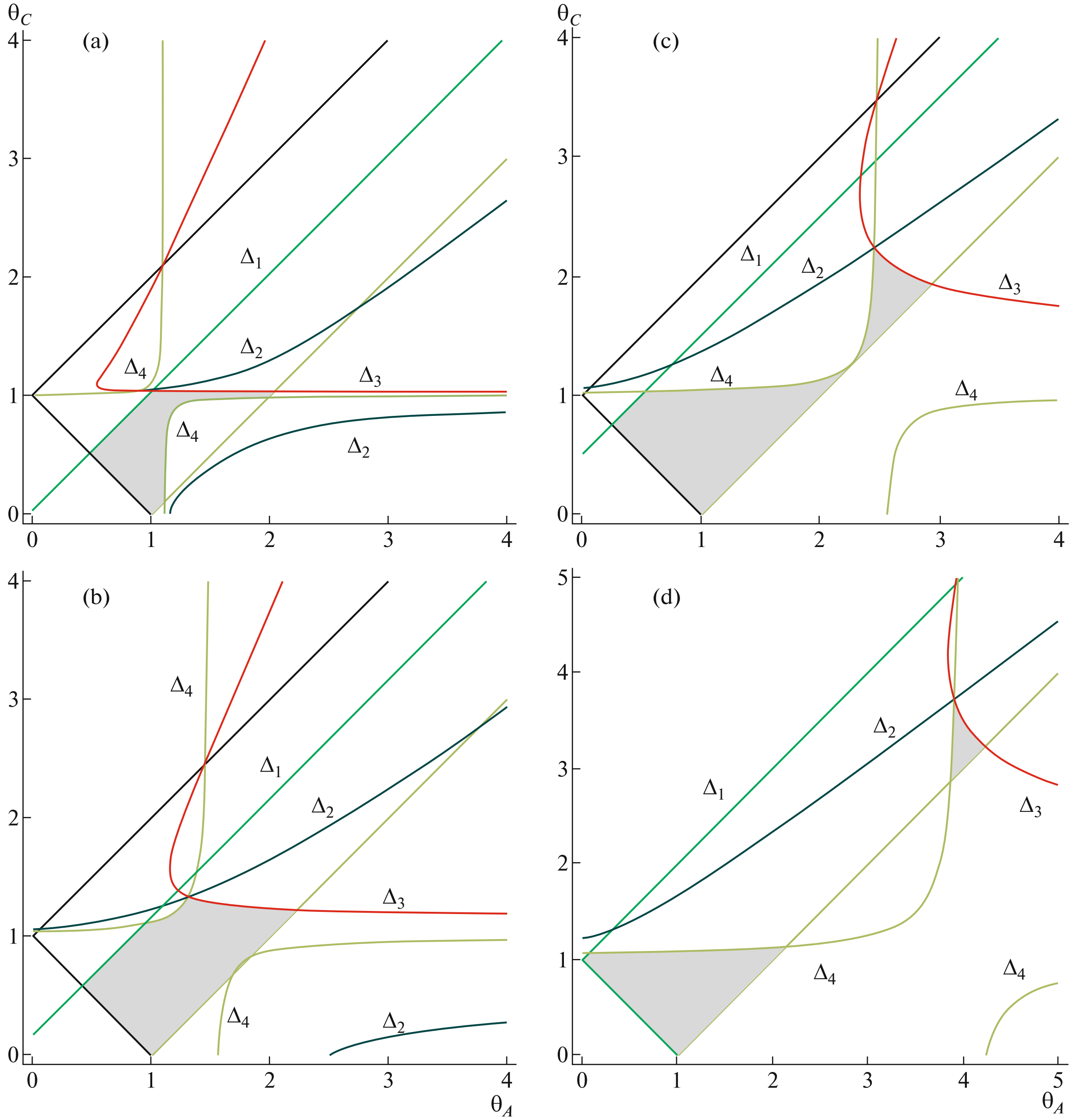

Fig. 1. Domain with satisfied asymptotic stability conditions: (a) $k = 0.2$, $h_1 = 0.1$; (b) $k = 0.5$, $h_1 = 0.5$; (c) $k = 0.5$, $h_1 = 1.5$; and (d) $k = 1.0$, $h_1 = 3.0$.

$$\begin{aligned}\Delta_3 &= A_1A_2A_3 - A_0A_3^2 - A_1^2A_4 = 3(1-\theta_C)\{k_1^2k_3^2\theta_C + k_1k_3^3\theta_A \\ &+ k_1^2\theta_C^2(\theta_A + \theta_C - 1) + k_1k_3\theta_C[(\theta_A + \theta_C - 1)(2\theta_A - 1) + 3\theta_C(1-\theta_C)] \\ &- k_3^2\theta_A(1-\theta_A)(\theta_A + \theta_C - 1)\} + k_1k_3h_1^2\theta_A^2 - 4h_1(1-\theta_C)(k_1\theta_C + k_3\theta_A)^2 \\ &+ k_1h_1\{k_1^2k_3\theta_C + k_1k_3^2\theta_A + (1-\theta_C)[k_1\theta_C(1-\theta_A+3\theta_C) + k_3\theta_A(1-\theta_A)]\} \\ &+ k_3h_1\theta_A^2[k_1\theta_C + k_3(3 - 3\theta_C + \theta_A)] > 0, \\ \Delta_4 &= \Delta_3A_4 > 0, \quad A_4 = k_1k_3 + 4(1-\theta_C)(1-\theta_A+h_1) > 0.\end{aligned} \tag{12}$$

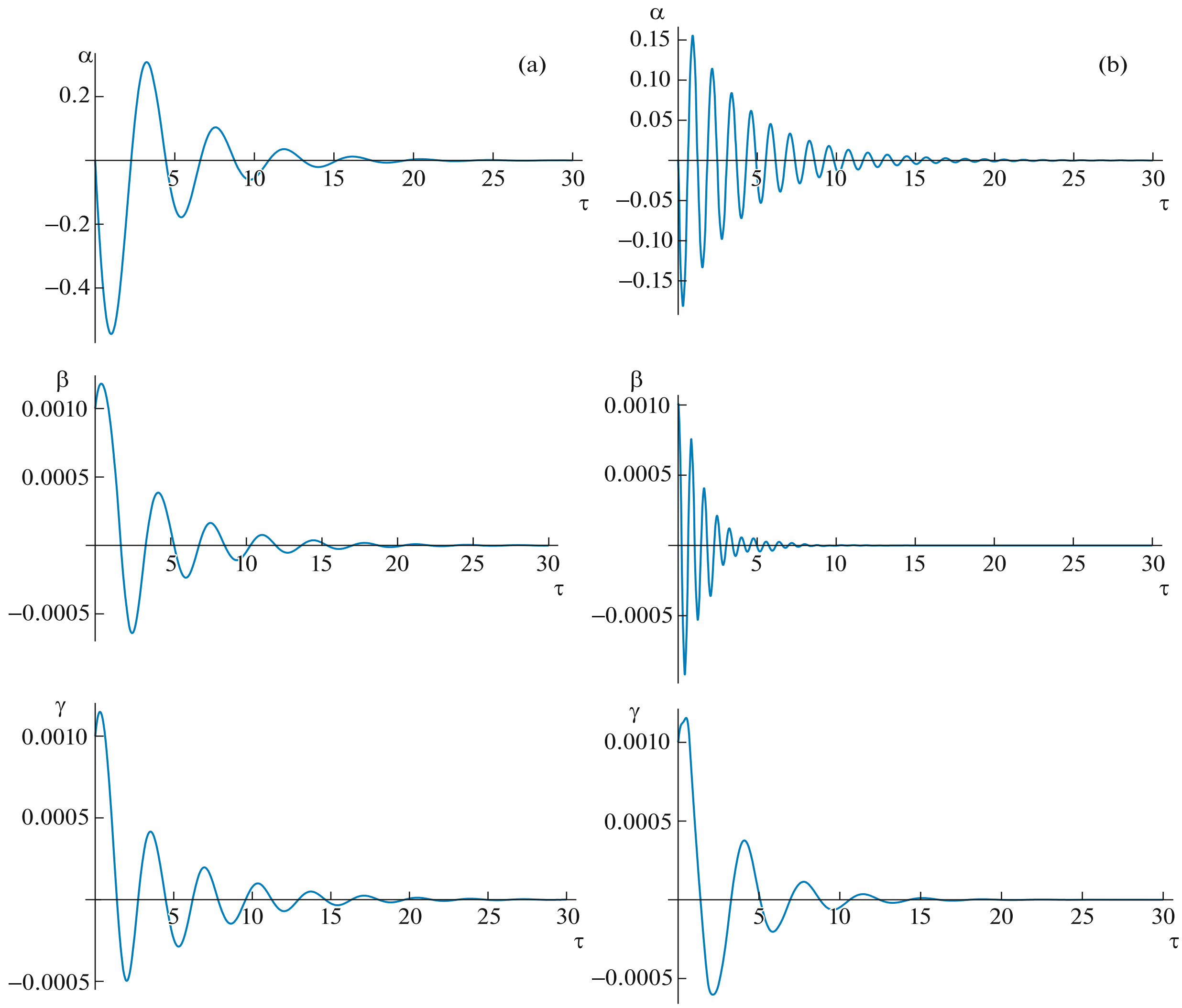

Fig. 2. Transition process: (a) $\theta_A = 0.8$, $\theta_C = 0.4$, $k = 0.5$, $h_1 = 1.0$ and (b) $\theta_A = 0.8$, $\boldsymbol{\theta}_C = 0.4$, $k = 0.5$, $h_1 = 25$.

Consider the special case when $k_1 = k_2 = k_3 = k$. Then conditions (12) simplify to

$$\begin{gathered} k > 0, \quad h_1 + 3(\theta_A - \theta_C) > 0, \\ \Delta_1 = k(\theta_C + \theta_A) > 0, \\ \Delta_2 = [h_1 - (1-\theta_C)]\theta_A^2 + [k^2 + (1-\theta_C)^2]\theta_A + [k^2 + (1-\theta_C)(1+3\theta_C)]\theta_C > 0, \\ \Delta_3 = 3k^2(1-\theta_C)\{\theta_A^3 + (3\theta_C - 2)\theta_A^2 + [k^2 + (1-\theta_C)(1-3\theta_C)]\theta_A \\ + \theta_C[k^2 + (1-\theta_C)(1+2\theta_C)]\} + h_1\theta_A^3 + h_1^2\theta_A^2 \\ + k^2h_1\theta_A - 9\theta_A\theta_C(1-\theta_C)h_1 + \theta_C h_1[k^2 + (1-\theta_C)^2] > 0, \\ \Delta_4 = \Delta_3 A_4 > 0, \quad A_4 = k^2 + 4(1-\theta_C)(1-\theta_A + h_1) > 0. \end{gathered} \tag{13}$$

The evolution of domains in which the necessary and sufficient stability conditions (13) hold was studied in the plane (θ_A, θ_C) depending on the parameters k and h_1. For actual bodies we need to take into account the triangle inequality conditions, which have to be satisfied by the parameters θ_A and θ_C: $1 + \theta_A \geq \theta_C$, $1 + \theta_C \geq \theta_A$, and $\theta_A + \theta_C \geq 1$. These conditions single out an infinite half-strip in the plane (θ_A, θ_C). Due to the inequality $h_1 + 3(\theta_A - \theta_C) > 0$ in conditions (13), the size of the half-strip depends on h_1 and, for

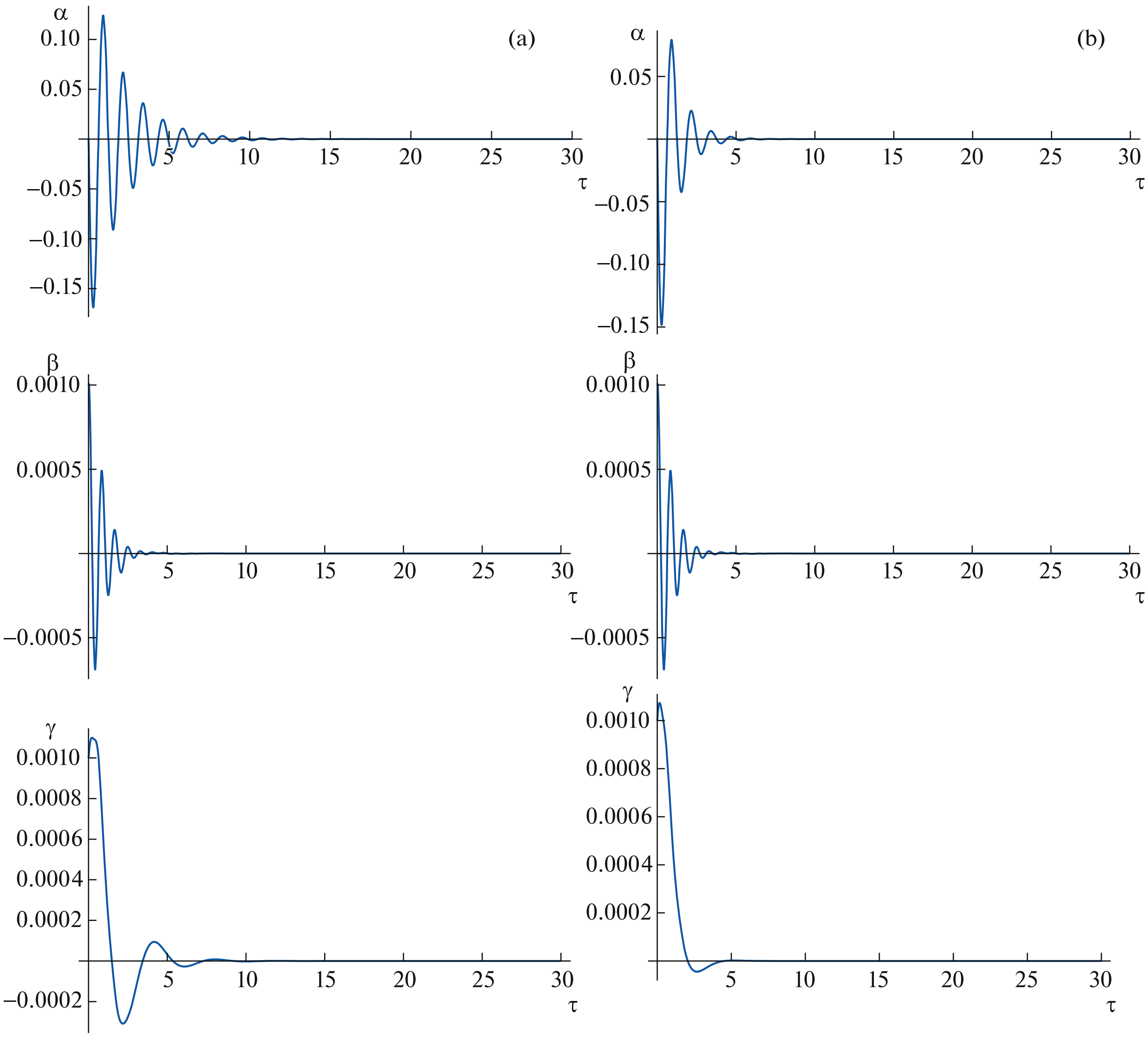

Fig. 3. Transition process: (a) $\theta_A = 0.8$, $\theta_C = 0.4$, $k = 1.0$, $h_1 = 25$ and (b) $\theta_A = 0.8$, $\theta_C = 0.4$, $k = 2.0$, $h_1 = 25.0$.

$h_1 = 3$, coincides with the first triangle inequality condition. Thus, conditions (13) were analyzed in the half-strip bounded by the straight lines

$$\theta_C = 1 - \theta_A, \quad \theta_C = 1 + \theta_A, \quad \theta_C = \theta_A - 1, \quad h_1 + 3(\theta_A - \theta_C) = 0. \tag{14}$$

Figure 1 presents the variations in the domains of satisfied asymptotic stability conditions (13) for various parameters k and h_1. These domains are shown in gray.

For small values of k and h_1 (Fig. 1a, $k = 0.2$, $h_1 = 0.1$), the stability domain approaches the domain with satisfied necessary and sufficient stability conditions for the solid satellite for $k = 0$ and $h_1 = 0$. This domain is bounded by the straight lines $\theta_C = 1 - \theta_A$, $\theta_C = \theta_A$, and $\theta_A = 1$ (see [1]).

For $k = 0.5$ and $h_1 = 0.5$, Fig. 1b shows in detail the curves from (13), which form the boundaries of the asymptotic stability domain for solution (9). The curve $\Delta_4 = 0$ is a hyperbole and the condition $\Delta_4 > 0$ holds in the domain lying between its branches.

As the parameter h_1 increases, the domain with satisfied stability conditions changes qualitatively. The domain with satisfied conditions (13) splits into two subdomains (Fig. 1c, $k = 0.5$, $h_1 = 1.5$). For $h_1 = 3$

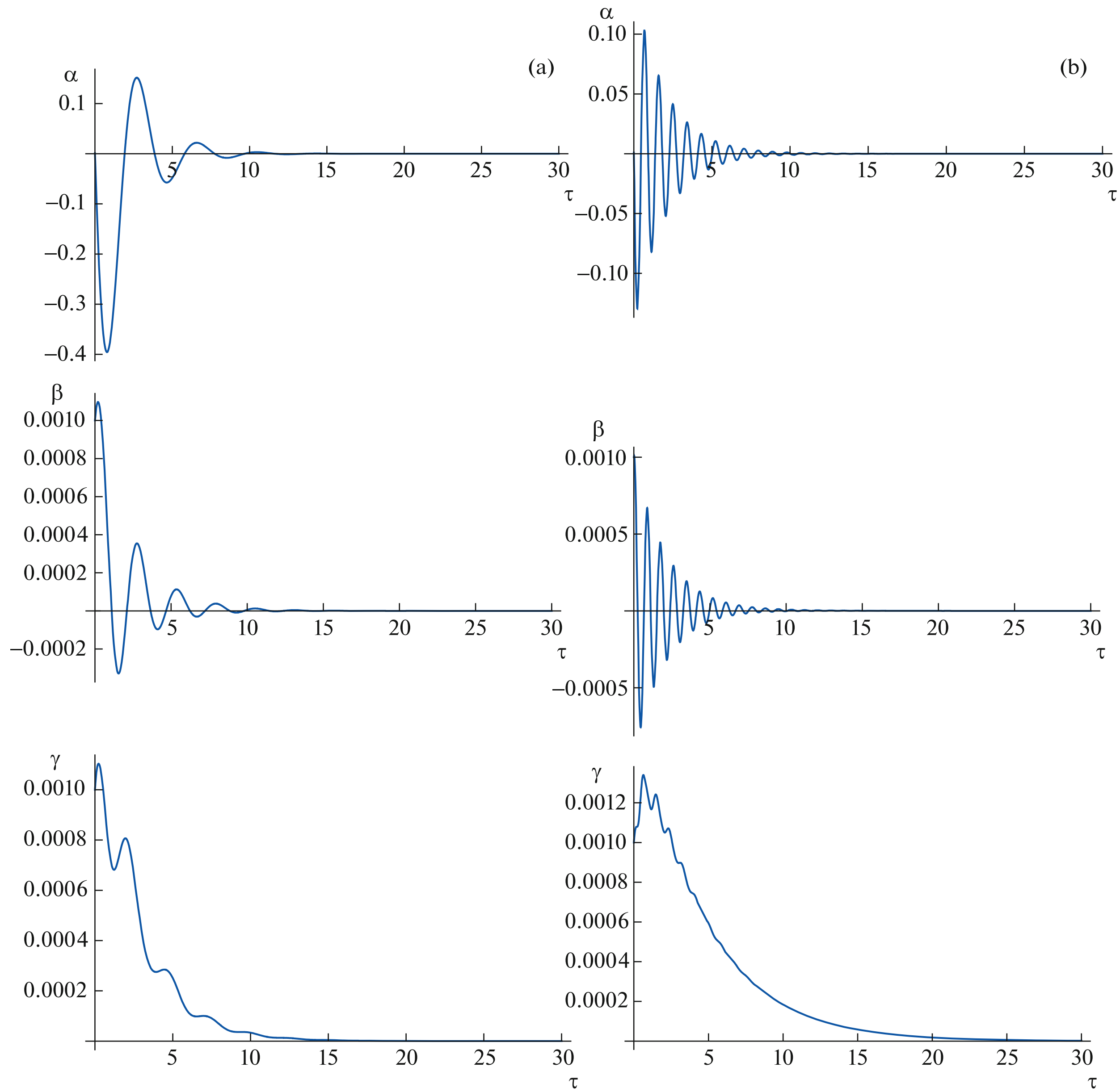

Fig. 4. Transition process: (a) $\theta_A = 0.24$, $\theta_C = 0.95$, $k = 1.0$, $h_1 = 5.0$ and (b) $\theta_A = 0.24$, $\theta_C = 0.95$, $k = 1.0$, $h_1 = 50.0$.

and $k = 1.0$ (Fig. 1d), this domain represents two curved triangles bounded by the straight lines $\theta_C = 1 - \theta_A$, $\theta_C = \theta_A - 1$ and the upper branch of the hyperbole $\Delta_4 = 0$ and by the straight line $\theta_C = \theta_A - 1$ and the curves $\Delta_3 = 0$ and $\Delta_4 = 0$. As the parameter h_1 increases further, the domains with satisfied necessary and sufficient asymptotic stability conditions (13) for solution (9) nearly do not change and remain similar in shape to the curved triangles described above.

4. ANALYSIS OF TRANSITION PROCESSES

Now we analyze transition processes described by the system of differential equations (4), (5) with various values of the aerodynamic parameter and damping coefficients. System (4), (5) is rewritten in a standard form convenient for numerical integration under the condition $k_1 = k_2 = k_3 = k$:

$$\dot{p} = [(1 - \theta_C)qr - 3(1 - \theta_C)a_{32}a_{33} - kp]/\theta_A = 0,$$
$$\dot{q} = (\theta_C - \theta_A)rp - 3(\theta_C - \theta_A)a_{33}a_{31} - k(q - 1) - h_1 a_{13} = 0,$$

$$\begin{aligned} \dot{r} &= [(\theta_A - 1)pq - 3(\theta_A - 1)a_{31}a_{32} - kr + h_1 a_{12}]/\theta_C = 0; \\ \dot{\alpha} &= [(q\cos\gamma - r\sin\gamma)/\cos\beta] - 1, \\ \dot{\beta} &= q\sin\gamma + r\cos\gamma, \\ \dot{\gamma} &= p - (q\cos\gamma - r\sin\gamma)\tan\beta. \end{aligned} \tag{15}$$

System (15) was numerically integrated with fixed parameter values satisfying conditions (13). The damping coefficient k ranged from 0.5 to 2.0, while the aerodynamic parameter h_1, from 1.0 to 50.0. The solutions of system (15) were investigated in the cases $\theta_A = 0.8$, $\theta_C = 0.4$ and $\theta_A = 0.24$, $\theta_C = 0.95$. In the second case, we took into account that the indicated parameters were found optimal for a satellite with an aerogyroscopic stabilization system in the study of the dynamics of this system in [5]. Figures 2–4 give examples of transition processes described by system (15) with parameter values from the above-indicated ranges. In the computations the initial values of the variables were set equal to 0.001.

The figures show that, for small values of the damping coefficient and the aerodynamic torque, the transition process in the system is slow, and the value of τ is greater than 20 (Fig. 2a, $k = 0.5$, $h_1 = 1.0$). As the damping coefficient increases, the transition to the equilibrium is reduced. The transition time is less than 10 for $k = 1.0$ and $h_1 = 25.0$ (Fig. 3a) and less than 6 for $k = 2.0$ and $h_1 = 25.0$ (Fig. 3b). As the aerodynamic torque grows, the oscillation frequency of the satellite with respect to α and β is increased (Fig. 2b, $k = 0.5$, $h_1 = 25.0$; Fig. 4b, $k = 1.0$, $h_1 = 50.0$). For large values of the damping coefficient and the aerodynamic torque, the transition process in the system is completed in one revolution of the satellite on the orbit (Fig. 3b, $k = 2.0$, $h_1 = 25.0$, $\tau \approx 6$).

For nearly axisymmetric cases ($\theta_A = 0.24$, $\theta_C = 0.95$), the transition time with respect to the roll angle γ exceeds those with respect to α and β and increases considerably with growing aerodynamic torque (Fig. 4a, $k = 1.0$, $h_1 = 5.0$; Fig. 4b, $k = 1.0$, $h_1 = 50.0$).

5. CONCLUSIONS

The dynamics of the rotational motion of a satellite influenced by gravitational, aerodynamic, and active damping torques was studied.

In the case when the axes of the coordinate system attached to the satellite coincide with the axes of the orbital coordinate system, necessary and sufficient conditions for the asymptotic stability of the corresponding zero equilibrium position were obtained. The variations in the domain within the parameter plane (θ_A, θ_C), where the asymptotic stability conditions for equilibrium hold true, were analyzed for various values of the aerodynamic parameter and the damping coefficients. It was shown that the stability domain is enlarged as h_1 grows from 0 to 3. For $h_1 > 3$, the size of the stability domain remains nearly unchanged.

The transition process to the zero equilibrium position in the system was numerically studied for various values of the aerodynamic parameter and the damping coefficients.

The results obtained in this paper can be used in the design of artificial Earth satellites with an aerodynamic attitude control system.

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