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Generation of a Longitudinal Current by a Transverse Electromagnetic Field in Collisional Degenerate Plasma

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Abstract—From the Vlasov—Boltzmann kinetic equation for a collisional degenerate plasma, the electron distribution function is constructed in the quadratic approximation in the electric field strength. A formula for calculating the electric current is derived. It is shown that nonlinearity leads to the rise of a longitudinal electric current directed along the wave vector. The longitudinal current is orthogonal to the known transverse classical current obtained in the linear analysis. When the collision frequency tends to zero, all results obtained for a collisional plasma pass into the corresponding results for a collisionless plasma. The case of small wavenumbers is considered. It is shown that, when the collision frequency tends to zero, the expression for the current passes into the corresponding expression for the current in a collisionless plasma. Graphic analysis of the real and imaginary parts of the current density is performed. The dependence of the electromagnetic field oscillation frequency and electron—plasma-particle collision frequency on the wavenumber is studied.

Keywords: Vlasov—Boltzmann equation, degenerate plasma, collision frequency, electromagnetic field, transverse and longitudinal current.

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INTRODUCTION

In this work, we derive formulas for calculating the electric current in a degenerate plasma.

When solving the Vlasov—Boltzmann equation describing the behavior of a degenerate collisional plasma, we took into account quantities proportional to the square of the external electric field strength. These quantities were taken into account in the expansion of the distribution function and the collision integral.

In this nonlinear approach, it turned out that the electric current has two nonzero components. One component of the electric field is directed along the electric field strength. This component is exactly the same as in the linear analysis. It is the transverse current. Therefore, in the linear analysis, we obtain the well-known expression for the electric current.

The second nonzero component of the electric current has the second order of smallness with respect to the electric field strength; it is directed along the wave vector. It is the longitudinal current.

The longitudinal current is the result of the nonlinear analysis of the interaction between the electromagnetic field and a plasma.

The nonlinear effects in plasma have been studied for a long time (see [1—9]).

In [3], the nonlinear current was studied, in particular, in problems concerning the probability of decay processes. It should be noted that, in [2], the existence of a nonlinear current along the wave vector was mentioned (see formula (2.9) in [2]).

In experimental work [6], the contribution of the normal component of the surface current in the signal of the second harmonic was found. In [7, 8], the contribution of the generation of the nonlinear surface current on the interaction of a laser pulse with metal was studied. Nowadays, collisional plasma is used in various issues [10—14]. In [15—17], the generation of a longitudinal current by a transverse electromagnetic field was studied. In [15], the cases of a classical and Fermi—Dirac quantum plasma; in [16], the case of a Maxwellian plasma; and, in [17], the case of a degenerate plasma were considered. In [18], the generation of a longitudinal current by a transverse electromagnetic field in a collisional Fermi—Dirac plasma at an arbitrary temperature (i.e., at an arbitrary degree of degeneracy of electron gas) was studied.

1. SOLUTION OF THE VLASOV–BOLTZMANN EQUATION

Consider the Vlasov–Boltzmann equation describing the behavior of a collisional plasma with the BGK (Bhatnagar–Gross–Krook) collision integral:

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+e\left(\mathbf{E}+\frac{1}{c}[\mathbf{v},\mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}}=\nu(f^{(0)}-f). \tag{1.1}$$

In Eq. (1.1) f is the plasma electron distribution function; $\mathbf{E}$ and $\mathbf{H}$ are the components of the electromagnetic field; c is the speed of light; ν is the effective electron–plasma-particle collision frequency; $f^{(0)} = f_{\rm eq}(\mathbf{r}, v)$ is the locally equilibrium Fermi distribution; $f_{\rm eq}(r, v, t) = \Theta(\mathscr{E}_0(\mathbf{r}, t) - \mathscr{E})$, where $\Theta(x)$ is the Heaviside step function: $\Theta(x) = 1, x > 0$, $\Theta(x) = 0, x < 0$; $\mathscr{E}_0(\mathbf{r}, t) = mv_0^2(\mathbf{r}, t)/2$ is the perturbed electron energy on the Fermi surface; $v_0(\mathbf{r}, t)$ is the perturbed electron velocity on the Fermi surface; $\mathscr{E} = mv^2/2$ is the electron energy; and $\mathbf{p}_0(\mathbf{r}, t) = m\mathbf{v}_0(\mathbf{r}, t)$ is the perturbed electron momentum on the Fermi surface.

Let $\mathbf{P} = \mathbf{p}/p_0 = \mathbf{v}/v_0$, where $\mathbf{P}$ is the dimensionless electron momentum, $p_0 = mv_0$ is the electron momentum on the Fermi surface, and v_0 is the electron velocity on the Fermi surface.

Assume that, in the plasma, there is an electromagnetic field corresponding to a running harmonic wave: $\mathbf{E} = \mathbf{E}_0e^{i(\mathbf{kr}-\omega t)}$, $\mathbf{H} = \mathbf{H}_0e^{i(\mathbf{kr}-\omega t)}$. The electric and magnetic fields are coupled by the equality $\mathbf{H} = -(ic/\omega)\mathrm{rot}\,\mathbf{E}$.

Suppose that the wave vector is orthogonal to the electric field: $\mathbf{kE} = 0$. We will assume, for definiteness, that the wave vector is directed along the x-axis and the electric field, along the y-axis, i.e., $\mathbf{k} = k(1, 0, 0)$ and $\mathbf{E} = E_y(x, t)(0, 1, 0)$. Therefore,

$$\mathbf{H}=\frac{ck}{\omega}E_y\cdot(0,0,1),\quad [\mathbf{v},\mathbf{H}]=\frac{ck}{\omega}E_y\cdot(v_y,-v_x,0),$$

$$e\left(\mathrm{E}+\frac{1}{c}[\mathbf{v},\mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}}=\frac{e}{\omega}E_y\left[kv_y\frac{\partial f}{\partial p_x}+(\omega-kv_x)\frac{\partial f}{\partial p_y}\right],$$

and $[\mathbf{v}, \mathbf{H}]\dfrac{\partial f_0}{\partial \mathbf{p}} = 0$, because $\dfrac{\partial f_0}{\partial \mathbf{p}} \sim \mathbf{v}$.

Let us transform the locally equilibrium electron distribution function in a degenerate plasma. We have

$$f_{\rm eq}(x,v,t)=\Theta\left[\frac{v_0^2(x,t)-v^2}{v_0^2}\right]=\Theta\left[P_0^2(x,t)-P^2\right]=\Theta\left[P_0(x,t)-P\right]=f_{\rm eq}(P,x,t).$$

Here, $\mathbf{P}_0(x, t) = \mathbf{p}_0(x, t)/p_0 = \mathbf{v}_0(x, t)/v_0$ is the dimensionless perturbed electron momentum (velocity).

Let us consider the linearization of the function $f_{eq}(\mathbf{P}, x, t)$ about the Fermi surface by setting $P_0(x, t) = P_0 + \delta P_0(x, t)$ (here, $P_0 = 1$, because, on the Fermi surface, $P_0 = p_0/p_0 = 1$):

$$f_{\rm eq}(P,x)=\Theta[P_0(x,t)-P]=\Theta[1-P+\delta P_0(x,t)]=f_0(P)+\delta(1-P)\delta P_0(x,t),$$

where $\delta(x)$ is the Dirac delta function and $f_0(P) = \Theta(1 - P)$.

Now Eq. (1.1) can be rewritten as

$$\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+\frac{eE_y}{\omega}\left[kv_y\frac{\partial f}{\partial p_x}+(\omega-kv_x)\frac{\partial f}{\partial p_y}\right]+\nu f=\nu f_0(P)+\nu\delta(1-P)\delta P_0(x,t). \tag{1.2}$$

We will seek the solution of Eq. (1.2) in the form

$$f=f_0(P)+f_1+f_2, \tag{1.3}$$

where

$$f_1\sim E_y\sim e^{i(kx-\omega t)},\quad f_2\sim E_y^2\sim e^{2i(kx-\omega t)}.$$

We find $\delta P(x, t)$ from the law of conservation of particle number:

$$\int(f_{\rm eq}-f)\frac{2d^3p}{(2\pi\hbar)^2}=0.$$

From this equation, we find that

$$\delta P_0(x,t) = \int [f - f_0(P)] d^3P \left[\int \delta(1-P) d^3P\right]^{-1}.$$

Note that $\int \delta(1-P)d^3P = 4\pi\int_0^\infty \delta(1-P)P^2 dP = 4\pi$. Therefore,

$$\delta P_0(x,t) = \frac{1}{4\pi}\int [f - f_0(P)] d^3P.$$

Now Eq. (1.2) can be transformed to the integral equation

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + \nu f = \nu f_0(P) - \frac{eE_y}{\omega}\left[kv_y\frac{\partial f}{\partial p_x} + (\omega - kv_x)\frac{\partial f}{\partial p_y}\right] + \nu\delta(1-P)\frac{1}{4\pi}\int [f - f_0(P)] d^3P. \tag{1.4}$$

In this problem, we have two length parameters: $l_1 = v_0/\omega$ and $l_2 = 1/k$. Assume that, at both lengths l_1 and l_2, the variation in the electron energy under the action of the electric field E is much smaller than the electron energy on the Fermi surface, $\mathcal{E}_0 = mv_0^2/2$, i.e., $\alpha_1 = |eE|v_0/(\mathcal{E}_0\omega)$ and $\alpha_2 = |eE|/(k\mathcal{E}_0)$ are considered small parameters. We will use the method of successive approximation, assuming that $\alpha_1 \ll 1$ and $\alpha_2 \ll 1$. Then, Eq. (1.4) by virtue of (1.3) is equivalent to the equations

$$\frac{\partial f_1}{\partial t} + v_x\frac{\partial f_1}{\partial x} + \nu f_1 = -\frac{eE_y}{\omega}\left[kv_y\frac{\partial f_0}{\partial p_x} + (\omega - kv_x)\frac{\partial f_0}{\partial p_y}\right] + \nu\delta(1-P)\frac{1}{4\pi}\int f_1 d^3P \tag{1.5}$$

and

$$\frac{\partial f_2}{\partial t} + v_x\frac{\partial f_2}{\partial x} + \nu f_2 = -\frac{eE_y}{\omega}\left[kv_y\frac{\partial f_1}{\partial p_x} + (\omega - kv_x)\frac{\partial f_1}{\partial p_y}\right] + \nu\delta(1-P)\frac{1}{4\pi}\int f_2 d^3P. \tag{1.6}$$

From Eq. (1.5), we find that

$$(\nu - i\omega + ikv_x)f_1 = -\frac{eE_y}{\omega}\left[kv_y\frac{\partial f_0}{\partial p_x} + (\omega - kv_x)\frac{\partial f_0}{\partial p_y}\right] + \nu\delta(1-P)A_1,$$

where

$$A_1 = \frac{1}{4\pi}\int f_1 d^3P. \tag{1.7}$$

Introduce the dimensionless parameters

$$\Omega = \frac{\omega}{k_0v_0}, \quad y = \frac{\nu}{k_0v_0}, \quad q = \frac{k}{k_0}.$$

Here, q is the dimensionless wavenumber, $k_0 = \frac{mv_0}{\hbar}$ is the Fermi wavenumber, and Ω is the dimensionless oscillation frequency of the electromagnetic field.

In the previous equation, let us pass to the dimensionless parameters:

$$i(qP_x - z)f_1 = -\frac{eE_y}{\Omega k_0p_0v_0}\left[qP_y\frac{\partial f_0}{\partial P_x} + (\Omega - qP_x)\frac{\partial f_0}{\partial P_y}\right] + y\delta(1-P)A_1, \tag{1.8}$$

where

$$z = \Omega + iy = \frac{\omega + iy}{k_0v_0}.$$

Note that $\frac{\partial f_0}{\partial P_x} \sim P_x$ and $\frac{\partial f_0}{\partial P_y} \sim P_y$. Therefore,

$$qP_y\frac{\partial f_0}{\partial P_x} + (\Omega - qP_x)\frac{\partial f_0}{\partial P_y} = \Omega\frac{\partial f_0}{\partial P_y}.$$

From Eq. (1.8), we find that

$$f_1 = \frac{ieE_y}{k_0 p_0 v_0} \cdot \frac{\partial f_0/\partial P_y}{qP_x - z} - iy \cdot \frac{\delta(1-P)}{qP_x - z} A_1. \tag{1.9}$$

Substituting (1.9) into Eq. (1.7) yields the equality

$$A_1\left(1 + iy\int \frac{\delta(1-P)d^3P}{qP_x - z}\right) = \frac{ieE_y}{k_0 p_0 v_0}\int \frac{\partial f_0/\partial P_y}{qP_x - z} d^3P.$$

It is easy to see that the integral on the right-hand side of this equality is zero. Therefore, $A_1 = 0$ and, according to (1.9), the function f_1 has been constructed and is defined by the equality

$$f_1 = \frac{ieE_y}{k_0 p_0 v_0} \cdot \frac{\partial f_0/\partial P_y}{qP_x - z}. \tag{1.10}$$

In the second approximation, substituting f_1, according to (1.10), into Eq. (1.6) yields the equation

$$(\nu - 2i\omega + 2ikv_x)f_2 = -\frac{ie^2E_y^2}{k_0 p_0 v_0 \omega}\left[kv_y \frac{\partial}{\partial p_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + (\omega - kv_x)\frac{\partial}{\partial p_y}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right)\right] + \nu\delta(1-P)A_2,$$

where

$$A_2 = \frac{1}{4\pi}\int f_2 d^3P. \tag{1.11}$$

Passing in this equation to dimensionless parameters, we obtain the equation

$$2i\left(pP_x - x + \frac{iy}{2}\right)f_2 = -\frac{ie^2E_y^2}{\Omega k_0^2 p_0^2 v_0^2}\left[qP_x \frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + (\Omega - qP_x)\frac{\partial}{\partial P_y}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right)\right] + y\delta(1-P)A_2.$$

Set

$$z' = \Omega + \frac{iy}{2} = \frac{\omega}{k_0 v_0} + i\frac{\nu}{2k_0 v_0} = \frac{\omega + i\nu/2}{k_0 v_0}.$$

From the last equation, we find

$$f_2 = -\frac{e^2E_y^2}{2k_0^2 p_0^2 v_0^2 \Omega}\left[qP_y \frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + \frac{\Omega - qP_x}{qP_x - z}\frac{\partial^2 f_0}{\partial P_y^2}\right]\frac{1}{qP_x - z'} - \frac{iy}{2} \cdot \frac{\delta(1-P)}{qP_x - z'} A_2. \tag{1.12}$$

To find A_2, substitute (1.12) into (1.11). From the resulting equation, we find A_2:

$$A_2 = -\frac{e^2E_y^2}{2k_0^2 p_0^2 v_0^2 \Omega} \cdot \frac{J_1}{4\pi + (iy/2)J_0}.$$

Here,

$$J_0 = \int \frac{\delta(1-P)d^3P}{qP_x - z'} = 2\pi\int_{-1}^{1} \frac{d\tau}{q\tau - z'} = \frac{2\pi}{q}\ln\frac{z' - q}{z' + q},$$

$$J_1 = \int\left[qP_y \frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + \frac{\Omega - qP_x}{qP_x - z}\frac{\partial^2 f_0}{\partial P_y^2}\right]\frac{d^3P}{qP_x - z'}.$$

Substituting A_2 into (1.12), we find the function f_2 in the explicit form:

$$f_2 = -\frac{e^2E_y^2}{2k_0^2 p_0^2 v_0^2 \Omega}\left[qP_y \frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + \frac{\Omega - qP_x}{qP_x - z}\frac{\partial^2 f_0}{\partial P_y^2}\right]\frac{1}{qP_x - z'} + \gamma\frac{e^2E_y^2}{2k_0^2 p_0^2 v_0^2 \Omega} \cdot \frac{\delta(1-P)}{qP_x - z'}, \tag{1.13}$$

where

$$\gamma = \frac{(iy/2)J_1}{4\pi + (iy/2)J_0}. \tag{1.14}$$

2. ELECTRIC CURRENT DENSITY

Let us find the electric current density

$$\mathbf{j} = e\int \mathbf{v} f \frac{2d^3p}{(2\pi\hbar)^3}. \tag{2.1}$$

From equalities (1.4)–(1.6), it is evident that the vector of the current density has two nonzero components:

$$\mathbf{j} = (j_x, j_y, 0).$$

Here, j_y is the transverse component of the current density:

$$j_y = e\int v_y f \frac{2d^3p}{(2\pi\hbar)^3} = e\int v_y f_1 \frac{2d^3p}{(2\pi\hbar)^3}.$$

This current is directed along the electric field; its density is determined only by the first approximation of the distribution function.

The second approximation of the distribution does not contribute to this current.

The transverse component of the current density is determined by the equality

$$j_y = \frac{2ie^2p_0^2}{(2\pi\hbar)^3 k_0} E_y(x,t)\int \frac{(\partial f_0/\partial P_y)P_y}{qP_x - z} d^3P.$$

This current is proportional to the first degree of the electric field strength.

According to the definition of the transverse component of the current density, we have

$$j_x = e\int v_x f \frac{2d^3p}{(2\pi\hbar)^3} = e\int v_x f_2 \frac{2d^3p}{(2\pi\hbar)^3} = \frac{2ev_0p_0^3}{(2\pi\hbar)^3}\int P_x f_2 d^3P.$$

Hence, by virtue of (1.6), we obtain

$$j_x = \frac{e^3E_y^2 m}{(2\pi\hbar)^3 k_0^2\Omega}\left[-\int\left[qP_y\frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + \frac{x - qP_x}{qP_x - z}\cdot\frac{\partial^2 f_0}{\partial P_y^2}\right]\frac{P_x d^3P}{qP_x - z'} + \gamma\int\frac{P_x\delta(1-P)d^3P}{qP_x - z'}\right]. \tag{2.2}$$

In the integral of the second term in brackets, the inner integral with respect to P_y is zero:

$$\int\limits_{-\infty}^{\infty}\frac{\partial^2 f_0}{\partial P_y^2}dP_y = \left.\frac{\partial f_0}{\partial P_y}\right|_{P_y=-\infty}^{P_y=+\infty} = 0.$$

In the first integral in the brackets in (2.2), the inner integral with respect to P_x is integrated by parts:

$$\int\limits_{-\infty}^{\infty}\frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right)\frac{P_x dP_x}{qP_x - z'} = z'\int\limits_{-\infty}^{\infty}\frac{\partial f_0/\partial P_y}{(qP_x - z)(qP_x - z')^2}dP_x.$$

As a result, equality (2.2) is substantially simplified:

$$j_x = \frac{e^3E_y^2 m}{(2\pi\hbar)^3 k_0^2\Omega}\left[-z'q\int\frac{P_y(\partial f_0/\partial P_y)d^3P}{(qP_x - z)(qP_x - z')^2} + \gamma\int\frac{P_x\delta(1-P)d^3P}{qP_x - z'}\right].$$

The inner integral with respect to P_y is integrated by parts:

$$\int\limits_{-\infty}^{\infty}P_y\frac{\partial f_0}{\partial P_y}dP_y = P_y f_0\Big|_{P_y=-\infty}^{P_y=+\infty} - \int\limits_{-\infty}^{\infty}f_0(P)dP_y = -\int\limits_{-\infty}^{\infty}f_0(P)dP_y.$$

As a result, the expression for the longitudinal current is reduced to

$$j_x = \frac{e^3E_y^2 m}{(2\pi\hbar)^3 k_0^2\Omega}\left[z'q\int\frac{f_0(P)d^3P}{(qP_x - z)(qP_x - z')^2} + \gamma\int\frac{P_x\delta(1-P)}{qP_x - z'}d^3P\right]. \tag{2.3}$$

The inner integral in the plane (P_y, P_z) is calculated in polar coordinates:

$$\int \frac{f_0(P)d^3P}{(qP_x - z')^2(qP_x - z)} = \pi\int\limits_{-1}^{1} \frac{(1 - P_x^2)dP_x}{(qP_x - z)(qP_x - z')^2}.$$

In addition,

$$\int \frac{P_x\delta(1 - P^2)d^3P}{qP_x - z'} = \int\limits_{-1}^{1}\int\limits_{0}^{2\pi}\int\limits_{0}^{\infty} \frac{\delta(P - 1)\mu P^3 d\mu d\chi dP}{q\mu P - z'} = 2\pi\int\limits_{-1}^{1} \frac{\mu d\mu}{q\mu - z'} = 2\pi\left[\frac{2}{q} + \frac{z'}{q^2}\ln\frac{z' - q}{z' + q}\right].$$

Equality (2.3) is reduced to a one-dimensional integral:

$$j_x = \frac{e^3E_y^2m}{(2\pi\hbar)^3k_0^2\Omega}\left[z'q\pi\int\limits_{-1}^{1} \frac{(1 - P_x^2)dP_x}{(qP_x - z)(qP_x - z')^2} + \gamma 2\pi\int\limits_{-1}^{1} \frac{\tau d\tau}{q\tau - z'}\right],$$

or

$$j_x = \frac{\pi e^3E_y^2mq}{(2\pi\hbar)^3k_0^2\Omega}\left[z'J_{12} + \gamma\frac{2}{q}J_{02}\right],$$

where

$$J_{12} = \int\limits_{-1}^{1} \frac{(1 - \tau^2)d\tau}{(q\tau - z)(q\tau - z')^2}, \quad J_{02} = \int\limits_{-1}^{1} \frac{\tau d\tau}{q\tau - z'} = \frac{2}{q} + \frac{z'}{q^2}\ln\frac{z' - q}{z' + q}.$$

Let us find the number density of plasma particles, corresponding to the degenerate Fermi distribution

$$N = \int \Theta(1 - P^2)\frac{2d^3p}{(2\pi\hbar)^3} = \frac{2p_0^3}{(2\pi\hbar)^3}\int \Theta(1 - P^2)d^3P = \frac{k_0^3}{3\pi^2},$$

where k_0 is the Fermi wavenumber, $k_0 = \dfrac{mv_0}{\hbar}$.

In the expression preceding the integral in (2.4), let us separate the plasma (Langmuir) frequency $\omega_p = \sqrt{4\pi e^2N/m}$ and the number density N and express the latter via the Fermi wavenumber. As a result, we obtain

$$j_x^{\text{long}} = \left(\frac{e\Omega_p^2}{k_0p_0}\right)\frac{3kE_y^2}{32\pi\Omega q^3}\left(z'J_{12} + \frac{2\gamma}{q}J_{02}\right), \tag{2.5}$$

where

$$\Omega_p = \frac{\omega_p}{k_0v_0} = \frac{\hbar\omega_p}{mv_0^2}$$

is the dimensionless plasma frequency.

The expression for γ can be simplified by reducing the integrals entering into γ to one-dimensional integrals (see (1.15)). Note that

$$4\pi + \frac{iy}{2}J_0 = 4\pi + \pi iy\int\limits_{-1}^{1} \frac{d\tau}{q\tau - z'} = 2\pi J_{01},$$

where

$$J_{01} = \int\limits_{-1}^{1} \frac{q\tau - \Omega}{q\tau - z'}d\tau = 2 + \frac{iy}{2q}\ln\frac{z' - q}{z' + q}.$$

In addition,

$$J_1 = \int\left[qP_y\frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right) + \frac{\Omega - qP_x}{qP_x - z}\cdot\frac{\partial^2 f_0}{\partial P_y^2}\right]\frac{d^3P}{qP_x - z'} = q\int P_y\frac{\partial}{\partial P_x}\left(\frac{\partial f_0/\partial P_y}{qP_x - z}\right)\frac{d^3P}{qP_x - z'}$$

$$= -q^2\int\frac{P_y[\partial f_0/\partial P_y]d^3P}{(qP_x - z)(qP_x - z')^2} = q^2\int\frac{f_0(P)d^3P}{(qP_x - z)(qP_x - z')^2} = \pi q^2\int\limits_{-1}^{1}\frac{(1-\tau^2)d\tau}{(q\tau - z)(q\tau - z')^2} = \pi q^2 J_{12}.$$

As a result, we find

$$\gamma = \frac{(iy/2)J_1}{4\pi + (iy/2)J_0} = \frac{iy}{4}q^2\frac{J_{12}}{J_{01}}, \quad z'J_{12} + \frac{2\gamma}{q}J_{02} = \left[z' + \frac{iy}{2}q\frac{J_{02}}{J_{01}}\right]J_{12}.$$

Represent equality (2.5) in the form

$$j_x^{\text{long}} = J(\Omega, y, q)\sigma_{l,\text{tr}}kE_y^2, \tag{2.6}$$

where $\sigma_{l,\text{ tr}}$ is the longitudinal and transverse conductivity,

$$\sigma_{l,\text{tr}} = \frac{e\Omega_p^2}{p_0k_0} = \frac{e\hbar}{p_0^2}\left(\frac{\hbar\omega_p}{mv_0^2}\right)^2 = \frac{e}{k_0p_0}\left(\frac{\omega_p}{k_0v_0}\right)^2,$$

and $J(\Omega, y, q)$ is the dimensionless current density,

$$J(\Omega, y, q) = \frac{3}{32\pi\Omega}\left[\Omega + \frac{iy}{2}\left(1 + q\frac{J_{02}}{J_{01}}\right)\right]J_{12}.$$

Here,

$$q\frac{J_{02}}{J_{01}} = \frac{2q + \left(\Omega + \frac{iy}{2}\right)\ln\frac{z'-q}{z'+q}}{2q + \frac{iy}{2}\ln\frac{z'-q}{z'+q}} = 1 + \frac{\Omega\ln\frac{z'-q}{z'+q}}{2q + \frac{iy}{2}\ln\frac{z'-q}{z'+q}}.$$

After introducing the transverse field

$$\mathbf{E}_{\text{tr}} = \mathbf{E} - \frac{\mathbf{k}(\mathbf{Ek})}{k^2} = \mathbf{E} - \frac{\mathbf{q}(\mathbf{Eq})}{q^2}, \quad \mathbf{k}\mathbf{E}_{\text{tr}}^2 = \frac{\omega}{c}[\mathbf{E},\mathbf{H}],$$

equality (2.6) can be rewritten in the invariant form:

$$\mathbf{j}^{\text{long}} = J(\Omega, y, q)\sigma_{l,\text{tr}}\mathbf{k}\mathbf{E}_{\text{tr}}^2 = J(\Omega, y, q)\sigma_{l,\text{tr}}\frac{\omega}{c}[\mathbf{E},\mathbf{H}].$$

Remark 1. It is evident from formula (2.5) (or (2.6)) that, at $y = 0$ (or $\nu = 0$), i.e., when a collisional plasma passes into a collisionless one ($z \to \Omega$, $z' \to \Omega$), this formula exactly passes into the corresponding formula from [15] from a collisionless plasma:

$$j_x^{\text{long}} = \sigma_{l,\text{tr}}kE_y^2\frac{3}{32\pi}\int\limits_{-1}^{1}\frac{(1-\tau^2)d\tau}{(q\tau - \Omega)^3}.$$

Now let us consider the case of small wavenumbers. From expression (2.6) for small wavenumbers, we obtain

$$j_x^{\text{long}} = -\sigma_{l,\text{tr}}kE_y^2\frac{1}{8\pi\Omega(\Omega + iy)}.$$

Remark 2. For $y = 0$ ($\nu = 0$), this formula passes exactly into the corresponding formula (1.15) from [15] for the longitudinal current in the case of small wavenumbers in a collisionless plasma.

Figure 1 shows the behavior of the real (Fig. 1a) and imaginary (Fig. 1b) parts of the longitudinal component of the dimensionless current density for $\Omega = 1$ as a function of the dimensionless wavenumber q. Curves *1*, *2*, and *3* correspond to the values of the dimensionless electron–plasma-particle collision frequency $y = 0.04$, 0.07, and 0.1. At small and large values of the parameter q, curves *1*, *2*, and *3* approach

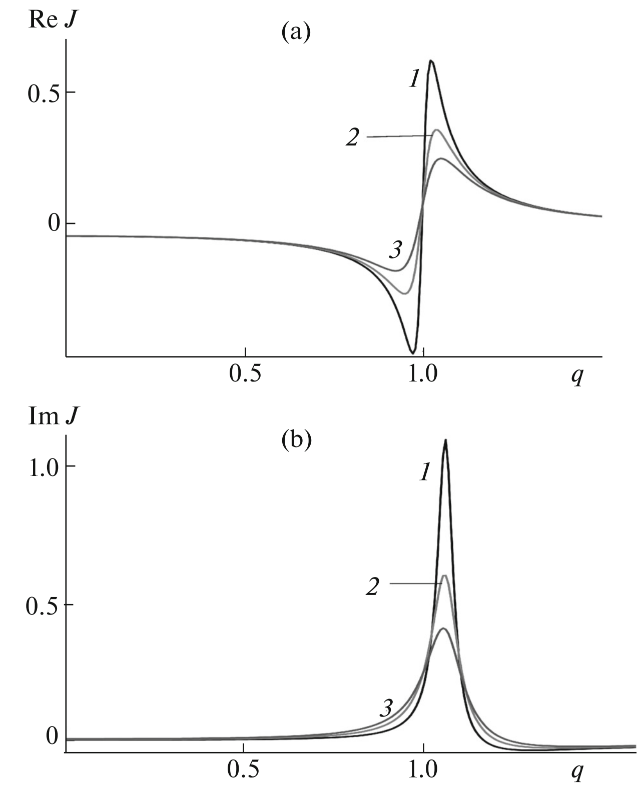
Re J
(a)
0.5
0
1
2
3
0.5
1.0
q
Im J
(b)
1.0
0.5
0
1
2
3
0.5
1.0
q

Fig. 1.

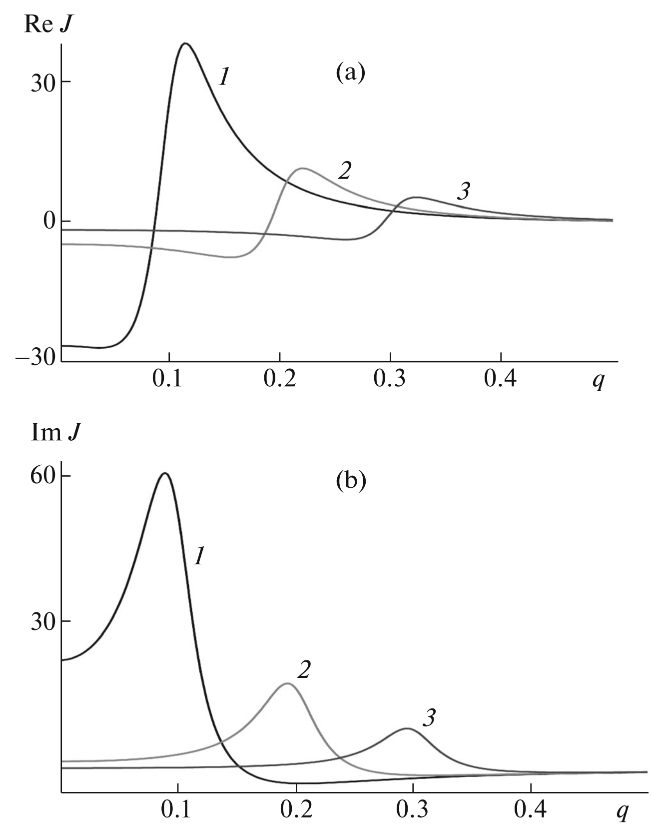
Re J
(a)
30
0
−30
1
2
3
0.1
0.2
0.3
0.4
q
Im J
(b)
60
30
1
2
3
0.1
0.2
0.3
0.4
q

Fig. 2.

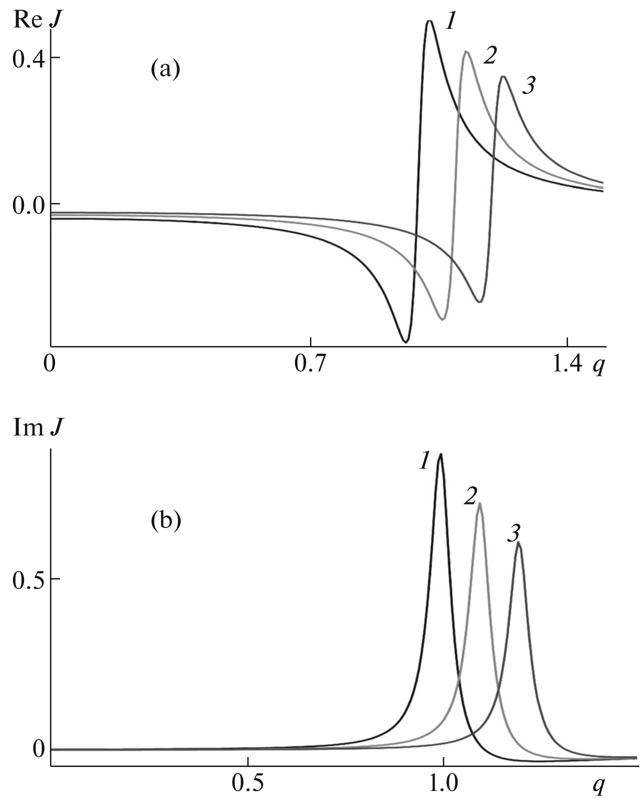

Fig. 3.

one another and become undistinguishable. At first, the real part has the minimum and, then, the maximum. With an increase in the dimensionless electron collision frequency, the imaginary part of the current density has one maximum.

Figure 2 shows the behavior of the real (Fig. 2a) and imaginary (Fig. 2b) parts of the longitudinal component of the current density as a function of the dimensionless wavenumber q for $y = 0.05$. Curves *1*, *2*, and *3* correspond to the values of the dimensionless oscillation frequency of the electromagnetic field $\Omega = 0.1$, 0.2, and 0.3. At first, the real part has the minimum and, then, the maximum. At large values of the dimensionless wavenumber, curves *1*, *2*, and *3* approach one another and become undistinguishable.

Figure 3 shows the behavior of the real (Fig. 3a) and imaginary (Fig. 3b) parts of the longitudinal component of the current density as a function of the dimensionless wavenumber q for $y = 0.05$. Curves *1*, *2*, and *3* correspond to the values of the dimensionless oscillation frequency of the electromagnetic field $\Omega = 1$, 1.1, and 1.2. At first, the real part has the minimum and, then, the maximum. The imaginary part has one maximum. With an increase in the dimensionless wavenumber q, curves *1*, *2*, and *3* approach one another and practically coincide.

CONCLUSIONS

In this work, we have considered the influence of the nonlinear character of the interaction of the electromagnetic field with a degenerate collisional plasma. It turned out that the presence of nonlinearity of the electromagnetic field in the Vlasov–Boltzmann equation leads to generation of an electric current orthogonal to the field.

REFERENCES

1. V. L. Ginzburg and A. V. Gurevich, "Nonlinear phenomena in a plasma located in an alternating electric field," Usp. Fiz. Nauk. **70** (2), 201–246 (1960).
2. L. M. Kovrizhkhykh and V. N. Tsytovich, "Effects of transverse electromagnetic wave decay in a plasma," Sov. Phys. JETP **20** (4), 978–983 (1965).

3. V. N. Tsytovich, "Nonlinear effects in plasmas," Usp. Fiz. Nauk **90** (3), 435–489 (1996).
4. V. N. Tsytovich, *Nonlinear Effects in Plasmas* (Leland, Moscow, 2014) [in Russian].
5. P. K. Shukla and B. Eliasson, "Nonlinear aspects of quantum plasma physics," Phys. Usp. **53**, 51–76 (2010).
6. N. N. Akhmediev, I. V. Mel'nikov, and L. J. Robur, "Second-harmonic generation be a reflecting metal surface," Laser Phys. **4** (6), 1194–1197 (1994).
7. S. G. Bezhanov and S. A. Uryupin, "Generation of nonlinear currents and low-frequency radiation upon interaction of a laser pulse with a metal," Quantum Electron. **43** (11), 1048–1054 (2013).
8. V. E. Grishkov and S. A. Uryupin, "Generation of nonlinear currents in the direction of propagation of a short laser pulse," *The 41st International (Zvenigorod) Conference on Plasma Physics and Controlled Fusion, February 10–14, 2014* (Zvenigorod, 2014), p. 315.
9. V. E. Grishkov and S. A. Uryupin, "Generation of a magnetic field in a weakly inhomogeneous plasma interacting with a short laser pulse," JETP Lett. **95** (12), 626–630 (2012).
10. M. Dressel and G. Grüner, *Electrodynamics of Solids: Optical Properties of Electrons in Matter* (Cambridge Univ. Press, Cambridge, 2003).
11. R. Fuchs and K. L. Kliewer, "Surface plasmon in a semi-infinite free-electron gas," Phys. Rev. B **3** (7), 2270–2278 (1971).
12. G. Brodin, M. Marklund, and G. Manfredi, "Quantum plasma effects in the classical regime," Phys. Rev. Lett. **100**, 175001-1–175001-4 (2008).
13. N. V. Gritsienko, A. V. Latyshev, and A. A. Yushkanov, "Plasma waves reflection from a boundary with specular accommodative boundary conditions," Comput. Math. Math. Phys. **50** (8), 1433–1446 (2010).
14. A. V. Latyshev and A. A. Yushkanov, "Electric field in the Smoluchowski problem in a metal with an arbitrary coefficient of specular reflection," Comput. Math. Math. Phys. **50** (3), 481–494 (2010).
15. A. V. Latyshev and A. A. Yushkanov, Generation of the longitudinal current by the transversal electromagnetic field in classical and quantum plasmas, arXiv:1503.02102 [physics.plasm-ph] Mar 6, 2015.
16. A. V. Latyshev and A. A. Yushkanov, "Generation of longitudinal electric current by transversal electromagnetic field in Maxwellian plasmas," arXiv:1503.04478 [physics.plasm-ph] Mar 15, 2015.
17. A. V. Latyshev and A. A. Yushkanov, "Generation of longitudinal electric current by transversal electromagnetic field in degenerate plasmas," arXiv:1503.08700v1 [physics.plasm-ph] Mar 30, 2015.
18. A. V. Latyshev and A. A. Yushkanov, "Generation of longitudinal electric current by the transversal electromagnetic field in collisional plasma," arXiv:1504.01412v2 [physics.plasm-ph] Apr 12, 2015.

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