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Parametric performance analysis of multiple reheat cycle for hydrogen fueled scramjet with multi-staged fuel injection *

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Multi-staged fuel injection is a good choice for hydrogen fueled scramjet engine to overcome the restriction of thermal choke and over-temperature. From a thermodynamic perspective, the multi-staged fuel injection is the practical application of reheat cycle. A parametric performance model has been developed for the hydrogen-fueled scramjet with multi-staged fuel injection to analyze its performance. The key parameters which affect the engine performance of scramjet with multi-staged fuel injection are total combustor area expansion ratio, reheat times and distribution ratio of area expansion. These parameters were carefully analyzed to provide some direct and transparent results for engine designers. The results showed that the specific thrust of scramjet can be greatly improved by increasing the total combustor area expansion ratio and reheat times, and/or choosing an appropriate distribution proportion of area expansion ratio. The effect of increasing the total combustor area expansion is most obvious for performance enhancement.

Keywords: reheat cycle; multi-staged fuel injection; hydrogen fueled scramjet; specific thrust; specific impulse.

Introduction

Supersonic combustion ramjet (scramjet) engines are expected to be most suitable for use as propulsion systems of trans-atmospheric vehicle applications, and thus many studies on such engines have been conducted in a few countries like USA, Russia and China [1–8]. Hydrogen fuel possesses superior characteristics to any other hydrocarbon fuel in terms of ignitability, low ignition delay and higher flame stability. These inherent advantages turn the hydrogen fueled scramjet engines received increased attention [9–12].

Such a propulsion system must operate under a wide range of Mach numbers, it requires more regulation methods to make the engine work efficiently and safely throughout the whole flight envelope. It was found that there is sufficient preponderance in multi-staged fuel injection [2, 6, 10, 13–15]. In the work [6], an expression was derived for the maximum admissible

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increment of entropy in a steady gas flow in a variable-section channel with energy supply and dissipation of kinetic energy. A condition of the transition through the velocity of sound was obtained for a quasi-one-dimensional flow. After that, publications [2,10] described a one-dimensional functional mathematical model of a hydrogen-driven combustion chamber for a scramjet considering fuel injection in three cross sections of the channel consisting of segments with weak and strong expansion. It was found that the angle between the velocity vectors of the gaseous hydrogen flow and the main gas flow can be fairly large in the case of distributed injection of the fuel and it allows effective control of the mixing process. A staged supersonic combustor with a strut for the first-stage injection and second-stage wall injectors at its divergent section was introduced in [13]. It was studied experimentally in a directly connected wind tunnel facility and it was found that the maximum thrust increment was augmented by 120%. A scramjet engine with two-staged hydrogen injection by one-dimension numeric method within the acceleration from Mach 4 to 7 was simulated in [14]. It was found that better thrust performance can be achieved as more fuel injected at the upstream fuel injector as possible, while ensuring the engine safety. It was found in [15] that better pressure distributions and higher thrust can be attained in a staged supersonic combustor.

One benefit of multi-staged fuel injection is that it can mitigate or avoid the combustor-inlet interaction and thereby prevent the inlet from unstating. Another more notable and substantial benefit of multi-staged fuel injection is that it can improve the thrust performance within the restrictions of thermal choke and over-temperature caused by intense heat release.

From a thermodynamic perspective, the multi-staged fuel injection is the practical application of reheat cycle. The concept of reheat cycle was first introduced into the steam power cycle system to remove the moisture carried by the steam at the final stages of the expansion process and further increase the output work of steam power cycle system under the restriction of turbine inlet temperature. Similar situation also was encountered in the operation of turbojet engine. The highest gas temperature in the turbojet is always at the first stage of the turbine and the ability of high-temperature resistance for the turbine is one of the primary restrictions on total engine thrust. To avoid the blade being burnt, the fuel injection must be restricted in a certain range. In this manner, the air captured by the inlet will not be fully utilized and it still has a great potential to improve the engine performance. Thus, the reheat cycle was introduced into turbojet and implemented through an afterburner. In a turbojet with an afterburner, more fuel is injected again downstream of the turbine to increase the temperature and pressure of airflow and engine performance. But it should be noted that the influence of reheat cycle on the cycle thermal efficiency is completely different for the steam turbine (Rankine cycle) and turbojet engine (Brayton cycle). For a Rankine cycle, reheat cycle increases the average temperature of heat addition and improves the cycle thermal efficiency, whereas for a Brayton cycle, reheat cycle increases the average temperature of heat rejection and lowers the cycle thermal efficiency.

A physical model of scramjet typically includes an inlet, an isolator, a combustor and a nozzle. A detailed description about the thermodynamic cycle of scramjet engine had been advanced in the work [16]. The results showed that for a hydrogen fueled scramjet with single-staged fuel injection, the maximum amount of heat addition is limited due to the restrictions of thermal choke and over-temperature. Because the calorific value of hydrogen fuel is much higher than for other hydrocarbon fuel, the maximum equivalence ratio of scramjet is limited to far less than one. To further utilize the captured airflow and improve the specific thrust performance under these restrictions, the reheat cycle should be employed for the scramjet according to the experience of its application on the turbojet. Thus, the maximum amount of fuel injection can be further increased and the thrust can be further improved without needing to capture more airflow.

This study was, therefore, motivated by the desire to establish a performance model for the hydrogen fueled scramjet with multi-staged fuel injection from the viewpoint of reheat

cycle analysis and further to provide some direct and transparent results to help the scramjet engine designer improve the engine performance and broaden the operation range. In this article, hence, a parametric performance model of multiple reheat cycle was presented. The key parameters which affect the engine performance were discussed respectively and their basic design principles were given.

Limitations analysis of single-staged fuel injection

Inlet unstart limitation for low flight Mach number

For a certain design level of scramjet engine, there is an inlet unstart boundary for the compression system (including inlet and isolator), which means that there is a maximum value for the pressure ratio between the exit and entrance pressure of the compression system. For a given flight Mach number, if the combustor entrance pressure (backpressure for inlet) is too high and the pressure ratio exceeds the inlet unstart boundary, inlet will unstart and the performance of engine will dramatically drop. The inlet unstart boundary is determined by many factors and it is found that as the flight Mach number decreases, the maximum allowed value of the pressure ratio decreases [17]. This means that it is easier to unstart for a scramjet at a low flight Mach number.

For a scramjet with single-staged fuel injection, the fuel is mostly injected at the entrance of the combustor. As the fuel increases, a shock or shock train will appear in the isolator and move forward due to the backpressure. When the fuel equivalence ratio reaches a large enough value, the high backpressure caused by the strong heat release will push the shock out the throat of the inlet and thus an inlet unstart occurs. When the scramjet works at a high flight Mach number, the allowed maximum fuel equivalence ratio is high enough to reach the stoichiometric value. However, when the scramjet works at a low flight Mach number, the inlet will unstart at a low fuel equivalence ratio. This means that due to the inlet unstart, the fuel equivalence ratio for a scramjet with single-staged fuel injection is far less than its stoichiometric fuel equivalence ratio at a low flight Mach number. It will result in the underutilization of the air captured by inlet and the loss of the thrust.

Over-temperature limitation for high Mach number

Due to the temperature limit of the material, the maximum temperature of a thermal cycle is limited, and so consequently, the heat quantity which can be added into the cycle is also limited. For a scramjet operating at a high flight Mach number, the total temperature of incoming flow is very high. After the strong compression by the inlet, the combustor entrance temperature becomes high enough. For a scramjet with single-staged fuel injection, the fuel is mostly injected at the entrance of the combustor. The localized strong fuel combustion causes a localized high temperature zone near the downstream of the fuel injectors, which may lead to the temperature excess. This means that due to the over-temperature limitation, the fuel equivalence ratio for a scramjet with single-staged fuel injection cannot reach its stoichiometric fuel equivalence ratio at a high flight Mach number. It also will result in the underutilization of the air captured by inlet and the loss of the thrust.

Performance model of multiple reheat cycle

Scheme of multiple reheat cycle

Figure 1 shows the basic physical model of multiple reheat cycle for a scramjet engine with multi-staged fuel injection. For the basic thermodynamic cycle of a scramjet with the single fuel injection, the heat addition is limited due to the restrictions like thermal choke and over-temperature. So, for a multiple reheat cycle, the combustor is divided into several segments and the fuel is added into each sub-combustor separately. Some divergent ducts connect

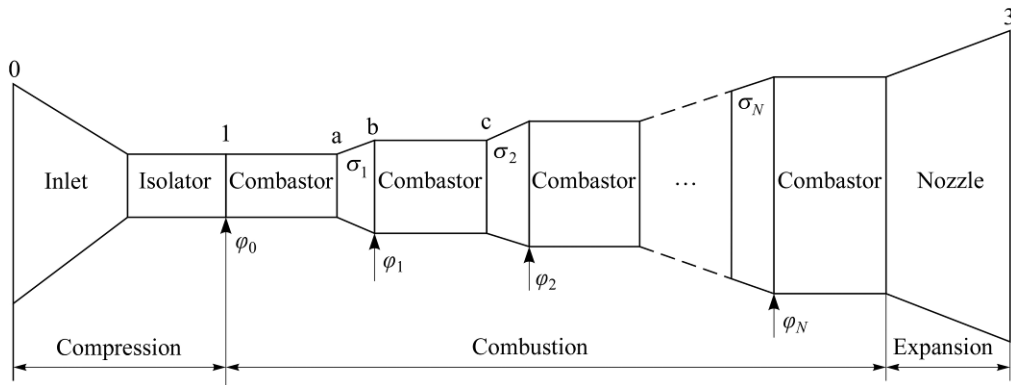


Fig. 1. The basic physical model of multiple reheat cycle.

the sub-combustors and are used as diffusers. In the diffuser, the heated supersonic airflow accelerates. Its temperature decreases and Mach number increases. After that, the airflow enters the next sub-combustor to absorb heat further with supersonic combustion. Thus, the whole heat addition process is split into multiple stages. Like the multiple reheat gas turbine cycle, the heat absorption for per unit of airflow will increase with the number of reheated times increasing, and more heat can be converted to thrust without needing to increase the airflow.

From the basic physical model of multiple reheat cycle shown in Fig. 1, the total fuel equivalence ratio can be defined as

$$\varphi_t = \sum_{j=0}^N \varphi_j, \tag{1}$$

and the total combustor area expansion ratio can be defined as

$$\sigma_t = \prod_{j=1}^N \sigma_j, \tag{2}$$

where N is the number of reheat times.

As an example, the T - s diagram and H - K diagram for the multiple reheat cycle of a scramjet are shown in Figs. 2 and 3 separately. The H - K diagram can reflect the change of the enthalpy and kinetic energy of the airflow and hence, is a productive tool for the analysis

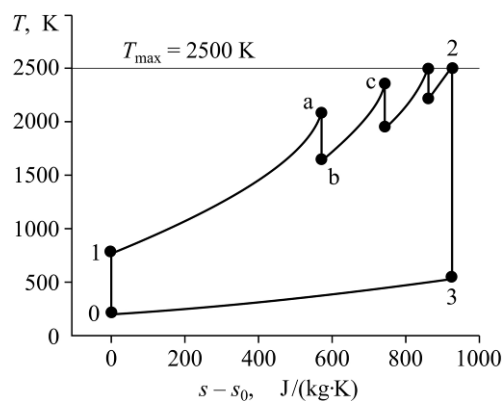


Fig. 2. T - s diagram for multiple reheat cycle of a scramjet.

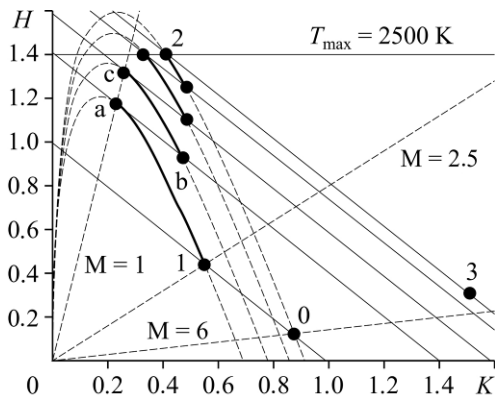


Fig. 3. H - K diagram for multiple reheat cycle of a scramjet.

of the thermodynamic process of supersonic flow [1]. In this example, $M_0 = 6$, $N = 3$, $\varphi_t = 0.39$ and $\sigma_t = 1.59$ were used. It can be seen from the figures that the first two heat addition processes were limited by the thermal choke and the last two were limited by the over-temperature.

Basic assumptions

In most cases of thermodynamic cycle analysis, some assumptions must be made to simplify the analysis process. In this study, some basic assumptions were given in the following.

1. For the physical model, it was assumed that the sub-combustor was a constant cross section duct [13, 18]. Under this assumption, the combustion process becomes constant area heating and the Rayleigh flow could be easily used to model it. Besides, it was assumed that the heat was added only in the constant cross-section combustor and in the divergent section the total enthalpy of airflow kept constant.

- 2. The compression and expansion processes were isentropic and adiabatic.
- 3. No mass addition, no heat transfer loss and no friction loss were considered.
- 4. Calorically perfect gas conditions of constant specific heat were used.

Performance model

The mathematical analysis presented here followed and developed from the basic approach presented in detail in [19–22].

1. Compression process.

For a scramjet, the Mach number of the compressed airflow is an important parameter and is determined by the engine inlet. For a specified M_1 , the compression static temperature-rise ratio can be written as

$$\psi = \frac{2 + (k - 1)M_0^2}{2 + (k - 1)M_1^2} \tag{3}$$

The maximum of compression static temperature-rise ratio can be given as

$$\psi_{\max} = \frac{2 + (k - 1)M_0^2}{k + 1} \tag{4}$$

2. Heat addition process for sub-combustor.

According to the Rayleigh flow, the entropy increase for a sub-combustor can be given by

$$\frac{s_{c,j} - s_{b,j}}{C_p} = \ln \left[\frac{M_{c,j}^2}{M_{b,j}^2} \left(\frac{1 + kM_{b,j}^2}{1 + kM_{c,j}^2} \right)^{(k+1)/k} \right], \quad j = 0, 1, \dots, N. \tag{5}$$

The local Mach number at the termination of the heat addition process can be obtained using the energy conservation equation between stations b and c .

$$C_p T_{b,j} \left(1 + \frac{k-1}{2} M_{b,j}^2 \right) + \varphi_j H_{PR} f_{st} = C_p T_{c,j} \left(1 + \frac{k-1}{2} M_{c,j}^2 \right), \quad j = 0, 1, \dots, N. \tag{6}$$

Static temperature ratio $T_{c,j}/T_{b,j}$ can easily be obtained through analysis of Rayleigh flow model. Substitute it into Eq. (6) and we get

$$1 + \frac{\varphi_j H_{PR} f_{st}}{C_p T_{b,j} \left(1 + \frac{k-1}{2} M_{b,j}^2 \right)} = \frac{M_{c,j}^2 (1 + kM_{b,j}^2)^2}{M_{b,j}^2 (1 + kM_{c,j}^2)^2} \cdot \frac{2 + (k-1)M_{c,j}^2}{2 + (k-1)M_{b,j}^2}, \quad j = 0, 1, \dots, N. \tag{7}$$

Consequently, the total entropy increase throughout the multiple heat addition process can be written as

$$\frac{s_2 - s_1}{C_p} = \sum_{j=0}^N \frac{s_{c,j} - s_{b,j}}{C_p} = \sum_{j=0}^N \ln \left[\frac{M_{c,j}^2 \left(\frac{1 + kM_{b,j}^2}{1 + kM_{c,j}^2} \right)^{(k+1)/k}}{M_{b,j}^2} \right] = \ln \left[\frac{M_2^2 \left(\frac{1 + kM_1^2}{1 + kM_2^2} \right)^{(k+1)/k}}{M_1^2} \right]. \quad (8)$$

3. Expansion process for divergent duct.

The expansion processes in the divergent duct is isentropic. Thus, according to mass conservation equation, the exit Mach number can be expressed as

$$q(M_{b,j}) = \frac{q(M_{a,j})}{\sigma_j}, \quad j = 1, 2, \dots, N. \quad (9)$$

4. Performance parameter.

The cycle thermal efficiency η_{th} can be written [13] as

$$\eta_{th} = 1 - \frac{C_p T_0 \left[\exp \left(\frac{s_2 - s_1}{C_p} \right) - 1 \right]}{\varphi_t H_{PR} f_{st}}. \quad (10)$$

Once the cycle thermal efficiency η_{th} is known, more traditional uninstalled performance parameters, such as specific thrust F_s and specific impulse I_{sp} can be further determined.

$$F_s = \sqrt{V_0^2 + 2\eta_{th}\varphi_t H_{PR} f_{st}} - V_0, \quad (11)$$

$$I_{sp} = \frac{F_s}{\varphi_t f_{st} g}. \quad (12)$$

Results and discussion

The motivation for introducing multi-staged fuel injection into a hydrogen fueled scramjet is to broaden the operation range of the engine under the restrictions of thermal choke and over-temperature. Hence, for the performance analysis of reheat cycle of scramjet, more attention should be paid to the maximum performance of the cycle. In the previous studies, it was found that the specific thrust of scramjet is the function of φ_t and ψ for a constant M_0 [13]. As mentioned in the Introduction, the restrictions determine the maximum amount of heat addition and further determine the maximum specific thrust. It should be noted that in this study, there are two maximum values of specific thrust which were analyzed. The first one is the maximum value of F_s for a given ψ corresponding to the maximum total fuel equivalence ratio. It was defined as the maximum specific thrust and denoted by $F_{s,\psi}$. The second one is the maximum value of $F_{s,\psi}$ corresponding to the optimum compression static temperature ratio and maximum total fuel equivalence ratio. It was defined as limit specific thrust and denoted by $F_{s,lim}$.

For a reheat cycle, there are a few factors influencing its performance mainly: total combustor area expansion ratio σ_t , reheat times N , and distribution ratio of area expansion $\chi = (\sigma_1 - 1)/(\sigma_t - 1)$ (for $N=2$). They were discussed separately in the following sections. In the following analysis, the values for $H_{PR}=140000$ kJ/kg, $T_0=217$ K, $T_{max}=2500$ K, $C_p=1.004$ kJ/(kg·K), $M_0 = 6$, and $k=1.4$ were used.

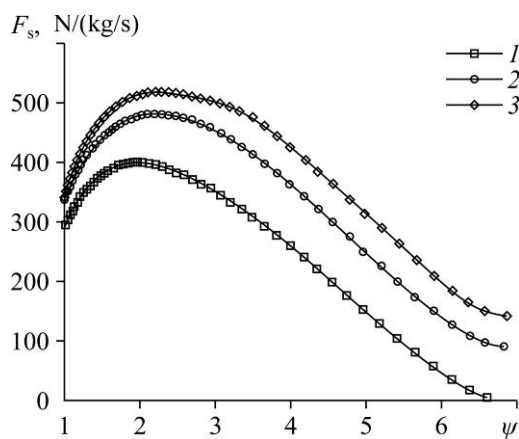


Fig. 4. Variation of maximum specific thrust with compression static temperature-rise ratio for different area expansion ratio. $\sigma = 1$ (1), 1.2 (2), and 1.4 (3).

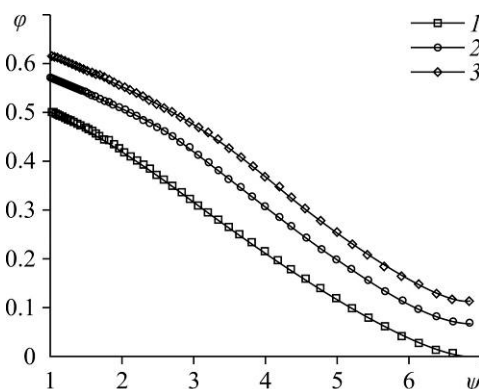


Fig. 5. Variation of fuel equivalence ratio with compression static temperature-rise ratio for different area expansion ratio. $\sigma = 1$ (1), 1.2 (2), and 1.4 (3).

Variation of area expansion ratio σ_t

As shown in Figs. 4–7, the performance of thermodynamic cycle with different σ_t were compared. In this analysis, $N = 1$ was chosen and so $\sigma_t = \sigma_1$. As shown in Fig. 4 and presented in [13], the maximum specific thrust $F_{s,\psi}$ increases at first as compression static temperature-rise ratio ψ increases for any σ_t . When compression static temperature-rise ratio increases to a certain degree, the maximum specific thrust decreases due to the limitation of the over-temperature limitation. This means that there is an optimum compression static temperature ratio ψ_{opt} for a given M_0 , N , and σ_t .

It can be seen in Fig. 4 that maximum specific thrust $F_{s,\psi}$ increases with σ_t for any ψ . The main reason is that the greater the area expansion ratio is, the lower entrance static temperature and/or the high entrance Mach number can be obtained for the sub-combustor. So, more heat can be added to the cycle and this result can be apparently seen in Fig. 5. Besides, it can be found from Fig. 4 that ψ_{opt} increases with σ_t . This takes place because a bigger combustor area expansion ratio leads to a lower temperature for the sub-combustor and allows more heat to be added in it. The results shown in Figs. 6 and 7 indicate that increase of area expansion ratio σ_t has little influence on the thermal efficiency and specific impulse of scramjet. This means that the increase of area expansion ratio is beneficial for scramjet engine.

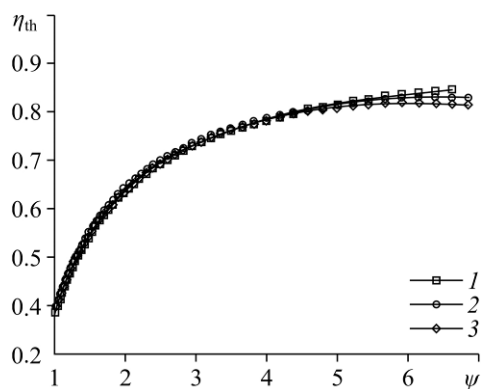


Fig. 6. Variation of thermal efficiency with compression static temperature-rise ratio for different area expansion ratio. $\sigma = 1$ (1), 1.2 (2), and 1.4 (3).

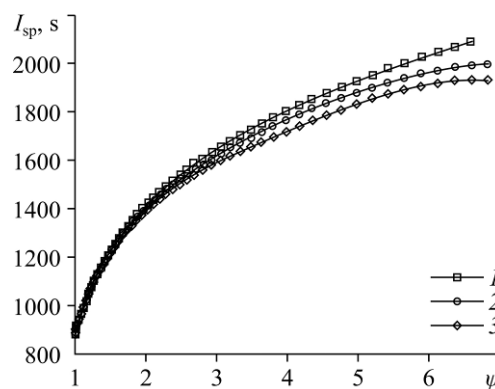


Fig. 7. Variation of specific impulse with compression static temperature-rise ratio for different area expansion ratio. $\sigma = 1$ (1), 1.2 (2), and 1.4 (3).

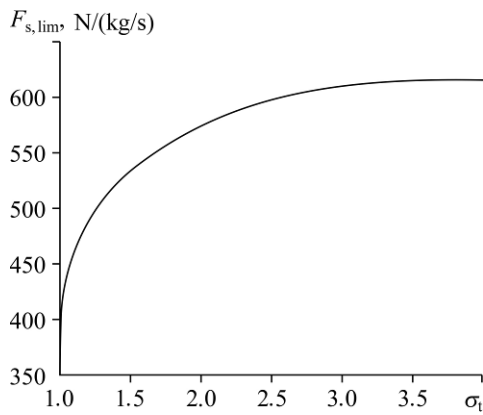


Fig. 8. Variation of limit specific thrust with total area expansion ratio.

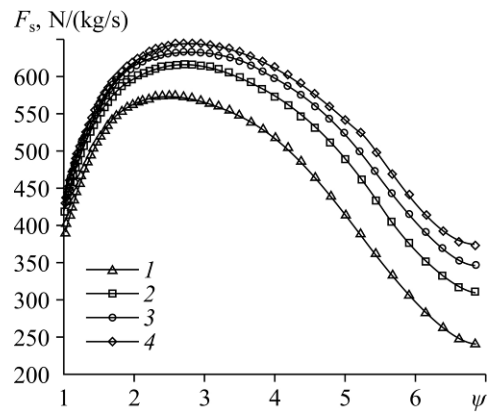


Fig. 9. Variation of maximum specific thrust with compression static temperature-rise ratio for different reheat times. $N = 1$ (1), 2 (2), 3 (3), and 4 (4).

As shown in Fig. 8, with a large area expansion, the limit specific thrust can be improved by 50 percent. However, the trend for increase of limit specific thrust $F_{s,lim}$ becomes slow with increase of area expansion ratio σ_t . Furthermore, the increase of combustor area expansion ratio will lead to a great increase of the windward area of the scramjet engine, which will cause a great deal of flight drag. Therefore, an appropriate area expansion ratio should be determined in the design process of the scramjet with multi-staged fuel injection.

Variation of reheat times N

As shown in Figs. 9–12, the performances of thermodynamic cycle with different N were compared. In this analysis, $\sigma_t = 2$ was chosen. The area expansion ratio is divided equally among all of reheat section and hence $\sigma_t = \sigma_t^{1/N}$. It can be seen in Fig. 9 that maximum specific thrust $F_{s,\psi}$ at first increases and then decreases as compression static temperature-rise ratio ψ increases for any N . It can also be found that the maximum specific thrust $F_{s,\psi}$ increases with N for any ψ and ψ_{opt} also increases with N . The corresponding maximum fuel equivalence ratio was shown in Fig. 10. The results shown in Figs. 11 and 12 indicate that the thermal efficiency and specific impulse of scramjet slightly increase with increasing reheat times N . This means that the increase of reheat times is favorable for improving the performance of scramjet with multi-staged fuel injection.

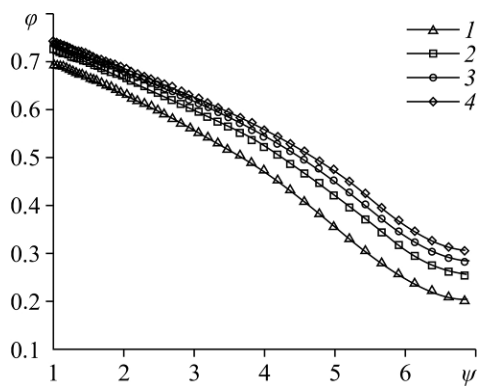


Fig. 10. Variation of fuel equivalence ratio with compression static temperature-rise ratio for different reheat times. $N = 1$ (1), 2 (2), 3 (3), and 4 (4).

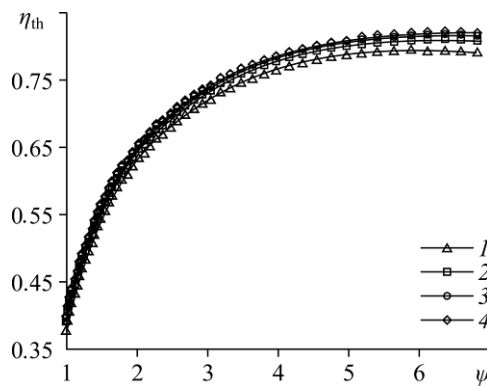


Fig. 11. Variation of thermal efficiency with compression static temperature-rise ratio for different reheat times. $N = 1$ (1), 2 (2), 3 (3), and 4 (4).

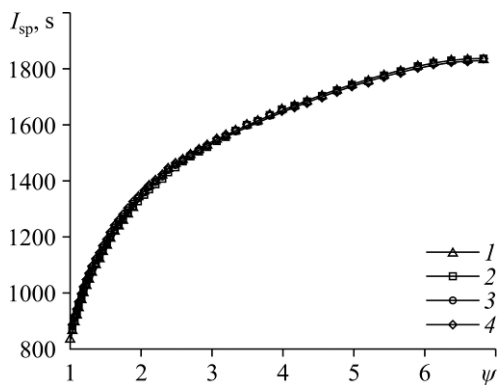


Fig. 12. Variation of specific impulse with compression static temperature-rise ratio for different reheat times. $N = 1$ (1), 2 (2), 3 (3), and 4 (4).

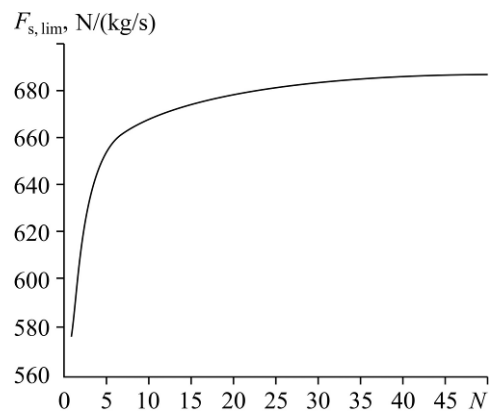


Fig. 13. Variation of limit specific thrust with reheat times.

As shown in Fig. 13, with large reheat times, the limit specific thrust can be improved by about 17 percent. However, the trend for increase of limit specific thrust $F_{s,lim}$ becomes slow with increase of reheat times N . Furthermore, the increase of reheat times N will lead to a great increase of the length and weight of the combustor. Therefore, an appropriate reheat times should be determined in the design process of the scramjet with multi-staged fuel injection.

Variation of distribution proportion of area expansion χ

As shown in Figs. 14–17, the performance of thermodynamic cycle at different χ were compared. In this analysis, $\sigma_t = 2$ and $N = 2$ were chosen. As in the previous case, the maximum specific thrust $F_{s,\psi}$ increases at first and then decreases as compression static temperature-rise ratio ψ increases for any σ_t .

According to the performance model of reheat cycle, the case $\chi = 0$ is identical with the case $\chi = 1$. So as shown in Fig. 14, maximum specific thrust $F_{s,\psi}$ increases at first and then decreases with χ for any ψ . It means that there is an optimum distribution proportion of area expansion χ_{opt} that makes $F_{s,\psi}$ maximum. This trend can also be apparently found in Fig. 18. Figure 15 illustrates the maximum fuel equivalence ratio as a function of ψ . It can be seen that

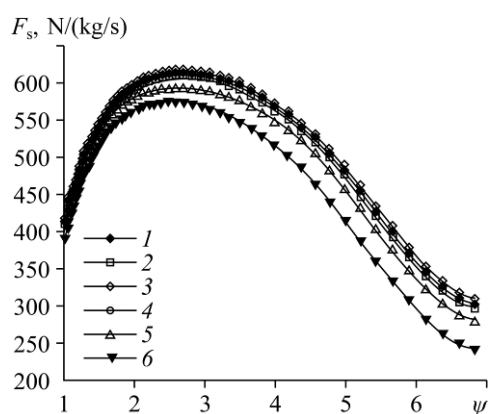


Fig. 14. Variation of maximum specific thrust with compression static temperature-rise ratio for different distribution proportion. $\sigma = 1.0$ (1), 1.2 (2), 1.4 (3), 1.6 (4), 1.8 (5), 2.0 (6).

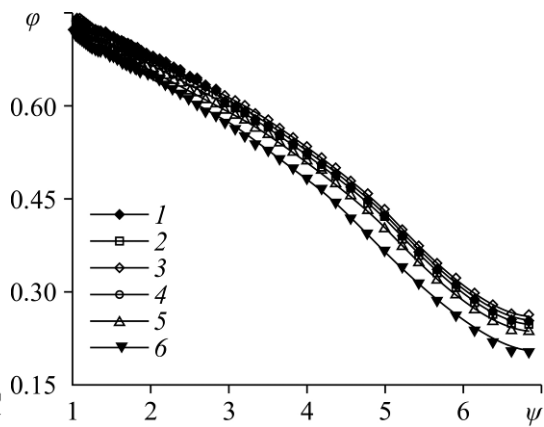


Fig. 15. Variation of fuel equivalence ratio with compression static temperature-rise ratio for different distribution proportion. $\sigma = 1.0$ (1), 1.2 (2), 1.4 (3), 1.6 (4), 1.8 (5), 2.0 (6).

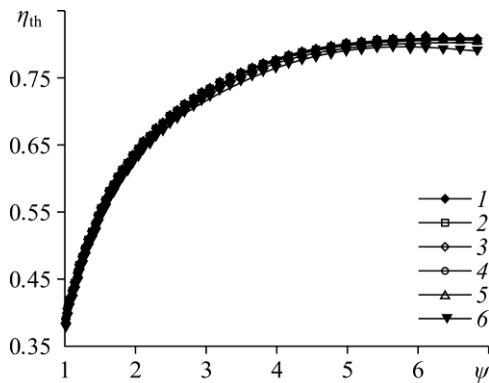


Fig. 16. Variation of thermal efficiency with compression static temperature-rise ratio for different distribution proportion. $\sigma = 1.0$ (1), 1.2 (2), 1.4 (3), 1.6 (4), 1.8 (5), 2.0 (6).

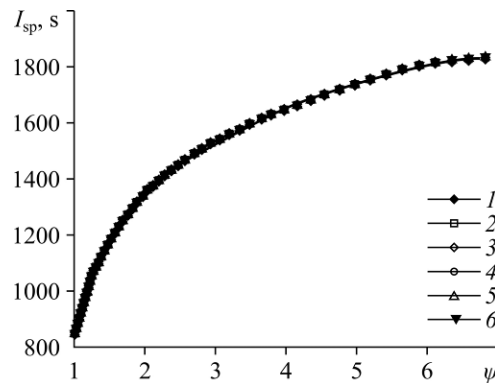


Fig. 17. Variation of specific impulse with compression static temperature-rise ratio for different distribution proportion. $\sigma = 1.0$ (1), 1.2 (2), 1.4 (3), 1.6 (4), 1.8 (5), 2.0 (6).

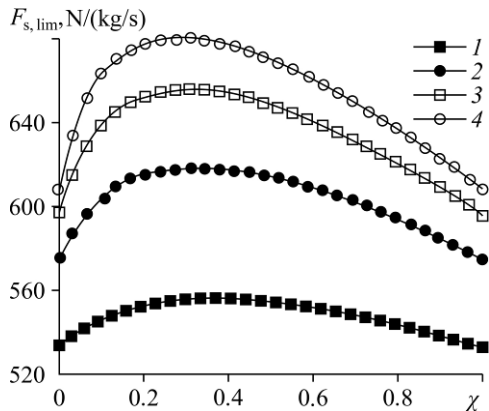


Fig. 18. Variation of limit specific thrust with distribution proportion. $\sigma_t = 1.5$ (1), 2 (2), 2.5 (3), 3 (4).

The increase of $F_{s,\psi}$ is mainly caused by the increase of heat addition. The results shown in Figs. 16 and 17 indicate that the alteration of distribution proportion of area expansion has little influence on the thermal efficiency and specific impulse of scramjet.

Depicted in Fig. 18 is the limit specific thrust $F_{s,lim}$ as a function of χ with σ_t as a parameter. It is seen that χ_{opt} decreases as σ_t increases.

Hence, for a scramjet engine with two-staged fuel injection, though the area expansion ratio for the first-stage should also be increased as the total area expansion ratio increases, the percentage of area expansion for the first-stage should be reduced for performance consideration. With an appropriate distribution of area expansion ratio, the limit specific thrust can be improved by about 10 percent.

As analyzed above, the specific thrust performance of scramjet can be improved by increasing the total combustor area expansion ratio and reheat times, and/or choosing an appropriate distribution proportion of area expansion ratio. The effect of increasing the total combustor area expansion is most obvious for performance enhancement.

Conclusion

The performance of hydrogen fueled scramjet engine is limited by the restrictions of thermal choke and over-temperature. Multi-staged fuel injection is a good choice to overcome these restrictions. From a thermodynamic perspective, the multi-staged fuel injection is the practical application of reheat cycle, which is a mature technology applied in the steam turbine and turbojet engine. In this paper, a parametric performance model has been developed for the hydrogen fueled scramjet with multi-staged fuel injection to analyze its performance. The thermal efficiency, specific thrust and specific impulse of scramjet engine can be obtained using this model.

The key parameters which affect the engine performance of scramjet with multi-staged fuel injection are total combustor area expansion ratio, reheat times, and distribution ratio of area expansion. These parameters were carefully analyzed to provide some direct and transparent results for engine designers. From analysis results, the following conclusions can be draw.

1. The specific thrust of scramjet can be greatly improved by increasing the total combustor area expansion ratio and reheat times, and/or choosing an appropriate distribution proportion of area expansion ratio. The effect of increasing the total combustor area expansion is most obvious for performance enhancement.

2. The limit specific thrust can be improved by about 50 percent for an area expansion ratio large enough. The increase of combustor area expansion ratio will lead to a great increase of the windward area and flight drag. Therefore, an appropriate area expansion ratio should be determined in the design process of the scramjet with multi-staged fuel injection.

3. The limit specific thrust can be improved by about 17 percent for reheat times large enough. The increase of reheat times will lead to a great increase of the length and weight of the combustor and the appropriate reheat times should be determined in the design process of the scramjet with multi-staged fuel injection.

4. With an appropriate distribution of area expansion ratio, the limit specific thrust can be improved by about 10 percent.

References

1. **W.H. Heiser and D.T. Pratt**, Hypersonic air breathing propulsion, AIAA, 1993.
2. **A.F. Latypov**, On organizing the working process in air-breathing engines, AIP Conf. Proc. 2017. Vol. 1893, No. 1, S. 1, P. 030014.
3. **F.S. Billig**, Research on supersonic combustion, J. Propul–Power, 1993, Vol. 9, P. 499–514.
4. **V.I. Zvegintsev**, Gas-dynamic problems in off-design operation of supersonic inlets (review), Thermophysics and Aeromechanics, 2017, Vol. 24, No. 6, P. 807–834.
5. **Y.P. Gounko and I.I. Mazhul**, Gas-dynamic design of a two-dimensional supersonic inlet with the increased flow rate factor, Thermophysics and Aeromechanics, 2012, Vol. 19, No. 3, P. 363–379.
6. **A.F. Latypov**, The condition of steady-state flow in a channel of variable cross section in the presence of heat supply and kinetic energy dissipation, Techn. Phys. Lett., 2012, Vol. 38, No. 11, P. 1013–1015.
7. **W. Huang, L. Yan, and J.G. Tan**, Survey on the mode transition technique in combined cycle propulsion systems, Aerospace Science and Technology, 2014, Vol. 39, P. 685–691.
8. **V.M. Fomin and A.F. Latypov**, From atmosphere to space, Science First Hand, 2011, No. 3, P. 33–41.
9. **W. Bao, J. Qin, W.X. Zhou, and D.R. Yu**, Effect of cooling channel geometry on re-cooled cycle performance for hydrogen fueled scramjet, Inter. J. Hydrogen Energy, 2010, Vol. 35, Iss. 13, P. 7002–7011.
10. **A.F. Latypov**, Functional mathematical model of a hydrogen-driven combustion chamber for a scramjet, J. Appl. Mech. Tech. Phys., 2015, Vol. 56, No. 5, P. 799–812.
11. **J. Qin, W. Bao, W.X. Zhou, and D.R. Yu**, Flow and heat transfer characteristics in fuel cooling channels of a recooling cycle, Inter. J. Hydrogen Energy, 2010, Vol. 35, Iss. 13, P. 10589–10598.
12. **W. Huang**, Design exploration of three-dimensional transverse jet in a supersonic crossflow based on data mining and multi-objective design optimization approaches, Inter. J. Hydrogen Energy, 2014, Vol. 39, P. 3914–3925.
13. **S. Tomioka, A. Murakami, K. Kudo, and T. Mitani**, Combustion tests of a staged supersonic combustor with a strut, J. Propul. Power, 2001, Vol. 17, Iss. 2, P. 293–300.
14. **R.F. Cao, J.T. Chang, W. Bao, M. Guo, D. Qin, and Z.G. Wang**, Analysis of combustion mode and operating route for hydrogen fueled scramjet engine, Inter. J. Hydrogen Energy, 2013, Vol. 38, Iss. 14, P. 5928–5935.
15. **S.R. Thomas and R.W. Guy**, Scramjet testing from Mach 4 to 20-present capability and needs for the nineties, AIAA-1990-1388, 1990.
16. **R.F. Cao, J.T. Chang, J.F. Tang, Z.Q. Wang, and D.R. Yu**, Study on combustion mode transition of hydrogen fueled dual-mode scramjet engine based on thermodynamic cycle analysis, Inter. J. Hydrogen Energy, 2014, Vol. 39, Iss. 36, P. 21251–21258.
17. **X.X. Shi, J.T. Chang, W. Bao, D.R. Yu, and B. Li**, Supersonic inlet buzz margin control of ducted rockets, in: Proc. Inst. Mech. Engng. Part G, J. Aerospace Engng, 2010, Vol. 224, No. 10, P. 1131–1139.

18. **F.R. Chavez and D.K. Schmidt**, Analytical aeropropulsive-aeroelastic hypersonic-vehicle model with dynamic analysis. *J. Guid. Control Dyn.*, 1994, Vol. 17, No. 6, P. 1308–1319.
19. **J.D. Mattingly, W.H. Heiser, and D.T. Pratt**, *Aircraft engine design*, 2nd Edition, AIAA, 2002.
20. **P.G. Hill and C.R. Peterson**, *Mechanics and thermodynamics of propulsion*, Addison–Wesley Publishing Co., New York, 1992, P. 155-164.
21. **A.H. Shapiro**, *The Dynamics and Thermodynamics of Compressible Flow*, Vol. 1, Roland Press Company, New York, 1953.
22. **J.D. Anderson**, *Modern Compressible Flow*, 3rd Edition, McGraw-Hill, New York, 2003.