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Structure of the gas-droplet wall jet injected through round holes into a transverse trench. Comparison of Eulerian and Lagrangian approaches*

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The flow structure and thermal efficiency of a gas-droplet wall jet, injected through inclined holes into a transverse trench, is analyzed numerically. The predictions are carried out using three-dimensional RANS equations in the following ranges of two-phase flow parameters: initial droplet size d_1 = 0–20 μm and their mass fraction M_{L1} = = 0 – 0.05. Gas turbulence is simulated using the model of Reynolds stress transport taking into account the two-phase character of the flow. The obtained simulation results are compared using the Eulerian and Lagrangian descriptions. The applicability of both approaches to describing the dynamics and heat transfer of a two-phase wall jet is shown.

Keywords: film cooling, thermal efficiency, gas-droplet wall jet, injection through the holes, trench, numerical simulation.

Introduction

A continuous increase in the gas temperature in front of a turbine is the main factor for the growth of thermal efficiency and capacity of the power plant. To date, such temperatures already exceed 1800 K and tend to further increase. This value is significantly higher than the operating temperatures of applied modern materials. In this regard, it becomes necessary to cool the working surfaces of the power plants, being exposed to a high-enthalpy gas flow. Thermal protection of the wall from the effects of high-enthalpy flows by means of wall jets remains an urgent and important task at the development of various power machines and installations for more than half a century (see monographs [1–3] and reviews [4–7]). A very effective and simple method of thermal protection for practical implementation is the injection of cooling gas into the near-wall region through a system of inclined holes in a protected surface. This method of surface protection is most common when cooling the blades of gas turbines and walls of combustion chambers [4–7]. However, it has significant drawbacks: significant non-uniformity of cooling in the transverse direction, small length of the zone with high thermal efficiency of the wall jet and noticeable decrease in thermal protection at large parameters of

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jet injection: ($m = \rho_2 U_2/\rho_1 U_1 > 1$ [2, 4], here ρ is the gas density, U is the averaged longitudinal velocity of gas, subscripts *1* and *2* correspond to the parameters of the main and secondary flows). Based on the foregoing, the search for new effective ways of cooler supply is still an urgent problem from both scientific and practical points of view.

One of the effective ways to increase significantly the thermal protection of the wall as well as to improve the uniformity of cooling distribution in the transverse direction is cooler supply through the holes in the transverse trench. Recently, this problem has been studied in experimental [7–9] and numerical [10, 11] works. Organization of the wall two-phase gas-droplet jets is also an effective way to increase thermal efficiency. The main mechanism for increasing the heat-protecting properties when using a two-phase system is the use of phase transition energy at evaporation of droplets in the immediate vicinity of the protected surface. The idea of using gas-droplet wall jets to cool the surface exists for about 50 years. Over the past few decades, a large number of works on the study of gas-droplet wall jets have been presented in the literature.

Experimental studies of the wall gas-droplet jets are presented in [12–15]. A significant increase in the efficiency of film cooling (up to 1.5–2 times) was shown for a gas-droplet wall jet in comparison with a single-phase jet with relatively low contents of liquid phase in the injected flow (up to 5–10% by weight of the secondary flow). It is rather difficult to study experimentally the structure and thermal efficiency of such flows. Large problems arise when transferring the results obtained under the laboratory conditions to real objects. For this reason, numerical simulation can be considered as a priority research method. For the numerical description of a two-phase flow, two main calculation methods are proposed. The first method is the Eulerian continuum description: the so-called two-fluid models. The second method includes the Lagrangian trajectory approach. Both of these methods have their pros and cons and complement each other. To describe the dynamics of the disperse phase in two-phase turbulent wall jet, the Lagrangian method is used more often [16–19] and the Eulerian approach [20, 21] is somewhat less common.

The model of two-phase wall jet based on the Lagrangian approach is presented in [16]. Here, the possibility of using various turbulence models to calculate the dynamics of a two-phase wall jet is studied. The influence of the main flow shape and temperature on uniform distribution of the wall jet at various ratios of shapes of the outlet holes is studied numerically in [17]. It is shown that the holes with different configurations of the outlet edge demonstrate noticeably better uniformity of film cooling in the direction transverse to the main flow in comparison with conventional inclined holes. Thermal efficiency of the wall jet for such holes $\eta = (T_{AW} - T_1)/(T_2 - T_1)$ is noticeably higher. Here, T_{AW} is the adiabatic wall temperature, T_1 and T_2 are the temperatures of the main and secondary flows. The work [19] deals with a numerical study with the use of Lagrangian approximation for the description of injection of a gas-droplet wall jet along a flat plate. A noticeable increase in thermal efficiency is obtained: more than 60 % with mass fraction of droplets M_{L1} = 6% and their initial diameter d_1 = 5 μm.

In [20], a model was developed and numerical study of the flow structure and thermal efficiency of a gas-droplet wall jet was performed. The indicated model uses the Eulerian approach; turbulence is described by the k-ε model taking into account the two-phase character of the flow. Here, the influence of the main parameters of the two-phase flow (droplet diameter and concentration) on the flow structure and thermal efficiency was analyzed. Later, the authors have developed a three-dimensional numerical model in the Eulerian approximation for calculating a two-phase turbulent wall flow and simulated the thermal efficiency of a gas-droplet wall jet being injected through inclined cylindrical holes into a transverse trench [21].

To describe gas phase turbulence in a two-phase flow, various modifications of two-parameter isotropic models are often used for engineering calculations [16, 18–20]. Such turbulence models, for example, the k-ε model have a number of serious limitations, when describing complex flows with separation zones and regions with flow recirculation [22]. One of the methods allowing partial consideration of the complex mixing processes and anisotropy of the components of gas velocity pulsations in separated flows is the use of the models of Reynolds stress transport (Second moment closure). This model is successfully used to describe the turbulent characteristics of the carrier phase in the wall single- [11] and two-phase jets [16, 17, 21].

This paper presents the results of comparing the capabilities of the Eulerian and Lagrangian methods for describing the effect of dispersed phase dynamics on the flow structure and thermal efficiency of a gas-droplet wall jet, blown through inclined holes into a trench transverse to the flow. The prospect of such a method of organizing a wall jet was previously shown for both single- [11] and two-phase gas-droplet [21] flows.

Eulerian approach

The Eulerian approach is used to describe flow dynamics and heat and mass transfer in the gas and disperse phases. The system of equations for simulating the disperse phase motion in the Eulerian continuum is obtained from the kinetic equation for the probability density function of particle distribution in a turbulent flow [23]. We should note that this method was originally developed to describe the flows with solid particles without interfacial heat transfer. In the present work, this approach is used to describe the two-phase flows with droplet evaporation. The solution uses three-dimensional Reynolds-averaged Navier–Stokes equations (RANS equations) written down considering the influence of particles on transport processes in gas. To describe gas turbulence, the Reynolds stress transport model [24], modified for the case of disperse phase presence, was used [25]. This approach made it possible to abandon the hypothesis of isotropic viscosity, strictly speaking, not applicable for calculating the separated flows. To predict the pulsations of disperse phase velocity, a differential model of Reynolds stress transport was used. Temperature pulsations and turbulent heat flux of particles were described in terms of models [25]. The main equations of the model and the technique of numerical implementation were considered in detail in [21], where the numerical algorithm was tested in comparison with the measurement data for the two-phase wall jets.

Lagrangian description

The gas phase is simulated in the same way as in the Eulerian approach. In the Lagrangian method, the effect of particles on the carrier gas flow is estimated using the approach of [26]. The transition from the results of calculating the particle trajectories to distributions of disperse phase parameters in the physical space (for example, when calculating droplet concentration) is performed by spatial or spatial-temporal averaging by all particle trajectories over the control volume of the Eulerian grid used to calculate the gas phase field [26]. Particle dynamics is usually calculated using the well-known Stochastic Separated Flow (SSF) model [27], which takes into account the stochastic effect of gas turbulence on particle motion or its various modifications (see, for example, [28]).

The effect of the following factors on the particle dynamics is taken into account: forces of drag, gravity, and Saffman and forces taking into account the pressure gradient of the carrier medium. One of the problems is the selection of the method for calculating the instantaneous (actual) velocity of the gas phase $\mathbf{U}_{S,i}$. It is necessary to know velocity $\mathbf{U}_{S,i}$ to calculate the drag force in the framework of the Lagrangian approach. In the current study, we use the well-known Continuous Random Walk approach [28]: $\mathbf{U}_{\mathrm{S},i} = \mathbf{U}_i + \mathbf{u}_{\mathrm{S},i}$, where $\mathbf{U}_i$ and $\mathbf{u}_{\mathrm{S},i}$ are the components of the averaged gas velocity (determined directly by the RANS calculation), and instantaneous gas velocity at the particle location point determined by approach [28]. There are a number of modified SSF models, which differ from the known model [27] mainly by the fact that they consider interfacial interaction as a continuous process. At that, the averaged phase velocity is used in the equations of motion. Random rms pulsations of disperse phase are calculated along the stochastic trajectory, which allows the preservation of the stochastic nature of disperse phase motion. The expression for calculating the instantaneous gas velocity has the form [28]:

$$u_{S,i}^{m} = a_{ij} u_{S,j}^{m-1} + b_{ij}\zeta_j + \mathbf{A}_i \Delta t, \quad \zeta_i \in N(0,\ 1),$$

here, superscript m refers to the current time step, $N(0, 1)$ is a random Gaussian value having distribution with an average value equal to zero and standard deviation equal to one, $\mathbf{A}$ is the vector of corrected bias, and Δt is the time step. The expression [29] is used to calculate the components of the corrected bias vector

$$\mathbf{A}_{ij} = \frac{1}{1+\mathrm{St_L}}\left[\frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\rho u_{ij}\right)\right], \quad \mathrm{St_L} = \frac{\tau}{1/7\left(k/\varepsilon\right)},$$

where $\tau = \dfrac{d_1 \rho_{L2}}{3\rho C_D \left|\mathbf{U}_i - \mathbf{U}_{\mathrm{L},i}\right|/4}$ is the time of dynamic relaxation of particles. The displacement **a** and diffusion **b** tensors are determined from the following relations given in [28]:

$$a_{ij} = \exp\left(-\frac{\Delta t}{\Omega_{\mathrm{L}}}\right)\delta_{ij}, \quad b_{ik}b_{jk} = \left(1 - a_{ii}a_{jj}\right)u_{ij},$$

where Ω_{L} is the turbulent time scale [30].

Numerical method

A schematic representation of the development of a two-phase wall jet and flow geometry are shown in Fig. 1 and correspond to those described in [21]. A numerical solution was obtained using the control volume method on staggered grids. The QUICK algorithm (Quadratic Upwind Interpolation for Convective Kinematics) was used for the convective terms of differential equations. For diffusion flows, the central difference scheme was used. The pressure field was corrected by the finite-volume consistent SIMPLEC (Semi-Implicit Pressure-Linked Equation Corrected) algorithm. The boundary, input, and output conditions were described in detail in [21]. For droplets on the walls, “elastic” boundary conditions were set: when after a contact with the bounding surface the particle returns to the flow without loss of momentum. The effects of droplet break up are not considered in this work due to the small Weber number We calculated by interfacial velocity and secondary flow parameters. The range of Weber numbers in this study is $\mathrm{We} = \rho(\mathbf{U}-\mathbf{U}_{\mathrm{L}})^2 d_1/\sigma = (0.3\text{–}1.5)\cdot 10^{-6}$, which is significantly

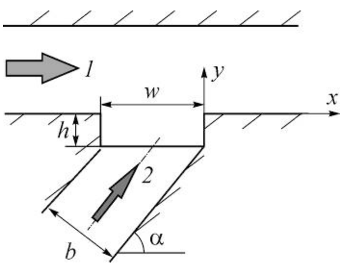

Fig. 1. The scheme of the development of the two-phase wall jet injected into the transverse trench.
1 — main single-phase flow of heated air,
2 —gas-droplet wall jet.

less than the critical Weber number $We_* = 7$ [16, 18]. The effect of drop-let coalescence can be also neglected due to the small volume fraction of particles ($\Phi_2 \le 10^{-4}$).

Results of numerical simulations and analysis of results

The diameter of holes for supplying a two-phase coolant is $b = 3.2$ mm, the length of the larger axis of the output ellipse equals two hole diameters and corresponds to the trench width ($w = 6.4$ mm). The calculations are performed for a single-row system of three holes located at an angle $\alpha = 30°$ in a trench with depth $h = 2.4$ mm and width $w = 6.4$ mm ($h/w = 0.375$, $h/b = 2$). The transverse pitch of the holes is $t = 10$ mm ($t/b = 3.13$). The distance x is counted from the rear edge of the trench, and transverse distance z is counted from the center line of the middle (second) hole. The air in the main flow with temperature $T_1 = 323$–573 K is considered dry. The temperature of the secondary two-phase flow is constant and equal to $T_{L1} = T_2 = 293$ K, and mass fractions of water droplets and steam are $M_{L1} = 0.01$–0.05 and $M_{V1} = 0.005$, respectively, with initial diameter of water droplets $d_1 = 1$–20 μm. The flat channel height is $H = 30$ mm ($H/b = 9.375$), the average velocity of the main single-phase air flow is $U_1 = 30$ m/s, and the velocity of the secondary gas-droplet flow is $U_2 = U_{L2} = 10$–30 m/s, the blowing ratio varies over a wide range $m = 0.25$–2.5. The Reynolds numbers based on the equivalent hydraulic diameter of the channel and parameters of the main and secondary flows are: $Re_1 = U_1 2H/\nu_1 = 7.8 \cdot 10^4$ and $Re_2 = U_2 b/\nu_2 = (1.5$–$12.5) \cdot 10^3$. In the present work, when using the Lagrangian approach for calculations, the number of particles equal to 25000 was chosen as the base. The calculation time by the Lagrangian method exceeds the corresponding value for the Eulerian method by an average of 10–20%. The results of all simulations shown in Figs. 2–5 are obtained on the axis of the middle hole for the adiabatic wall condition.

The transverse profiles of the averaged longitudinal velocity component of the gas (Fig. 2*a*) and disperse (Fig. 2*b*) phases in several cross sections along the flat channel behind the rear edge of the trench are presented in Fig. 2 for mass loading of droplets at the inlet $M_{L1} = 0.02$. According to Fig. 2*a*, the gas phase velocity profiles are almost independent of the method for calculating the disperse phase dynamics by the Eulerian (*2*) and Lagrangian (*3*) approaches. The gas velocity in a two-phase flow almost coincides with the gas velocity in a single-phase flow, which is explained by an extremely small value of droplet concentration and the small effect of particles on the gas phase. This is consistent with recent calculations of the authors for two-phase gas-dispersed separated [31] and swirling [32] flows behind a sudden expansion of the tube. The profiles of longitudinal velocity of droplets (see Fig. 2*b*) calculated by the Eulerian (*2*) and Lagrangian (*3*) approaches have qualitatively similar forms. The difference in calculation results for these two approaches does not exceed 12%. It should be noted that the slip velocity of phases increases downstream, especially in the near-wall zone.

Distributions of droplet sizes and their mass fractions over the channel height are shown in Fig. 3. It can be seen that transverse distributions of particle sizes and their number along the channel length have a qualitatively similar form. We should note that initially, as moving along the channel length, the height of the region of two-phase jet existence increases due to the mixing processes: from $y/b \approx 0.3$ at $x/b = 0$ to $y/b \approx 0.43$ at $x/b = 10$. Further, due to droplet evaporation, the height of the two-phase flow region decreases, which is explained by heating of the upper part of the wall jet when it is mixed with the main heated flow ($y/b \approx 0.2$ at $x/b = 20$). Predictions for both approaches give close values for the height of a two-phase jet: the difference does not exceed 5%. In contrast to the Eulerian method, the Lagrangian

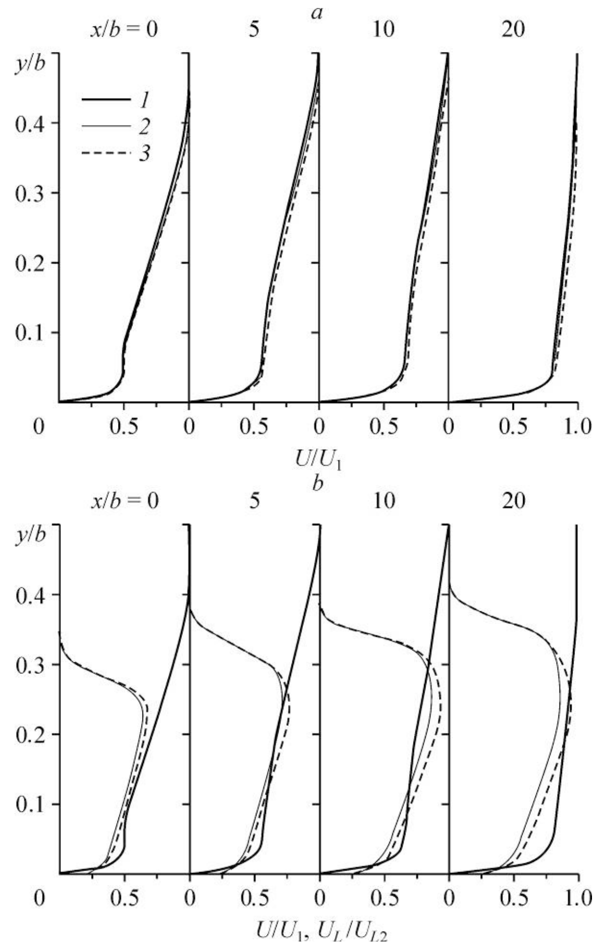

Fig. 2. Profiles of the averaged longitudinal velocity of the gas (*a*) and disperse (*b*) phases along the channel height in the two-phase wall jet injected into the transverse channel.

a: *1* — single-phase air flow, *2*, *3* — gas in two-phase flow in Eulerian and Lagrangian approximations, respectively; *b*: *1* — gas in two-phase flow (Euler approach), *2* — droplets (Eulerian approximation), *3* — droplets (Lagrangian description);

$M_{L1} = 0.02$, $d_1 = 10$ μm , $H = 30$ mm, $U_1 = 30$ m/s, $U_2 = 11.8$ m/s, $m = 0.5$, $T_1 = 373$ K, $T_2 = 293$ K.

approach predicts a slightly larger particle size and their mass concentration (by 10–20%). Thus, we can conclude that according to the Lagrangian description, droplets evaporate slower than according to the Eulerian model.

The normalized temperature profiles of gas $\Theta = (T - T_{AW})/(T_1 - T_2)$ (*a*) and dispersed $\Theta_L = (T_L - T_1)/(T_{L2} - T_1)$ (*b*) phases are presented in Fig. 4 for mass loading of dispersed phase $M_{L1} = 5$ %. Here T, T_L, and T_{L2} are the temperatures of gas and droplets at the considered point and the temperature of droplets in the inlet cross section, respectively. According to Fig. 4*a*, the influence of the methods for calculating the disperse phase temperature by the Eulerian (*2*) and Lagrangian (*3*) approaches on the gas temperature is minimal. Figure 4*b* shows the transverse distribution of droplet temperature for $M_{L1} = 0.02$; for other values of droplet concentration, similar results are obtained. It can be noted that the maximum difference in the results of droplet temperature calculations reaches 15 %. The droplet temperature predicted using the Eulerian approach is higher than the corresponding value obtained by the Lagrangian method.

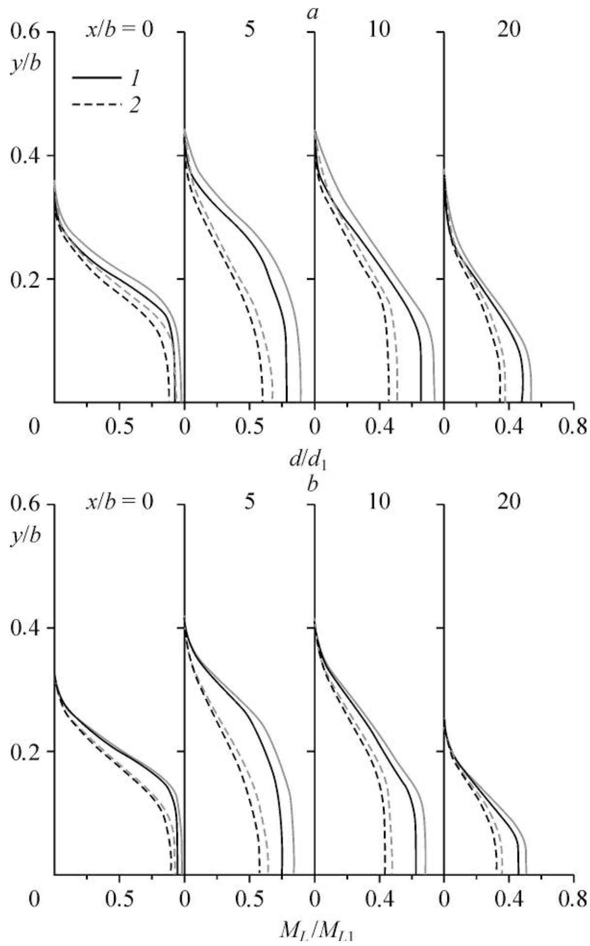

Fig. 3. Distributions of droplet diameters (*a*) and mass fraction of disperse phase (*b*) over the channel cross section with varying droplet concentration.
Gray lines — Lagrangian approach, black lines — Eulerian description; M_{L1} = 0.02 (*1*), 0.05 (*2*).

The droplets in the Lagrangian description warm up and evaporate slower, which is consistent with the data in Fig. 3.

The results of numerical studies on thermal efficiency parameter $\eta = (T_{AW} - T_1)/(T_2 - T_1)$ of a two-phase gas-droplet wall jet for various models for calculating the dispersed phase dynamics are shown in Fig. 5. We should note that the results of numerical simulations using the Eulerian approach (line *2*) were taken by the authors from their previous work [21]. The presence of evaporating droplets of liquid, even at relatively low mass concentrations not exceeding 5 % of the secondary flow mass, leads to the fact that the efficiency of the wall jet can be more than doubled in comparison with blowing a single-phase coolant. At that, the area of cooling intensification extends over large distances and, which is especially important for practical purposes, this effect increases with a distance from the point of injection. As can be seen from the analysis of the data shown in Fig. 5, the difference in calculation results for both approaches is extremely small and does not exceed 3 %.

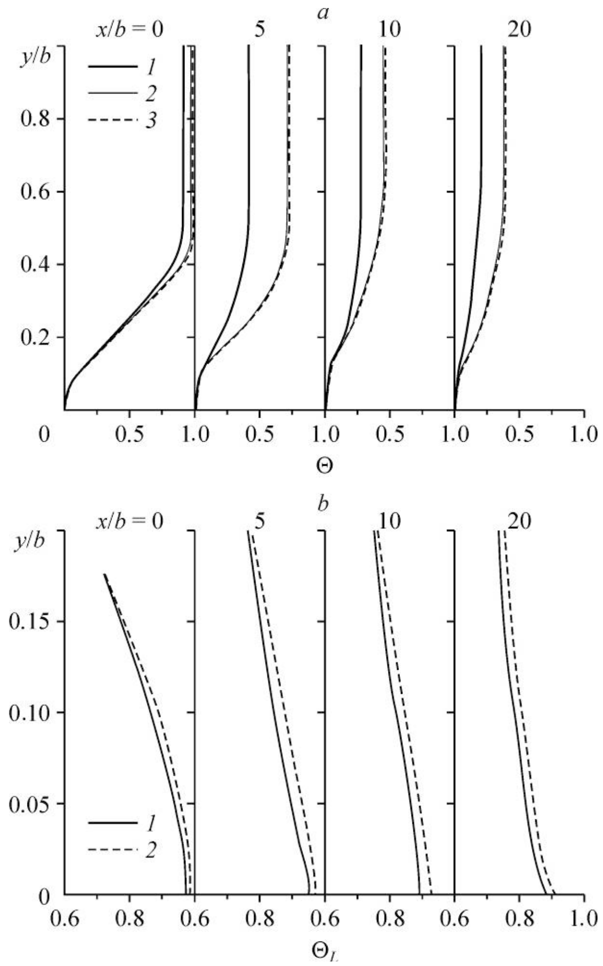

Fig. 4. Dimensionless temperature profiles of the gas (*a*) and dispersed (*b*) phases in the gas-droplet wall jet.

a: $M_{L1} = 0.05$, *1* — $M_{L1} = 0$ (single-phase flow), *2* — Eulerian description, *3* — Lagrangian approach; *b*: $M_{L1} = 0.05$, *1*, *2* — Eulerian and Lagrangian approaches, respectively.

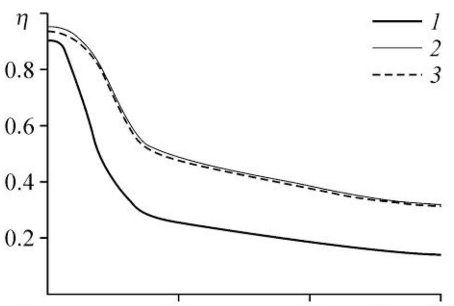

Fig. 5. Thermal efficiency of the wall single-phase (*1*) and gas-droplet (*2* and *3*) wall jets along the channel length.

$M_{L1} = 0.05$, $d_1 = 10$ μm ; *1* — $M_{L1} = 0$ (single-phase jet), *2* and *3* — Eulerian and Lagrangian approaches, respectively.

Conclusion

It is found that the influence of the method for simulating the dynamics of droplets on the averaged parameters of the carrier gas phase (velocity and temperature) of a two-phase wall jet is negligible. In numerical simulations using the Lagrangian and Eulerian approaches, the components of longitudinal averaged particle velocity differ slightly and this difference is about 10 %. Basically, the difference between the results obtained by the Eulerian and Lagrangian approaches is manifested in predictions of droplet concentration. The Eulerian method predicts a lower particle concentration as compared to the Lagrangian description both in the recirculation region and in the flow core (the difference reaches 15–20 %). The difference in results of calculating the thermal efficiency of the two-phase wall jet obtained by both approaches is extremely small and does not exceed 3 %. The presented work is the first step in studying the capabilities of the Eulerian and Lagrangian descriptions when simulating wall gas-droplet jets. It is necessary to carry out further numerical and experimental studies in a wider range of changes in the disperse phase parameters.

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