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Numerical study of shock wave interaction in steady flows of a viscous heat-conducting gas with a low ratio of specific heats*

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Specific features of shock wave interaction in a viscous heat-conducting gas with a low ratio of specific heats are numerically studied. The case of the Mach reflection of shock waves with a negative angle of the reflected wave with respect to the free-stream velocity vector is considered, and the influence of viscosity on the flow structure is analyzed. Various issues of nonuniqueness of the shock wave configuration for different Reynolds numbers are discussed. Depending on the initial conditions and Reynolds numbers, two different shock wave configurations may exist: regular configuration interacting with an expansion fan and Mach configuration. In the dual solution domain, a possibility of the transition from regular to the Mach reflection of shock waves is considered.

Key words: three-shock configuration with a negative reflection angle, nonuniqueness of the numerical solution of the Navier–Stokes equations, transition between regular and Mach reflection, dual solution domain, viscous effects, interaction of shock waves with an expansion fan.

Introduction

Shock wave interaction has been studied for many years [1–5]. Two types of shock wave reflection are well known: regular reflection (Fig. 1*a*) consisting of an incident shock (IS) and reflected shock (RS) and irregular reflection consisting of three or more waves. Since the time when Mach discovered irregular reflection at the end of the 19th century, numerous theoretical, experimental, and numerical investigations of both regular and irregular reflection configurations have been performed. The latter type is often called the Mach reflection. The Mach reflection (Fig. 1*b*) is characterized by the presence of the third shock called the Mach stem (MS) and contact discontinuity (CD) emanating from the triple point (T). In the middle of the 20th century, von Neumann proposed two-shock and three-shock theories [2] based on the laws of conservation of parameters across oblique shocks to describe the flow parameters near the point of shock wave confluence: point R (Fig. 1*a*) in the case of regular reflection and point T (Fig. 1*b*) in the case of the Mach reflection. The current interest in shock wave reflection

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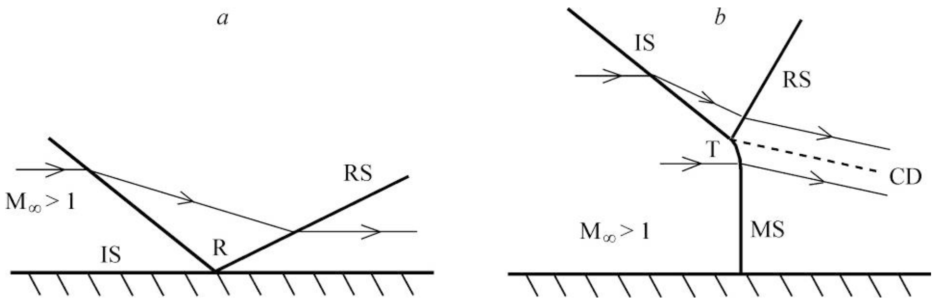

Fig. 1. Regular (*a*) and Mach (*b*) configurations of shock waves.

is primarily caused by applied problems in the field of aeromechanics encountered, e.g., in design of inlets for supersonic flying vehicles [6, 7]. However, despite a large number of publications dealing with shock wave reflection, there are some new topics to be studied.

Many new interesting specific features of flows with reflection and interaction of shock waves have been recently discovered. One of such features is a hysteresis between regular and Mach reflection configurations, which is observed in the domain of parameters where the von Neumann theory predicts that both regular and Mach reflection may exist (dual solution domain). This phenomenon was predicted theoretically by Hornung et al. [8]; later on, it was confirmed by numerical [9−11] and experimental [11−13] investigations. The cited studies were performed to study reflection of shock waves between two wedges located in a steady supersonic flow. Similar nonuniqueness of the shock wave configuration was detected numerically and experimentally by Khotyanovsky et al. [14] who studied the transition from regular reflection of shock waves interacting with an expansion fan EF (Fig. 2*a*) to the Mach reflection. However, viscosity and heat transfer were ignored in those investigations, and the Reynolds number was not varied in those experiments. All those studies were performed for gases with the ratio of specific heats $\gamma = 1.4$. Based on those results, it may be assumed that viscosity does not produce any significant effect on the flow structure and on the change in the shock reflection type in the transition from regular to Mach reflection.

The influence of viscous effects on the flow structure near the point of shock wave intersection in the case of irregular reflection was numerically studied in [15]. Similarity of the flow fields for different Reynolds numbers in a small vicinity of the point of shock wave intersection in the case of the Mach reflection was detected. Nevertheless, viscosity did not exert any significant effect on flow reconstruction at scales comparable with the characteristic scale of the problem either.

The above-mentioned numerical and experimental studies were performed at sufficiently high free-stream Mach numbers $M_\infty > 2.2$. Investigations at lower Mach numbers ($M_\infty < 2.2$) were usually performed for the purpose of resolving the so-called von Neumann paradox

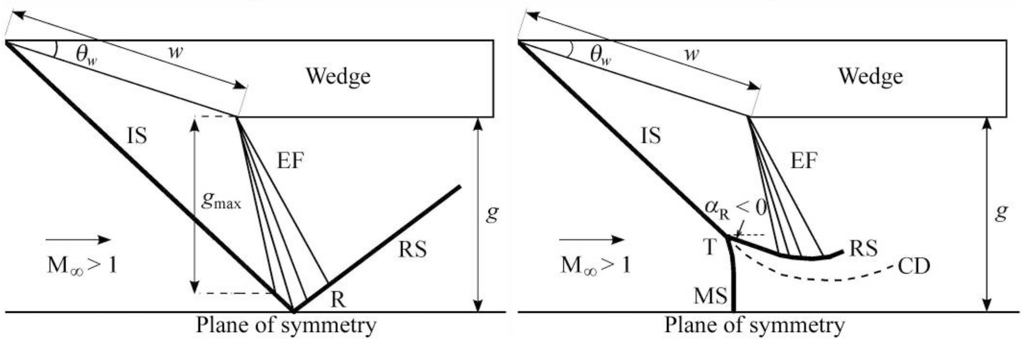

Fig. 2. Two-shock configuration interacting with an expansion fan (*a*)
and three-shock configuration with a negative angle of shock wave reflection (*b*).

associated with failure to provide a correct mathematical description of the irregular three-shock configuration. In experiments, the three-shock configuration was formed in the domain of parameters where the von Neumann theory does not predict the existence of this configuration. This discrepancy is usually called the von Neumann paradox or the triple point paradox. To obtain a solution within the framework of gas dynamics, Guderley proposed a new irregular configuration in 1947 [3], which included a fourth centered expansion wave emanating from the point of intersection of shock waves. However, the first numerical confirmation of this solution was provided only half a century later by Vasil'ev and Kraiko [16] who considered diffraction of shock waves on a wedge by solving the Euler equations. Numerical simulations [17, 18] based on the Navier–Stokes equations and Direct Simulation Monte Carlo (DSMC) method revealed the global influence of viscous effects at scales comparable with the characteristic scale of the problem: the slope of the reflected wave differed considerably from the theoretical prediction. All these studies of shock wave reflection were performed for gases with the ratio of specific heats $\gamma = 1.4$ or $\gamma = 1.66$.

It was demonstrated analytically [19] and numerically [20] that the three-shock von Neumann theory [2] predicts a possibility of the Mach reflection of shock waves with a negative angle of the reflected wave with respect to the free-stream velocity vector for small values of the ratio of specific heats ($\gamma < 1.4$). Following the definition of [19, 20], we will use the term "three-shock configuration with a negative reflection angle." It should be noted that this configuration can be formed only at low ratios of specific heats. For this reason, it was observed neither in experiments nor in numerical simulations.

The three-shock configuration with a negative reflection angle was numerically simulated in [20] on the basis of Navier–Stokes equations with the use of the STAR-CCM+ software package. In that study, Gavrenkov and Gvozdeva [20] considered a steady two-dimensional flow between two symmetric wedges in a supersonic gas flow with a low ratio of specific heats. The calculated results showed that the three-shock configuration with a negative reflection angle was unstable. To the best of our knowledge, such a shock wave configuration was obtained for the first time in [19, 20], and it was not actually studied by other researchers. The formation of a three-shock configuration with a negative reflection angle is an interesting and poorly studied fundamental question. The goal of the present work is a numerical study of new features of shock wave configurations in gas flows with a low ratio of specific heats.

1. Comparison of shock polar for different ratios of specific heats

The qualitative and quantitative analysis of interaction of oblique shock waves within the framework of the inviscid theory is often performed by means of geometrical construction of shock polar on a plane that shows the relationship between the pressure and the angle of flow deflection behind the oblique shock. The shock polars are constructed on the basis of the Rankine–Hugoniot relations and actually express the mass, momentum, and energy conservation laws on the oblique shocks. In this work, the shock polars are constructed on the basis of the pressure normalized to the free-stream pressure, i.e., p/p_∞, and the flow deflection angle is measured in degrees.

Figure 3 shows the shock polars for the cases corresponding to three-shock configurations for different ratios of specific heats. The shock polar of the incident shock is constructed from the point (0,1) corresponding to the free-stream parameters. The point A corresponds to the flow parameters behind the normal shock, and the polar portion from the point (0,1) to the point A corresponds to all possible combinations of the pressure and the angle of flow deflection behind the oblique shock. After that, the shock polar of the reflected shock is constructed from the point D corresponding to the flow parameters behind the incident shock. This point is determined by the angle of flow deflection behind the incident shock IS (see Fig. 2), which, for instance, can be determined by the wedge generating the incident shock. The point B(C)

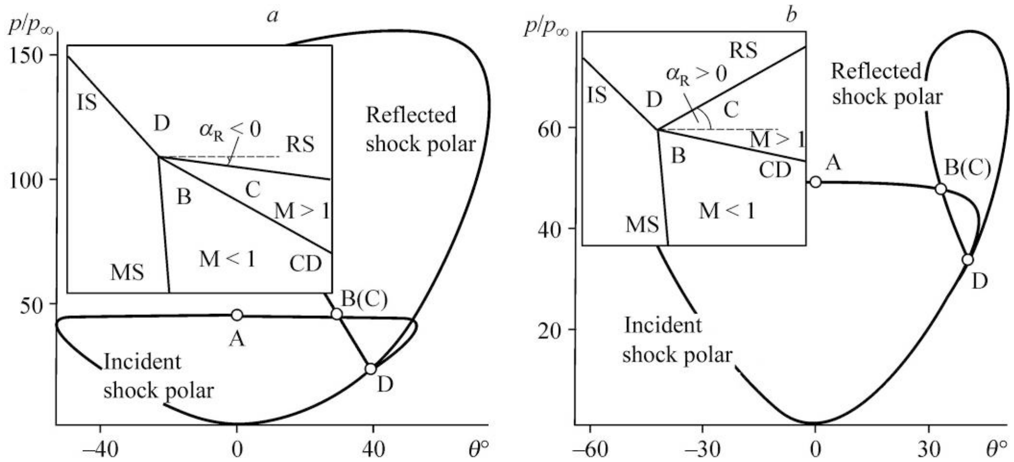

Fig. 3. Shock polars.

$M_\infty = 6.5$ (*a*, *b*), $\theta_w = 40°$ (*a*, *b*), and $\gamma = 1.2$ (*a*) and 1.4 (*b*);
α_R is the reflected shock angle with respect to the free-stream direction.

of intersection of the shock polars corresponds to the parameters behind the shock wave intersection point from which the contact discontinuity CD emanates. The pressure and the flow deflection angle do not experience jumps on the contact discontinuity CD. Knowing the angle of flow deflection, it is possible to calculate the angles of the reflected shock RS and Mach stem MS and then all other flow parameters.

Let us consider the problem of shock wave reflection between two symmetric wedges for flow parameters corresponding to those used to construct the shock polars in Figs. 3*a* and 3*b*. We assume that a three-shock configuration is formed (see, e.g., Fig. 2*b*). In this case, the Mach stem in the plane of symmetry is the normal shock, i.e., the parameters behind the MS correspond to the points A on the shock polars (Fig. 3) for both cases. As we go from the plane of symmetry to the wedge, the Mach stem becomes slightly curved, which corresponds to the motion along the incident shock polar from the point A to the point B(C). According to the analysis of the shock polars in Fig. 3*a*, the angle of flow deflection behind the reflected shock (point B) in the case of a three-shock configuration with a negative reflection angle ($\alpha_R < 0$) is smaller than the angle of flow deflection behind the incident shock (point D). A similar pattern is also observed in the case of the Mach reflection with a positive angle ($\alpha_R > 0$) of the reflected shock (Fig. 3*b*). In both cases, the behavior of flow deflection from the flow behind the incident shock to the flow behind the reflected shock is qualitatively similar. According to the inviscid solution, a subsonic flow should occur behind the Mach stem MS in both cases, whereas a supersonic flow should be observed in the region behind the reflected shock. Thus, according to the analysis of the shock polars, the three-shock configuration with a negative reflection angle formed between two symmetric wedges does not differ qualitatively from the case of the Mach reflection with a positive angle of the reflected shock (see Figs. 3*a* and 3*b*). It should be noted that this conclusion was drawn only on the basis of the shock polar analysis, i.e., the nonideal features of the gas (its viscosity and thermal conductivity) and particular sizes of the wedge were ignored.

2. Problem formulation and numerical methods

We consider interaction of shock waves generated by two symmetric wedges with an apex angle $\theta_w = 40°$, which are located in a steady supersonic ($M_\infty = 6.5$) gas flow with the ratio of specific heats $\gamma = 1.2$. Such a formulation corresponds to one of the test cases considered in [20] except for the wedge geometry. In [20], a domain with an increasing cross-sectional area (diverging channel) was placed behind the trailing edge of the wedge, whereas a constant-section channel is used in the present work.

Numerical simulations of shock wave reflection are performed on the basis of two-dimensional Euler and Navier–Stokes equations with the use of the Ansys Fluent software package. The second-order upwind scheme is used to solve the Euler equations, and the third-order MUSCL (Monotonic Upstream-Centered for Conservation Lows) scheme was used to solve the Navier-Stokes equations. The fluxes through the control volume faces are calculated by the AUSM solver [21]. The computational domain geometry is shown in Fig. 2. The distance between the wedges is chosen in such a way that the first characteristic of the expansion fan EF crosses the incident shock IS in the region between the plane of symmetry and the wedge, as is shown in Fig. 2*a*. Free-stream parameters are set on the left (inflow) boundary. As the problem is symmetric, the symmetry condition is imposed on the lower boundary. The wedge is used only as a generator of the incident shock wave; therefore, the wedge surface is subjected to the no-slip condition. The outflow boundary of the computational domain is located sufficiently far from the trailing edge of the wedge to obtain a supersonic flow on this boundary. This is a "free" boundary, i.e., all variables are extrapolated from the computational domain. The numerical solution is integrated in time until the steady state is reached, with the use of an implicit or explicit scheme depending on the initial flow field. Criteria of convergence of the numerical solution are taken to be standard residual monitors (within 10^{-14}) and reaching an unchanged position of the shock wave on the plane of symmetry.

In this work, we actually solve two problems of the flow between two symmetric wedges. The first problem is aimed at calculating the viscous structure of the three-shock configuration with a negative reflection angle. This problem is solved by using an implicit scheme, and the initial conditions correspond to a uniform supersonic flow with free-stream parameters or to a steady numerical solution of the Euler equations. Running ahead of the story, we can say that one of two different shock wave configurations may be formed depending on the initial conditions: regular reflection configuration interacting with the expansion fan or three-shock configuration with a negative reflection angle. This nonuniqueness of the shock wave structure leads to a question concerning the transition from one state to another, which is the second problem considered in this paper. To study the transition from the two-shock to three-shock configuration, pressure perturbations are inserted into the flow. The pressure is increased in a small (ring-shaped) region of the free stream, slightly upstream from the place of shock wave reflection, and then the computation with an explicit scheme is continued until the steady state is reached.

The system of equations of motion of mechanics of continuous media is closed by the equation of state for an ideal gas. A power-law dependence of viscosity on temperature with a power-law exponent of 0.66 is used in the computations. The thermal conductivity is calculated for the Prandtl number of 0.72. The viscous computations are performed for comparatively low Reynolds numbers in the laminar flow regime.

The simulations are performed on a structured rectangular grid. To estimate the accuracy of the numerical solution, we use three different schemes: schemes of the first and second orders with upwind differences and the third-order MUSCL scheme. The convergence of the numerical solutions is verified by comparing the results computed on two grids with a two-fold difference in the cell size: 1) $\Delta x \sim 1.4{\cdot}10^{-3}w$, $\Delta y \sim 10^{-3}w$; 2) $\Delta x \sim 7{\cdot}10^{-4}w$, $\Delta y \sim 5{\cdot}10^{-4}w$, where w is the length of the windward face of the wedge (see Fig. 2). The results of all computations with the use of different schemes and grids agree well with each other, i.e., numerical errors are small and do not affect the final result.

3. Three-shock configuration with a negative reflection angle

The results of simulations based on the Navier–Stokes equations are shown in Fig. 4. The initial data for these computations are taken to be a steady numerical solution of the Euler equations with a three-shock configuration. The bold solid curve in Fig. 4*a* is the sonic line, and the streamlines are indicated by the arrows. The Mach configuration shown in this figure is stable and does not change with time. The expansion fan emanating from the trailing edge of the wedge is refracted on the reflected shock and interacts with the contact discontinuity (mixing layer). As a result of this interaction, the contact discontinuity becomes curved, and a "virtual" nozzle is formed [22], where the subsonic flow that passed through the Mach stem is again accelerated to a supersonic velocity. The points A, B, C, D, and E in the flow field correspond to the points on the plane (θ, p/p_∞) shown in Fig. 4*b*. The point A corresponds to conditions behind the normal shock (flow parameters on the plane of symmetry). The point B is the "reversal" point of the numerical values behind the Mach stem. The point C corresponds to one more "reversal" point of the numerical data. The point D is the point with the maximum pressure in the entire considered region. The parameters at the point E behind the reflected shock approach the parameters predicted by the three-shock theory. At this point, the expansion fan emanating from the trailing edge of the wedge starts to affect the flow. Such motion of the numerical data in the plane (θ, p/p_∞) actually illustrates the effect of viscosity on the three-shock configuration with a negative reflection angle. A qualitatively similar behavior of the numerical data was also observed in the case considered in [15], where viscous effects on the flow structure in the case of the Mach reflection of shock waves at $\gamma = 5/3$ were considered.

The flow field in the vicinity of the triple point is shown in Fig. 5. The solid curves in Fig. 5*a* indicate the shock wave positions predicted by the three-shock theory. The computed shock wave positions near the triple point are in good agreement with the theoretical prediction. The flow field near the triple point (Fig. 5*b*) contains a small region where the flow deflection angle becomes negative, i.e., the flow changes its direction. This region corresponds to the numerical data in the vicinity of the point C in Fig. 4*b*.

The simulations of a three-shock configuration with a negative reflection angle in a steady gas flow allow us to conclude that this configuration is stable and has no qualitative differences from the classical case of the Mach reflection with a positive angle of the reflected wave. It should be noted that similar results were obtained in [23], where a numerical simulation (based on the Euler equations) of a supersonic jet coming from the nozzle was performed. A decrease in the ratio of specific heats leads to an interesting feature

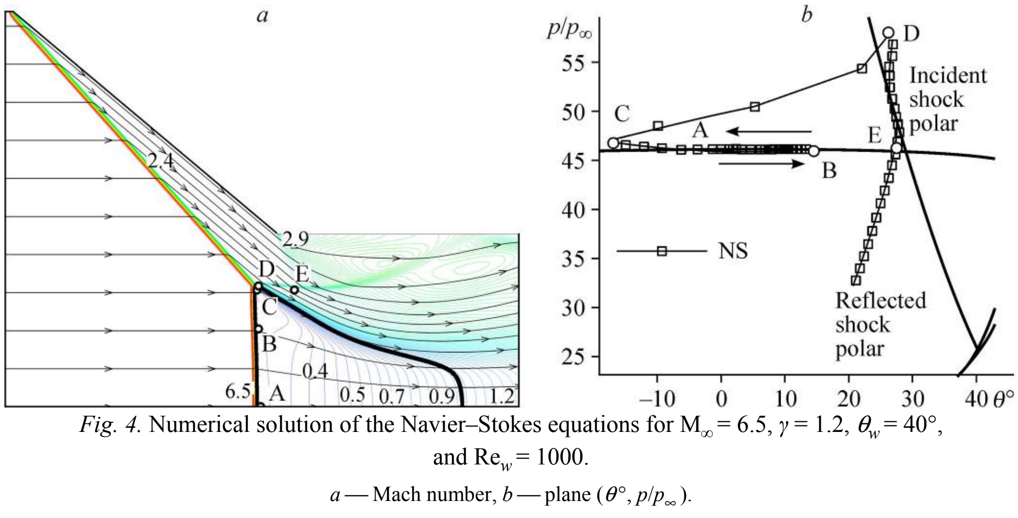

Fig. 4. Numerical solution of the Navier–Stokes equations for $M_\infty = 6.5$, $\gamma = 1.2$, $\theta_w = 40°$, and $Re_w = 1000$.

a — Mach number, *b* — plane ($\theta°$, p/p_∞).

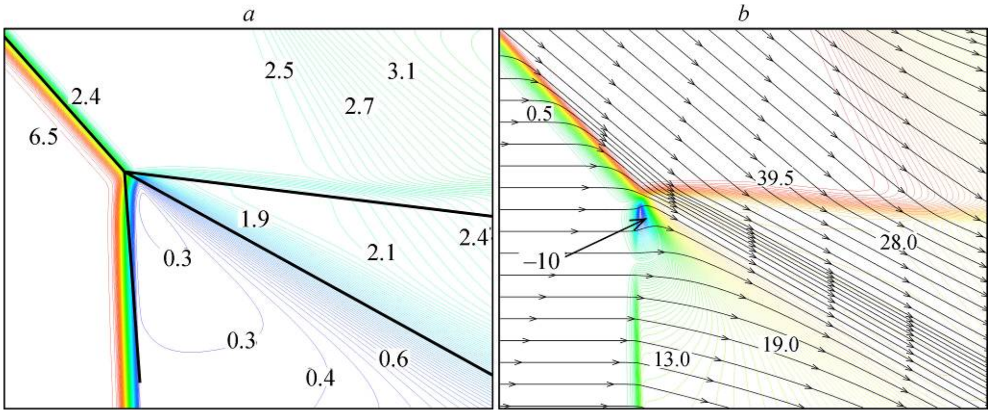

Fig. 5. Flow field in the vicinity of the triple point.

$M_\infty = 6.5$, $\gamma = 1.2$, $\theta_w = 40°$, and $Re_w = 1000$; *a* — Mach number, *b* — angle of flow deflection.

of the flow: negative slope of the reflected shock. Another interesting feature induced by a combination of viscosity with a low ratio of specific heats is the nonuniqueness of the shock wave configuration. As was noted above, the initial flow field in the considered case corresponds to the steady numerical solution of the Euler equations with a three-shock configuration. Nevertheless, if the initial flow field is taken to be a uniform supersonic flow with free-stream parameters, then the steady solution is regular reflection of shock waves interacting with the expansion fan (Fig. 6*a*). According to the analysis of the shock polars (Fig. 3*a*), only a three-shock configuration is possible in this case, but the expansion fan changes

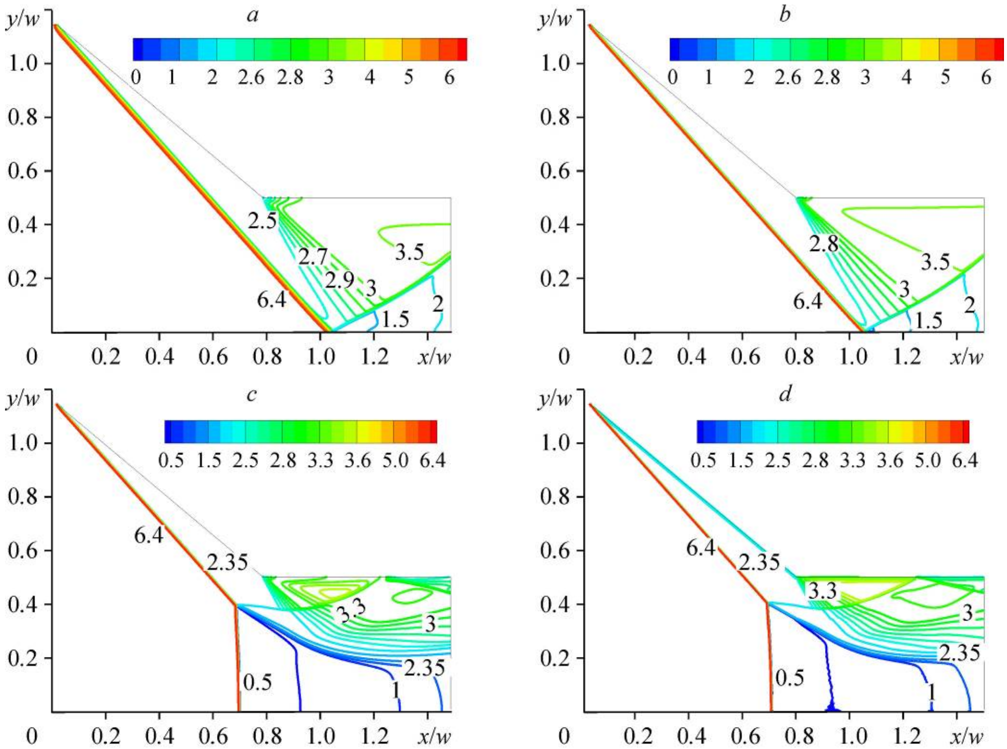

Fig. 6. Results of computations for different Reynolds numbers as compared to computations based on the Euler equations.

$M_\infty = 6.5$, $\gamma = 1.2$, $\theta_w = 40°$, $Re_w = 1000$ (*a*), 3000 (*b*), 4000 (*c*), numerical solution of the Euler equations (*d*).

the pressure, the flow deflection angle, and the Mach number of the flow in the vicinity of the point of reflection of shock waves; actually, it “allows” the shock to reflect in a regular manner. In what follows, we consider specific features of the formation of two-shock and three-shock configurations.

4. Effect of viscosity on nonuniqueness of the shock wave configuration

In this Section, we consider the interaction (reflection) of shock waves between two symmetric wedges (see Fig. 2) for the case corresponding to the solution B in Fig. 3*a*.

A series of computations is performed for different Reynolds numbers from $\mathrm{Re}_w = 500$ to $\mathrm{Re}_w = 5000$, as well as an additional computation based on the Euler equations. The Reynolds number is varied by changing the free-stream pressure (which, in turn, affects the gas density). The initial flow field in all computations is a uniform supersonic flow with free-stream parameters. At low Reynolds numbers ($\mathrm{Re}_w = 1000$ and 3000, see Figs. 6*a* and 6*b*), a two-shock configuration interacting with the expansion fan is observed. Flow fields for different Reynolds numbers are not presented because shock wave configurations for all cases with two-shock reflection are similar; the only difference is a greater thickness of the shock at lower Reynolds numbers.

For comparatively high Reynolds numbers $\mathrm{Re}_w = 4000$ (Fig. 6*c*), a three-shock configuration with a negative reflection angle is formed. A similar configuration is also observed in the inviscid case (Fig. 6*d*); moreover, the shock wave configurations in these simulated cases with three-shock configurations almost coincide. In fact, the effect of viscosity is attenuated with increasing Reynolds number, and the numerical solution of the Navier–Stokes equations approaches the numerical solution of the Euler equations. In addition, an attempt is made to delay the transition from the two-shock to three-shock configuration. The numerical solution with regular reflection obtained at a low Reynolds number is taken as the initial flow field for the subsequent computation with a higher Reynolds number. However, no significant delays between different types of reflection are found.

As was noted in the previous Section, the numerical solution of the Euler equations (Fig. 6*d*) is used for simulating a three-shock configuration with a negative reflection angle at $\mathrm{Re}_w = 1000$; an irregular configuration is obtained in the viscous case (Fig. 4*a*). The results reported in the present Section show that a regular configuration (Fig. 6*a*) interacting with the expansion fan is formed if other initial conditions are used. Thus, we can state that the numerical solution of the Navier–Stokes equations is not unique and depends on the initial conditions.

Let us consider the distributions of the Mach numbers, flow deflection angles, and pressures near the point of shock wave reflection for $\mathrm{Re}_w = 1000$ (Figs. 7*a*–7*c*) in the case of the two-shock configuration. These distributions are constructed for a constant coordinate y/w, i.e., parallel to the plane of symmetry. It is clearly seen that a nonuniform flow is formed behind the incident shock; all parameters of this flow change continuously because of the influence of the expansion fan emanating from the trailing edge of the wedge. As a result, the flow parameters ahead of the reflected wave near the reflection point differ from the parameters behind the incident shock. In this case, the polar R of the reflected shock (see Fig. 7*d* or Fig. 3*a*) should emanate from the point D1 determined by the flow parameters upstream of the reflected shock wave rather than from the point D (Fig. 7*d*). These parameters can be estimated from the distributions in Figs. 7*a* and 7*c*: the flow deflection angle is $\theta_{\mathrm{D1}} \approx 36°$, the pressure is $p_{\mathrm{D1}}/p_\infty \approx 22$, and the Mach number is $\mathrm{M}_{\mathrm{D1}} \approx 2.6$ (the values are given for $y = 0.0125w$). Based on these estimates, it is possible to construct a new polar R1 of the reflected shock (Fig. 7*d*). The presence

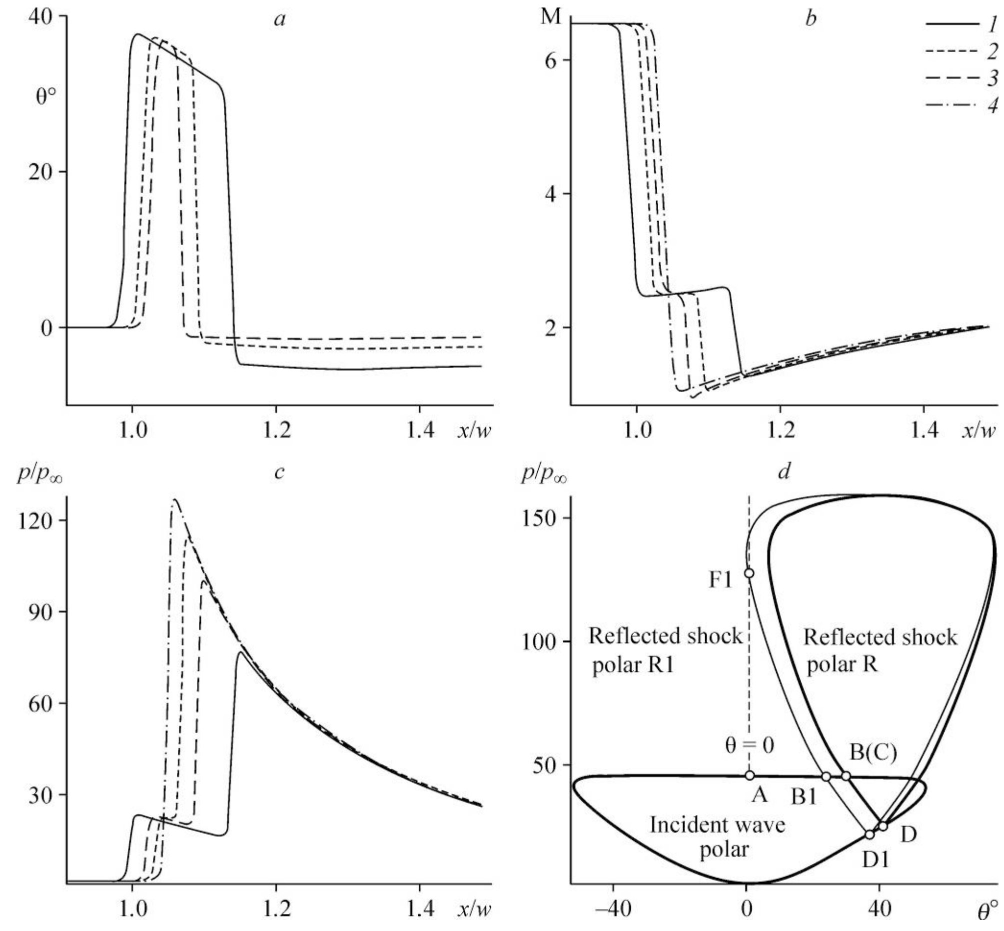

Fig. 7. Distributions of gas-dynamic parameters near the point of shock wave reflection. $M_\infty = 6.5$, $\gamma = 1.2$, $\theta_w = 40°$, $Re_w = 1000$; *a*, *b*, *c* — $y = 0.05w$ (*1*), $0.025w$ (*2*), $0.125w$ (*3*), 0 (*4*), *d* — shock polars in the plane ($\theta°$, p'/p_∞).

of the expansion fan reduces the angle of flow deflection behind the incident shock (Fig. 7*a*); therefore, the point D1 is located to the left of the point D. Reduction of the flow deflection angle leads to a minor shift of the new polar R1, which, in turn, starts to intersect the pressure axis at the point F1 (two-shock solution). In fact, this is an explanation for the emergence of regular reflection. It should be noted that viscosity produces a stabilizing effect in this case: the regular configuration is stationary (independent of time) and does not transform to the irregular configuration as it happens in the inviscid case. On the other hand, it is clearly seen that the Mach reflection is possible under these conditions (see the solution B1 in Fig. 7*d*). It is well known that the transition from one state to the other may occur, for example, due to insertion of a perturbation into the free stream (see, e.g., [24, 25]). In the case considered here, a sufficiently intense perturbation initiates a transition from the state F1 to the state B, then the Mach stem increases in size, and the expansion fan affects only the reflected shock, as it was demonstrated in the previous Section. The transition from one state to the other due to a perturbation of the free-stream pressure is considered below.

In the case with $Re_w = 1000$, the transition occurs only if the perturbation intensity is very high. Figure 8 shows the transition initiated by inserting a local perturbation (1300-fold increase in pressure as compared to the free-stream value) in the domain of the free stream ahead of the incident shock. After the perturbation passes through the computational domain (Figs. 8*a*–8*c*), a small region with a subsonic flow is formed near the plane of symmetry (Fig. 8*d*), which is responsible for the subsequent transition from regular to irregular reflection (Figs. 8*e*–8*f*).

At high Reynolds numbers, the transition can be initiated by significantly less intense perturbations. In particular, at $Re_w = 2000$, a local increase in pressure by a factor of 1.4 is sufficient for the transition to the three-shock configuration. This transitional process is not shown in this paper because it is qualitatively identical to the process illustrated in Fig. 8. As the Reynolds number increases, the influence of viscous effects on the formation of the flow structure becomes less and less significant, and their contribution to stabilization of the two-shock structure becomes appreciably smaller.

The results described in this paper allow us to conclude that there exists a certain critical Reynolds number below which the numerical solution of the Navier–Stokes equations is not unique. If the Reynolds number is higher than the critical value, a unique shock wave configuration is observed.

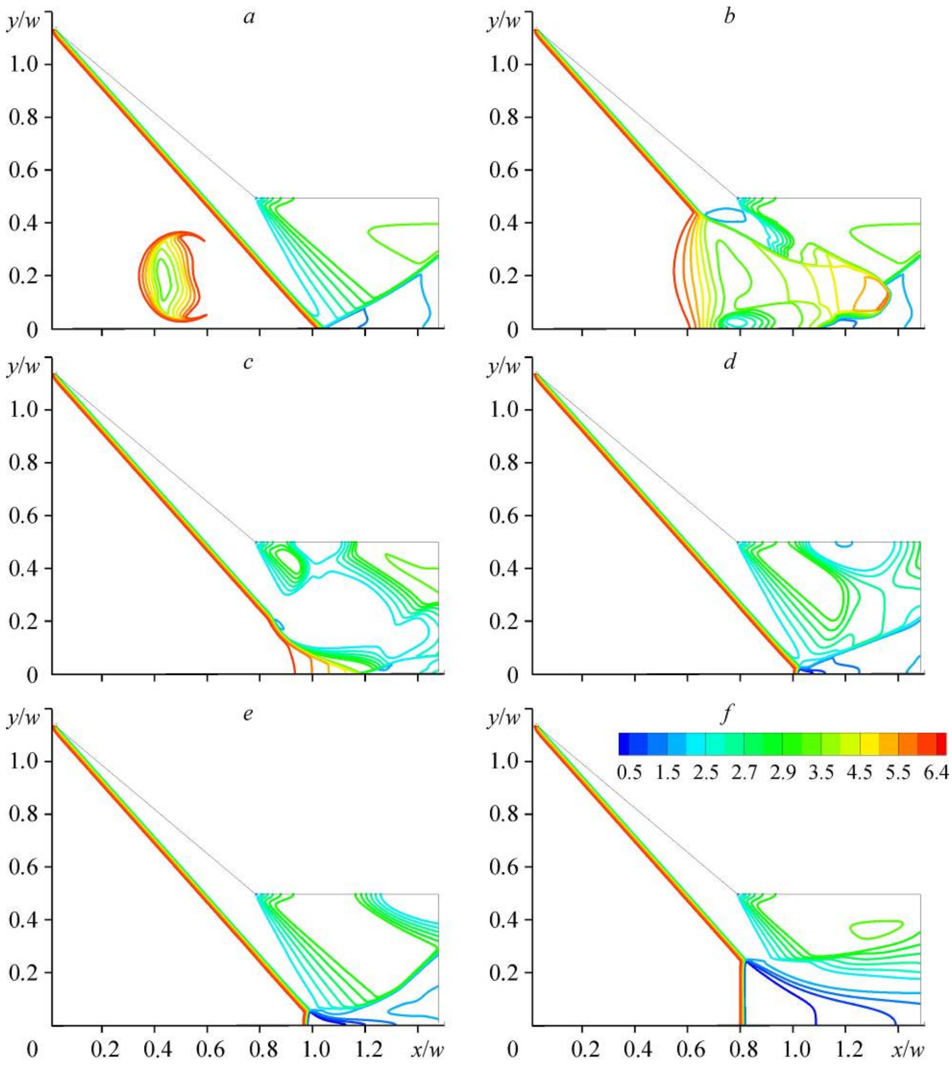

Fig. 8. Transition from regular reflection with an expansion fan to irregular reflection.
$M_\infty = 6.5$, $\gamma = 1.2$, $\theta_w = 40°$, $Re_w = 1000$.

Conclusions

Reflection of shock waves between two symmetric wedges in a steady flow of a viscous heat-conducting gas with a low ratio of specific heats is numerically studied. According to the inviscid three-shock theory, the case considered in this study corresponds to a three-shock configuration with a negative reflection angle. It is numerically demonstrated that two different shock wave configurations can be formed depending on the initial flow field: regular reflection pattern interacting with the expansion fan and three-shock configuration with a negative reflection angle. Both configurations are steady, and no instability is observed. Despite specific features of the three-shock configuration with a negative reflection angle, it is qualitatively similar to the Mach reflection with a positive angle of the reflected shock.

The nonuniqueness of the shock wave configuration is observed only in the range of low Reynolds numbers, where viscosity produces a significant effect on the flow structure. A transition from the two-shock to three-shock configuration is possible, for example, due to a pressure perturbation introduced into the free stream. As the Reynolds number increases, only a three-shock configuration with a negative reflection angle is observed.

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