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# Longitudinal control of aircraft dynamics based on optimization of PID parameters

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Recent years many flight control systems and industries are employing PID controllers to improve the dynamic behavior of the characteristics. In this paper, PID controller is developed to improve the stability and performance of general aviation aircraft system. Designing the optimum PID controller parameters for a pitch control aircraft is important in expanding the flight safety envelope. Mathematical model is developed to describe the longitudinal pitch control of an aircraft. The PID controller is designed based on the dynamic modeling of an aircraft system. Different tuning methods namely Zeigler–Nichols method (ZN), Modified Zeigler–Nichols method, Tyreus–Luyben tuning, Astrom–Hagglund tuning methods are employed. The time domain specifications of different tuning methods are compared to obtain the optimum parameters value. The results prove that PID controller tuned by Zeigler–Nichols for aircraft pitch control dynamics is better in stability and performance in all conditions. Future research work of obtaining optimum PID controller parameters using artificial intelligence techniques should be carried out.

**Key words**: pitch control dynamics, PID controller, optimum parameters.

## Introduction

Wright brothers conducted thousands of experiments in gliders in developing successful airplane. After successful invention, the aircraft soon started with designing of dynamic characteristics. The dynamic performance such as stability and control characteristics of an airplane are termed as aircraft flying qualities. In general, airplanes with poor flying qualities will be difficult to fly and could be dangerous. This paper focuses in designing PID controller and optimum values for general aviation aircraft. General aviation flights range from gliders and powered parachutes to corporate jet flights. General aviation covers a large range of activities, both commercial and non-commercial, including flying clubs, flight training, agricultural aviation, light aircraft manufacturing and maintenance [1–3]. The main objective of the paper is to design PID controller for pitch control general aviation aircraft. To obtain optimum parameters value from different closed loop tuning methods such as Zeigler–Nichols method (ZN), Modified Zeigler–Nichols method, Tyreus–Luyben tuning, Astrom–Hagglund tuning methods are employed. The classical approach root locus method is employed in determining the ultimate gain constant and period of oscillation of aircraft dynamics. Simulation is performed by using Matlab–Simulink model. The approach of the work illustrates time domain specifications of the system performance as affected due to parameter variations [4–5].

## 1. Mathematical model of an aircraft

The standard notation [6–9] for describing the motion of, and the aerodynamic forces and moments acting upon, flight vehicle is indicated in Fig. 1. The variables $x$, $y$, and $z$ represent coordinates, with origin at the center of mass of the vehicle. The $x$-axis lays in the symmetry plane of the vehicle and points toward the nose of the vehicle. The $z$-axis also is taken to lie in the plane of symmetry, perpendicular to the $x$-axis, and pointing approximately down. The $y$-axis completes a right-handed orthogonal system, pointing approximately out the right wing. The variables $u$, $v$, and $w$ represent the instantaneous linear velocity components in the directions of the $x$, $y$, and $z$-axes, respectively. The variables $X$, $Y$, and $Z$ represent aerodynamic force components of rotational velocity about the $x$, $y$, and $z$-axes, the variables $p$, $q$, and $r$ represent the instantaneous angular rates around the $x$, $y$, and $z$-axes, respectively.

The equations of motion for a flight vehicle are obtained from Newton's second law, which states that the addition of all the forces acting on a vehicle will be the same measure as that of momentum of vehicle; and the aggregate of the moments acting on the vehicle will be same as that of measure of angular momentum. The force equation can be expressed as follows,

$$\sum F = \frac{d}{dt}(mv), \tag{1}$$

where $F$ represents force components such as $F_x$, $F_y$, and $F_z$ on three axes: $x$, $y$, and $z$. When working out the acceleration of each mass element ($m$), $v$ represents velocity contributions from both linear ($u$, $v$, and $w$) and rotational rates ($p$, $q$, and $r$) about $x$, $y$, and $z$-axes, respectively. The force components are composed of contributions due to sum of aerodynamic, propulsive and gravitational force acting on the airplane. The moment equation can be expressed as follows,

$$\sum M = \frac{d}{dt}(H), \tag{2}$$

where $M$ represents moment components such as $L$, $M$, and $N$ on the respective three axes $x$, $y$, and $z$-axes. Also $H$ represents moment of momentum components such as $H_x$, $H_y$, and $H_z$ along $x$, $y$, and $z$-axes, respectively.

## 2. Longitudinal equation

The aerodynamic forces and moments can be expressed as a function of all the motion variables. The complete set of equation of motion is given in equation (3), equation (4) and equation (5).

$$\left[\frac{d}{dt} - X_u\right]u + g_0 \cos\theta_0 - X_w w = \\ = X_{\delta_e}\delta_e + X_{\delta_T}\delta_T, \tag{3}$$

$$-Z_u u + \left[(1 - Z_{\dot{w}})\frac{d}{dt} - Z_w\right]w - \left[u_0 + Z_q\right]q + \\ + g_0 \sin\theta_0 = Z_{\delta_e}\delta_e + Z_{\delta_T}\delta_T, \tag{4}$$

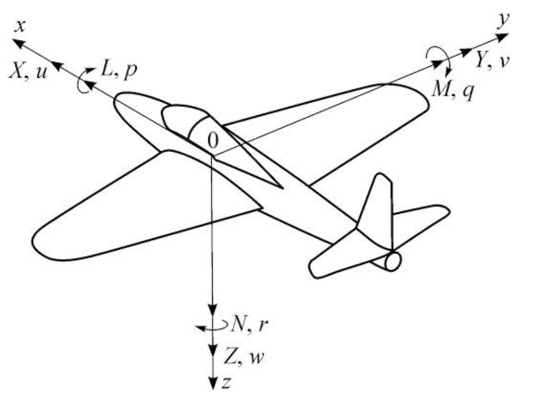


*Fig. 1.* Force, moments, and velocity components in a body fixed coordinate.

$$-M_u u - \left[ (M_{\dot{w}}) \frac{d}{dt} - M_w \right] w + \left[ \frac{d}{dt} - M_q \right] q = M_{\delta_e} + M_{\delta_T} \delta_T, \tag{5}$$

where $X_w$, $Z_w$, $M_w$, and $M_{\dot{w}}$ are called stability derivatives, that are evaluated at the reference flight condition. The control variables $\delta_e$ and $\delta_T$ correspond to perturbations from trim in the elevator and thrust (throttle) settings. Also $X_{\delta_e}$, $Z_{\delta_e}$, $M_{\delta_e}$ correspond to elevator settings for $X$ force, $Z$ force and pitching moment. The variables $X_{\delta_T}$, $Z_{\delta_T}$, $M_{\delta_T}$ correspond to throttle settings for $X$ force, $Z$ force and pitching moment [10–14]. The dot above the variables denotes in equations (3)–(8) the derivative of these variables. The relation of stability derivative for longitudinal motions to dimensionless derivatives aerodynamic coefficients is shown in Table 1. The stability coefficients $C_{x_\alpha}, C_{x_0}, C_{x_u}$ are related to the corresponding to $X$ force coefficients with respect to angle of attack, reference value and change speed. The variables $C_{z_0}, C_{z_u}, C_{z_\alpha}, C_{z_{\dot{\alpha}}}, C_{z_q}$ are called stability coefficients correspond to $Z$ force with respect to reference value, change speed, angle of attack and pitch rate. The stability coefficients $C_{m_u}$, $C_{m_\alpha}$, $C_{m_{\dot{\alpha}}}$, $C_{m_q}$ are related to the corresponding to pitching moment coefficients. The term $I_y$ is the mass moment of inertia of the body about the y axis. The terms $m$, $\bar{c}, u_0$ and $Q$ are the mass, the wing mean aerodynamic chord, reference flight speed and the dynamic pressure of the vehicle.

The nonlinear equations of motion can be linearized by using small disturbance theory. Small disturbance theory is applied considering small deviations about steady flight conditions. The following equations (1), (2), and (3) represent linearized equations of pitch control dynamics of an aircraft.

$$\Delta X = \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \delta_e} + \frac{\partial X}{\partial \delta_T} \delta_T, \tag{6}$$

$$\Delta Z = \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \dot{w}} \dot{w} + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial \delta_e} \delta_e + \frac{\partial Z}{\partial \delta_T}, \tag{7}$$

$$\Delta M = \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w} + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \delta_e} \delta_e + \frac{\partial Z}{\partial \delta_T} \delta_T. \tag{8}$$

Table 1

**Stability derivatives for longitudinal motions**

| Variable | $X$ | $Z$ | $M$ |
|---|---|---|---|
| $U$ | $X_u = \frac{QS}{mu_0}\left[2C_{x_0} + C_{x_u}\right]$ | $Z_u = \frac{QS}{mu_0}\left[2C_{z_0} + C_{z_u}\right]$ | $M_u = \frac{QS\bar{c}}{I_y u_0} C_{m_u}$ |
| $w$ | $X_w = \frac{QS}{mu_0} C_{x_\alpha}$ | $Z_w = \frac{QS}{mu_0} C_{z_\alpha}$ | $M_w = \frac{QS\bar{c}}{I_y u_0} C_{m_\alpha}$ |
| $\dot{w}$ | $X_{\dot{w}} = 0$ | $Z_{\dot{w}} = \frac{QS}{mu_0^2} C_{z_{\dot{\alpha}}}$ | $M_{\dot{w}} = \frac{QS\bar{c}^2}{I_y u_0} C_{m_{\dot{\alpha}}}$ |
| $Q$ | $X_q = 0$ | $Z_q = \frac{QS}{mu_0} C_{z_q}$ | $M_q = \frac{QS\bar{c}^2}{I_y u_0} C_{m_q}$ |

In these equations, the control variables $\delta_{\mathrm{e}}$ and $\delta_{\mathrm{T}}$ correspond to perturbations from trim in the elevator and thrust (throttle) settings. Note that the $Z$ force and pitching moment $M$ are assumed to depend on both the rate of change of angle of attack $w$ and the pitch rate $q$, but the dependence of the $X$ force on these variables is neglected. The linearized longitudinal equations give valuable information of dynamic characteristics of airplane motion.

Equation (9) gives the transfer function for the change in the pitch rate to the change in elevator deflection angle.

$$\frac{\Delta q(s)}{\Delta \delta_{\mathrm{e}}(s)} = \frac{-\left(M_{\delta_{\mathrm{e}}} + \dfrac{M_{\dot{\alpha}} Z_{\delta_{\mathrm{e}}} +}{u_0}\right)s - \left(\dfrac{M_{\alpha} Z_{\delta_{\mathrm{e}}}}{u_0} + \dfrac{Z_{\alpha} M_{\delta_{\mathrm{e}}}}{u_0}\right)}{s^2 - \left(M_q + M_{\dot{\alpha}} + \left(Z_{\alpha}/u_0\right)\right)s + \left(\left(Z_{\alpha} M_q/u_0\right) - M_{\alpha}\right)}. \tag{9}$$

### 3. PID controller

The PID controller is closed loop feedback mechanism widely used in many industries and flight control systems. The PID controller calculates an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting pitch control inputs. The PID controller parameters are called three-term control such as the proportional, the integral, and derivative values denoted P, I, and D [15−17]. Tuning the three parameters in the PID controller algorithm, the controller can provide control action designed for specific flight requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set point, and the degree of system oscillation. The structure is also known as parallel form and is represented by,

$$G(s) = K_{\mathrm{P}} + K_{\mathrm{I}}(1/s) + K_{\mathrm{D}}s = K_{\mathrm{P}}\left(1 + 1/(T_{\mathrm{I}}s) + T_{\mathrm{D}}s\right) \tag{10}$$

where $K_{\mathrm{P}}$ is the proportional gain, $K_{\mathrm{I}}$ is the integral gain, $K_{\mathrm{D}}$ is the derivative gain, $T_{\mathrm{I}}$ is the integral time constant, and $T_{\mathrm{D}}$ is the derivative time constant.

The simple block diagram of general aviation aircraft with actuator dynamics and PID controller is shown in Fig. 2. The proportional term provides the error signal through the constant gain factor. The integral term is to reduce steady-state, and the derivative term is to improve transient response. The effect of variation of parameter for closed loop response is given in Table 2. The combination of PID controller performs better compared to independent operations. The selection of gains for the PID controllers can be determined by various closed loop tuning methods.

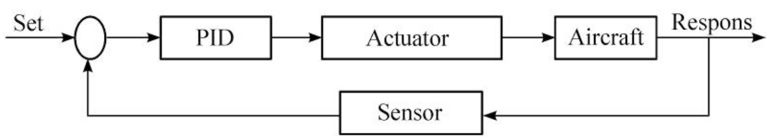


*Fig. 2*. Block diagram of PID controller.

**Table 2**

| **Parameters affecting system dynamics** | | | |
|---|---|---|---|
| **Closed loop response** | **Proportional gain ($K_{\mathrm{P}}$)** | **Integral gain ($K_{\mathrm{I}}$)** | **Derivative gain ($K_{\mathrm{D}}$)** |
| Rise time | Decrease | Decrease | Small change |
| Overshoot | Increase | Increase | Decrease |
| Settling time | Small change | Increase | Decrease |
| Steady-state error | Decrease | Eliminate | No change |

### 4. Aircraft dynamics without controller effect

In general, the non-linear aircraft model is complex, and the complexity arises from the mathematical model of dynamics given in equation (7).

$$G(s) = \frac{120s + 253.8}{s^4 + 13.17s^3 + 44.64s^2 + 129.4s}. \quad (11)$$

Figure 3 shows step response of system without controller. The rise time is 1.1 secs, and settling time is high in the range of 9.3 secs. The simulation is carried out using Matlab–Simulink model [12].Though overshoot is less but the response leads to oscillation for longer period. This leads the aircraft difficult to fly and makes the performance unstable in nature. Table 3 shows the values of parameters of dynamic response of aircraft without PID controller.

### 5. Tuning methods

The selection of gains for PID controller can be determined by various tuning methods [18–21]. The gains are determined in terms of two parameters, $k_{pu}$, called the ultimate gain, and $T_u$, the period of the oscillation that occurs at the ultimate gain. From Fig. 4, the ultimate gain can be obtained as 1.87, and the period of oscillation can be determined as 1.22.

#### *Case 1: Ziegler–Nichols* (*ZN*) *method*

Ziegler and Nichols (1942) first proposed a trial and error tuning method. This method most widely used method for tuning of PID controllers. This method does not require process model. This method is applicable for closed loop flight control systems. The values of $K_P$, $K_I$, and $K_D$ can be determined from Table 4 as 1.122, 1.833, and 0.1711. The step response of aircraft dynamics using ZN method is shown in Fig. 5.

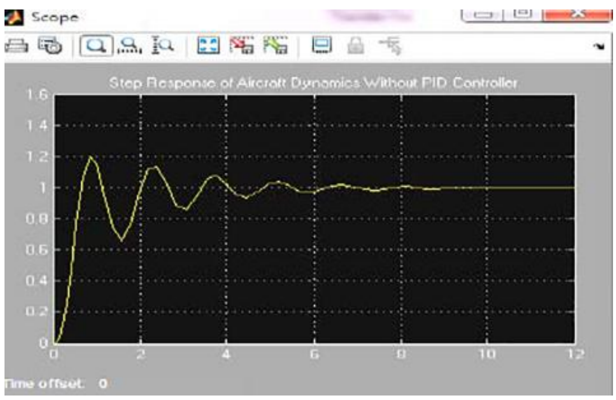


*Fig. 3*. Step response of aircraft dynamics without controller.

**Table 3**

| Effect of closed loop response without controller | | | | | |
|---|---|---|---|---|---|
| Parameters | Rise Time ($t_r$) in secs | Settling Time ($t_s$) in secs | Delay Time ($t_d$) in secs | Overshoot (% $M_p$) in % | Transient |
| Values | 1.1 | 9.3 | 0.46 | 20 | Oscillation |

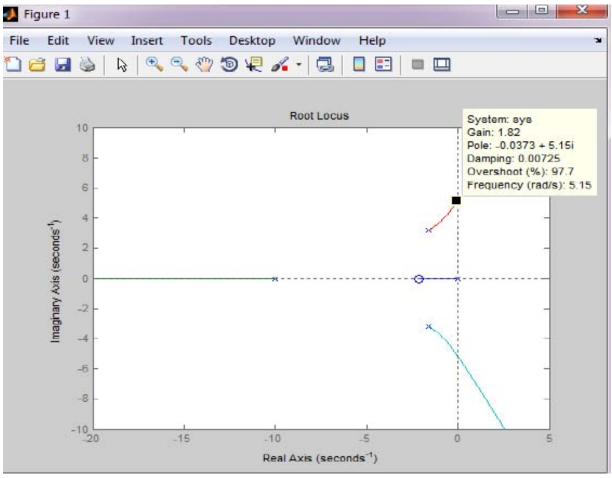


*Fig. 4*. Root locus for aircraft system.

**Table 4**

| Different Tuning Methods | | | |
|---|---|---|---|
| **Methods** | **Proportional Gain ($K_P$)** | **Integral Gain ($K_I$)** | **Derivative Gain ($K_D$)** |
| ZN Method | 0.6 $k_{pu}$ | $2k_{pu}/T_u$ | $k_{pu}\,T_u$ / 8 |
| Modified ZN Method | 0.33 $k_{pu}$ | $T_u$ / 2 | $T_u$ / 3 |
| Tyreus–Luyben Method | 0.45 $k_{pu}$ | 2.2 $T_u$ | $T_u$ / 6.3 |
| Astrom–Hagglund Method | 0.32 $k_{pu}$ | 0.94 | 0 |

***Case 2: Modified Ziegler–Nichols (ZN) method***

For some control loops, the measures of oscillation provided by 1/4 decay ratio and the corresponding large overshoots for set point changes are undesirable, therefore, more conservative methods are often preferable such as modified ZN settings These modified settings are shown in Table 4. The values of $K_P$, $K_I$, and $K_D$ can be calculated as 0.6171, 0.61, and 0.406. The step response of aircraft dynamics using modified ZN method is shown in Fig. 6.

***Case 3: Tyreus–Luyben method***

The Tyreus–Luyben method is similar to the Ziegler–Nichols method but the final controller settings are different. This method only proposes settings for $P_I$ and PID controllers. The values of $K_P$, $K_I$, and $K_D$ can be calculated as 0.8415, 2.684 and 0.193. The step response of aircraft dynamics using Tyreus–Luyben method is shown in Fig. 7.

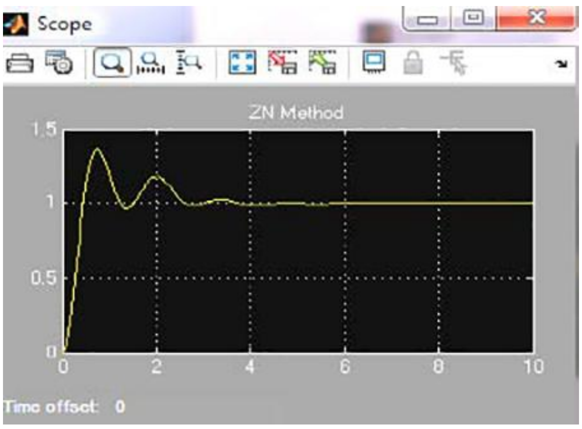


*Fig. 5*. Step response of ZN method.

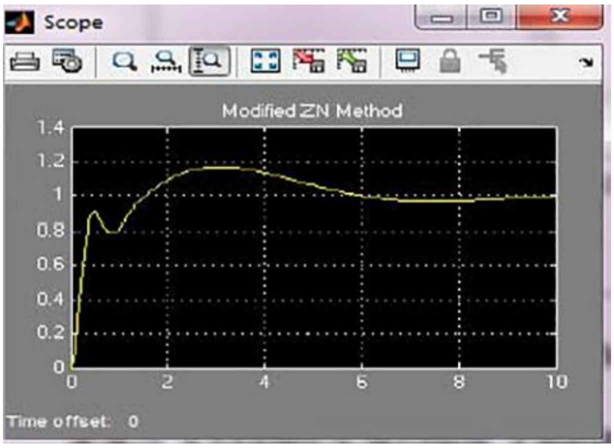


*Fig. 6*. Step response of modified ZN method.

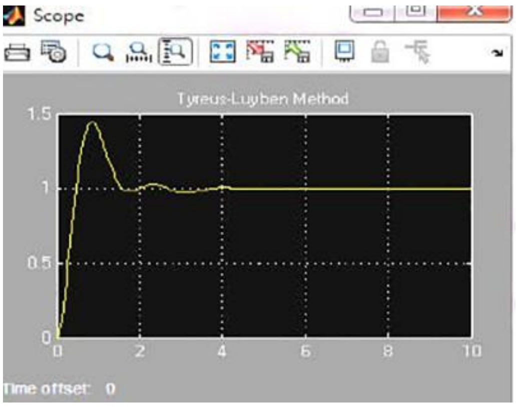


*Fig. 7*. Step response of Tyreus–Luyben method.

### *Case 4: Astrom–Hagglund tuning method*

This method is proposed by Astrom and Hagglund. They used nonlinear relay feedback. The ultimate gain and period of oscillation can be obtained from the limit cycle oscillation. The values of $K_P$, $K_I$, and $K_D$ can be estimated as 0.5984, 0.94, and 0. The step response of aircraft dynamics using Tyreus Astrom–Hagglund method is shown in Fig. 8.


*Fig. 8*. Step response of Astrom–Hagglund method.

**Table 5**

**Comparison of Different Tuning Methods**

| **Parameters** | **ZN Method** | **Modified ZN Method** | **Tyreus–Luyben Tuning** | **Astrom–Hagglund Tuning** |
|---|---|---|---|---|
| Delay Time $T_d$ in secs | 0.3 | 0.2 | 0.45 | 0.62 |
| Rise Time $T_r$ in secs | 0.6 | 0.4 | 0.9 | 1 |
| Settling Time $T_s$ in secs | 3.8 | 5.5 | 3.8 | 5.8 |
| Peak Overshoot $M_p$ in % | 37 | 18 | 46 | 50 |
| Transient behavior | Smooth | Smooth | Smooth | Oscillatory |

The different tuning methods are compared, and the results are shown in Table 5. The delay and rise time give a measure of how fast the system responds to a step input. Rise time is less in Modified ZN method compared to other methods. The settling time is less in ZN method and Tyreus–Luyben method. Peak overshoot is less in Modified ZN method. Astrom–Hagglund method response is oscillatory. This leads to unstable dynamics of aircraft. Compared to all methods, Modified ZN method shows good in time response characteristics. Though Modified ZN method is better but it has high settling time as 5.5 secs. From the standpoint of aircraft control system design, the required characteristic is that the system has to respond rapidly for any change in input. This helps the flight to fly in safe envelope. By considering this, ZN method gives optimal gain values of PID controller parameters.

### 6. Results and discussion

The longitudinal equations developed in equations (6) to (8) are simple, and the coefficients in the differential equations are made up of the aerodynamic stability derivatives, mass and inertia characteristics of the airplane. These equations can be written as a first order differential equation, called the state space equations and are represented in equation (12).

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\boldsymbol{\eta}, \tag{12}$$

where $\mathbf{x}$ is the state vector, $\boldsymbol{\eta}$ is the control vector, and the matrices $\mathbf{A}$ and $\mathbf{B}$ contain aircraft dimensional stability derivatives. The stability coefficients are calculated using the values obtained from Table 1. The longitudinal state space matrix is given in equation (13).

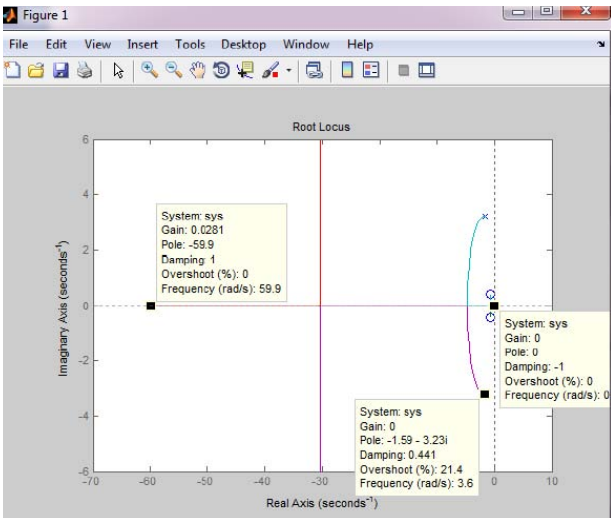


*Fig. 9.* Root locus of PID controller with aircraft dynamics response.

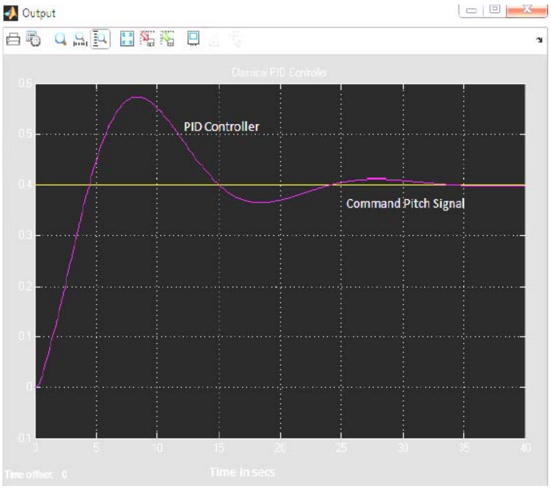


*Fig. 10*. PID controller response.

$$\mathbf{A} = \begin{bmatrix} -63.17 & -203.14 & -776.4 & 0.0000 \\ 1.00000 & 0.0000 & 0.0000 & 0.0000 \\ 0.00000 & 1.0000 & 0.0000 & 0.000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix} \quad (13)$$

The eigenvalues of the longitudinal transport are given in equations (9) and (10).

$$\lambda_{1,2} = 0,\, 60 \quad (14)$$

$$\lambda_{3,4} = -1.585 \pm i\, 3.22. \quad (15)$$

The root locus of closed loop PID controller for aircraft dynamics is shown in Fig. 9. The roots are real, there is of course no period, and only parameter is the time to double or half. When the modes are oscillatory, it is envelope ordinate that doubles or halves. Since the envelope may be regarded as an amplitude modulation, then we may think of the doubling or halving as applied to the variable amplitude. The stability of the airplane is governed by the real parts of the eigenvalues, roots of the characteristics equation.

The step response of the PID controller is shown in Fig. 10. The delay time is 3 secs, rise time is 8 secs, settling time is 33 secs, and peak overshoot is 59 %. The response is oscillatory in nature.

## Conclusions

The parameters of a control system may have tendency to vary due to changing environment conditions, and this variation in parameter affects the desired performance of a control system. Hence, in this paper, optimum gain values of PID controller parameters are obtained for controlling non-linear longitudinal pitch control of an aircraft. In recent years, many techniques are developed using PID controller to control aircraft dynamics. Compared to various tuning methods, Zeigler–Nichols Method gives optimum gain values of PID controllers. The tuned parameter values of PID controller can effectively eliminate the dangerous oscillations and provide smooth operation. This optimum value works efficiently for nonlinear dynamics of pitch control aircraft where safety is high priority.

## *Nomenclature*

$g$ — acceleration due to gravity,
$M_q$ — dimensional variation of pitching moment with pitch rate,
$M_u$ — dimensional variation of pitching moment with speed,
$M_\alpha$ — dimensional variation of pitching moment with angle of attack,
$M_{\dot{\alpha}}$ — dimensional variation of pitching moment with rate of change angle of attack,
$q$ — perturbed pitch rate,
$S$ — reference wing area,
$T$ — thrust,
$u$ — perturbed velocity along $X$,
$X_q$ — dimensional variation of $X$ force with pitch rate,
$X_{tu}$ — dimensional variation of $X$ force due to thrust with speed,
$X_u$ — dimensional variation of $X$ force with speed,
$X_\alpha$ — dimensional variation of $X$ force with angle of attack,
$w$ — perturbed velocity along $Z$,
$Z_q$ — dimensional variation of $Z$ force with pitch rate,
$Z_u$ — dimensional variation of $Z$ force with speed,
$Z_\alpha$ — dimensional variation of $Z$ force with angle of attack,
$Z_{\dot{\alpha}}$ — dimensional variation of $Z$ force with rate of change angle of attack,
$\alpha$ — perturbed angle of attack,
$\varsigma_p$ — damping ratio the phugoid,
$\varsigma_{sp}$ — damping ratio the short period,
$\theta$ — disturbed pitch attitude angle,
$\theta_1$ — steady state pitch attitude angle,
$\rho$ — air density,
$\omega_{np}$ — frequency of phugoid,
$\omega_{nsp}$ — frequency of short period.

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