

# Dynamical Analysis of a Charged Spherical Star in $R - \beta R_c \tanh(R/R_c) + \xi RT$ Gravity

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**Abstract**—We illustrate the dynamical instability of charged spherical fluid configuration with anisotropic conditions in nonminimally coupled  $f(R, T)$  theory of gravitation, where  $R$  is the Ricci scalar and  $T$  is the trace of the energy momentum-tensor. We investigate both modified field equations and continuity equations to provide some extra degrees of freedom under the constraints of a specific considered form of  $f(R, T)$  gravity. We examine how small changes in geometric and material profiles affect the fluid's collapsing structure via a perturbation scheme. We study the unstable eras using Newtonian ( $\mathbb{N}$ ) and post-Newtonian (pN) approximations by applying some significant constraints on the collapse equation. Our findings demonstrate that the stiffness parameter  $\Gamma$  has a substantial influence on the identification of unstable phases for our charged stellar geometry. We conclude that some correction terms that are dark source terms, which appear due to  $f(R, T)$  gravity, lead to an unstable structure/phase throughout the evolutionary process.

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## 1. INTRODUCTION

The general theory of relativity (GR) is a significant theory able to study the evolution of our mysterious cosmos. It is a geometric theory that replacing the Newtonian principle of gravitational study. Gravitation may be examined from the viewpoint of the GR fundamental and essential components. The main building block of GR has been well identified in our modern understanding of universal gravitation. In the field of cosmology and astrophysics, there are many other serious challenges surrounding dynamical systems under the influence of gravity.

During the fluid model evolution, compact self-gravitating objects move via different transitions. Either in Newtonian ( $\mathbb{N}$ ) gravity or GR, with anisotropic distributions of any fluid, the research of cosmic inflation is generally considered. New observations from many data sources, like type Ia supernovae, cosmic microwave background (CMB) [1–3], large-scale structure, etc., provide a conclusive proof of the universe's accelerated expansion [4, 5]. It is concluded that our cosmos consists of almost 94 to 96% of enigmatic dark constituents, according to observations of our interstellar evolution [6, 7]. These mysterious constituents are usually referred

to as unobservable and mysterious forms of matter, i.e., dark energy (DE) and dark matter (DM). In our stellar dynamics, the former induces contraction influence while the latter is indeed an unseen form of matter whose existence has been eventually discovered just because of its gravitational effects on luminous structures. The most recent large-scale astrophysical evidence has verified its physical existence.

It is still thought that this nature of the entire cosmos is due to the influence of DE that probably carries a high negative extra pressure. The available evidence on the universe's early or late acceleration and indeed the presence of DM have provided gravitational theories with a substantial platform. One key to understanding this phenomenon is by assuming that Einstein's gravity concept of GR starts to break down at large scales, and gravity is defined by a more general action. One of the most well-known ways of obtaining such action is where the typical Einstein Hilbert-action is replaced with a function of the Ricci scalar curvature  $R$ . Modified  $f(R)$  cosmology [8] may provide the best description of nature as well as the physical presence of a cosmic expansion at late/early periods.

Criteria have also been formulated for empirical manifestation of realistic cosmic models, and weak field restrictions arising from classical experiments of GR for the solar system framework, which seems to rule out several models proposed to date [9]. Feasible

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models are being established by performing solar system tests, as can be seen in [10]. A few of the physically attractive concepts dealing with the existence of DE use the idea of modified gravity theories MGTs. Many researchers believe that the technique of MGTs could be constructive and helpful for explaining the existence and characteristics of the dark sector of the entire cosmos and to provide certain corrections to the research (see for more detail [11–22]). In this view, Capozziello and de Laurentis [23] adequately addressed several other terms and conditions for any MGT to be a good fit for observational cosmology, particularly scalar-tensor gravity as well as  $f(R)$  theories. Using new  $f(R)$  dark energy models, Nojiri and Odintsov [24] reviewed cosmic inflation of our entire cosmos.

The extended  $f(R, T)$  theory is a modification of  $f(R)$  gravity, where certain quantum phenomena or any exotically non-ideal material configurations eventually show a  $T$  dependence. It is well established that once a compact stellar object is in a hydrostatic stage, it can consequently need to be in unstable. Harko et al. studied arbitrary couplings in both geometry and matter [25], even this model was indeed massively expanded. The astrophysical and general-relativistic effects regarding a nonminimal matter-geometry coupling have also been explored extensively in the literature [26], and taking into account the Palatini model of nonminimal coupling [27]. A full expansion of EHA was strongly suggested in this scenario.

In relativistic fluid distribution, Chandrasekhar [28] eventually found an instability parameter whose failure corresponds to the collapse of a compact supernova of our cosmos. He also investigated the crashing super-massive interstellar object's stability framework via the adiabatic index  $\Gamma$ . Furthermore, the influence on various ranges of  $\Gamma$  on different structural parameters in stable and consistent epochs of compact structures in MGT was extensively studied. Herrera et al. [29] generalized the findings of Chandrasekhar's work for anisotropic spherical self-gravitating internal structures. He also investigated the nonadiabatic nature of the celestial structure stability at a certain stage. Chan et al. [30] investigated several realistic fluids while inside radiating stars to modify their effects. Chandrasekhar's finding was later extended to both N and pN eras. The pressure anisotropy and radiation density both have substantial impacts, as well as viscosity, on the structural instability of nonadiabatic fluid configurations, and shearing viscous fluids configuration has also been discussed [29–35].

Schmidt et al. [36] looked into the spherical implosion, utilising vast and large-scale computations in combination with quantitative simplifications for

the Hu as well as Sawicki  $f(R)$  framework [10]. Some scholars studied the possible implications and influence of  $f(R)$  theory in their theoretical work [37–39]. As to different fluid configurations, Yousaf et al. studied  $f(R)$  theory for some spherical and cylindrical space-times [40, 41]. The involvement of astrophysical MGT models is investigated by Bhatti and Yousaf [42] by calculating the structure scalars and mass-radius relationships. Herrera et al. also investigated the (in)stability limits for a spherical structure, particularly when combined with anisotropic pressure. Capozziello et al. [43] evaluated the Jeans instability with weak-field approximation in  $f(R)$  cosmology for an appropriate collapsing configuration. They also studied an Einstein model with some other functions of  $f(R)$  gravity. Capozziello also examined [44] how higher-order gravitational theories can be used to explore topics like quintessence.

Many scientists were fascinated by the discovery of dark physical dimensions of the cosmos through the newly evolved  $f(R, T)$  theory of gravity. Harko and his collaborators [45] adequately addressed cosmological implications of modified gravity by choosing particular types of  $f(R, T)$  models. A key role of some other important MGT models in the understanding of cosmic acceleration was investigated by Yousaf et al. They worked with various astrophysical frameworks in MGT cosmology [15, 46]. Yousaf et al. examined the consequences of  $f(R, G)$  gravity [47] and  $f(R, T, R_{\alpha\delta}T^{\alpha\delta})$  [48, 49] in this specific direction as well as effects of  $f(R, G)$  on cosmic inflation and dark parameters. In the context of  $f(R)$  gravity, Bamba et al. [38] examined a curvature singularity that develops during stellar collapse and, in particular, figured out how the curvature singularity is formed and estimated how long it would take for it to occur by adding  $R^\mu$  ( $1 < \mu \ll 2$ ) to a valid  $f(R)$  gravity model. They also investigated [50] a particular  $f(G)$  type arrangement that might be used to handle finite-time future singularities possible in late time accelerated periods.

Bhatti et al. [51, 52] studied the complexity of compact symmetric fluid configurations in the presence of MGTs with some scalar term. They employed orthogonality breakdown in the Riemann tensor, and one of the scalar functions was used to determine the complexity factor. They also revealed the degrees of freedom and complexity factor specific physical factors such as the energy density inhomogeneity and anisotropic pressure. They also investigated alternatives to black holes, i.e., gravastars or string-like stars in the context of some MGTs for different matter configurations with and without electromagnetic effects [53, 54]. Several novel techniques were used for discussing the quintessence and rapid expansion

of our universe and comparing them to the data. The second method is to update a gravitational theory by assessing the action of  $f(R)$  gravity [55].

In two separate situations with bulk viscosity of matter, Sahoo et al. [56] looked at a spatially homogeneous anisotropic cosmos in  $f(R, T)$  gravity. To find an exact solution of the field equations, they assumed a time-varying deceleration parameter that produces an accelerating cosmos. They also showed gravitational models with varied  $f(R)$  functions as well as fixed  $T$  dependence that results in a subclass of  $f(R, T) = f(R) + f(T)$  gravitational models [57]. The study of different symmetric self-gravitating anisotropic compact fluid structures in various extended theories of gravity was carried out by utilizing different mathematical techniques [58–61]. Moraes et al. [62] investigated the presence of cosmic bodies and inferred those viable conditions, particularly in minimally coupled  $R + \beta T$  gravity, with  $\beta = \text{const}$ .

Guha and Ghosh [63] discussed dynamical instability of spherical matter compositions in  $f(R, T)$  gravity using a significant gravitational model in  $\mathbb{N}$  and pN domains under shear-free conditions. Bhatti et al. [64, 65] investigated the dynamic instability constraints with and without electric effects under the influence of  $f(R, T)$  gravity for compact cylindrically symmetric stars with anisotropic fluids. They studied the dynamic behavior of their considered cylindrically symmetric structures in a nonminimally coupled four-parametric gravitational model, i.e.,  $f(R, T) = R + \alpha R_c [1 - (1 + \phi)^{-n}] + \lambda RT$  by establishing the expression of the adiabatic index  $\Gamma$  in  $\mathbb{N}$  and pN eras. Here,  $\phi = R^2/R_c^2$ ,  $\alpha > 0$ ,  $n \in R^+$ , and  $0 < \lambda \ll 1$ , also  $R_c = \text{const}$ .

The major goal of this research is to focus on the instability of a charged spherically symmetric non-static fluid assuming  $f(R, T)$  gravity via a perturbation scheme, to obtain approximate solutions of our highly nonlinear established field and continuity equations. In  $f(R, T)$  astrophysical gravity model, the source term plays a vital role, determining the variance of  $T_{\alpha\beta}^{(m)}$ , i.e., the usual matter stress tensor, w.r.t. the metric. The main parameter in this dynamical study is the stiffness factor or the adiabatic index  $\Gamma$ , whose value ultimately determines the (in)stability in a wide range. Thus an astrophysical model under consideration goes into an unstable phase at a value of  $\Gamma$  [28] that should be less than  $4/3$  in a  $\mathbb{N}$  perfect fluid. Under the assumption of MGT gravity, we study the dynamical stability of a spherically symmetric geometry under magnetic effects. In this article, we concentrate on the stability of our considered geometry in an  $f(R, T)$  gravitational model i.e.,  $f(R, T) = R - \beta R_c \tanh(R/R_c) + \xi RT$  [66]. This

model shows significant findings for  $\beta > 0$  and  $0 < \xi \ll 1$ , where  $\xi$  is a correction parameter to  $f(R)$  gravity, and  $R_c$  is a positive constant, showing a specific values of the Ricci scalar  $R$ .

The following is the structure of this article. In Sec. 2, we look at field equations and conservation laws in  $f(R, T)$  gravity which is nonminimally coupled. Our systematic analysis in Sec. 3 concerns the gravitational model and a perturbation scheme. The collapse equation is also established in the same section by using perturbed parts of the conservation equations, surface pressures in the respective directions and the Harrison et al. equation of state (EoS) [67]. Sections 4 and 5 present expressions of  $\Gamma$  in  $\mathbb{N}$  and pN epochs, and we find the (in)stability criteria for considered charged compact systems under some physical restrictions. The last section contains conclusions and an Appendix.

## 2. SPHERICAL GEOMETRY AND BASICS OF $f(R, T)$ GRAVITY

We consider the action of  $f(R, T)$  gravity [45], obtained by generalizing the action of GR and coupling it with ordinary a matter Lagrangian  $\mathbb{L}_m$  and that of the electromagnetic field  $\mathbb{L}_{em}$ . As a result, the action for this MGT may be written as

$$\mathbb{I}_{\text{modified}} = \frac{1}{2} \int \sqrt{-g} d^4x \frac{f(R, T)}{8\pi} + \int \sqrt{-g} d^4x (\mathbb{L}_m + \mathbb{L}_{em}).$$

In general, the associated field equations can be written as

$$\frac{1}{f_R} \{8\pi G(T_{\zeta\varrho} + E_{\zeta\varrho}) - f_T(\Theta_{\zeta\varrho} + T_{\zeta\varrho} + E_{\zeta\varrho})\} = R_{\zeta\varrho} + (g_{\zeta\varrho}\square - \nabla_{\zeta}\nabla_{\varrho}) - \frac{f}{2f_R}g_{\zeta\varrho}. \quad (1)$$

Here,  $\square = \nabla_{\zeta}\nabla^{\zeta}$ ,  $\nabla_{\zeta}$  is the covariant derivative associated with the Levi-Civita relationship of  $g_{\zeta\varrho}$ ,  $f_T = \partial f/\partial T$ ,  $f_R = \partial f/\partial R$ , where  $f$  is a function of  $R$  and  $T$  while  $\Theta_{\zeta\varrho}$  is given by

$$\Theta_{\zeta\varrho} = g^{\eta\eta} \frac{\delta T_{\eta\eta}}{\delta g^{\zeta\varrho}} = -2(T_{\zeta\varrho} + E_{\zeta\varrho}) + g_{\zeta\varrho}(\mathbb{L}_m + \mathbb{L}_{em}) - 2g^{\eta\eta} \frac{\partial^2 (\mathbb{L}_m + \mathbb{L}_{em})}{\partial g^{\zeta\varrho} \partial g^{\eta\eta}}.$$

Here,  $E_{\zeta\varrho}$  is the electromagnetic tensor (ET) which can be written as

$$E_{\zeta\varrho} = \frac{1}{4\pi} \left\{ F_{\zeta}^{\gamma} F_{\varrho\gamma} - \frac{1}{4} F^{\gamma\alpha} F_{\gamma\alpha} g_{\zeta\varrho} \right\}. \quad (2)$$

The ET estimates the influence of an electric charge on a relative comoving flow rate. Here,  $F_{\zeta\varrho} = \phi_{\zeta,\varrho} - \phi_{\varrho,\zeta}$ , while  $\phi_\zeta$  is the 4-potential presented as  $\phi_\zeta = \phi(t, r)\delta_\zeta^0$ . In terms of the Maxwell field tensor, the Maxwell equations are given as  $F_{;\varrho}^{\zeta\varrho} = \mu_0 J^\zeta$ , where  $J^\zeta = \sigma U^\zeta$ , also  $U^\zeta = |g_{\zeta\zeta}|^{-1/2}\delta_0^\zeta$ ;  $J_\zeta$  stands for the 4-current, and the magnetic permeability is  $\mu_0$ . We will choose  $\mathbb{L}_m = \rho$ , and  $\mathbb{L}_{em} = \mathcal{F}$  as well as  $8G\pi = 1$  for simplicity. As a result,  $\Theta_{\zeta\varrho}$  becomes

$$\Theta_{\zeta\varrho} = -2(T_{\zeta\varrho} + E_{\zeta\varrho}) - (\rho + \mathcal{F})g_{\zeta\varrho}. \quad (3)$$

A three-dimensional spherically symmetric demarcation surface is specified to correlate with two specific type of areas, (1) inner and (2) outer space-time regions. For the interior space-time region, we have a spherically symmetric line element written diagonally as

$$dS_-^2 = A^2(t, r)dt^2 - \left( B^2(t, r)dr^2 + C^2(t, r)d\theta^2 + \sin^2\theta C^2(t, r)d\phi^2 \right), \quad (4)$$

while the exterior region is described as

$$dS_+^2 = \left( 1 - 2M\frac{1}{r} + Q^2\frac{1}{r^2} \right) dv^2 + 2drdv - r^2(t, r)(d\theta^2 + \sin^2\theta d\phi^2). \quad (5)$$

The total mass, the total charge of compact system, and the retarded time are denoted by  $M$ ,  $Q$ , and  $\nu$ , respectively. Our systematic investigation regarding the stability of charged spherically symmetric compact systems shows that some extra curvature ingredients are found due to our considered MGT. The energy and matter concentrations in gravitational dynamics are characterized by  $T_{\zeta\varrho}$ , i.e., the energy-momentum tensor (EMT), and each nonzero constituent of the EMT relates to dynamic parameters with certain physical effects. By metric variation in EHA, the modified field equations MFEs may be computed as

$$G_{\zeta\varrho} = T_{\zeta\varrho}^{\text{eff}} = \frac{1}{f_R} \left\{ (f_T + 1) \left( T_{\zeta\varrho}^{(m)} + E_{\zeta\varrho} \right) - g_{\zeta\varrho} (\rho + \mathcal{F}) f_T + (\nabla_\zeta \nabla_\varrho - g_{\zeta\varrho} \square) f_R + \frac{1}{2} (f - Rf_R) g_{\zeta\varrho} \right\}. \quad (6)$$

Here,

$$\mathcal{F} = \frac{1}{16\pi} F_{\zeta\varrho} F^{\zeta\varrho} = -\frac{s^2}{8\pi C^2},$$

while

$$s = -4\pi \int \sigma C A dr.$$

Thus the nonzero components of the ET can be obtained by using Eq. (2) as

$$E_{00} = -\frac{s^2 A^2}{8\pi C^2}, \quad E_{11} = -\frac{3s^2 B^2}{8\pi C^2}, \quad E_{22} = -\frac{s^2}{8\pi}.$$

In our calculations,  $G_{\zeta\varrho}$  is the usual Einstein tensor. We consider  $T_{\zeta\varrho}^{(m)}$  as the EMT of a usual matter configuration,

$$T_{\zeta\varrho}^{(m)} = (\rho + P_t) V_\zeta V_\varrho - P_t g_{\zeta\varrho} + (P_r - P_t) \mathcal{S}_\zeta \mathcal{S}_\varrho,$$

where the 4-vectors can be related as  $V_\zeta = \delta_\zeta^0 A$ ,  $\mathcal{S}_\zeta = \delta_\zeta^1 B$ ,  $V^\zeta V_\zeta = 1$ ,  $\mathcal{S}^\zeta \mathcal{S}_\zeta = -1$ . Also,  $P_r$  and  $P_t$  represent surface pressures in radial and tangential directions, respectively, while  $V_\zeta$  is the velocity 4-vector. After lengthy calculations, the metric variation in the EHA can be used to generate the MFEs as

$$\mathcal{G}_{00} = \frac{1}{f_R} \{ \rho^{(\text{eff})} + Q_1 \} + \varphi_{00}, \quad \mathcal{G}_{01} = \frac{\varphi_{01}}{f_R}, \quad (7)$$

$$\mathcal{G}_{11} = \frac{1}{f_R} \{ P_r^{(\text{eff})} + Q_2 \} + \varphi_{11}, \quad (8)$$

$$\mathcal{G}_{22} = \frac{1}{f_R} \{ P_t^{(\text{eff})} + Q_3 \} + \varphi_{22}, \quad (9)$$

where

$$\rho^{(\text{eff})} = \rho + \frac{1}{2}(f - Rf_R),$$

$$P_\varphi^{(\text{eff})} = P_\varphi - \frac{1}{2}(f - Rf_R) + (\rho + P_\varphi) f_T.$$

In the above expression,  $\varphi = r, t$ , respectively. Also, the terms related to charges  $Q_1, Q_2, Q_3$  turn out to be

$$Q_1 = (1 - f_T) E_{00} - A^2 \mathcal{F} f_T,$$

$$Q_2 = (1 - f_T) E_{11} - B^2 \mathcal{F} f_T,$$

$$Q_3 = (1 - f_T) E_{22} - C^2 \mathcal{F} f_T.$$

For simplicity, we use the notations  $\varphi_{00}, \varphi_{01}, \varphi_{11}, \varphi_{22}$ , given by

$$\varphi_{00} = \frac{1}{B^2} \left( \frac{f_R''}{f_R} - \frac{B\dot{B}}{A^2} \frac{f_R}{f_R} - \frac{B'}{B} \frac{f_R'}{f_R} \right) - \frac{2}{C f_R} \left( \frac{\dot{C} f_R}{A^2} - f_R' \frac{C'}{B^2} \right),$$

$$\varphi_{01} = \frac{1}{f_R} \left( \dot{f}_R - \dot{f}_R \frac{A'}{A} - f_R' \frac{\dot{B}}{B} \right),$$

$$\varphi_{11} = \frac{1}{A^2 f_R} \left( \ddot{f}_R - \dot{f}_R \frac{\dot{A}}{A} - \frac{A A'}{B^2} f_R' \right) + \frac{2 f_R'}{C f_R} \frac{\dot{C} f_R}{A^2} - \frac{2 f_R'}{C f_R} \frac{C'}{B^2},$$

$$\varphi_{22} = \frac{1}{f_R} \left\{ \frac{1}{A^2} \ddot{f}_R - \left( \frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right) \frac{f_R}{A^2} - \frac{\dot{B}}{B} \frac{f_R}{A^2} \right\}$$

$$- f_R'' \frac{1}{B^2} - \frac{f_R'}{B^2} \left( \frac{A'}{A} + \frac{C'}{C} \right) + \frac{f_R'}{B^2} \frac{B'}{B} \Big\}.$$

The Ricci curvature invariant for our considered charged spherically symmetric anisotropic geometry is

$$R = \frac{4}{B^2} \left\{ \frac{C'}{C} \left( \frac{C'}{2C} - \frac{B'}{B} + \frac{A'}{A} \right) + \frac{B^2 \dot{C}}{A^2 C} \left( \frac{\dot{A}}{A} - \frac{\dot{C}}{2C} - \frac{\dot{B}}{B} \right) + \frac{1}{2} \left( \frac{2C''}{C} - \frac{A'B'}{AB} + \frac{A''}{A} \right) - \frac{B^2}{2C^2} - \frac{B^2}{2A^2} \left( \frac{\ddot{B}}{B} + \frac{2\dot{C}}{C} - \frac{\dot{A}\dot{B}}{AB} \right) \right\}, \quad (10)$$

where the dot denotes a time derivative, and the prime a derivative in the radial coordinate.

### 2.1. Bianchi Identities

In  $f(R, T)$  gravity, the conservation laws are critical for maintaining the (in)stability criterion. In our systematic analysis, the covariant divergence of the energy-momentum tensor provides continuity equations together with the electromagnetic tensor, which is written as

$$\begin{aligned} \nabla^\varphi (T_{\varphi\tau}^{(m)} + E_{\varphi\tau}) &= \frac{f_T}{1 - f_T} \left[ \Theta_{\varphi\tau} - (T_{\varphi\tau}^{(m)} + E_{\varphi\tau}) \right] \nabla^\varphi \ln f_T \\ &+ \nabla^\varphi \Theta_{\varphi\tau} - \frac{1}{2} g_{\varphi\tau} \nabla^\varphi (T + E). \end{aligned} \quad (11)$$

We wrote the continuity equations for our locally anisotropic charged compact geometry in the considered MGT by varying  $\varphi$  and fixing the free index  $\tau$  as  $\tau = 0, 1$ . Thus the two conservation equations are

$$\begin{aligned} \dot{\rho} - \rho \left( \frac{2\dot{A}}{A^3} - \frac{\dot{C}}{CA^2} - \frac{\dot{B}}{B^3} f_T - \frac{2\dot{C}}{C} f_T \right) - \frac{2\dot{A}}{A^3} Q_1 \\ + \frac{\dot{B}}{B^3} Q_2 + \frac{2\dot{C}}{C} Q_3 + (1 + f_T) \left( \frac{\dot{B}}{B^3} P_r + \frac{2\dot{C}}{C} P_t \right) \\ + Z_1(t, r) = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} (\rho + P_r) f_T' + (1 + f_T) P_r' - \frac{2B'}{B} (\rho + P_r) f_T \\ - \frac{A'}{A^3} \rho + \frac{2C'}{C^3} P_t + f_T \rho' - \frac{A'}{A^3} Q_1 \\ - \frac{2B'}{B} Q_2 + Z_2(t, r) = 0. \end{aligned} \quad (13)$$

Here, the extra curvature terms  $Z_1(t, r)$  and  $Z_2(t, r)$ , which appear due to the MGT, are given in the Appendix.

### 3. $f(R, T)$ GRAVITY

#### AND THE PERTURBATION APPROACH

This section presents a feasible study of a specific  $f(R, T)$  gravitational model and modification of the physical parameters of a spherical charged structure via a relevant perturbation approach. The set of nonlinear MFEs in our considered gravitational theory is extremely complicated, which makes finding general solutions quite challenging. To minimize this problem, we will use a perturbation method to get a less intricate set of nonlinear equations. The perturbation framework helps us to divide the resulting field equations into perturbed and nonperturbed parts, making easier the exploration. Each of the physical variables in the perturbation scheme is initially in a state of static equilibrium, but as time passes, the perturbed physical values exhibit radial and temporal dependence. The adiabatic index  $\Gamma$ , which illustrates the domain for dynamical (in)stability, is established by first-order perturbations in the dynamical equations.

A viable gravitational model is characterized by a set of parameters whose fluctuations are compatible with the observable data in general. As a result, cosmological scenarios are chosen depending on their viability, to be satisfied to extract a constant matter dominant epoch, also pass the solar system tests and offer a reliable high-curvature setup that may be used to replicate the standard GR. The model  $f(R) = R + \alpha R^2$  has been identified as being vital for inflation in the early universe from the 1980s. The inflationary phase ends when the quadratic term  $R^2$  is equal to the linear quantity  $R$ . However, for late-time cosmic expansion, this strategy is ineffective. We continue our dynamic analysis for (in)stability criteria by establishing an inequality for  $\Gamma$  for spherical charged compact fluid configurations in a particular  $f(R, T)$  model. We discuss the dynamic (in)stability of a locally anisotropic charged stellar object for a specific functional form of nonminimally coupled  $f(R, T)$  gravity with  $f(R, T) = R - \beta R_c \tanh(R/R_c) + \xi RT$ . This functional form of  $f(R, T)$  leads to significant results only for  $\beta > 0$  and  $0 < \xi < 1$ . The free parameter values are  $R_c = H_0^2 \sim 8\rho_c/(3m_p^2) \approx 10^{-84} \text{ GeV}^2$ ,  $\beta \simeq \mathcal{O}(1)$ , and  $\xi \simeq \mathcal{O}(1)$ . Here,  $R_c$  is a constant in this gravitational function, and  $\xi RT$  represent a correction term to  $f(R)$  gravity. In this model  $R_c$  is simply of the same order as the today's Ricci scalar, while  $H_0$  is today's Hubble constant,  $\rho_c$  is the critical density, approximately equal to  $10^{-29} \text{ g/cm}^3 \sim 4.50 \times 10^{-47} \text{ GeV}^{-4}$ . In this manuscript, we ignore the second and higher-order terms of  $\epsilon$ . In this scheme,  $1 \gg \epsilon > 0$ , and we have perturbations as [68]

$$f(R, T) = R_0 - \beta R_c \tanh\left(\frac{R_0}{R_c}\right) + \xi R_0 T_0$$

$$+ \epsilon \left[ \Psi e_4 - \beta R_c \tanh \left( \frac{\Psi e_4}{R_c} \right) + \xi R_0 \bar{T} + \xi \Psi e_4 T_0 \right], \tag{14}$$

$$f_R(R, T) = 1 - \beta \sec h^2 \left( \frac{R_0}{R_c} \right) + \xi T_0 - \epsilon \left[ \beta \sec h^2 \left( \frac{\Psi e_4}{R_c} \right) + \xi \bar{T} \right], \tag{15}$$

$$f_T(R, T) = \xi R_0(r) - \epsilon \xi \Psi(t) e(r), \tag{16}$$

$$A(t, r) = A_0(r) + \epsilon \Psi(t) a(r), \tag{17}$$

$$B(t, r) = B_0(r) + \epsilon \Psi(t) b(r), \tag{18}$$

$$C(t, r) = C_0(r) + \epsilon \Psi(t) c(r), \tag{19}$$

$$R(t, r) = R_0(r) + \epsilon \Psi(t) e(r), \tag{20}$$

$$P_r(t, r) = P_{r0}(r) + \epsilon \bar{P}_r(t, r), \tag{21}$$

$$P_t(t, r) = P_{t0}(r) + \epsilon \bar{P}_t(t, r), \tag{22}$$

$$m(t, r) = m_0(r) + \epsilon \bar{m}(t, r), \tag{23}$$

$$T(t, r) = T_0(r) + \epsilon \bar{T}(t, r), \tag{24}$$

$$\rho(t, r) = \rho_0(r) + \epsilon \bar{\rho}(t, r), \tag{25}$$

$$Q_i(t, r) = Q_i^{(s)}(r) + \epsilon \Psi(t) q_i(r), \quad i = 1, 2, 3. \tag{26}$$

Here, both  $\epsilon$  and  $\xi$  are very small.

### 3.1. Static Configuration

Here we deal with static parts of the Ricci curvature invariant  $R$  and the conservation laws. Firstly, we explore the static part of  $R$  which can be expressed as

$$R_0(r) = \frac{2}{C_0} \left\{ \frac{1}{C_0 B_0^2} - \left( \frac{B'_0}{B_0} + \frac{A'_0}{A_0} \right) \frac{2}{B_0^2} + \frac{1}{B_0^2} \left( \frac{A''_0}{A_0} + \frac{A'_0 B'_0}{A_0 B_0} \right) - \frac{1}{C_0} \right\}. \tag{27}$$

Also, we can have static parts of the conservation laws with the help of Eqs. (14)–(25) in (12), (13) as

$$\begin{aligned} \dot{\rho}_0 + Z_1^{(s)} &= 0, \\ \xi (\rho_0 + P_{r0}) R'_0 + \xi \rho'_0 R_0 + (1 + \xi R_0) P'_{r0} \\ - \frac{2\xi B'_0}{B_0} (\rho_0 + P_{r0}) R_0 - \frac{A'_0}{A_0^3} (\rho_0 + Q_1^{(s)}) \\ + \frac{2C'_0}{C_0^3} P_{t0} - \frac{2B'_0}{B_0} Q_2^{(s)} + Z_2^{(s)} &= 0. \end{aligned}$$

### 3.2. Nonstatic Configuration

This subsection accommodates the perturbed parts of the conservation equations and  $R$ . So,

we single out the perturbed parts of the continuity equations using Eqs. (14)–(25) in (12), (13) as

$$\begin{aligned} \dot{\bar{\rho}} + \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 \right. \\ \left. - (1 + \xi R_0) \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) - \frac{2a}{A_0^3} Q_1^{(s)} \right. \\ \left. - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\} \dot{\Psi} = 0, \tag{28} \end{aligned}$$

$$\begin{aligned} \xi (\bar{\rho} + \bar{P}_r) R'_0 + (1 + \xi R_0) \bar{P}'_r - \frac{b'\Psi}{B_0} Q_2^{(s)} \\ - \frac{2\xi \Psi R_0}{B_0} (\rho_0 + P_{r0}) \left( b' - \frac{bB'_0}{B_0} \right) - \frac{2c'\Psi}{C_0^3} P_{t0} \\ - \frac{2C'_0}{C_0^3} \bar{P}_t + \frac{a'\Psi}{A_0^3} (\rho_0 + Q_1^{(s)}) + Y_1 = 0. \tag{29} \end{aligned}$$

Here, we found the dark source terms (DS terms)  $Y_1$ , which appear due to the considered MGT, given as

$$\begin{aligned} Y_1 = \xi \Psi e' (\rho_0 - P_{r0}) + 6 \Psi \frac{cC'_0}{C_0^4} P_{t0} + \xi \Psi e \rho'_0 \\ + \bar{\rho}' \xi R_0 + 2 \xi \Psi e (\rho_0 + P_{r0}) \frac{B'_0}{B_0} + \xi \Psi e P'_{r0} \\ - (\rho_0 + q_1 \Psi) \frac{A'_0}{A_0^3} - a \Psi (\rho_0 + Q_1^{(s)}) \frac{A'_0}{A_0^4} \\ - \frac{\Psi B'_0}{B_0} \left( 2q_2 - bQ_2^{(s)} \right) + \bar{Z}_2, \end{aligned}$$

while for the nonstatic part of  $R$  we can write

$$\frac{\partial^2 \Psi(t)}{\partial t^2} - Z_4^2(r) \Psi(t) = 0, \tag{30}$$

where  $Z_4^2(r)$  is some lengthy expression given in the Appendix. The most general solution to the above second-order differential equation is

$$\Psi(t) = C_1 e^{Z_4 t} + C_2 e^{-Z_4 t}, \tag{31}$$

The constants  $C_1$  and  $C_2$  in this situation are arbitrary. Thus Eq. (31) exhibits two unconnected reactions, one is stable while the other is unstable. We would like to understand more about the unstable spectrum of collapsing fluid compositions that are somewhat compact.

As a result, we assume that this relativistic compact structure was in a static state, characterized by  $\Psi(-\infty) = 0$ , a long time ago, and that it subsequently entered the present condition as time proceeded, continuing to collapse and also moving on by shrinking its radius. Only  $C_1 = -1$  and  $C_2 = 0$ , which represent diminishing over time, may yield such a result. The generalized solution also includes oscillating and nonoscillating features, which correspond to stable and unstable behaviors, respectively. Although we

are interested in instabilities in this compact material arrangement, we choose the nonoscillating approach that meets our needs. If  $Z_4^2(r) < 0$  is true, we will see oscillations, indicating that the system implodes at one point but bursts at another point.

This sort of dynamical arrangement of expansion and contraction happens as a result of oscillations generated by  $Z_4^2(r)$ , which is difficult for a star to maintain for the rest of its existence. To reach its ultimate destination, which might be a white dwarf, neutron star, or black hole, the star should undergo collapse in its final phases of existence, this it demands the presence of  $Z_4^2(r) > 0$ . We need to keep the disturbances to restrict positive significant quantities such that  $Z_4^2(r) > 0$  since we are seeking a meaningful solution to Eq. (30) describing a collapse. Consequently, the above solution to the partial differential equation indicates that both stable and unstable fluid configurations may be accommodated.

At the initial phase of this dynamic analysis or at a very large past time, i.e.,  $\Psi(-\infty) = 0$ , we assume that our locally anisotropic charged gravitating system is in a static equilibrium. It is worth noting that the metric variables have the same temporal dependence, which means that at  $t = -\infty$  our charged symmetric star entered into a collapsing scenario. The goal of this paper is to investigate the (in)stability of a charged gravitating compact system in the context of a nonminimally coupled MGT gravitational model. Thus the solution of Eq. (30) can be written as

$$\Psi(t) = -e^{Z_4 t}, \tag{32}$$

with  $Z_4^2(r) > 0$ . To continue our study, we consider the Harrison et al. [67] EoS which is a relationship among the surface pressures,  $\Gamma$  and the energy density of our gravitating star and is given by

$$\bar{P}_r = \frac{\bar{\rho} P_{r0}}{\rho_0 + P_{r0}} \Gamma. \tag{33}$$

Equation (28) yields the value of  $\bar{\rho}$  as

$$\begin{aligned} \bar{\rho} = & - \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 \right. \\ & - (1 + \xi R_0) \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) - \frac{2a}{A_0^3} Q_1^{(s)} \\ & \left. - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\} \Psi. \end{aligned} \tag{34}$$

By utilizing Eq. (34) in (33), we have the perturbed part of the surface pressure  $\bar{P}_r$  in the radial direction as

$$\bar{P}_r = -\frac{\Psi P_{r0}}{\rho_0 + P_{r0}} \Gamma \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 \right.$$

$$\begin{aligned} & \left. - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 - (1 + \xi R_0) \right. \\ & \left. \times \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) \right. \\ & \left. - \frac{2a}{A_0^3} Q_1^{(s)} - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\}. \end{aligned} \tag{35}$$

From Eq. (9), we compute the nonstatic part of the pressure  $\bar{P}_t$  in the tangential direction as

$$\begin{aligned} \bar{P}_t = & \frac{\xi_1}{A_0^2} \left\{ \frac{b}{B_0^2} + \frac{c}{C_0^2} + \frac{2\beta e}{R_c} \sec h^2 \left( \frac{e\Psi}{R_c} \right) \right. \\ & \left. \times \tanh \left( \frac{e\Psi}{R_c} \right) \right\} \ddot{\Psi} + \xi \xi_1 \xi_3 R_0 \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 \right. \\ & \left. - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 - (1 + \xi R_0) \right. \\ & \left. \times \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) \right. \\ & \left. - \frac{2a}{A_0^3} Q_1^{(s)} - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\} \Psi \\ & + \left( \frac{2b}{B_0} P_{t0} - \xi \xi_3 e(\rho_0 + P_{t0}) + Z_3 \right) \Psi \xi_1. \end{aligned} \tag{36}$$

For simplicity, we have introduced the notations  $\xi_1$  and  $\xi_3$ :

$$\begin{aligned} \xi_1 = & \left[ 1 - \beta \sec h^2 \left( \frac{R_0}{R_c} \right) + \xi T_0 \right] \\ & \times \left[ 1 + \xi R_0 - \beta \sec h^2 \left( \frac{R_0}{R_c} \right) - \xi T_0 \right]^{-1}, \\ \xi_3 = & \left[ 1 + \beta \sec h^2 \left( \frac{R_0}{R_c} \right) + \xi T_0 \right]^{-1}. \end{aligned}$$

The collapse equation is constituted with the help of the nonstatic parts of the continuity equations and the perturbed parts of surface pressures in the respective directions. It is worthy to note that the collapse equation helps us to study the (in)stability of the compact system under consideration in the N and pN approximations. We use Eqs. (34)–(36) in (29), which yields

$$\begin{aligned} & -\xi R_0' \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 \right. \\ & \left. - (1 + \xi R_0) \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) - \frac{2a}{A_0^3} Q_1^{(s)} \right. \\ & \left. - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\} \Psi - \xi R_0' \frac{\Psi P_{r0}}{\rho_0 + P_{r0}} \\ & \times \Gamma \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 \right. \end{aligned}$$

$$\begin{aligned}
 & - (1 + \xi R_0) \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) \\
 & - \left. \frac{2a}{A_0^3} Q_1^{(s)} - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\} \\
 & + \Psi \Gamma (1 + \xi R_0) \left[ \frac{P_{r0}}{\rho_0 + P_{r0}} \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \rho_0 \right. \right. \\
 & - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 - \left. \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) \right. \\
 & \times (1 + \xi R_0) \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) - \frac{2a}{A_0^3} Q_1^{(s)} \\
 & \left. \left. - \frac{b}{B_0^3} Q_2^{(s)} - \frac{2c}{C_0} Q_3^{(s)} + \bar{Z}_1 \right\} \right]' - \frac{2\xi \Psi R_0}{B_0} \\
 & \quad \times \left( \rho_0 + P_{r0} \right) \left( b' - \frac{b B_0'}{B_0} \right) \\
 & - \frac{2C_0'}{C_0^3} \left[ \frac{\xi_1}{A_0^2} \ddot{\Psi} \left\{ \frac{b}{B_0^2} + \frac{c}{C_0^2} + \frac{2\beta e}{R_c} \sec^2 h^2 \left( \frac{e\Psi}{R_c} \right) \right. \right. \\
 & \times \left. \left. \tanh \left( \frac{e\Psi}{R_c} \right) \right\} + \xi \xi_1 \xi_3 R_0 \left\{ \left( \frac{c}{C_0 A_0^2} - \frac{2a}{A_0^3} \right) \right. \right. \\
 & \times \left. \left. \rho_0 - \xi R_0 \left( \frac{b}{B_0} - \frac{c}{C_0} \right) \rho_0 - \frac{2c Q_3^{(s)}}{C_0} \right. \right. \\
 & - (1 + \xi R_0) \left( \frac{b}{B_0^3} P_{r0} + \frac{2c}{C_0} P_{t0} \right) - \frac{2a Q_1^{(s)}}{A_0^3} \\
 & \left. \left. - \frac{b Q_2^{(s)}}{B_0^3} + \bar{Z}_1 \right\} \Psi + \left( \frac{2b}{B_0} P_{t0} - \xi \xi_3 e (\rho_0 \right. \right. \\
 & \left. \left. + P_{t0}) + Z_3 \right) \Psi \xi_1 \right] + \frac{a' \Psi}{A_0^3} \left( \rho_0 + Q_1^{(s)} \right) \\
 & - \frac{2c' \Psi}{C_0^3} P_{t0} - \frac{b' \Psi}{B_0} Q_2^{(s)} + Y_1 = 0. \tag{37}
 \end{aligned}$$

4.  $\mathcal{N}$ -APPROACH

Here, we explore the collapse equation in an  $\mathcal{N}$ -era, for which we modify this equation under the approximation  $A_0 = 1 = B_0$ , and  $C_0 = r$ . Thus the expression for  $\Gamma$  helps us to understand the behavior and the stability criteria for a charged gravitating spherically symmetric fluid configuration under some significant constraints. Also, the constraints on the energy density and pressure components, i.e.,  $\rho_0, P_{r0}$ , and  $P_{t0}$  are  $P_{r0} \ll \rho_0$ , and  $P_{t0} \ll \rho_0$ . Furthermore, we consider in the  $\mathcal{N}$ -era,  $P_{r0}/\rho_0 \rightarrow 0$ , and  $P_{t0}/\rho_0 \rightarrow 0$ . The collapse equation under these constraints yields for  $\Gamma$  the inequality

$$\Gamma < \frac{2 \left( \xi b - \frac{c'}{r^3} \right) P_{t0} + a' \rho_0 + \Pi_1 + Y_{1\mathcal{N}}}{P_{r0} (\xi R_0' E_1 + \beta W_1) + \Pi_2}, \tag{38}$$

where

$$\begin{aligned}
 \Pi_1 &= \xi (E_1 + W_1) \left( 2\xi_1 \xi_3 R_0 \frac{\rho_0}{r^3} - \xi_4 \rho_0 R_0' \right) \\
 &+ E_2 - 2b \xi R_0 (\rho_0 + P_{r0}), \\
 \Pi_2 &= (1 + \xi R_0) \{ P_{r0} (E_1 + W_1) \}' , \\
 \frac{1}{\xi_3} &= 1 + \beta \sec^2 h^2 \left( \frac{R_0}{R_c} \right) + \xi T_0, \\
 \xi_4 &= b + \frac{ac}{r} + \xi b R_0.
 \end{aligned}$$

In an  $\mathcal{N}$ -era, the metric and material variables as well as DS-terms play a vital role in determining the dynamical stability range of our charged spherical geometry. Also,  $\Gamma$  shows a dependence on  $P_r, P_t, \rho$ , etc. This locally anisotropic charged spherical compact star will remain stable as long as it adheres to the  $\Gamma$  restrictions, which require that all terms should be positive. The following requirements must be met to meet the aforementioned criteria:

$$\begin{aligned}
 P_{r0} &< P_{t0}, \quad \xi \xi_3 > \xi_4, \\
 \xi b &> \frac{c'}{r^3}, \quad 2b \xi R_0 \rho_0 < a' \rho_0,
 \end{aligned}$$

and also

$$\begin{aligned}
 \xi (E_1 + W_1) \left( 2\xi_1 \xi_3 R_0 \frac{\rho_0}{r^3} - \xi_4 \rho_0 R_0' \right) \\
 > 2b \xi R_0 (\rho_0 + P_{r0}),
 \end{aligned}$$

where

$$\begin{aligned}
 E_1 &= -\frac{1}{\rho_0} \left( 2a Q_1^{(s)} + \frac{2c Q_3^{(s)}}{r} + b Q_2^{(s)} \right), \\
 E_2 &= a' Q_1^{(s)} + b' Q_2^{(s)}, \\
 W_1 &= \frac{c}{r} - 2a + \xi R_0 \left( b + \frac{c}{r} \right).
 \end{aligned}$$

The anisotropic charged spherical star system's stability zones are created by considering that every factor in the collapse calculation should be favorable. As a result, Eq. (40) for  $\Gamma$ , which determines the stability condition, must be used to formulate the corresponding hydrodynamic equation. As a result, we observe that as long as the inequality (40) holds, this symmetric configuration would remain in a stable phase. Consequently, the adiabatic index demonstrates that the instability range of (40), as determined by the pressure components and the antigravitational force, depends on the pressure components and the anti-gravitational force. The radial properties of the energy density, anisotropy, and  $f(R, T)$  additional curvature components ultimately determine the variable values. The dynamic (un)stable phase of this charged compact fluid structure is disturbed due to some DS terms, so that the (in)stability in the  $\mathcal{N}$ -era is affected by these corrective terms.



For  $f(R)$  gravity in an  $\mathbb{N}$ -domain, we modify the collapse equation with  $A_0 = 1 = B_0$ , and  $C_0 = r$ . Thus the expression for  $\Gamma$  helps us to understand the behavior and stability criteria for the configuration under study with some significant constraints. Also, constraints on the energy density and pressure components  $\rho_0, P_{r0}$ , and  $P_{t0}$  are  $P_{r0} \ll \rho_0$ , and  $P_{t0} \ll \rho_0$ . Furthermore, we consider, in an  $\mathbb{N}$ -era,  $P_{r0}/\rho_0 \rightarrow 0$  and  $P_{t0}/\rho_0 \rightarrow 0$ . The results found in Eq. (37) are well consistent with those in  $f(R)$  gravity in the  $\mathbb{N}$  regime with the usual limit  $\xi \rightarrow 0$  as in [69, 70]. The collapse equation under the above constraints yields an inequality for the adiabatic index  $\Gamma$ :

$$\Gamma < \frac{-2\frac{c'}{r^3}P_{t0} + \Pi_3 + \beta q_1 + 2q_2 + Z_{2\mathbb{N}}^{(s)}}{\beta P_{r0} \left(\frac{c}{r} - 2a\right) + \Pi_4}, \quad (39)$$

where

$$\begin{aligned} \Pi_3 &= +a'Q_1^{(s)} + b'Q_2^{(s)} + a'\rho_0 + \frac{6c}{r^4P_{t0}} \\ &\quad + 2e\rho'_0 + 2\beta e'\rho_0, \\ \Pi_4 &= \left[ -\frac{P_{r0}}{\rho_0} \left( 2aQ_1^{(s)} + \frac{2cQ_3^{(s)}}{r} + bQ_2^{(s)} \right) \right. \\ &\quad \left. + P_{r0} \left( \frac{c}{r} - 2a \right) \right]'. \end{aligned}$$

The results found in Eq. (37) reduce to those of GR in the  $\mathbb{N}$  regime with the usual limit  $\xi \rightarrow 0$  and  $\beta \rightarrow 0$ . The results obtained here are well consistent with the findings of Herrera et al. [32] for anisotropic spherical self-gravitating internal structures in standard GR. Consequently, the collapse equation under the above constraints yields the inequality for the adiabatic index  $\Gamma$

$$\Gamma < \frac{\frac{6c}{r^4P_{t0}} + \Pi_5 + Z_{2\mathbb{N}}^{(s)}}{\{P_{r0}(\Pi_6 + c/r - 2a)\}'}, \quad (40)$$

where

$$\begin{aligned} \Pi_5 &= -2\frac{c'}{r^3}P_{t0} + a'Q_1^{(s)} + b'Q_2^{(s)} + a'\rho_0 \\ &\quad + 2e\rho'_0 + 2q_2, \\ \Pi_6 &= -\frac{1}{\rho_0} \left( 2aQ_1^{(s)} + \frac{2cQ_3^{(s)}}{r} + bQ_2^{(s)} \right). \end{aligned}$$

### 5. pN APPROACH

Distinct components of specific gravitational systems may cause particular problems in real-world applications. This includes the nonlinear nature of the equations of motion and lack of a background geometry for considering physically significant variables such as energy. To arrive at physically appropriate

results, certain approximation procedures are used. Linearized gravity, which ignores nonlinear parts of spacetime metrics, is an example of an approximation approach that eventually gives some beneficial acceptable conclusions. As a result of this method, linearized field equations expressing weak gravitational fields may be easily managed while relativistic gravitational theory's  $\mathbb{N}$  and p $\mathbb{N}$  constraints are viewed as weak-field approximations.

In this section, the dynamical stability of charged spherically symmetric stars in a p $\mathbb{N}$  era is considered in nonminimally coupled gravity, i.e., in  $f(R, T)$  gravity. The relativistic consequences may be investigated up to  $O\left(\frac{m_0}{r} + \frac{Q^2}{2r^2}\right)$  in this scenario by using post Newtonian approaches such as

$$\begin{aligned} A_0 &= 1 - \frac{m_0}{r} + \frac{Q^2}{2r^2}, & B_0 &= 1 + \frac{m_0}{r} - \frac{Q^2}{2r^2}, \\ C_0 &= r, \end{aligned}$$

where  $Q = Q_1 = Q_2 = Q_3$ . From these expressions we find

$$\begin{aligned} \frac{A'_0}{A_0} &= \frac{2}{r} \left\{ \frac{Q^2 - m_0r}{2m_0r - Q^2 - 2r^2} \right\}, \\ \frac{B'_0}{B_0} &= \frac{2}{r} \left\{ \frac{Q^2 - m_0r}{2m_0r - Q^2 + 2r^2} \right\}. \end{aligned}$$

Under these assumptions, our established collapse equation modifies and helps in computing the inequality for  $\Gamma$ , namely,

$$\Gamma < \frac{\Pi_7 + \xi \left( 2\xi_1\xi_3\frac{R_0}{r^3} - R'_0 \right) E_3 + E_4 + Y_{2(\text{pN})}}{\frac{R'_0P_{r0}}{\rho_0 + P_{r0}} (\beta W_2\rho_0 - \xi W_3 + \xi E_3) + \Pi_8}, \quad (41)$$

where

$$\begin{aligned} \Pi_7 &= \xi \left( 2\xi_1\xi_3\frac{R_0}{r^3} - R'_0 \right) (W_2\rho_0 - W_3) \\ &\quad - \frac{2c'}{r^3}P_{t0} + W_4 + \xi_1W_5, \\ \Pi_8 &= (1 + \xi R_0) \left\{ \frac{P_{r0}}{\rho_0 + P_{r0}} (\rho_0W_2 - W_3 + E_3) \right\}'. \end{aligned}$$

The following constraints on the physical quantities in the case of positive-definite terms in  $\Gamma$  must be met:

$$\begin{aligned} \xi\xi_1\xi_3\frac{R_0}{r^3} &> \xi R'_0, & W_2 &> W_3, \\ \xi \left( 2\xi_1\xi_3\frac{R_0}{r^3} - R'_0 \right) (W_2\rho_0 - W_3) &> \frac{2c'}{r^3}P_{t0}, \\ \frac{\xi R'_0P_{r0}}{\rho_0 + P_{r0}} (W_2\rho_0 - W_3 + E_3) &> \left( \frac{P_{r0}}{\rho_0 + P_{r0}} (\rho_0W_2 - W_3 + E_3) \right)'. \end{aligned}$$

These constraints must be met for charged stellar object under study to have a stable structure. It can be shown from Eq. (43) that  $\Gamma$  significantly depends on the stellar mass and radius as well as the electromagnetic field composition and some DS terms due to the MGT. To study the dynamic stability in a pN era, we examine the material and metric variables that have a key role for that:  $W_2, W_3, W_4, W_5, E_3, E_4$ , given as follows:

$$\begin{aligned}
 W_2 &= \frac{c}{r} \left( 1 + \frac{2m_0}{r} - \frac{Q^2}{r^2} \right) - 2a \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) + \frac{c\xi R_0}{r} + b\xi R_0 \left( 1 - \frac{3m_0}{r} + \frac{3Q^2}{2r^2} \right), \\
 W_3 &= (1 + \xi R_0) \left\{ b \left( 1 - \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) + \frac{2c}{r} P_{t0} \right\} - \bar{Z}_{1\text{pN}}, \\
 W_4 &= -2\xi R_0(\rho_0 + P_{r0}) \left( 1 - \frac{m_0}{r} + \frac{Q^2}{2r^2} \right) \times \left\{ b' - \frac{2b}{r} \left( \frac{Q^2 - m_0 r}{2m_0 r - Q^2 + 2r^2} \right) \right\}, \\
 W_5 &= \xi_1 \left\{ 2b \left( 1 - \frac{m_0}{r} + \frac{Q^2}{2r^2} \right) P_{t0} - \xi \xi_3 e (\rho_0 + P_{t0}) + Z_{3\text{pN}} \right\}, \\
 E_3 &= -2a \left( 1 + \frac{3m_0}{r} \right) Q + \frac{c}{r} Q_c - b \left( 1 - \frac{3m_0}{r} \right) Q, \\
 E_4 &= a'(\rho_0 + Q) \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) + b' \left( 1 - \frac{m_0}{r} \right) Q.
 \end{aligned}$$

One can regain the results of  $f(R)$  gravity in both  $\mathbb{N}$  and pN regimes in the usual limit  $\xi = 0$ . The expression of  $\Gamma$  in GR is retrieved from the results of  $f(R, T)$ -gravity by substituting  $\beta = 0 = \xi$ .

One can utilize the usual limit  $\beta \rightarrow 0$  to discuss the dynamic stability of a charged spherically symmetric star in a pN era in the framework of  $f(R)$  gravity. The relativistic consequences may be investigated up to  $O\left(\frac{m_0}{r} + \frac{Q^2}{2r^2}\right)$  in this scenario by using the post-Newtonian approach as described above, with  $Q = Q_1 = Q_2 = Q_3$ . Under these conditions on the metric coefficients  $A_0, B_0, C_0$ , the results in Eq. (38) reduce to  $f(R)$  gravity in a pN regime with the usual limit  $\xi \rightarrow 0$  which are compatible with [69], thus the inequality for adiabatic index  $\Gamma$  becomes

$$\Gamma < \frac{-\frac{2c'}{r^3} P_{t0} + \Pi_9 + E_4 + Y_{2(\text{pN})}}{\frac{\beta W_2 \rho_0 R_0' P_{r0}}{\rho_0 + P_{r0}} + \Pi_{10}}, \quad (42)$$

where

$$\begin{aligned}
 \Pi_9 &= W_4 + \left( 1 + \beta \sec h^2 \left( \frac{R_0}{R_c} \right) \right)^{-1} W_5, \\
 \Pi_{10} &= \left\{ \frac{P_{r0}}{\rho_0 + P_{r0}} (\rho_0 W_2 - W_3 + E_3) \right\}'.
 \end{aligned}$$

Also, some intermediate terms,  $W_2, W_3, W_4, W_5, E_3, E_4$  are given as follows:

$$\begin{aligned}
 W_2 &= \frac{c}{r} \left( 1 + \frac{2m_0}{r} - \frac{Q^2}{r^2} \right) - 2a \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right), \\
 W_3 &= b \left( 1 - \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) + \frac{2c}{r} P_{t0} - \bar{Z}_{1\text{pN}}, \\
 W_5 &= \xi_1 \left\{ 2b \left( 1 - \frac{m_0}{r} + \frac{Q^2}{2r^2} \right) P_{t0} + Z_{3\text{pN}} \right\}, \\
 E_3 &= -2a \left( 1 + \frac{3m_0}{r} \right) Q - b \left( 1 - \frac{3m_0}{r} \right) Q + \frac{c}{r} Q, \\
 E_4 &= a'(\rho_0 + Q) \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) + b' \left( 1 - \frac{m_0}{r} \right) Q, \\
 Y_{1\text{pN}} &= \frac{6c}{r^4} P_{t0} + \frac{2}{r} \left[ (\rho_0 + 2\beta q_1) \left( 1 + \frac{2m_0}{r} - \frac{Q^2}{r^2} \right) - a \left( \rho_0 + Q_1^{(s)} \right) \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) \right] \\
 &\quad \times \left\{ \frac{Q^2 - m_0 r}{Q^2 - 2m_0 r + 2r^2} \right\} + 2\beta e \rho_0' - \frac{2}{r} \\
 &\quad \times \left\{ \frac{Q^2 - m_0 r}{2m_0 r - Q^2 + 2r^2} \right\} \left( 2q_2 - bQ_2^{(s)} \right) + Z_{2\text{pN}}^{(s)}.
 \end{aligned}$$

The expression for the adiabatic index  $\Gamma$  in GR can be retrieved from the results of  $f(R, T)$  gravity in Eq. (38) by substituting  $\beta = 0 = \xi$ , and the one in standard GR can be retrieved in the following form [32]:

$$\Gamma < \frac{-\frac{2c'}{r^3} P_{t0} + W_4 + W_5 + E_4 + Y_{2(\text{pN})}}{\left\{ \frac{P_{r0}}{\rho_0 + P_{r0}} (\rho_0 W_2 - W_3 + E_3) \right\}'}, \quad (43)$$

where the quantities  $W_2, W_3, W_4, E_3, E_4$  are given as previously, and

$$\begin{aligned}
 W_5 &= 2b \left( 1 - \frac{m_0}{r} + \frac{Q^2}{2r^2} \right) P_{t0} + Z_{3\text{pN}}, \\
 Y_{2\text{pN}} &= \frac{2}{r} \left\{ \frac{Q^2 - m_0 r}{Q^2 - 2m_0 r + 2r^2} \right\} \\
 &\quad \times \left[ \rho_0 \left( 1 + \frac{2m_0}{r} - \frac{Q^2}{r^2} \right) - a \left( \rho_0 + Q_1^{(s)} \right) \right] \\
 &\quad \times \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) + \frac{6c}{r^4} P_{t0}
 \end{aligned}$$

$$-\frac{2}{r} \left\{ \frac{Q^2 - m_0 r}{2m_0 r - Q^2 + 2r^2} \right\} \left( 2q_2 - bQ_2^{(s)} \right) + Z_{2\mathbb{P}\mathbb{N}}^{(s)}.$$

The terms  $W_2$ ,  $E_3$ ,  $E_4$  are identical in both  $f(R)$  gravity and GR.

If the conditions in both  $\mathbb{N}$  and  $\mathbb{p}\mathbb{N}$  eras are met, the system will achieve a stable profile. These results also indicate that the energy density inhomogeneity and anisotropic pressure affect the system's dynamic stability. Furthermore, Sharif and Bhatti discovered instability limits for cylindrical collapse with the expansion-free criterion [71], in which the adiabatic index  $\Gamma$  plays no role, and without the expansion-free constraint [72], in which it plays a critical part for continuing a systematic study of dynamic stability. We have investigated the dynamic stability of a charged spherically symmetric fluid configuration with anisotropic pressure in non-minimally coupled  $f(R, T)$  gravity, determined the instability range for fluid distributions in the absence of an expansion-free state and conceived that it depends on the adiabatic index, which is well consistent with [58, 69, 72, 73] and includes static terms in the configuration.

### 5.1. Graphic Interpretation

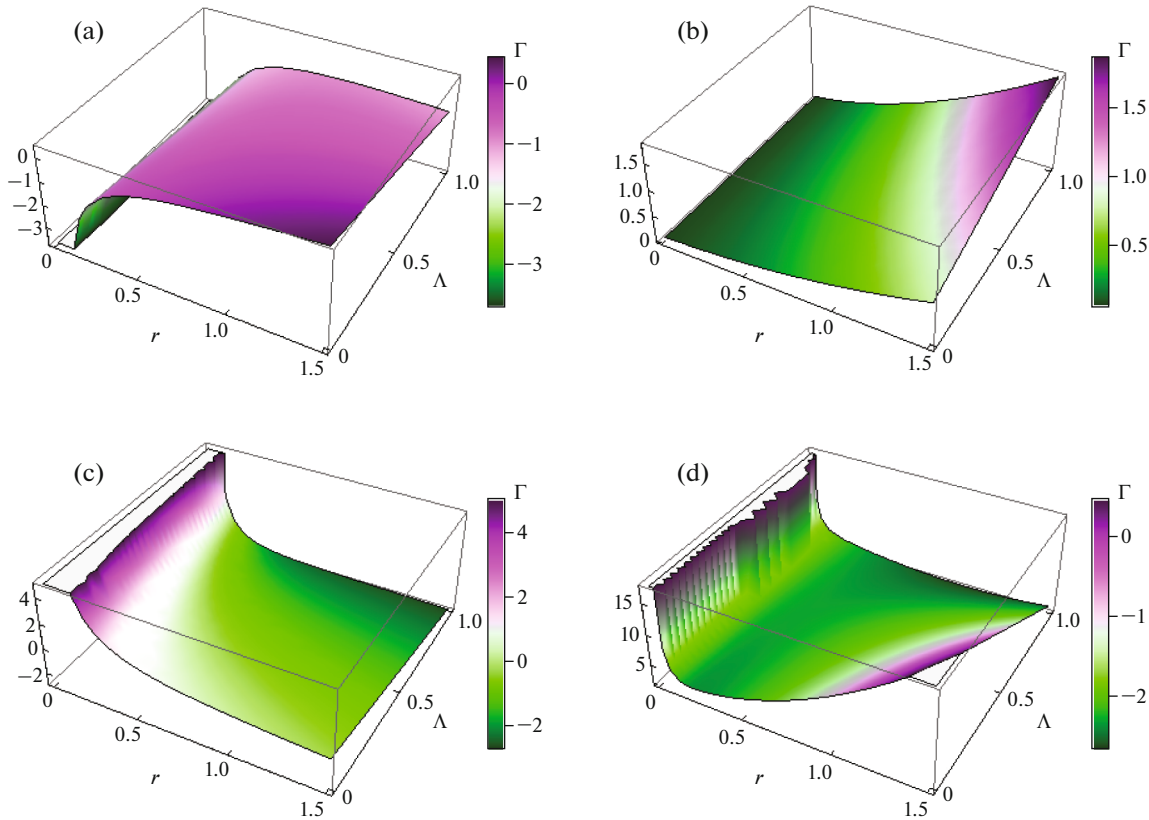
We study the dynamic stability of electrically charged spherical compact structures in which the matter sector represents an anisotropic configuration, in a specific hyperbolic mathematical formalism of  $f(R, T)$  gravity,  $f(R, T) = R - \beta R_c \tanh(R/R_c) + \xi RT$ . This functional shape of gravity, under parametric restrictions like  $\beta > 0$  and  $0 < \xi < 1$ , may provide relativistic consequences. Thus, it is possible that the physical entity  $R_c$  and the Hubble constant have a relationship. It is important to note that in this paper, the collapse equation is explicitly obtained from the non-conservation equations by utilizing the Harrison et al. [67] equation of state in Newtonian ( $\mathbb{N}$ ) and post-Newtonian ( $\mathbb{p}\mathbb{N}$ ) realms. We started by considering a hydrostatic profile for a charged stellar structure, which enters into a nonstatic phase with a linear perturbation parameter. For this anisotropic compact structure to start collapsing, the mathematical approach gives some significant stability limitations. In this study of a symmetric collapsing configuration, the adiabatic index ( $\Gamma$ ) determines the stability constraints.

The stable (unstable) behavior of our system is determined in the  $\mathbb{N}$  and  $\mathbb{p}\mathbb{N}$  eras via schematic diagrams. For both eras, we displayed all of the illustrative findings that were significant for a range of the parameter  $\xi$  and particular parametric values of  $\beta$ . This demonstrates the importance of the stiffness parameter  $\Gamma$  and supports Chandrasekhar's conclusions presented in [28]. We also describe how matter

and the electric charge affect various elements in this scenario for the specific hyperbolic form of  $f(R, T)$  gravity. It indicates the importance of the adiabatic index and validates Chandrasekhar's results, i.e., the system remains stable for  $\Gamma > 4/3$  and unstable for  $\Gamma < 4/3$ . Schematic diagrams of our studied  $f(R, T)$  gravitational model for stable and unstable behavior are presented in Figs. 1 and 2 for the  $\mathbb{N}$  and  $\mathbb{p}\mathbb{N}$  eras, respectively.

A basic summary of what we have studied is given below:

- According to the established results in (37) and (38), the existence of an anisotropy in fluid pressure appears to have a significant impact on the stability profile of the considered class of electrically charged spherical systems in  $\mathbb{N}$  and  $\mathbb{p}\mathbb{N}$  eras. We found that when the denominator's absolute values are taken, the effective pressure anisotropy diminishes the stability regions.
- Due to  $f(R, T)$  gravity, the extra curvature terms in the  $\mathbb{N}$  and  $\mathbb{p}\mathbb{N}$  eras always arise, which drastically affects the stability restrictions. We found that the system enters into a hydrostatic equilibrium if the stiffness parameter is equal to the right-hand sides of Eqs. (37) and (38).
- The schematic representations denote an unstable (for  $\Gamma < 4/3$ ) or stable (for  $\Gamma > 4/3$ ) stellar structure. We obtain only unstable regions for the parameter values  $\beta = 0.3$ ,  $Q = 0$  and the range of  $\xi$  from 0 to 1, as indicated in Fig. 1a, in the  $\mathbb{N}$  era.
- For the sets of parameters  $\beta = 0.5$ ,  $Q = 0.5$  (Fig. 1b) and  $\beta = 0.7$ ,  $Q = 0.7$  (Fig. 1c) both stable and unstable states are found, but the ranges of the adiabatic index are slightly different in both cases. In Fig. 1d, we observe only a stable behavior for the parametric values  $\beta = 1$ ,  $Q = 1$ .
- In Fig. 1b, we see a very small stable region as compared to an unstable one in the  $\mathbb{N}$  era for specific parametric values. In Fig. 1d, we observe that the index range begins from 2, which is larger than the standard value of the stiffness parameter for a stable stellar fluid configuration, i.e.,  $\Gamma > 4/3$ .
- We continued our study in the  $\mathbb{p}\mathbb{N}$  domain and observe only unstable graphical representations in Fig. 2a for the same parameter values as considered in the  $\mathbb{N}$  era, i.e.,  $\beta = 0.3$ ,  $Q = 0$  and the range of  $\xi$  from 0 to 1.



**Fig. 1.** Schematic diagrams for the unstable and stable behavior of a charged spherically symmetric compact object for different values of model parameters in the N domain.

- In the Figs. 2b, 2c, 2d, we observe both stable and unstable states for the sets of parameters  $(\beta = 0.5, Q = 0.5)$ ,  $(\beta = 0.7, Q = 0.7)$  and  $(\beta = 1, Q = 1)$ , respectively. Also, the ranges of the adiabatic index are slightly different in all scenarios for the considered nonminimally coupled  $f(R, T)$  gravity.
- Moreover, in Fig. 2b, we see a very small stable region as compared to the unstable one, however, Fig. 2d shows a very small unstable region for the considered values of the parameters in the pN domain.
- According to the hyperbolic mathematical formalism of  $f(R, T)$  gravity, these outcomes are consistent with those found in GR as well as in the metric  $f(R)$  theory.

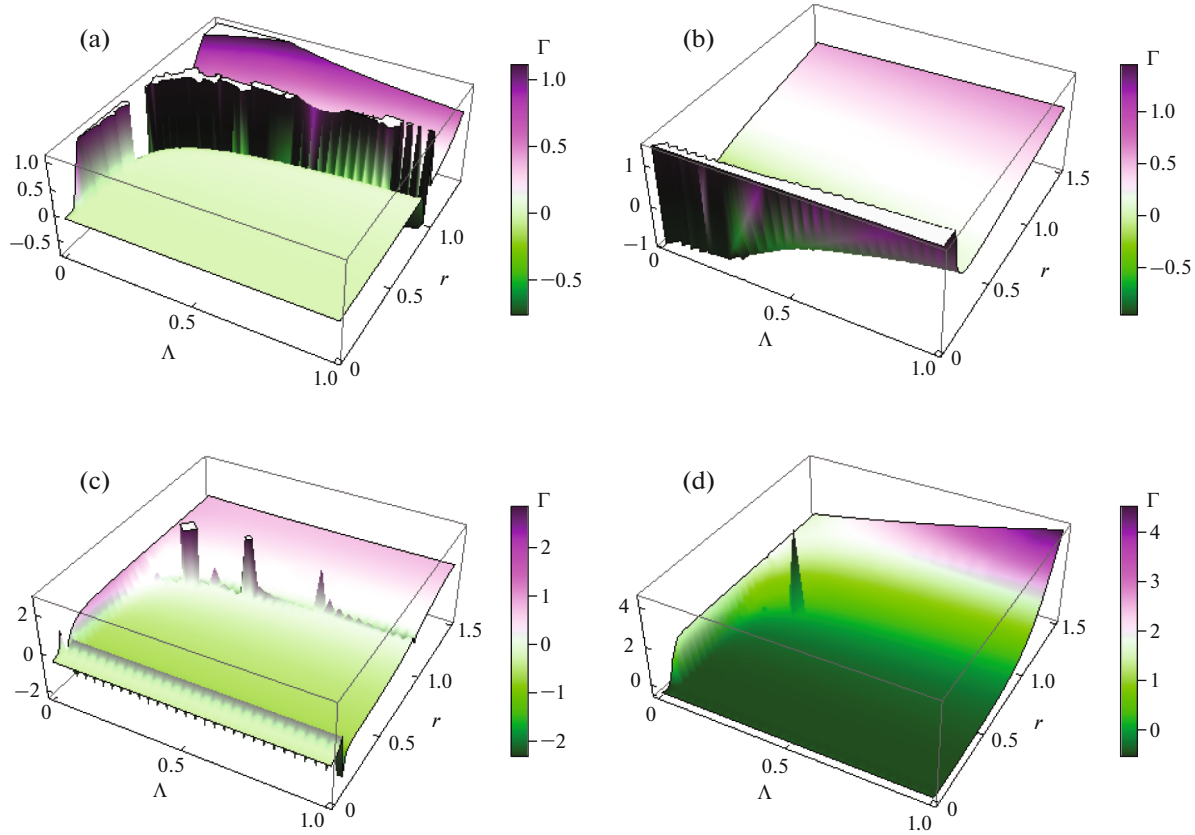
### 6. FINAL REMARKS

The consequences of massive collapse of compact fluids and investigation of (in)stability of cosmic geometries are new problems of GR and modified theories of gravity. A compact geometry’s collapse is reliant on the fuel supply, and the loss or depletion of all fuel causes gravitational collapse because the internal

gravitational pull then overcomes the outward drawn forces. In this work, we investigated the mentioned problem, i.e., the stability of compact geometries, by considering a restricted class of nonstatic locally anisotropic spherical matter structures in the context of modified  $f(R, T)$  gravity [45]. We continue our comprehensive investigation of dynamical stability to get some constrained conclusions for this charged geometry.

It is interesting to note that the MFEs computed in  $f(R, T)$  gravitational theory have an extra term of  $T$ , i.e., a scalar force, as compared to  $f(R)$  gravity [55], and this increment indicates that the relativistic theory of gravity is more generalized under this assumption. To arrive at dynamic continuity equations, the covariant divergence of the effective EMT is used. To fix this concern, we can choose a perturbation technique as considered by Herrera et al. [29, 32].

Examination of the stability spectrum of modified gravity theories reveals a gravitational impact in current time, which is one of the known universe’s expansions. This study explored the entire influence of the  $f(R, T)$  cosmological framework on the dynamic stability of our analyzed compact objects. For a dynamic analysis, a nonminimally coupled  $f(R, T)$  model can only have the general form  $f(R, T) =$



**Fig. 2.** Schematic diagrams for the unstable and stable behavior of a charged spherically symmetric compact object for varying values of the model parameters in the pN domain.

$f(R) + \xi RT$ , where  $\xi$  is a nonminimal coupling parameter and is considered as a correction to  $f(R)$  gravity. Using this nonminimally coupled model, we have here  $f(R) = R - \beta R_c \tanh(R/R_c)$ . This model provides significant conclusions for  $\beta > 0$ ,  $1 \gg \xi > 0$  using  $R_c$  as a constant quantity storing the values of the Ricci scalar  $R$  effective order.

To overcome the later mentioned problem, we utilize a relevant perturbation scheme that provides approximate solutions of our calculated expressions of motion. We perturbed our considered modified gravitational  $f(R) + \beta RT$  mode, the conservation equations and extra ingredients which appear due to the MGTs.

To create a collapse equation, we employed a perturbation technique to distinguish static and non-static components of the MFEs and continuity equations. With the help of non-static parts of the conservation equations, surface pressures in different directions, and the Harrison et al. [67] equation of state, we created a collapse equation that demonstrates a correlation among the three physical parameters: (a) pressures, (b) energy density, and (c) the stiffness parameter  $\Gamma$ . We obtained stability criteria for a charged spherically symmetric fluid in N and pN

epochs using the perturbed parts of continuity equations by establishing the expression for  $\Gamma$ . To continue our theoretical investigation, we impose some substantial conditions on the physical entities to make them positive and retain the same criteria within N and pN eras.

Many stellar systems, including clusters and galaxies as well as their constituent stars have densities higher than the universe's average densities. As a consequence, these systems are now categorized as nonlinear ones. To acquire a thorough picture of their structural history, we have to examine their linear, nearly linear, or quasilinear iterations. In the literature, several mathematical approaches to dynamical aspects of nonlinear celestial systems have been presented. These methods will not, of course, explain all the unanswered problems of nonlinear stellar structure, but they may yield some interesting results. Since the gravitational field expressions in  $f(R, T)$  gravity are so complex, establishing a general solution is challenging. The development of linear perturbations may always be used to investigate gravitational changes by removing such contradictions. By executing a linear perturbation in dynamic equations against a static configuration that

correspond to an instability spectrum, the relevant collapse equation may be produced. Fixed or comoving coordinates, such as in Eulerian or Lagrangian techniques, might be used to anticipate the dynamical analysis. Because the universe is nearly uniform at large scale, we used the comoving coordinates [74].

We continue our systematic investigation of dynamic stability of locally anisotropic charged stellar object by considering a nonminimally coupled cosmic  $f(R, T)$  model given as  $f(R, T) = R - \beta R_c \times \tanh\left(\frac{R}{R_c}\right) + \xi RT$ . This functional form of  $f(R, T)$  gravity has significant results only for  $\beta > 0$  and  $0 < \xi < 1$ ,  $R_c$  being a positive constant, and  $\xi RT$  demonstrates a correction term to  $f(R)$  gravity. We use a linear perturbation method to construct dynamical equations in this paper. The stability of a collapsing charged compact spherically symmetric structure with anisotropic environment has been investigated, with  $\Gamma$  determining instability ranges in both  $\mathbb{N}$  and  $p\mathbb{N}$  domains. This highlights the importance of such an index in our research, which backs up Chandrasekhar's first conclusions from 1964. Here is a brief overview of what we explored:

- It is worth noting that the presence of anisotropy in the fluid pressure has a considerable influence on the instability of the class of relativistic structures under study in  $\mathbb{N}$  and  $p\mathbb{N}$  eras, as reflected by the inequalities (40) and (43), respectively. By keeping the absolute values of the denominator, the effective pressure anisotropy tends to reduce the stability regions or eliminate hindrances for the mechanism to operate in the collapse stage.
- Since many of the additional curvature elements in Eqs. (40) and (43) arise in the numerator with a negative sign, they are attributable to  $f(R, T)$  gravity. We can see that the existence of these variables tends to lower the stability range. This equation contains effective interpretations of matter quantities, implying that the factors coming from the relationship of matter and geometry have a considerable impact on the stability constraints owing to their nonattractive nature.
- We have established two meaningful relations, one for the  $\mathbb{N}$  regime and the other for the  $p\mathbb{N}$  regime, for the adiabatic index, (40) and (43). The system must fulfill these specifications to enter an unstable window. Thus non-compliance with these restrictions will result in stable configurations.
- We found that the system achieves a state of hydrostatic equilibrium for this axial geometry

if the stiffness parameter is equal to the right-hand sides of Eqs. (40) and (43).

- As a result, the inequalities (40) and (43) reveal that the radial allocation of surface pressures in the respective directions, and also the extra degrees of freedom created as a direct consequence of minimally coupled  $f(R, T)$  gravitational theory, affect the dynamic instability ranges. These results for our axially symmetric star are similar to those obtained using metric  $f(R)$  gravity and GR.

By specifying such conditions as  $\xi \rightarrow 0$  and  $\beta \rightarrow 0$ , one may readily acquire and relate findings in the backdrop of GR.

### Appendix

The terms appearing due to MGT are given as

$$\begin{aligned}
 Z_1 = & \left\{ \frac{1}{f_R A^2} \left( \frac{1}{2} (f - R f_R) + Q_{11} + \varphi_{00} f_R \right) \right\}_{,0} \\
 & - \frac{2\dot{A}}{A^3} \left( \frac{1}{2} (f - R f_R) + \varphi_{00} f_R \right) - \frac{1}{B^2} \left\{ \frac{\varphi_{01}}{f_R} \right\}_{,1} \\
 & + \frac{\dot{B}}{A^2 B} \varphi_{01} - \frac{\dot{B}}{2B^3} f_R \varphi_{11} - \frac{\dot{B}}{2B^3} (f - R f_R) \\
 & - \frac{2\dot{C}}{C A^2} f_R \varphi_{00} + \frac{\dot{C}}{C A^2} (f - R f_R) + \frac{2\dot{C}}{C} f_R \varphi_{22} \\
 & - \frac{\dot{C}}{C} (f - R f_R) + \frac{1}{f_R (1 + f_T)} \\
 & \times \left\{ 2\rho A^2 \dot{f}_T + \frac{1}{2} A^2 \dot{T} f_T + \dot{Q}_{11} f_T \right\},
 \end{aligned}$$

where  $Q_{11} = -\rho A^2 + \frac{s A^2}{8\pi C^2}$ . Also,

$$\begin{aligned}
 Z_2 = & \frac{1}{A^2} \left\{ \frac{\varphi_{01}}{f_R} \right\}_{,0} + \frac{1}{B^2} \left( \frac{1}{2} (f - f_R R) + Q_2 \right. \\
 & \left. + \varphi_{11} f_R \right)_{,1} - \frac{2B\dot{B}}{A^2} \left( \frac{\varphi_{01}}{f_R} \right) - \frac{2B'}{B} \left( P_r - \frac{1}{2} \right. \\
 & \left. \times (f - f_R R) + Q_2 + \varphi_{11} f_R \right) \\
 & + \frac{2\dot{C}}{C A^2} \left( \frac{\varphi_{01}}{f_R} \right) - \frac{1}{A^2} \left( \frac{\varphi_{01}}{f_R} \right) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \\
 & + \left( \frac{2C'}{C B^2} - \frac{A'}{A B^2} \right) \left( P_r - \frac{1}{2} (f - f_R R) \right. \\
 & \left. + (\rho + P_r) f_T + Q_2 + \varphi_{11} f_R \right) \\
 & - \frac{A'}{2A^3} (2\varphi_{00} f_R + f - f_R R) \\
 & + \frac{2C'}{C^3} \left( (\rho + P_t) f_T - \frac{1}{2} (f - f_R R) + Q_3 \right)
 \end{aligned}$$

$$+ \varphi_{22} f_R) + \frac{1}{1 + f_T} \left\{ (\rho + 2P_r) B^2 \frac{f'_T}{f_R} + \frac{1}{2} B^2 T' \frac{f_T}{f_R} + Q'_{22} \frac{f_T}{f_R} \right\},$$

where  $Q_{22} = B^2 \left( \rho + 3P_r + \frac{3s^2}{8\pi C^2} \right)$ . Also,

$$\begin{aligned} Z_3 = & \frac{\Psi}{\xi_3 B_0^2} \left\{ \left( \frac{2A'_0}{A_0} - \frac{B'_0}{B_0} + \frac{C'_0}{C_0} \right) \left( \frac{a}{A_0} \right)' \right. \\ & \left. + \left( \frac{A'_0}{A_0} - \frac{B'_0}{B_0} + \frac{2C'_0}{C_0} \right) \left( \frac{c}{C_0} \right)' \right\} \\ & - \frac{1}{2} \left[ \Psi e - \beta R_c \tanh \left( \frac{\Psi e}{R_c} \right) + \xi R_0 \bar{T} \right. \\ & \left. + \xi \Psi e T_0 + R_0 \left( \beta \sec h^2 \left( \frac{\Psi e}{R_c} \right) - \xi \bar{T} \right) \right. \\ & \left. - \Psi (1 + \xi T_0) \right] + \frac{\Psi}{\xi_3 B_0^2} \left[ \left( \frac{a}{A_0} \right)'' + \left( \frac{c}{C_0} \right)'' \right] \\ & + 2\Psi^2 \beta \frac{e^2}{R_c^2} \sec h^2 \left( \frac{\Psi e}{R_c} \right) \\ & \times \left( 2 \tanh \left( \frac{\Psi e}{R_c} \right) + \sec h^2 \left( \frac{\Psi e}{R_c} \right) \right) \\ & + \left( 2\beta \sec h^2 \left( \frac{R_0}{R_c} \right) \left( \frac{R''_0}{R_c} \tanh \left( \frac{R_0}{R_c} \right) \right) \right. \\ & \left. + 2 \frac{R_0'^2}{R_c^2} \tanh^2 \left( \frac{R_0}{R_c} \right) + \frac{R_0'^2}{R_c^2} \sec h^2 \left( \frac{R_0}{R_c} \right) \right) \\ & + \xi T_0'' \frac{2\Psi b}{B_0^3} - \frac{1}{B_0^2} \left( 2\beta \sec h^2 \left( \frac{\beta e}{R_c} \right) \right. \\ & \left. \times \left( \frac{e'' \Psi}{R_c} \tanh \left( \frac{\Psi e}{R_c} \right) + 2 \frac{e'^2 \Psi}{R_c^2} \tanh^2 \left( \frac{\Psi e}{R_c} \right) \right) \right. \\ & \left. + \frac{e'^2 \Psi}{R_c^2} \sec h^2 \left( \frac{\Psi e}{R_c} \right) + \xi \bar{T}'' \right) - \frac{\Psi \left( \frac{b}{B_0} \right)'}{\xi_3 B_0^2} \\ & \times \left( \frac{A'_0}{A_0} + \frac{C'_0}{C_0} \right) - \frac{1}{B_0^2} \left( \frac{A'_0}{A_0} - \frac{B'_0}{B_0} + \frac{C'_0}{C_0} \right) \\ & \times \left[ 2\beta \frac{R'_0}{R_c} \sec h^2 \left( \frac{R_0}{R_c} \right) \tanh \left( \frac{R_0}{R_c} \right) + \xi T_0' \right. \\ & \left. - 2\beta \frac{e' \Psi}{R_c} \sec h^2 \left( \frac{e \Psi}{R_c} \right) \tanh \left( \frac{e \Psi}{R_c} \right) - \xi \bar{T}' \right] \\ & - \left( \frac{a}{A_0} - \frac{aA'_0}{A_0^2} - \frac{b}{B_0} + \frac{bB'_0}{B_0^2} + \frac{c}{C_0} - \frac{cC'_0}{C_0^2} \right) \\ & \times \frac{\Psi}{B_0^2} \left( 2\beta \frac{R'_0}{R_c} \sec h^2 \left( \frac{R_0}{R_c} \right) \right. \\ & \left. \times \tanh \left( \frac{R_0}{R_c} \right) + \xi T_0' \right) + \frac{\xi \xi_1 \ddot{\Psi}}{A_0^2} \end{aligned}$$

We considered  $Z_4^2(r)$  to simplify some lengthy computations, given as

$$\begin{aligned} Z_4^2 = & \frac{C_0 A_0^2 B_0}{b C_0 + 2 B_0 c} \left[ \frac{e}{2} - \frac{2c}{C_0^3} \left\{ A_0'' \left( \frac{a}{A_0} + \frac{2b}{B_0} \right) \right. \right. \\ & \left. \left. - \frac{1}{B_0} (a' B'_0 + a'' + A_0' b' - B_0' A_0') \left( \frac{a}{A_0} + \frac{3b}{B_0} \right) \right\} \right. \\ & \left. + \frac{2}{C_0} \left( a' + c' A'_0 - A_0' \left( \frac{a}{A_0} + \frac{2b}{B_0} + \frac{c}{C_0} \right) \right) \frac{A_0}{C_0} \right. \\ & \left. \times \left( c'' - \frac{B_0' c'}{B_0} + \frac{3b}{B_0} + \frac{c}{C_0} \right) \left( c'' - \frac{b'}{B_0} - \frac{B_0' c'}{B_0} \right. \right. \\ & \left. \left. - \frac{b'}{B_0} + \frac{2}{C_0^2} \left( c' - \frac{cb}{C_0 B_0} \right) \right) \right]. \end{aligned}$$

In Newtonian domain, taking a flat background, and in pN domain, considering a Schwarzschild exterior, also applying some physical constraints in both domains, we get the values of  $Z_{3\text{N}}$ ,  $Y_{1\text{N}}$ , and  $Y_{1\text{pN}}$ :

$$\begin{aligned} Z_{3(\text{N})} = & -\frac{1}{2} \left[ e - \beta R_c \tanh \left( \frac{e}{R_c} \right) + \xi R_0 \bar{T} + \xi e T_0 \right. \\ & \left. + R_0 \left( \beta \sec h^2 \left( \frac{e}{R_c} \right) - \xi T_0 \right) - (1 + \xi T_0) \right] \\ & + \frac{1}{\xi_3} \left[ (a)'' + \left( \frac{c}{r} \right)'' \right] + \frac{1}{\xi_3} \left\{ \frac{1}{r} (a)' + \frac{2}{r} \left( \frac{c}{r} \right)' \right\} \\ & + 2\beta \frac{e^2}{R_c^2} \sec h^2 \left( \frac{e}{R_c} \right) \times \left( 2 \tanh \left( \frac{e}{R_c} \right) \right. \\ & \left. + \sec h^2 \left( \frac{e}{R_c} \right) \right) + \left( 2\beta \sec h^2 \left( \frac{R_0}{R_c} \right) \right. \\ & \left. \times \left( \frac{R''_0}{R_c} \tanh \left( \frac{R_0}{R_c} \right) + 2 \frac{R_0'^2}{R_c^2} \tanh^2 \left( \frac{R_0}{R_c} \right) \right. \right. \\ & \left. \left. + \frac{R_0'^2}{R_c^2} \sec h^2 \left( \frac{R_0}{R_c} \right) \right) + \xi T_0'' \right) 2b - \xi T_0'' \\ & - 2\beta \sec h^2 \left( \frac{\beta e}{R_c} \right) \left( \frac{e''}{R_c} \tanh \left( \frac{e}{R_c} \right) \right. \\ & \left. + 2 \frac{e'^2}{R_c^2} \tanh^2 \left( \frac{e}{R_c} \right) + \frac{e'^2}{R_c^2} \sec h^2 \left( \frac{e}{R_c} \right) \right) - \frac{b'}{\xi_3 r} \\ & - \frac{2b}{r} \left[ \left( 2\beta \frac{R'_0}{R_c} \sec h^2 \left( \frac{R_0}{R_c} \right) \tanh \left( \frac{R_0}{R_c} \right) + \xi T_0' \right) \right. \\ & \left. - 2\beta \frac{e'}{R_c} \sec h^2 \left( \frac{e}{R_c} \right) \tanh \left( \frac{e}{R_c} \right) - \xi T_0' \right] \\ & - \left( a - b + \frac{c}{r} - \frac{c}{r^2} \right) \left( 2\beta \frac{R'_0}{R_c} \sec h^2 \left( \frac{R_0}{R_c} \right) \right. \\ & \left. \times \tanh \left( \frac{R_0}{R_c} \right) + \xi T_0' \right) + \xi \xi_1, \\ Y_{1\text{N}} = & \xi e' \left( \rho_0 - P_{r0} \right) + 6 \frac{c}{r^4 P_{t0}} + 2e\rho'_0 + \bar{\rho}' \xi R_0 \end{aligned}$$

$$\begin{aligned}
& + 2\xi e\rho_0 + \xi eP'_{r0} - 2\beta e'\rho_0 + \beta q_1 + 2q_2 + Z_{2N}^{(s)}, \\
& Y_{1pN} = \xi e' \left( \rho_0 - P_{r0} \right) + 6\frac{c}{r^4} P_{t0} + 2\beta e\rho'_0 \\
& + \bar{\rho}' \xi R_0 + 4\xi \frac{e}{r} \left( \rho_0 + P_{r0} \right) \left\{ \frac{Q^2 - m_0 r}{2m_0 r - Q^2 + 2r^2} \right\} \\
& + \xi eP'_{r0} + \frac{2}{r} \left\{ \frac{Q^2 - m_0 r}{Q^2 - 2m_0 r + 2r^2} \right\} \left[ \left( \rho_0 + 2\beta q_1 \right) \right. \\
& \left. + \xi eP'_{r0} \left( 1 + \frac{2m_0}{r} - \frac{Q^2}{r^2} \right) - a \left( \rho_0 + Q_1^{(s)} \right) \right. \\
& \left. \times \left( 1 + \frac{3m_0}{r} - \frac{3Q^2}{2r^2} \right) \right] - \frac{2}{r} \left( 2q_2 - bQ_2^{(s)} \right) \\
& \times \left\{ \frac{Q^2 - m_0 r}{2m_0 r - Q^2 + 2r^2} \right\} + Z_{2pN}^{(s)}.
\end{aligned}$$

### CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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