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Constraining $f(R,T)$ Gravity from the Dark Energy Density Parameter Ω_Λ

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Abstract—$f(R,T)$ gravity is a widely used extended theory of gravity introduced by Harko et al., which is a straightforward generalization of $f(R)$ gravity. The action in this extended theory of gravity incorporates well-motivated functional forms of the Ricci scalar R and the trace of the energy momentum tensor T. The present manuscript aims at constraining the most widely used $f(R,T)$ gravity model of the form $f(R+2\lambda T)$ to understand its coherency and applicability in cosmology. We communicate here a novel method to find a lower bound on the model parameter $\lambda \gtrsim -1.9\times10^{-8}$ through the equation relating the cosmological constant (Λ) and the critical density of the universe $(\rho_{\rm cr})$.

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1. INTRODUCTION

The cosmological constant (Λ) problem is one of the major unsolved mysteries concerned with the dissimilarity between the tiny observed value of the cosmological constant and the extremely large value of zero point energy. Based on the Planck energy cutoff along with other factors, the disaccord is as high as 120 orders of magnitude [1], a predicament often quoted as [2] "the worst theoretical prediction in the history of physics." After the discovery of the expansion of the universe by E. Hubble in 1929 [3], it was expected that the rate of expansion must be slowing down owing to the attractive nature of gravity. Nonetheless, measurements of the intrinsic brightness of distant Type Ia supernovae [4, 5] showed that the expansion is in fact accelerating. This mysterious component which fuels the expansion at an ever increasing rate accounts for nearly 70% of the energy budget of the universe and is termed Dark Energy (DE). There are three different kinds of DE models. These are: quintessence $(-1<\omega<0)$, phantom energy $(\omega<-1)$ and the cosmological constant $(\omega=-1)$, where ω represents the equation-of-state (EoS) parameter. Current observations suggest $\omega\approx-1$ [6].

Due to the lack of any observational evidence for the existence of any DE candidates (for a detailed reference of various DE candidates, one may refer to [7]), researchers were inspired to modify the geometric sector of the field equations, where the Ricci scalar R in the action is replaced by various generic functions of $f(R)$ [8], $f(\mathcal{T})$ [9], where $\mathcal{T}$ is the torsion scalar, $f(R,T)$ [10] where T is the trace of the energy-momentum tensor, and $f(G)$ [11], where G is the Gauss–Bonnet invariant.

Due to some fascinating features of $f(R,T)$ gravity and robustness is solving the cosmological issues, it is often employed in the literature [12]. $f(R,T)$ gravity is also reported to clearly narrate the transition from the matter-dominated to late-time accelerated phase of expansion [13]. $f(R,T)$ gravity models have been applied to scalar field models [14], anisotropic models [15, 16], dark matter [17], dark energy [18], bouncing cosmology [19, 20], gravitational waves [21–23], super-Chandrasekhar white dwarfs [24], massive pulsars [25, 26], wormholes [27–39], baryogenesis [40, 41], Big-Bang nucleosynthesis [42], growth rate of matter fluctuations [43] and in varying speed of light scenarios [44]. In [45], the authors investigated the causes of irregular energy density in $f(R,T)$ gravity.

The present paper reports a pioneering method for constraining the model parameters of $f(R,T)$ gravity from the equation relating the cosmological constant (Λ) and the critical density of the universe $(\rho_{\rm cr})$. According to the Friedmann solutions [46], the critical density is a particular density at which the universe is flat or Euclidean, and as a result, the curvature parameter vanishes [47]. The ratio of the current value

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of the density of the universe to the current value of critical density is called the density parameter $\Omega_0 = \rho/\rho_{\rm cr}$. This is a very important cosmological parameter which determines the evolution and the ultimate fate of the universe [47]. For $\Omega_0 = 1$, the universe is flat, while for $\Omega_0 > 1$ the universe is closed, and it is open for $\Omega_0 < 1$. Since the current observations suggest $\Omega_0 \simeq 1$, this indicates that the universe is approximately flat and apparently infinite, ergo favors the inflationary paradigm.

The paper is organized as follows: In Section 2 we provide a summary of $f(R,T)$ gravity. In Section 3 we introduce the framework to constrain the model parameter of $f(R,T)$ gravity. In Section 4 we present our conclusions.

2. OVERVIEW OF $f(R,T)$ GRAVITY

The action in $f(R,T)$ gravity is given by

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\,[f(R,T) + \mathcal{L}_m]\,d^4x, \quad (1)$$

where $\mathcal{L}_m$ denotes the matter Lagrangian. The stress-energy-momentum tensor of matter fields reads

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \quad (2)$$

Varying the action (1) with respect to the metric yields

$$\Pi_{\mu\nu} f^1_{,R}(R,T) + f^1_{,R}(R,T)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R,T) = (T_{\mu\nu} + \Theta_{\mu\nu}) + \kappa^2 T_{\mu\nu} - f^1_{,T}(R,T), \quad (3)$$

where

$$\Pi_{\mu\nu} = g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu, \quad (4)$$

$$\Theta_{\mu\nu} \equiv g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}, \quad (5)$$

and $f^i_{,X} \equiv d^i f/dX^i$. The field equations (3) reduce to the standard GR form when $f(R,T) \equiv R$.

Contracting Eq. (3) with the inverse metric $g^{\mu\nu}$, one obtains the trace of the field equations as

$$3\Box f^1_{,R}(R,T) + f^1_{,R}(R,T)R - 2f(R,T) = \kappa^2 T - (\Theta + T)f^1_{,T}(R,T). \quad (6)$$

Consider a spatially flat FLRW metric as

$$ds^2 = dt^2 - a(t)^2[dx^2 + dy^2 + dz^2], \quad (7)$$

where $a(t)$ represents the scale factor. Assuming the universe to be dominated by a perfect fluid, the matter Lagrangian density can be assumed as $\mathcal{L}_m = -p$. Applying this to Eqs. (3) and (6), we obtain

$$\frac{\kappa^2 + f^1_{,T}(R,T)}{f^1_{,R}(R,T)}\rho + \frac{1}{f^1_{,R}(R,T)}\Big[p f^1_{,T}(R,T) - 3\dot{R}H f^2_{,R}(R,T) + \frac{1}{2}(f(R,T) - R f^1_{,R}(R,T))\Big] = 3H^2, \quad (8)$$

$$\frac{\kappa^2 + f^1_{,T}(R,T)}{f^1_{,R}(R,T)}p + \frac{1}{f^1_{,R}(R,T)}\Big[\ddot{R} f^2_{,R}(R,T) + \dot{R}^2 f^3_{,R}(R,T) - \frac{1}{2}\left(f(R,T) - R f^1_{,R}(R,T)\right) - p f^1_{,T}(R,T) + 2H\dot{R} f^1_{,R}(R,T)\Big] = -3H^2 - 2\dot{H}, \quad (9)$$

where dots represent time derivative, H is the Hubble parameter, ρ is the density, and p the pressure with $T = \rho - 3p$.

We set the $f(R,T)$ functional form to be

$$f(R,T) = R + 2\lambda T. \quad (10)$$

Substituting (10) into (8), we obtain the first modified Friedmann equation as

$$H^2 = \frac{8\pi G}{3}(8\pi + 3\lambda)\rho - \frac{2}{3}\lambda\omega\rho, \quad (11)$$

where $\omega = p/\rho$ is the EoS parameter. Current observations suggest $\omega \simeq -1$ [6].

3. THE FRAMEWORK TO CONSTRAIN λ

In this section we will propose a framework to put bounds on the model parameter λ from the equation relating the cosmological constant (Λ) and the critical density of the universe ($\rho_{\rm cr}$).

We start by substituting $\omega = -1$ in (11) to obtain

$$\frac{(8\pi)^2 G}{3}\rho + \lambda\rho\left(8\pi G + \frac{2}{3}\right) = H^2. \quad (12)$$

Since $8\pi G \ll \frac{2}{3}$, we neglect the $8\pi G$ term, which simplifies the above equation to obtain

$$H^2 = \frac{1}{3}\rho\left[2\lambda + (8\pi)^2 G\right]. \quad (13)$$

The Friedmann equation in standard GR with the cosmological constant Λ is given by [48, 49]

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3}. \quad (14)$$

Since the left-hand sides of Eqs. (13) and (14) are the same, we can equate them to obtain

$$\Lambda c^2 = 2\lambda\rho + \left[(8\pi)^2 G - 8\pi G\right]. \quad (15)$$

Equation (15) can further be simplified to the reduced form

$$\Lambda \approx \frac{\rho}{c^2}\left[2\lambda + 192\pi G\right]. \quad (16)$$

Now, the cosmological constant Λ is defined as [50]

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda, \quad (17)$$

where H_0 is the present value of the Hubble parameter, and Ω_Λ is the dark energy density parameter. As mentioned in the introduction, at the present epoch the total density parameter $\Omega_0 = \rho/\rho_{\rm cr} \simeq 1$ [51], we can therefore substitute ρ with the critical density $\rho_{\rm cr}$ in (16). This further yields

$$3\left(\frac{H_0}{c}\right)^2 \Omega_\Lambda approx \frac{3H_0^2}{8\pi G c^2}\left[2\lambda + 192\pi G\right]. \quad (18)$$

Now, the critical density $\rho_{\rm cr}$ can be defined as [46]

$$\rho_{\rm cr} = \frac{3H_0^2}{8\pi G}. \quad (19)$$

Simplifying (18) we finally obtain

$$\Omega_\Lambda = \frac{1}{8\pi G}\left(2\lambda + 192\pi G\right). \quad (20)$$

Observational Constraint: the Planck satellite data reported $\Omega_\Lambda = 0.6889 \pm 0.0056$ [51]. This imposes a lower bound on the model parameter $\lambda \gtrsim -1.9 \times 10^{-8}$.

4. CONCLUSIONS

$f(R,T)$ gravity is a modified theory of gravity where the Ricci scalar R in the action is replaced by a generic function of R and T, where T denotes the trace of the energy-momentum tensor. As a result, the emergent theory can resolve the major cosmological enigmas such as the current accelerated phase of the universe without requiring dark energy. $f(R,T)$ gravity is also reported to clearly narrate the transition from matter-dominated to late-time accelerated phase of the universe expansion [13].

However the functional form of $f(R,T)$ can be arbitrary, and any number of model parameters can be included, which can be fine-tuned to fit the observations. Hence we aimed in this paper to constrain the model parameter λ for the simplest minimal coupled model $f(R,T) = R + 2\lambda T$ through the equation relating the cosmological constant (Λ) and the critical density of the universe ($\rho_{\rm cr}$). We report a lower bound on $\lambda \gtrsim -1.9 \times 10^{-8}$.

From the analysis we establish the idea that the parameter λ is trivial and has no significant importance in cosmological models. It will be interesting to apply this method to constrain the model parameters of other $f(R,T)$ gravity models and to investigate their cosmological viability.

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