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Electromagnetic Origin of Particle Masses and Gravity: a Relativistic Theory

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Abstract—The theory of direct particle interaction proposed in our previous works is further developed. In this theory, the electromagnetic interaction is primary whereas the emergence of particle masses and gravity are its consequences. The equation of motion is generalized to arbitrary velocities of the selected particle (rather than small ones). With the introduction of an effective metric, a correspondence between this theory and General Relativity (GR) is established. It is shown that the theory reproduces GR effects associated with the Schwarzschild and Friedmann metrics: planetary perihelion shift, light deflection by a massive body, gravitational redshift and cosmological redshift. However, a general correspondence with GR is achieved under some restrictions (at sufficiently low speeds, small components of the gravitational potential, but sufficiently large accelerations). In the theory, there is a parameter with the dimension of acceleration, approximately equal to 7×10^{-10} m/s^2 and close to the parameter a_0 in modified Newtonian dynamics (MOND). Perhaps our theory can serve as a theoretical basis for MOND.

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1. INTRODUCTION

In our papers [1, 2], a theory was proposed in which the origin of mass and gravity are consequences of the electromagnetism. We took as a basis the theory of direct electromagnetic interaction between particles, constructed by Tetrode [3] and further developed by Wheeler and Feynman [4, 5]. In [3], both retarded and advanced electromagnetic interactions were considered. Moreover, in the equation of motion of charged particles, both the retarded and advanced interactions entered symmetrically. This presented a significant difficulty, since the advanced interaction is not observed in known experiments. Another difficulty of the theory [3] was that the equation of motion did not contain the force of radiative friction. The subsequently developed version of the theory [4] was to a large extent equivalent to Maxwell's electrodynamics and became known as the Wheeler–Feynman absorber theory. The key idea of [4] can be ascribed to Mach's principle in its general formulation: the interaction of particles in a certain local region is associated with the dynamics of all particles in the Universe. The Universe was considered in [4] as an *absolute* absorber, due to which only retarded forces remained in the equation of motion and there appeared the radiation friction force.

In our papers [1, 2] the requirement of absolute nature of the absorber was weakened: the Universe was considered as an absolute absorber only approximately. The non-absolute nature of the absorber leads to important new results: the emergence of particle masses and gravity are consequences of the electromagnetic interaction. Moreover, the cosmological coincidences (the known numerical relations involving the number of particles N inside the Hubble sphere) were obtained in [2] as a consequence of the theory. However, when deriving the equation of motion of charged particles, a number of approximations were used, among which was the assumption that the velocities of the particles in question were small. In this paper, we propose a more consistent relativistic version of the theory.

Our theory is based on the variational principle in the background of Minkowski space. The action of the system of electromagnetically interacting particles has the form

$$S = -\sum_i \sum_{k<i} \frac{e_i e_k}{c} \times \int\int u_i^\mu u_{k\mu} \delta(s^2(i,k)) ds_i ds_k, \qquad (1)$$

where $u_i^\mu = dx_i^\mu/ds_i$ is the 4-velocity of particle number i, e_i is its charge, c is the speed of light, and the delta function of the squared interval between events on the world lines of particles i and k may be presented in the form

$$\delta(s^2(i,k))$$

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$$= \frac{1}{2r_{ik}}[\delta(ct_{ik} - r_{ik}) + \delta(ct_{ik} + r_{ik})]. \quad (2)$$

In our theory, the mass is not introduced initially, it is an emergent property of particles: the corresponding term in the equation of motion appears as a consequence of the advanced interaction. In the nonrelativistic limit, instead of Newton's second law $m\vec{a} = \vec{F}$, we get an equation of the form $0 = \frac{1}{2}\vec{F}^{\text{ret}} + \frac{1}{2}\vec{F}^{\text{adv}}$, and after taking into account Mach's principle, this equation is converted to the form $0 = \vec{F}^{\text{ret}} - \text{const} \cdot \vec{a}$, where the constant is determined by all particles of the Universe and plays the role of mass of the particle in question.

For convenience, we can introduce a quantity corresponding to the electromagnetic field potential. Consider some point of the world line of particle i. The expression for the potential created at this point by another particle k can be written as

$$A_\mu(i,k) = e_k \int u_{k\mu}\delta(s^2(i,k))ds_k. \quad (3)$$

However, the "field" is, in this theory, an auxiliary construction, while real is, by assumption, a direct interaction of particles which could be in principle described without invoking this notion. The potential determined by Eq. (3) is half-retarded and half-advanced:

$$A_\mu(i,k) = \frac{1}{2}A_\mu^{\text{ret}}(i,k) + \frac{1}{2}A_\mu^{\text{adv}}(i,k), \quad (4)$$

where $A_\mu^{\text{ret}}(i,k)$ is the Lienard–Wichert retarded potential, while $A_\mu^{\text{adv}}(i,k)$ is a similar advanced potential, which differs from the retarded one in that it is determined by the existence and motion of particle k in the future with respect to the moment of its impact on particle i rather than in the past. As a result of varying the action (1) (see the details in [1, 2]), we obtain the set of equations of motion (one for each particle) of the simple form

$$0 = \frac{e_i}{c}u_i^\nu \sum_{k\neq i}\left(\frac{1}{2}F_{\mu\nu}^{\text{ret}}(i,k) + \frac{1}{2}F_{\mu\nu}^{\text{adv}}(i,k)\right). \quad (5)$$

A further development of the theory in [1, 2] was to *express the advanced interactions through the retarded ones* and to arrive at such an equation of motion of particle i that would include only retarded forces. In other words, we sought to construct a deterministic theory where the acceleration of any particle at any given time would be determined by the motion of all particles in the past rather than in the future. This can be done only in some approximations. In this paper we continue to develop the theory in the spirit of classical determinism. To express the advanced interactions through the retarded ones, it is necessary to take into account Mach's principle, i.e., the influence of the entire Universe on the interaction of any two particles i and j. To do that, the theory must initially contain some ideas about the Universe as a whole. Let us recall the related postulates that we adopted in [2]:

1. Basic assumptions about the Universe. The Universe is homogeneous and isotropic on large scales, of the order of the distance R, roughly equal to the radius of the Hubble sphere in the Standard cosmological model. Moreover, the Universe is static, and the mean density of matter in it does not change with time. On scales smaller than R, there are superclusters of galaxies whose positions relative to each other do not change with time. There is no expansion of the Universe in this model. The theory is based on the background Minkowski space, and there are reference frames in which large "units" of matter, such as superclusters of galaxies, can be considered to be at rest. We will call such reference frames Newtonian, following Dicke's paper [6].

2. A finite radius of action of the electromagnetic interaction. For each particle i, there is a sphere of radius R, to be called *an R-sphere*. The interaction of particle i with other particles inside the R-sphere obeys Eq. (5), whereas the interaction with particles outside the R-sphere is stochastic and, being averaged over time (on a certain time scale), yields zero. The number of particles N inside the R-sphere of any particle i can be regarded constant. When considering systems of small size (of the order of the Solar system or galaxy), we can assume that all particles of this system have one common R-sphere.

The introduction of a finite radius of the electromagnetic interaction and the concept of the R-sphere allows us to develop the method of accounting for the advanced interaction described by Wheeler and Feynman [4]. Consider particle i, for which we want to obtain an equation of motion in which the leading interactions are expressed in terms of the lagging ones. Let us rewrite Eq. (5) as

$$\begin{aligned} 0 &= \frac{e_i u_i^\nu}{c}\sum_{k\neq i} F_{\mu\nu}^{\text{ret}}(i,k) \\ &+ \frac{e_i u_i^\nu}{c}\left(\frac{1}{2}F_{\mu\nu}^{\text{ret}}(i,i) - \frac{1}{2}F_{\mu\nu}^{\text{adv}}(i,i)\right) \\ &- \frac{e_i u_i^\nu}{c}\sum_{k}\left(\frac{1}{2}F_{\mu\nu}^{\text{ret}}(i,k) - \frac{1}{2}F_{\mu\nu}^{\text{adv}}(i,k)\right). \end{aligned} \quad (6)$$

Here, the first term is the usual retarded interaction of classical electrodynamics, while the second term gives the force of radiation friction in accordance with Dirac's paper [7]. In the third term, summing is carried out over all particles, including i. The expression in parentheses of this term, as noted in [4], is a solution of Maxwell's equations without sources, but it is uniquely determined by the motion of all particles in the Universe. Drawing an analogy with field electrodynamics, it can be said that it describes electromagnetic "waves" coming into the R-sphere from outside and passing through it without re-emission.

To calculate the sum in the third term in (6), we considered in [2] two types of processes, called S-processes and P-processes. In the first type: an isolated particle i has a delayed effect on another particle k. This effect contributes to the acceleration of particle k, resulting in that k has a reciprocal advanced effect on particle i (we take into account the linearity of the equations and the possibility to single out, from the total acceleration of particle k, a part induced by i). The corresponding contribution to the sum in the third term in (6) will be denoted as $S_{\mu\nu}(i,k)$. The second type: some particle j exerts a retarded influence on another particle k. This effect contributes to the acceleration of k, and, as a result, k exerts an advanced influence on the selected particle i. The corresponding contribution to the sum in the third term in (6) is denoted by $P_{\mu\nu}(i,k,j)$. The calculations were carried out in [2] in the Newtonian reference frame, in which particle i, at the initial time $t=0$, is located at the origin, and it was assumed that the world lines of all particles are specified at $t\in(-\infty;0)$.

In the present paper, we restrict ourselves to considering S-processes and calculate $S_{\mu\nu}(i,k)$ for the relativistic case, and, instead of considering P-processes (and more complex chains of interactions), we use the hypothesis that the advanced interactions can be expressed through the retarded ones. In fact, we have already used this hypothesis in [1], but there we restricted ourselves to considering the nonrelativistic approximation.

Let us start the calculation of $S_{\mu\nu}(i,k)$. To begin with, consider an idealized situation where only two particles i and k are present in the entire space. Let the world line of particle k intersect the future light cone of particle i at point x_k^μ (which corresponds to the three-dimensional radius vector $\vec r_k$ and the time instant t_k). Let $w_{k\mu}$ be the 4-acceleration acquired by particle k due to the delayed action of particle i. Being accelerated, particle k at the instant t_k emits electromagnetic "waves." The divergent spherical wave corresponding to the retarded interaction will sooner or later leave the R-sphere of particle i. But $-1/2$ from the field strengths in this wave enters into the expression for $S_{\mu\nu}(i,k)$. This means that, from outside of the R-sphere of particle k comes a converging spherical wave corresponding to the advanced interaction. At the time $t=t_k$, this wave collapses at the point x_k^μ, and at $t=0$ this wave passes through particle i. This gives the following calculation algorithm for $S_{\mu\nu}(i,k)$:

Step 1. Calculate the retarded influence on particle k at the instant t_k from i. The result is well known: it is the retarded field of a particle moving by a specified law, which has the following covariant form:

$$F_{\mu\nu}^{\rm ret}(k,i)=\frac{e_i}{(x_k^\sigma u_{i\sigma})^2}(w_{i\mu}x_{k\nu}-w_{i\nu}x_{k\mu})+\frac{e_i(x_k^\sigma w_{i\sigma}-1)}{(x_k^\sigma u_{i\sigma})^3}(u_{i\nu}x_{k\mu}-u_{i\mu}x_{k\nu}),\tag{7}$$

where $w_{i\mu}=du_{i\mu}/ds_i$ is the 4-acceleration of particle i.

Step 2. Calculate the acceleration acquired by particle k due to the retarded interaction calculated at Step 1. Since this interaction is purely retarded, we can use the equation of motion of conventional electrodynamics. Neglecting the radiative friction as compared to the external force acting on k, we have:

$$w_{k\mu}=\frac{e_ie_k}{m_kc^2(x_k^\sigma u_{i\sigma})^2}\left(w_{i\mu}x_{k\sigma}u_k^\sigma-u_k^\sigma w_{i\sigma}x_{k\mu}\right)+\frac{e_ie_k(x_k^\sigma w_{i\sigma}-1)}{m_kc^2(x_k^\sigma u_{i\sigma})^3}\left(u_k^\sigma u_{i\sigma}x_{k\mu}-u_{i\mu}x_{k\sigma}u_k^\sigma\right).\tag{8}$$

Step 3. Apply to the point with the radius vector $\vec r_k$ one more vector equal to $\vec r_k$, and calculate, at the end of this vector, the retarded field $f_{\mu\nu}$ created by particle k at the expense of the acceleration calculated at step 2. The required field acting on the particle i at the moment $t=0$ is $S_{\mu\nu}(i,k)=-\frac12 f_{\mu\nu}$. After transformations we get:

$$S_{\mu\nu}(i,k)=\frac{e_ie_k^2}{2m_kc^2(x_k^\sigma u_{i\sigma})^2(x_k^\sigma u_{k\sigma})^2}\times\Big(x_{k\sigma}u_k^\sigma\left(w_{i\mu}x_{k\nu}-w_{i\nu}x_{k\mu}\right)+\left(x_k^\sigma w_{i\sigma}-1\right)\left(u_{i\nu}x_{k\mu}-u_{i\mu}x_{k\nu}\right)\Big).\tag{9}$$

Substituting (9) into (6) and using that $u_i^\rho w_{i\rho}=0$, we obtain that, in the equation of motion of particle i, emerges the following term:

$$-\frac{e_iu_i^\nu}{c}S_{\mu\nu}(i,k)=-\frac{e_i^2e_k^2}{2m_kc^3x_k^\sigma u_{k\sigma}x_k^\lambda u_{i\lambda}}w_{i\mu}-\frac{e_i^2e_k^2u_i^\nu(x_k^\sigma w_{i\sigma}-1)}{2m_kc^3(x_k^\sigma u_{k\sigma})^2(x_k^\sigma u_{i\sigma})^2}\times\left(u_{i\nu}x_{k\mu}-u_{i\mu}x_{k\nu}\right).\tag{10}$$

The term containing $w_{i\mu}$, after its transfer to the left-hand side of the equation of motion (6), will play the role of a product of the mass of particle i by its 4-acceleration vector and the speed of light.

We have considered an idealized situation where only two particles i and k are present in the whole space. But in [2] we have shown that the expression for $S_{\mu\nu}(i,k)$ does not change if there are many other particles inside the R-sphere of the particle i besides k. In this case, the main contribution to the sum $\sum_{k\neq i} S_{\mu\nu}(i,k)$ is made by distant particles, those whose sufficiently large part of radiation leaves the R-sphere of particle i.

Next, we must find the sum (10) over all particles $k \neq i$. Let us first consider only distant particles located at distances of the order of R from particle i. Let us pass from (10) to an approximate expression following from the symmetry of the R-sphere. Due to this symmetry, one can put $x_k^\sigma u_{k\sigma} = R$. In the first term in (10) one can also put $x_k^\lambda u_{i\lambda} = R$ (the latter can be checked by direct integration by the angle between the three-dimensional vectors $\vec{v}_i$ and $\vec{r}_k$). Therefore, the first term is presented as a product of the 4-acceleration by a relativistic invariant equal to a product of mass by the speed of light. The second term (10) has a more complex structure, and we will thus far leave it in its general form. Then by summing (10) over all distant particles $k \in R$ we obtain the expression:

$$-\frac{e_i u_i^\nu}{c}\sum_{k\in R} S_{\mu\nu}(i,k) = -\sum_{k\in R}\frac{e_i^2 e_k^2}{2m_k c^3 R^2} \times \left(w_{i\mu} + \frac{(1 - x_k^\sigma w_{i\sigma})(x_{k\mu} - u_{i\mu}u_{i\nu}x_k^\nu)}{(x_k^\sigma u_{i\sigma})^2}\right). \quad (11)$$

The expression $k \in R$ under the sign of sum means summing over distant particles (those at distances of order R), which may be approximately replaced by integrating over the angles in spherical coordinates. From (11) follows an expression for the mass of an arbitrary particle i

$$m_i = \sum_{k\neq i}\frac{e_i^2 e_k^2}{2m_k c^4 R^2}. \quad (12)$$

Within this paper, we will suppose that all particles have charges of the same absolute value e and the same mass m, which in this case is determined by the expression

$$m = \frac{e^2\sqrt{N}}{\sqrt{2}c^2 R}. \quad (13)$$

Let us now consider a small region of the Universe, whose linear dimensions are much smaller than the characteristic scale R, and calculate the contribution of these "nearby" particles to the third term in (6). To do that, we use the approach outlined in [1] and generalize it. Let us denote the sum of advanced potentials at some point of the world line of particle i, created by all other particles $k \neq i$, by the symbol $\tilde{A}_\mu^{\text{adv}}$. Let $\{K\}$ be the set of intersection points of the future light cone of particle i with the world lines of all particles $k \neq i$, and K be the point from this set belonging to the world line of particle k. In accordance with the deterministic nature of the developed theory, the value $\tilde{A}_\mu^{\text{adv}}$ can be represented as a function of many variables: the retarded potentials created at all points K by all other particles j, $j \neq k$ for the point K. Suppose that the desired function is decomposable into a Taylor series with tensor coefficients, and in some approximation one can restrict oneself to the linear term in this decomposition. Then for the quantity $\tilde{A}_\mu^{\text{adv}}$ one can write

$$\tilde{A}_\mu^{\text{adv}} = \sum_K\sum_{j\neq k} Q_{\mu\sigma}A^{ret\,\sigma}(k,j), \quad (14)$$

where $Q_{\mu\sigma}$ are the tensor coefficients of the decomposition. Note that particle i is included in the set of particles $j \neq k$ (in [1], instead of the set of particles $j \neq k$, only one particle i was considered). Due to the isotropy and homogeneity of the Universe, the tensor expansion coefficients were chosen in [1] in the simple form

$$Q_{\mu\sigma} = q\eta_{\mu\sigma}, \quad (15)$$

where q is a certain number, and $\eta_{\mu\sigma}$ is the metric tensor of Minkowski space. The results of our paper [2] have confirmed this assumption and, in addition, have allowed us to calculate q. It has turned out that $q \approx 1/\sqrt{N}$.

Let us single out from the set of all particles k a subset of "nearby" particles (the contribution of "distant" particles to the equation of motion has already been taken into account). We introduce two tensors (analogs of the 4-potential A_μ and the electromagnetic field tensor $F_{\mu\nu}$), which do not include the charge of particle j:

$$G_\mu^{\text{ret}}(k,j) = \frac{1}{e_j}A_\mu^{\text{ret}}(k,j) = \int u_{j\mu}\delta(s^2(k,j))ds_j = \frac{u_{j\mu}}{(x_j^\sigma - x_k^\sigma)u_{j\sigma}}, \quad (16)$$

$$G_{\mu\nu}^{\text{ret}}(k,j) = \frac{\partial G_\nu^{\text{ret}}(k,j)}{\partial x_j^\mu} - \frac{\partial G_\mu^{\text{ret}}(k,j)}{\partial x_j^\nu}. \quad (17)$$

Then Eq. (14) is rewritten as

$$\tilde{A}_\mu^{\text{adv}} = \frac{e_j}{\sqrt{N}}\sum_K\sum_{j\neq k}\eta_{\mu\sigma}G^{\text{ret}\,\sigma}(k,j). \quad (18)$$

Let us substitute to (6) the expression (11) and the advanced electromagnetic field tensor $\tilde{F}_{\mu\nu}^{\text{adv}}$, calculated through the potential (18). Taking into account the expression (13) for the mass, we find that the equation of motion of particle i takes the form

$$m_i c\left(w_{i\mu}+\sum_{k\in R}\frac{(1-x_k^\sigma w_{i\sigma})(x_{k\mu}-u_{i\mu}u_{i\nu}x_k^\nu)}{(x_k^\sigma u_{i\sigma})^2}\right)$$
$$=\frac{e_i u_i^\nu}{c}\sum_{k\neq i}F_{\mu\nu}^{\rm ret}(i,k)+f_\mu^{\rm rad}$$
$$+\frac{e_i^2 u_i^\nu}{c\sqrt{N}}\sum_{n\neq i}G_{\mu\nu}^{\rm ret}(n,i)$$
$$+\frac{e_i u_i^\nu}{c\sqrt{N}}\sum_{n\neq i}\sum_{j\neq i,n}e_j G_{\mu\nu}^{\rm ret}(n,j), \qquad (19)$$

where the sum in n is the sum over "nearby" particles, and $f_\mu^{\rm rad}$ is the radiative friction force. For clarity, we have divided all particles j into $j=i$ and all others $j\neq i,n$. The term corresponding to $j=i$ does not depend on the sign of the charge of particle i (it is proportional to the square of the charge). In the nonrelativistic approximation considered in [1, 2], this term describes the attraction force and can be interpreted as the gravitational force. Let us note that the last two terms of the equation contain the tensor $G_{\nu\mu}^{\rm ret}(n,j)$ rather than $G_{\nu\mu}^{\rm ret}(j,n)$. This equation of motion is the basic equation of our theory, which we will further analyze.

Note that Eq. (19) may be considered as the equation of motion of not only a single particle, but also of a macroscopic body if we assume that all particles of this body have the same velocity. This assumption is valid if we consider averaging of velocities over a sufficiently large period of time, which allows us to neglect the thermal motion of particles of the body. The tensor $G_{\mu\nu}^{\rm ret}(n,j)$ can be presented as

$$G_{\mu\nu}^{\rm ret}(n,j)=\frac{w_{i\mu}(x_{n\nu}-x_{j\nu})-w_{i\nu}(x_{n\mu}-x_{j\mu})}{((x_n^\sigma-x_j^\sigma)u_{j\sigma})^2}$$
$$+\frac{(x_n^\sigma-x_j^\sigma)w_{j\sigma}-1}{[(x_n^\sigma-x_j^\sigma)u_{j\sigma}]^3}$$
$$\times\left[u_{j\nu}(x_{n\mu}-x_{j\mu})-u_{i\mu}(x_{n\nu}-x_{j\nu})\right]. \qquad (20)$$

The coordinates $x_{n\mu}$ are coordinates of the points K belonging to the future light cone of the selected particle i. Due to this circumstance, the current version of the theory is not fully deterministic. Indeed, to calculate the acceleration of particle i, we need to know the positions of the surrounding n particles in the future. However, we can circumvent this difficulty for two practically important cases: (a) considering the motion of charged particles in the gravitational field of one massive body, and (b) considering the Universe as a whole, when some sufficiently large structural elements are assumed to be stationary in the Newtonian reference frame. In both these cases, massive bodies can be approximately considered to be at rest, and the coordinates of their constituent particles $x_{n\mu}$ to be known in the future.

Consider the left-hand side of Eq. (19). At "large" accelerations $\vec{a}$ of particle i, the first term dominates in the outer brackets, and the left-hand side passes into the usual product of mass by acceleration. A sufficient condition for the acceleration $\vec{a}$ to be large can be written as follows:

$$a>a_0=\frac{c^2}{R}\approx 7\times 10^{-10}\ {\rm m/s^2}. \qquad (21)$$

To verify that, it is necessary to take into account the inequality $c^2/R>vc/R$. It implies that if the absolute value of the acceleration is much larger than c^2/R, then it is also much larger than the similar terms containing vc/R. Here we notice an interesting fact: the acceleration determined by Eq. (21) has the same order of magnitude as the acceleration a_0 in Modified Newtonian Dynamics (MOND), see [8–14]. For galaxies whose main part of the mass is concentrated in the nucleus, the acceleration a_0 sets the characteristic distance scale

$$r_0\approx\sqrt{\frac{GM}{a_0}}, \qquad (22)$$

where M is the mass of the particular galactic nucleus, and G is the gravitational constant. At distances from the nucleus larger than r_0, deviations from Newton's laws in the observed rotation curves are obtained. To explain the rotation curves of the galaxies, the Dark Matter hypothesis has been introduced, an alternative to which is MOND. To date, MOND is an empirical theory for which no fundamental basis has yet been found. The fact that our theory has a characteristic acceleration scale (21) suggests that perhaps our theory can serve as a theoretical basis for MOND.

In the current version of the theory, it has not yet been possible to reproduce the rotation curves of galaxies predicted by MOND. Perhaps this is related to the unresolved issues that we will list at the end of the paper. We will note thus far that the very fact of emergence, in this theory, of a parameter with the dimension of acceleration close to a_0 of MOND is of interest. In the remaining part of this paper we will consider the case of large accelerations, $a\gg a_0$.

2. CORRESPONDENCE WITH GENERAL RELATIVITY

In this section we discuss the question of how the current version of our theory relates to GR. Consider the motion of a body whose total electric charge e_i is zero (but the mass m_i is not zero, since the mass is proportional to the sum of the squares of charges of the particles that make up the body). The mass of a macroscopic body in this paper will be considered to be equal to the mass of a particle (13) multiplied by the number of particles in this body. We also assume that the acceleration of the body is quite large ($a \gg a_0$). Then, for a macroscopic body i, the equation of motion takes the form

$$m_i c w_{i\mu} = \frac{m_i c R u_i^\nu}{N} \sum_n \left(\frac{w_{i\mu} x_{n\nu} - w_{i\nu} x_{n\mu}}{(x_n^\sigma u_{i\sigma})^2} + \frac{(x_n^\sigma w_{i\sigma} - 1)(u_{i\nu} x_{n\mu} - u_{i\mu} x_{n\nu})}{(x_n^\sigma u_{i\sigma})^3} \right), \tag{23}$$

where we have put for brevity $x_{i\mu} = 0$.

Let us compare this equation with the equation of motion in the theory of gravity developed by Granovsky and Pantyushin [15, 16], Piragas and Zhdanov [17, 18] and Turygin [19]. The theory developed in these papers is a theory of direct gravitational interaction of particles in background Minkowski space, and its conclusions approximately coincide with those of GR in the first order with respect to the gravitational constant G. However, in this theory, gravity is considered as a separate interaction, not related to the electromagnetic one. In this section, we modify the gravitational potential introduced in these papers according to the idea of the induced nature of gravity, and establish a connection with our theory. Let us first consider the gravitational potential of the kind introduced in [15] and [19]:

$$h_{\mu\nu}(i,n) = \sum_{n\neq i} \frac{G m_n}{c^2} \times \int (2u_{n\mu} u_{n\nu} - \eta_{\mu\nu}) \delta(s^2(i,n)) ds_n, \tag{24}$$

where G is the gravitational constant, $\eta_{\mu\nu}$ is the Minkowski metric (a more general case of the background metric was considered in [19]). The potential defined by Eq. (24) is half-retarded and half-advanced. Let us now introduce another potential, whose some properties will coincide with those of the potential (24), and pay attention to the above equation (18) for the advanced electromagnetic potential $\tilde{A}_\mu^{\mathrm{adv}}$. The sources for it are the surrounding particles $k \neq i$, however, instead of the charge and 4-velocity e_k and $u_{k\mu}$ of particle k, this potential includes the characteristics e_i and $u_{i\mu}$ of particle i. Let us introduce a similar gravitational potential instead of (24): we take the *advanced* part of (24) and replace the 4-velocity $u_{n\mu}$ of particles n with the 4-velocity of particle i, without replacing the coordinates $x_{n\mu}$. The new potential will take the form

$$\varphi_{\mu\nu}(i,n) = \sum_{n\neq i} \frac{G m_n}{2c^2} \cdot \frac{2u_{i\mu} u_{i\nu} - \eta_{\mu\nu}}{(x_i^\sigma - x_n^\sigma) u_{i\sigma}}, \tag{25}$$

where x_n^σ are the coordinates of the intersection point of of the future light cone of particle i and the world line of particle n.

The gravitational potential $\varphi_{\mu\nu}$ defined in this way identically satisfies the condition similar to that of the Lorentz gauge,

$$\frac{\partial}{\partial x^\mu} \left(\varphi_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \varphi \right) = 0, \tag{26}$$

where $\varphi = \varphi_{\mu\nu} \eta^{\mu\nu}$, and the derivatives of $\varphi_{\mu\nu}(x,n)$ are calculated at an arbitrary point of space with the coordinates x_μ, not necessarily lying on the world line of particle i (the 4-velocity $u_{i\mu}$ is then not subject to differentiation). Also, this potential identically satisfies the equation

$$\eta_{\alpha\beta} \frac{\partial^2}{\partial x_\alpha \partial x_\beta} \left(\varphi_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \varphi \right) = \frac{8\pi G}{c^4} \tilde{T}_{\mu\nu}, \tag{27}$$

where $\tilde{T}_{\mu\nu}$ is a tensor which is a modification of the energy-momentum tensor $T_{\mu\nu}$ of a system of point masses. The tensor $\tilde{T}_{\mu\nu}$ *for particle* i at an arbitrary point X of space is defined as

$$\tilde{T}_{\mu\nu}(X) = \sum_{n\neq i} \int m_n \delta^4(X - X_n) u_{i\mu} u_{i\nu} ds_n, \tag{28}$$

where $\delta^4(X - X_n)$ is the four-dimensional delta function of the coordinate differences $x_\sigma - x_{n\sigma}$ of an arbitrary point X and a point X_n on the world line of particle n.

Let us introduce, for particle i, the effective metric $g_{\mu\nu}$ defined by

$$g_{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}. \tag{29}$$

Then, for the contravariant metric tensor, the following relation holds:

$$g^{\mu\nu} = \eta^{\mu\nu} - \varphi^{\mu\nu}. \tag{30}$$

We would like to stress that for each particle i there is its own effective metric $g_{\mu\nu}$. There is no contradiction in this, since the effective metric serves in our theory as an auxiliary quantity, not a characteristic of space. Note that if the velocities of all particles are negligible as compared to the speed of light, then the effective metrics for them are approximately the same (and then we can talk about a single metric).

In what follows we will suppose that the metric $g_{\mu\nu}$ is introduced for a selected particle i (the index i in the notation $g_{\mu\nu}$ is omitted for brevity). We define the Christoffel symbols for the metric $g_{\mu\nu}$ as

$$\Gamma^{\lambda}_{\mu\nu} = \frac{g^{\lambda\sigma}}{2}\left(\frac{\partial g_{\sigma\nu}}{\partial x^{\mu}} + \frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right). \qquad (31)$$

The thus defined metric $g_{\mu\nu}$ identically satisfies the harmonic coordinate conditions (the de Donder–Fock conditions) which can be written as

$$\Gamma^{\lambda}_{\mu\nu} g^{\mu\nu} = 0. \qquad (32)$$

Further on, one can introduce the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R and show that for $g_{\mu\nu}$ the equation similar to Einstein's equation is identically fulfilled (differing only in that the right-hand side contains, instead of the energy-momentum tensor $T_{\mu\nu}$, the modified tensor (28)):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4}\tilde{T}_{\mu\nu}. \qquad (33)$$

In the effective metric $g_{\mu\nu}$, the interval $d\hat{s}$ and the velocity 4-vector $\hat{u}^{\mu}_i$ of particle i are expressed in terms of s and the 4-vector u^{μ}_i in the Minkowski metric using the relations

$$d\hat{s}^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = ds^2 + \varphi_{\mu\nu}dx^{\mu}dx^{\nu}, \qquad (34)$$

$$d\hat{s} = ds\left(1 + \frac{1}{2}\varphi_{\mu\nu}u^{\mu}u^{\nu}\right), \qquad (35)$$

$$\hat{u}^{\mu} = u^{\mu}\left(1 - \frac{1}{2}\varphi_{\mu\nu}u^{\mu}u^{\nu}\right). \qquad (36)$$

In GR, the equation of motion for monopole objects is the geodesic equation

$$g^{\mu\lambda}\hat{w}_{i\lambda} = -\Gamma^{\mu}_{\alpha\beta}\hat{u}^{\alpha}_i \hat{u}^{\beta}_i. \qquad (37)$$

But it can be shown that Eq. (37) coincides, in the first order in the constant G, with the equation of motion (23) in our theory if it holds

$$\frac{G m_i m_n}{2c} = \frac{e_i^2}{c\sqrt{N}}. \qquad (38)$$

Since in the current version of the theory all particles have the same absolute value of charge e and the same mass $m = \frac{e^2\sqrt{N}}{\sqrt{2}c^2 R}$, Eq. (38) is rewritten in the form

$$G = \frac{4c^4R^2}{e^2N\sqrt{N}}. \qquad (39)$$

This equation differs from Eq. (32) in our paper [2] only by a numerical coefficient of the order of unity, which is in both cases approximate. Thus we have shown that in our theory massive electrically neutral particles move in the same way as is predicted by GR, in the case of small velocities, small gravitational potentials, but not small accelerations (the acceleration must be larger by order of magnitude than a_0 determined by Eq. (21)). In particular, our theory describes the effect of planetary perihelion shift.

The effect of a massive body on the electromagnetic interaction of two charged bodies. One of the classical GR tests is the deflection of light rays by massive bodies. Let us look how this effect can be described in our theory. To do that, we rewrite the equation of motion (19) in the form

$$m_i c\left(\frac{du_{i\mu}}{ds_i} + \sum_{k\in R}\frac{(1 - x_k^{\sigma}w_{i\sigma})(x_{k\mu} - u_{i\mu}u_{i\nu}x_k^{\nu})}{(x_k^{\sigma}u_{i\sigma})^2}\right)$$
$$= \frac{e_i u_i^{\nu}}{c}\sum_{j\neq i}\left(F^{\rm ret}_{\mu\nu}(i,j) + \sum_{n\neq i,j}\frac{e_j}{\sqrt{N}}G^{\rm ret}_{\mu\nu}(n,j)\right)$$
$$+ f^{\rm rad}_{\mu} + \frac{e_i^2 u_i^{\nu}}{c\sqrt{N}}\sum_{n\neq i}G^{\rm ret}_{\mu\nu}(n,i). \qquad (40)$$

Consider the tensor in parentheses in the right-hand side of (40), denoting it by $\hat{F}_{\mu\nu}(i,j)$:

$$\hat{F}_{\mu\nu}(i,j) = F^{\rm ret}_{\mu\nu}(i,j) + \sum_{n\neq i,j}\frac{e_j}{\sqrt{N}}G^{\rm ret}_{\mu\nu}(n,j). \qquad (41)$$

This tensor consists of two terms, the first of which describes a direct electromagnetic influence of the charge j on the charge i, while the second one describes an indirect effect of j on i through an intermediate particle n, i.e., the effect of particles n in electromagnetic interactions of particles j and i. It is natural to assume that the deflection of light by a massive body can be described as a contribution of a third body into the electromagnetic interaction of two charged bodies. Let us show that it is really the case in our theory. It is easy to verify that the tensor $\hat{F}_{\mu\nu}(i,j)$ identically satisfies the equation

$$\nabla_{\nu}\hat{F}_{\mu\nu}(i,j) = \frac{1}{\sqrt{-g}}\frac{(\sqrt{-g}\hat{F}_{\mu\nu}(i,j))}{\partial x^{\nu}}$$
$$= -\frac{4\pi}{c}\sum_{j\neq i}\int e_j\delta^4(X_i - X_j)\hat{u}_{j\mu}d\hat{s}_j, \qquad (42)$$

where ∇_{ν} is a covariant derivative, and g is the determinant of the metric tensor $g_{\mu\nu}$. In the right-hand side there is a product of $-4\pi/c$ by the 4-vector of current density created by point charges. Note that $\hat{u}_{j\mu}d\hat{s}_j = u_{j\mu}ds_j$ due to (35) and (36). But Eq. (42) is nothing else but the second pair of Maxwell's equations in space with the effective metric $g_{\mu\nu}$. This means that in space with the metric $g_{\mu\nu}$, light propagates along null geodesics determined by the equation

$$k^{\nu}\nabla_{\nu}k^{\mu} = 0, \tag{43}$$

where k^{μ} is the wave 4-vector. This, in turn, means that our theory reproduces the effects of deflection of light rays and gravitational redshift of the Schwarzschild metric in GR if in the Newtonian reference frame the velocity of particle i associated with the radiation receiver is much smaller than the speed of light (and the massive body n is at rest). Similarly, our theory reproduces the cosmological redshift (at least for galaxies, to which the distance[1] l is much smaller than R). In terms of direct particle interaction in Minkowski space, this effect is explained by taking into account secondary electromagnetic waves from massive bodies n distributed uniformly in space on large scales. The case where l is, by order of magnitude, not much smaller than R (distant galaxies) is left for a further study.

Thus, in the case of small velocities, small gravitational potentials but not small accelerations, all classical GR tests are reproduced in our theory: the planetary perihelion shifts, deflection of light rays by massive bodies, the gravitational redshift and the cosmological redshift. The constructed theory is relativistic, despite the fact that the coincidence of its conclusions with GR is proved only under specified restrictions (in particular, low velocities). The listed GR effects have been well tested only for velocities much smaller than the speed of light, so we can believe that our theory is in agreement with GR. In addition, cosmological coincidences are consequences of the theory, as noted in [2].

In conclusion, let us enumerate the main questions that have remained unresolved in the current version of the theory.

1. We assumed that all particles have the same mass (13). This restriction is not critical if we consider macroscopic bodies whose mass is approximately determined by the number of nucleons in their atomic nuclei (and the proton and neutron masses are almost the same). However, in particle physics, the question of the mass spectrum of particles is one of the fundamental unsolved questions. Perhaps our theory can be useful for solving this question because it contains a new idea about the mechanism of the origin of mass.

2. We considered massive electrically neutral bodies (denoted above by the symbol n), which were assumed to be at rest. The assumption of the immobility of massive bodies is satisfactory if we restrict ourselves to the effects associated with the Schwarzschild or Friedmann metrics (in the second case, massive bodies are uniformly distributed in space and are at rest from the viewpoint of Minkowski space, unless an effective metric is introduced). The question of whether it is possible to build a deterministic theory (or a theory suitable for calculations) in a more general case, remains open.

3. The equation of motion (19) takes into account the "nearby" bodies separated from particle i by distances much smaller than R. Therefore, the cosmological redshift in the current version of the theory is correctly described only for close galaxies. There remains the question of what will happen if this restriction is removed. The question of the very nature of the appearance of the finite radius R of the electromagnetic interaction also remains open.

4. The theory is largely based on the assumption that the sum of the advanced potentials, denoted as $\tilde{A}_{\mu}^{\mathrm{adv}}$, can be represented as a function of many variables—retarded potentials at points K. Moreover, we admit that this function is decomposable into a Taylor series (in the neighborhood of zero) and restrict ourselves to linear terms. Presumably, due to these restrictions the current version of the theory has not yet been able to reproduce the rotation curves of galaxies predicted by MOND. It seems to be of interest to investigate higher-order terms in this decomposition. Moreover, it may turn out that in the general case it is necessary to consider not a function at points K, but a functional representing the sum of integrals over segments of world lines of particles $k \neq i$. This is indirectly evidenced by the fact that in the nonrelativistic case, considered in [2], finite segments of the world lines of particles $k \neq i$ participate in the gravitational interaction.

[1] One means the three-dimensional distance in Euclidean space between the points of emission and absorption of light.

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